11-1-2008

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Comments:
Technical Report: UTEP-CS-08-41
Published in *Applied Mathematical Sciences*, 2009, Vol. 3, No. 22, pp. 1091-1096.

Applied Mathematical Sciences, Vol. x, 200x, no. xx, xxx - xxx

A Paradox of Altruism: How Caring about Future Generations Can Result in Poverty for Everyone (Game-Theoretic Analysis)

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Abstract

Political and social activists are rightfully concerned about future generations: whenever a country borrows money, or an environmental situation worsens, this means, in effect, that we impose an additional burdens on future generations. There is clearly a conflict between the present generation's actions and interests and the welfare of the future generations. There exists a mathematical toolbox that provides solutions to many well-defined conflict situations: namely, the toolbox of game theory. It therefore seems reasonable to apply game theory techniques to the conflict between the generations. In this paper, we show that we need to be very cautious about this application, because reasonable game theoretic techniques such as Nash bargaining solution can lead to disastrous "solutions" such as universal poverty. In other words, seemingly reasonable altruism can lead to solutions which are as disastrous as extreme selfishness.

The development of appropriate techniques – techniques which would lead to a reasonable resolution of this inter-generational conflict – thus remains an important open problem.

Mathematics Subject Classification: 91A12

Keywords: Nash bargaining solution

1 Inter-Generational Conflict: A Problem

When society makes large-scale long-term plans, the planners usually try their best to take into account not only the interests of the current generation, but also the interests of future generations. There is a justifiable worry that if we spend too much of our resources (if we spend too much of the financial reserves, if we deplete our fuel and/or environmental resources), we thus make life more difficult for future generations.

Sometimes this difficulty is very clear: for example, when a country borrows money, this simply means that the burden of paying off this debt is passed to future generations. In general, there is often a conflict between the present generation's actions and interests and the welfare of future generations.

Journalists and politicians do an excellent job of attracting our attention to this conflict, but attention is not enough: we need to develop reasonable quantitative techniques that would allow us to resolve this conflict when making long-term large-scale decisions.

2 Game Theory: A Well-Tested Approach to Conflict Resolution

In real-life decision making, it is normal to have several participants with different interests. In other words, it is reasonable to have conflicts. The need to provide quantitative methods for resolving conflicts has been known for almost a century. A whole toolbox of different techniques for resolving such conflicts have been proposed and successfully used; these techniques are collectively (and somewhat misleadingly) known as *game theory*; see, e.g., [1, 2].

Game theory originally started with conflicts in which the interests of different sides are exactly opposite (they are called "zero-sum games"), and evolved into *cooperative* situations where all participants can benefit from collaboration.

One of the first solutions to cooperative conflict situations was proposed in 1950 by the (future Nobel prize winner) John F. Nash [3]. To derive a solution – which is now known as the Nash bargaining solution – Nash proposed several reasonable requirements and showed that the only alternative that satisfies all these requirements is the alternative for which the product $\prod_{i=1}^{N} u_i$ of the gains $u_1, \ldots, u_N$ of all the participants is the largest possible [1, 2, 3].

Let us therefore apply Nash bargaining solution to the inter-generational conflict situation.

3 Inter-Generational Conflict: A Formal Description

In order to apply Nash bargaining solution (or any other game-theoretic technique) to the problem of inter-generational conflict, let us describe this problem in precise mathematical terms.

For simplicity, let us assume that we are talking about only one type of resources. Under this simplifying assumption, at any moment of time, a single number characterizes the current amount of available resources. This can be money, this can be area covered by forests, this can be the amount of cattle.

In general, such resource, if unused, grows: money can be invested in a bank, forests grow and expand, cattle multiplies. For simplicity, let us assume that we have a fixed growth rate ε. So, if at present, we had r resources and we do not use any of them, then by the time of the next generation, we will have $r \cdot (1+\varepsilon)$ resources.

In the beginning, at generation 0, we have a certain amount r_0 of available resources. Each generation can spend some of these resources and leave the rest for the future generations. Let r_i denote the amount of resources with which the i-th generation starts, and let k_i be the portion of these resources that is used by this generation. In these terms, the gain of the i-th generation is equal to $u_i = k_i \cdot r_i$.

The i-th generation spends the k_i-th portion of the resources and leaves the remaining amount $r_i \cdot (1-k_i)$ for the next generation. By the time these remaining resources reach the next generation, they increase by a factor of $1+\varepsilon$, to the amount $r_{i+1} = r_i \cdot (1-k_i) \cdot (1+\varepsilon)$. What we need to decide is how much resources should each generation spend, i.e., what are the values $k_0, \ldots, k_N$.

Thus, we arrive at the following precisely formulated problem:

- We are given the number $r_0 > 0$ and an integer N.
- For each selection of $N+1$ real numbers $k_0, k_1, \ldots, k_N \in [0,1]$, we sequentially define the values $r_1, \ldots, r_N$ by using the formula $r_{i+1} = r_i \cdot (1-k_i) \cdot (1+\varepsilon)$.
- The objective is to find the values $k_0, \ldots, k_N$ for which the product $\prod\limits_{i=0}^{N} u_i$ attains the largest possible value, where $u_i \stackrel{\text{def}}{=} k_i \cdot r_i$.

4 Inter-Generational Conflict: Nash Solution to the Formalized Problem

Let us show how to solve this optimization problem. For that, let us first find an explicit expression for r_i in terms of r_0 and the coefficients $k_0, k_1, \ldots$.

For $i = 1$, we have $r_1 = r_0 \cdot (1 - k_0) \cdot (1 + \varepsilon)$. Similarly, for $i = 2$, we have $r_2 = r_1 \cdot (1 - k_1) \cdot (1 + \varepsilon)$. Substituting the above expression for r_1 in terms of r_0, we conclude that

$$r_2 = (r_0 \cdot (1 - k_0) \cdot (1 + \varepsilon)) \cdot (1 - k_1) \cdot (1 + \varepsilon) = r_0 \cdot (1 - k_0) \cdot (1 - k_1) \cdot (1 + \varepsilon)^2.$$

By induction over i, we can prove that, in general,

$$r_i = r_0 \cdot (1 - k_0) \cdot (1 - k_1) \cdot \ldots \cdot (1 - k_{i-1}) \cdot (1 + \varepsilon)^i.$$

Thus, we have

$$u_i = k_i \cdot r_i = k_i \cdot r_0 \cdot (1 - k_0) \cdot (1 - k_1) \cdot \ldots \cdot (1 - k_{i-1}) \cdot (1 + \varepsilon)^i.$$

We are interested in the largest possible value of the product $p \stackrel{\text{def}}{=} \prod\limits_{i=1}^{N} u_i$ over all possible values $k_i \in [0, 1]$. This largest possible value cannot be attained for $k_i = 0$ or $k_i = 1$ (for $i < N$). Indeed:

- If $k_i = 0$, then $u_i = k_i \cdot r_i = 0$ and thus, $\prod\limits_{i=1}^{N} u_i = 0$.
- If $k_1 = 1$, then $r_{i+1} = r_i \cdot (1 - k_i) \cdot (1 + \varepsilon) = 0$ and thus, $u_{i+1} = k_{i+1} \cdot r_{i+1} = 0$ and also $\prod\limits_{i=1}^{N} u_i = 0$.

So, the maximum is attained for $k_i \in (0, 1)$ and thus, at the point where the maximum is attained, we have $\dfrac{\partial p}{\partial k_i} = 0$.

Substituting the above expressions for u_i into the formula for the product, we conclude that for each parameter k_i, the product p contains several factors depending on k_i:

- the factor k_i in the term u_i, and
- the factor $(1 - k_i)$ in each of the following terms $u_{i+1}, \ldots, u_N$ (there are $N - i$ such terms).

Thus, with respect to k_i ($i < N$), the product p has the form

$$p = \text{const} \cdot k_i \cdot (1 - k_i)^{N-i},$$

where const denotes the product of all the factors that do not depend on k_i.

For $i = N$, the product is simply proportional to k_N and thus, its largest value is attained when the value $k_N \in [0, 1]$ is the largest possible – i.e., when $k_N = 1$.

For $i < N$, as we have mentioned earlier, the maximum of the product p is attained when $k_i \in (0, 1)$, i.e., when the derivative with respect to k_i is equal to 0. Differentiating the above expression with respect to k_i and equating the derivative to 0, we conclude that

$$(1 - k_i)^{N-i} - k_i \cdot (N - i) \cdot (1 - k_i)^{N-i-1} = 0.$$

Dividing both sides by $(1 - k_i)^{N-i-1}$, we get

$$1 - k_i - (N - i) \cdot k_i = 0,$$

i.e., that $(N + 1 - i) \cdot k_i = 1$ and

$$k_i = \frac{1}{N + 1 - i}.$$

In particular:

- for $i = 0$, we have $k_0 = \frac{1}{N + 1}$;
- for $i = 1$, we have $k_1 = \frac{1}{N}$, etc.

It is worth mentioning that for $i = N$, this general formula leads to the optimal value $k_N = 1$ – and is thus applicable for all $i = 0, \ldots, N$.

5 Discussion and Open Problem

In the intended application to an inter-generational conflict, N is the number of generations that we take into account. The larger N we take, the better. Ideally, we should consider as many generations as possible, i.e., we should consider $N \to \infty$.

However, in the limit $N \to \infty$, we have $k_i = \frac{1}{N + 1 - i} \to 0$ and thus, $u_i = k_i \cdot r_i \to 0$. Thus, in the seemingly optimal solution, when we take care of the interests of future generations, we end up with a worst-possible scenario where each generation lives in poverty (or at least does not benefit at all from the analyzed resource).

In other words, seemingly reasonable altruism can lead to solutions which are as disastrous as extreme selfishness.

The development of appropriate techniques – techniques which would lead to a reasonable resolution of this inter-generational conflict – thus remains an important open problem.

ACKNOWLEDGEMENTS. This work was supported in part by NSF grant HRD-0734825 and by Grant 1 T36 GM078000-01 from the National Institutes of Health.

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