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Maybe the Usual Students' Practice of Cramming For a Test Makes Sense: A Mathematical Analysis

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MAYBE THE USUAL STUDENTS' PRACTICE OF CRAMMING FOR A TEST MAKES SENSE: A MATHEMATICAL ANALYSIS

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Abstract: We always teach students that cramming for a test is a bad idea, that they should study at the same speed throughout the semester – but many still cram. We ourselves are not that different: when we prepare papers for a conference, we often "cram" in the last days before the deadline instead of working with a regular speed for the whole time before the conference. The ubiquity of cramming makes us think that maybe it is not necessarily always a bad idea. And indeed, a simple model of a study process shows that an optimal solution often involve some cramming – to be more precise, a study schedule, in which in some periods we study much more intensely than in other periods, is often more efficient than studying at the same speed.

Keywords: cramming, optimal study schedule, mathematical model of a study schedule, regular speed vs. bursts

How we teach students to study. From elementary school to PhD, we always teach students that the best way to study is to study with a regular speed:

- if we need to prepare a homework, we should start working on it when it was assigned (and not a few days before it is due);
- when we need to study for a test, we should study this material at a regular speed throughout the semester – and not a few days before the test, etc.

But do students listen to us? Some do, but, in our experience, the majority of students ignore our suggestions and study the old-fashioned way: \

- first, a rather slow period, t
- then a period of very intensive cramming.

Do we ourselves follow our suggestions? Not really.

- Conference organizers know that most abstracts arrive at the last minute, and usually, many of them are clearly finalized at the last minute – since they often have silly typos that could have been easily caught if they were prepared based on a regular-speed schedule.
- Granting agencies know that most proposals are submitted at the last minute.

So maybe there is something in cramming? The ubiquity of such behavior, shown both

- by not-very-experience students and
- by their well-experienced instructors,

shows that maybe there is some advantage in cramming.

Maybe our advice to follow a regular study schedule is not always a good idea.

Let us analyze this problem. To analyze this problem, let us formulate it in precise terms. Let us denote the overall study period, between the moment when the task is assigned and the moment when the task is due, by n. During this period, there is a certain overall amount of time T that we can devote to this particular study task.

- This amount can distributed equally between different days – so that, as we always recommend, a student studies for the same time T/n during all n days.

- This amount can also be distributed unequally – so that some days, the students study less, and some days, they study much more.

Which is the best study schedule?

What does "the best" mean? First thing we need to do is to understand what "the best" means. In general, this may be a difficult task, but for studying, this is easy to describe – at least in the first approximation: we want to maximize the overall amount of the material learned by a student. This amount can be described by adding the amounts of material that a student learns during each of n days.

Each study schedule can be characterized by the amounts of time t(1), …, t(n) that the student takes of each of the n days. These amounts should add up to the overall time T allocated for the study: t(1) + … + t(n) = T. Let m(t) denote the amount of material that a student learns during a day when he/she studies for time t. Then,

- in the first day, the student learns the amount m(t(1)),
- in the second day, the student learns the amount m(t(2)), etc.

Overall, the amount of the material that the student have learned in n days is equal to the sum m(t(1)) + … + m(t(n)).

In these terms, the problem of selecting the best study schedule takes the following form:

- given the numbers n and T and the function m(t), f
- find the values t(1), …, t(n) for which the sum

  m(t(1)) + … + m(t(n))

  attains the largest possible value under the constraint that

  t(1) + … + t(n) = T.

At first glance, it may look like the function m(t) is simple, but it is not. What is the function m(t)? At first glance, the situation looks straightforward: the more we study, the more we learn. So, it seems like the amount m(t) that we learn during time t should be simply proportional to the study time: m(t) = c * t for some constant c. In this simplified description, the ratio m(t)/t is the same for all t.

In reality, the situation is not so straightforward.

- First, when a student sits down to study, he or she does not immediately start learning at the same speed – the students need some time to switch to the study mode.
- So, for small t, the relative productivity m(t)/t is low.
- After a certain time, the productivity increases – but only up to a certain point, because for large values t, the student gets tired and his/her productivity drops again.

In other words, in real life, the ratio m(t)/t is not constant:

- first, it is small, close to 0,
- then it increases and t
- then it start decreasing again.

So what is the optimal study schedule: analysis of the problem. The ratio m(t)/t starts at a small value for small t, then increases and then starts decreasing again. Thus, for a certain value t = s, this ratio attains its largest possible value; let us denote it by M = m(s)/s.

For every other time-per-day t, the ratio m(t)/t does not exceed M. Thus, for each i-th day, the amount of material learned during this day does not exceed the product M * t(i). By adding up all the amounts of material m(t(1)) + … + m(t(n)) learned during all these days, we conclude that the overall amount of material learned does not exceed the product M * (t(1) + … + t(n)), i.e., does not exceed the value M * T.

On the other hand, if every day in which we actually study, we spend the "optimal" time s on studying, we learn exactly the amount M * s = M * t(i), and thus, the overall amount of material learned is equal exactly to the sum M* (t(1) + … + t(n)) = M * T.

Thus, we arrive at the following conclusion.

Optimal study schedule: conclusion. The optimal study schedule means that:

- on some days, we practically do not study at all, while
- on the days on which we do study, we spend the same amount of time s on studying.

Discussion. The above result means that the regular-speed study schedule – when we study the same amount of material every day – is indeed not the optimal way to prepare for the test. A better way is to allocate a few days for an intensive study.

Of course, this does not mean that cramming for the test is always a good idea. When students cram, it is not because they select – as we suggest – the optimal time-per-day, but because they:

- do not plan well,
- run out of time, and
- start studying all day long.

This improvised schedule, with the amount t(i) close to 24 hours, may not be optimal for the students – and, as the experience of many students show, is often not optimal indeed.

This idea can be applied to other activities as well. Our main interest is to analyze studying, but the same analysis can be applied to other activities. Instead of studying, we can take any other task that requires intensive many-days activity, be it

- working on a thesis or dissertation,
- working on a project at work, or
- writing a paper or a book.

Many writers tried their best to follow a regular schedule. Many articles and books giving advice to graduate students recommend to study regularly. Many business articles emphasize the need for regular work and regular planning. This is the ideal we all strive to, but this is not how many of us work. Inevitably:

- A graduate student working on a thesis or a dissertation works much more intensively at the end, when the deadline is approaching.
- Researchers working on a paper for a conference usually work much more intensively a few weeks (or even a few days) before the deadline.
- Even at work, many of us tend to work less intensively at first, and much more intensively when the deadline is approaching.

Our analysis shows that this may be more efficient than working at a regular speed. From this viewpoint, the way to increase productivity is:

- *not* to work more regularly, but rather
- to find a time-per-day when your per-hour productivity is the largest
- and work that amount every day allocated for this task.

The number of such days can be determined by dividing the overall amount of time allocated for this time by this optimal amount s.

This way, we may succeed even more.

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References

1. Ambrose, S. A., Bridges, M. W., DiPietro, M., Lovett, M. C., and Norman, M. K. How Learning Works: Seven Research-Based Principles for Smart Teaching, Jossey-Bass/Wiley, San Francisco, 2010.

2. Gillen-O’Neal, C., Huynh, V. W., and Fuligni, A. J. To study or to sleep? The academic costs of extra studying at the expense of sleep, Child Development 84(1) 133–142, 2013.