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Testing Shock Absorbers: Towards a Faster Parallelizable Algorithm

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***Abstract*—Cars are equipped with shock absorbers, which are designed to smooth out the shocks on the road. In practice, there is a need to test them. To test the shock absorbers, we need to estimate the values of the shock absorber's parameters. If we did not have any measurement errors, then two measurements would be sufficient to determine the parameters. However, in reality, there are measurement errors. Usually, in engineering practice, it is assumed that the errors are normally distributed with 0 mean, so we can use least squares method to test it. In practice, we often only know the upper bound on the measurement errors, so we have interval uncertainty. In principle, the problem of determining the parameters of the shock absorber under interval uncertainty can be solved by reducing it to several linear programming problems. However, linear programming problems take a reasonably long time $O(n^{3.5})$. A natural way to speed up computations is to parallelize the algorithm. However, it is known that linear programming is provably the most difficult problem to parallelize. So instead, we propose a new algorithm for finding ranges for shock absorber's parameters, an algorithm which is not only faster but also easy-to-parallelize.**

I. Formulation of the Problem

Practical problem: testing shock absorbers. Shock absorbers make the car ride smoother. This not only makes the car ride mode comfortable for passengers, it also decreases the wear and tear on the cargo and the car itself.

Because shock absorbers are important, it is necessary to test them – so that we will be able to predict the shock absorber's reaction to different road conditions.

Shock absorbers: mathematical description. Shock absorbers try to smooth the vertical motions $x(t)$ of the car. In general, the motion of the shock absorber can be described by Newton's law

$$m \cdot \frac{d^2x}{dt^2} = f(t) + F\left(x, \frac{dx}{dt}\right),$$

where m is the mass, $\frac{d^2x}{dt^2}$ is the acceleration, $f(t)$ is an external vertical force, and F is the additional force created within the shock absorber.

For example, if we simply add a spring, then we get a force that follows Hooke's law $F = -b \cdot x$: the force is proportional to the (vertical) displacement. If we add viscosity, then we have an additional force which is proportional to vertical velocity: $F = -a \cdot \frac{dx}{dt}$.

In general, the dependence of the force F on the vertical displacement and vertical velocity can be non-linear. However, in practice, both the vertical displacement and the vertical velocity are relatively small – i.e., small in comparison with the horizontal displacement and velocity. Therefore, we can expand the dependence $F\left(x, \frac{dx}{dt}\right)$ in Taylor series in x and $\frac{dx}{dt}$ and keep only linear terms in this expansion – thus ignoring quadratic and higher order terms. As a result, we get

$$F\left(x, \frac{dx}{dt}\right) = F_0 - b \cdot x - a \cdot \frac{dx}{dt}$$

for some coefficients F_0, a, and b.

On the ideally smooth road, when there is no vertical displacement (i.e., $x = 0$), no vertical motion $\frac{dx}{dt} = 0$, and no external vertical force, the shock absorbers should not change anything, i.e., we should have $F = 0$. Substituting $x = 0$ and $\frac{dx}{dt} = 0$ into the above general expression for the force, we conclude that $F_0 = 0$ and thus, that

$$F\left(x, \frac{dx}{dt}\right) = -b \cdot x - a \cdot \frac{dx}{dt}.$$

Substituting this formula for F into the general Newton's law expression, and moving linear terms to the left-hand side, we conclude that

$$m \cdot \frac{d^2x}{dt^2} + a \cdot \frac{dx}{dt} + b \cdot x = f(t).$$

Thus, to describe the reaction of the shock absorbers on arbitrary road conditions – i.e., on arbitrary force function $f(t)$ – it is sufficient to find the corresponding parameters m, a, and b.

How to predict the reaction of the shock absorber on the given force $f(t)$. In the case of discrete time, when we only consider moments $t_1 < t_2 < \ldots < t_n$, a general force function $f(t)$ can be described by its values $f(t_1), \ldots, f(t_n)$ at these moments of time. A general force function can therefore be represented as the sum of n different force functions, each of which acts only at the corresponding moment of time:

$$f(t) = \sum_{i=1}^{n} f(t_i) \cdot \delta(t - t_i),$$

where $\delta(s) = 0$ for $s \neq 0$ and $\delta(0) = 1$. Since the equations relating $x(t)$ and $f(t)$ are linear, the effect of the force $f(t)$ is equal to the sum of the effects of different force components, and the effect of each force component is proportional to $f(t_i)$. Let $R(t)$ denote the reaction of the shock absorber on the unit force applied at moment 0. Then,

- the reaction to a unit force applied at moment t_i is
$$R(t - t_i),$$
- the reaction to a force $f(t_i)$ applied at moment t_i is
$$R(t - t_i) \cdot f(t_i),$$
and
- the overall reaction is equal to the sum
$$\sum_{i=1}^{n} R(t - t_i) \cdot f(t_i).$$

In the continuous case, instead of the sum, we get an integral

$$x(t) = \int R(t - s) \cdot f(s)\, ds.$$

So, to predict the reaction of the shock absorber on an arbitrary force $f(t)$, it is sufficient to find the reaction function $R(t)$.

The function $R(t)$ describes the reaction of the shock absorber to the impulse force that acts for a short period of time: we apply a force and measure the reaction.

From the above differential equation, we can conclude that the reaction function has the form

$$R(t) = A \cdot \exp(-k \cdot t) \cdot \cos(\omega \cdot t + \varphi)$$

for some amplitude A, frequency ω, phase φ, and the *damping factor* k.

When we apply an impulse force, the displacement is proportional to $R(t)$. Thus, we arrive at the following idea.

How to determine the reaction of the shock absorber. We apply an instantaneous unit force, and measure the resulting displacements $x(t_i)$ at different moments of time

$$t_1 < t_2 < \ldots < t_n.$$

In the ideal case, when the measurements are absolutely accurate, the measured values are equal to

$$x_i = A \cdot \exp(-k \cdot t_i) \cdot \cos(\omega \cdot t_i + \varphi).$$

So, the question is: how to find the values A, k, ω, and φ from these measurement results.

Need to take uncertainty into account. In practice, measurements are never 100% accurate. As a result, the measured values x_i are only approximately equal to the actual displacements $x(t_i)$. Thus, instead of the exact equality, we only have an approximate equality

$$x_i \approx A \cdot \exp(-k \cdot t_i) \cdot \cos(\omega \cdot t_i + \varphi).$$

Frequency is usually easiest to determine. There are many techniques for determining frequency ω – and the corresponding phase φ. It is also relatively easy to find maxima and minima within each cycle, i.e., the moments t_i when $\cos(\omega \cdot t_i + \varphi) \approx 1$. For these moments of time, we have

$$x_i \approx \pm A \cdot \exp(-k \cdot t_i),$$

hence

$$|x_i| \approx A \cdot \exp(-k \cdot t_i).$$

Thus, the main remaining problem is to estimate the values A and k based on the corresponding values x_i.

Case of interval uncertainty. Often, the only information that we know about the measurement error $x_i - x(t_i)$ is the upper bound Δ_i on this error; see, e.g., [5]. Please note that the measurement accuracy may be different at different parts of the scale, so the values Δ_i may be different for different measurements i.

In this case, after the measurement, the only information that we have about the actual (unknown) value $x(t_i)$ is that this value belongs to the interval $[\underline{x}_i, \overline{x}_i]$, where we denote $\underline{x}_i \stackrel{\text{def}}{=} x_i - \Delta_i$ and $\overline{x}_i \stackrel{\text{def}}{=} x_i + \Delta_i$. As a result, the only information that we have about the actual value of the quantity $|x(t_i)| = A \cdot \exp(-k \cdot t_i)$ is that this value belongs to the interval $[\underline{X}_i, \overline{X}_i]$, where we denoted $\underline{X}_i \stackrel{\text{def}}{=} \max(0, |x_i| - \Delta_i)$ and $\overline{X}_i \stackrel{\text{def}}{=} |x_i| + \Delta_i$.

Please note that we cut off the range at 0 from below, since the absolutely value is always non-negative.

In general, different values A and k are consistent with these inequalities. Our objective is to find the range of possible values of A and k. Thus, we arrive at the following problem.

Formulation of the problem. We are given the values x_i and Δ_i. Based on these values, we compute $\underline{X}_i = \max(0, |x_i| - \Delta_i)$ and $\overline{X}_i = |x_i| + \Delta_i$.

Our objective is to find the smallest $\underline{A}$ and the largest $\overline{A}$ values of A and the smallest $\underline{k}$ and the largest $\overline{k}$ values of $k \geq 0$ among all the values of A and k that satisfy the constraints

$$\underline{X}_i \leq A \cdot \exp(-k \cdot t_i) \leq \overline{X}_i$$

for all n measurements $i = 1, \ldots, n$.

Case of fuzzy uncertainty. In some cases, in addition to the guaranteed upper bounds Δ_i on the measurement error $x_i - x(t_i)$, we also have smaller bounds about which we are not 100% certain. In other words, for different degrees of uncertainty α from the interval $(0, 1)$, we have a bound $\Delta_i(\alpha)$ – and thus, an interval $[x_i - \Delta_i(\alpha), x_i + \Delta_i(\alpha)]$ that contains $x(t_i)$ with a given degree of uncertainty. These intervals form a *fuzzy set* containing all this knowledge; see, e.g., [3], [4].

Case of fuzzy uncertainty can be reduced to interval uncertainty. For fuzzy uncertainty, instead of single bounds $\underline{A}$, $\overline{A}$, $\underline{k}$, and $\overline{k}$ corresponding to absolute certainty, we also want to find similar bounds corresponding to different levels α.

Thus, to solve the problem corresponding to fuzzy uncertainty, we need to solve several interval problems – corresponding to different values α. So, the case of fuzzy uncertainty can be reduced to the case of interval uncertainty. Because of this reduction, in the following text, we will concentrate on the case of interval uncertainty.

Simplification of the problem. The main difficulty with the above formulation of the interval-related problem is that the constraints non-linearly depend on the unknown k. To simplify the problem, we can use the fact that logarithm is a strictly increasing function and thus, an inequality $p \le q$ is equivalent to $\ln(p) \le \ln(q)$. By talking logarithms of all the sides of the above constraints, we thus get the following equivalent form of the above constraint:

$$\underline{y}_i \le z - k \cdot t_i \le \overline{y}_i,$$

where we denote $\underline{y}_i \stackrel{\text{def}}{=} \ln(\underline{X}_i)$, $\overline{y}_i \stackrel{\text{def}}{=} \ln(\overline{X}_i)$, and $z \stackrel{\text{def}}{=} \ln(A)$.

Here, $z = \ln(A)$ hence $A = \exp(z)$. Since $\exp(z)$ is an increasing function, the value A attains its maximum or minimum whenever z attains its maximum or minimum. So, once we have computed the bounds $\underline{z}$ and $\overline{z}$ for z, we can find the bounds for A as $\underline{A} = \exp(\underline{z})$ and $\overline{A} = \exp(\overline{z})$. Thus, to solve our problem, it is sufficient to find the bounds of k and z under the above constraints.

New formulation of the problem. We are given the values x_i and Δ_i. Based on these values, we compute

$$\underline{X}_i = \max(0, |x_i| - \Delta_i),$$

$\overline{X}_i = |x_i| + \Delta_i$, $\underline{y}_i = \ln(\underline{X}_i)$, and $\overline{y}_i = \ln(\overline{X}_i)$.

Our objective is to find the smallest $\underline{z}$ and the largest $\overline{z}$ values of z and the smallest $\underline{k}$ and the largest $\overline{k}$ values of $k \ge 0$ among all the values of z and k that satisfy the constraints

$$\underline{y}_i \le z - k \cdot t_i \le \overline{y}_i$$

for all n measurements $i = 1, \ldots, n$.

Then, we compute $\underline{A} = \exp(\underline{z})$ and $\overline{A} = \exp(\overline{z})$.

How this problem is solved now. In order to find $\underline{z}$, we must find the smallest possible value of z under the above constraints. Both the objective function (in this case, z) and the constraints are linear in terms of the unknowns z and k. Such problems – of optimizing a linear function under linear constraints – are *linear programming* problems; see, e.g., [2].

Similarly, the problem of finding the largest possible value z and the problems of finding the smallest and the largest possible values of k are examples of linear programming problems.

There exist efficient algorithms for solving linear programming problems; see, e.g., [2] and references therein. By applying these algorithms to our problem, we can thus find the desired bounds for the shock absorber problem. This approach – and similar approaches – are described, e.g., in [1].

Limitations of the existing approach. In general, known algorithms for solving linear programming problem take time that grows with the problem size as $O(n^{3.5})$. While this dependence is polynomial, it still grows very fast for large n. It is therefore desirable to find faster algorithms.

In general, one way to speed up computations is to parallelize them, i.e., to divide the computations between several computers working in parallel. Alas, linear programming is known to be the provably hardest problem to parallelize (to be precise, P-hard); see, e.g., Section 7.2 of [6] and references therein. Thus, a new algorithm is needed.

What we do in this paper. In this paper, we propose a new faster and easy-to-parallelize algorithm for solving the above problem.

II. Analysis of the Problem

Finding bounds for k. In the above inequality, if we add $k \cdot t_i$ to all three sides, we conclude that

$$\underline{y}_i + k \cdot t_i \le z \le \overline{y}_i + k \cdot t_i.$$

The value k is possible if there exists a value z that satisfies all these inequalities. In other words, the value k is possible if there exists a value z which is

- larger than or equal to all the lower bounds $\underline{y}_i + k \cdot t_i$ and
- smaller than or equal to all the upper bounds $\overline{y}_i + k \cdot t_i$.

Such a value exists if and only if the largest of the lower bounds is smaller than or equal to the smallest of the upper bounds:

$$\max_i(\underline{y}_i + k \cdot t_i) \le \min_i(\overline{y}_i + k \cdot t_i).$$

In this case, every lower bound is smaller than or equal to every upper bound. Vice versa, if every lower bound is smaller than every upper bound, then the largest of the lower bounds is smaller than or equal to the smallest of the upper bounds.

Thus, the value k is possible if and only if the following inequality holds for every i and j:

$$\underline{y}_i + k \cdot t_i \le \overline{y}_j + k \cdot t_j.$$

For $i = j$, this is always true, since we have $\underline{y}_i \le \overline{y}_i$. Thus, it is sufficient to consider only pairs for which $i \ne j$.

By moving all the terms proportional to k to the left-hand sides and all the others to the right-hand side, we get the equivalent inequality

$$k \cdot (t_i - t_j) \le \overline{y}_j - \underline{y}_i.$$

When $i > j$, we have $t_i > t_j$, so $t_i - t_j > 0$, and we can divide both side by this positive value, resulting in

$$k \le \frac{\overline{y}_j - \underline{y}_i}{t_i - t_j}.$$

When $i < j$, then $t_i - t_j < 0$, so division by $t_i - t_j$ changes the sign of the inequality:

$$k \ge \frac{\overline{y}_j - \underline{y}_i}{t_i - t_j},$$

or, equivalently,

$$k \ge \frac{\underline{y}_i - \overline{y}_j}{t_j - t_i}.$$

Therefore, the value k is possible if and only if it is

- smaller than or equal to all the upper bounds $\dfrac{\underline{y}_i - \overline{y}_j}{t_j - t_i}$ and
- larger than or equal to all the lower bounds $\dfrac{\overline{y}_j - \underline{y}_i}{t_i - t_j}$.

This is equivalent to k being larger than or equal to the largest of the lower bounds and smaller than or equal to the smallest of the lower bounds:

$$\max_{i>j} \frac{\underline{y}_i - \overline{y}_j}{t_j - t_i} \le k \le \min_{i<j} \frac{\overline{y}_j - \underline{y}_i}{t_i - t_j}.$$

Thus, we have the desired expression for the lower and the upper endpoints for k:

$$\underline{k} = \max\left(0, \max_{i>j} \frac{\underline{y}_i - \overline{y}_j}{t_j - t_i}\right); \quad \overline{k} = \min_{i<j} \frac{\overline{y}_j - \underline{y}_i}{t_i - t_j}.$$

Please note that since $k \ge 0$, we cut off the lower bound at 0.

Finding bounds for z. Similarly, the value z is possible if and only if there exists k for which

$$\underline{y}_i + k \cdot t_i \le z; \quad z \le \overline{y}_i + k \cdot t_i.$$

By moving $\underline{y}_i$ to the right-hand side of the first inequality and $\overline{y}_i$ to the right-hand side of the second inequality, we get an equivalent inequality

$$z - \overline{y}_i \le k \cdot t_i \le z - \underline{y}_i.$$

Dividing all three sides of this inequality by a positive number $t_i > 0$ (we can always assume that we start counting time with 0), we conclude that

$$\frac{z - \overline{y}_i}{t_i} \le k \le \frac{z - \underline{y}_i}{t_i}.$$

Similarly to the above case, such a value k exists if and only if all the lower bounds are smaller than or equal to all the upper bounds:

$$\frac{z - \overline{y}_i}{t_i} \le \frac{z - \underline{y}_j}{t_j}.$$

Similarly to the previous case, it is sufficient to consider only pairs $i \ne j$. In this case, by moving all the terms proportional to z to one side and all the other terms to the other side, we get an equivalent inequality

$$z \cdot \left(\frac{1}{t_i} - \frac{1}{t_j}\right) \le \frac{\overline{y}_i}{t_i} - \frac{\underline{y}_j}{t_j}.$$

When $i < j$, we have $t_i < t_j$, hence $\dfrac{1}{t_i} > \dfrac{1}{t_j}$, so

$$\frac{1}{t_i} - \frac{1}{t_j} > 0,$$

and we can divide both sides of the inequality by this difference without changing the sign of the inequality. As a result, we get the inequality

$$z \le \frac{\dfrac{\overline{y}_i}{t_i} - \dfrac{\underline{y}_j}{t_j}}{\dfrac{1}{t_i} - \dfrac{1}{t_j}} = \frac{\overline{y}_i \cdot t_j - \underline{y}_j \cdot t_i}{t_j - t_i}.$$

When $i > j$, then $\dfrac{1}{t_i} - \dfrac{1}{t_j} < 0$, and thus, when we divide both sides of the inequality by this number, the sign of the inequality changes. So, we get

$$z \ge \frac{\overline{y}_i \cdot t_j - \underline{y}_j \cdot t_i}{t_j - t_i} = \frac{\underline{y}_j \cdot t_i - \overline{y}_i \cdot t_j}{t_i - t_j}.$$

Therefore, the value z is possible if and only if it is smaller than or equal to all the upper bounds and larger than or equal to all the lower bounds This is equivalent to z being larger than or equal to the largest of the lower bounds and smaller than or equal to the smallest of the lower bounds:

$$\max_{i>j} \frac{\underline{y}_j \cdot t_i - \overline{y}_i \cdot t_j}{t_i - t_j} \le z \le \min_{i<j} \frac{\overline{y}_i \cdot t_j - \underline{y}_j \cdot t_i}{t_j - t_i}.$$

Thus, we have the desired expression for the lower and the upper endpoints for z:

$$\underline{z} = \max_{i>j} \frac{\underline{y}_j \cdot t_i - \overline{y}_i \cdot t_j}{t_i - t_j}; \quad \overline{z} = \min_{i<j} \frac{\overline{y}_i \cdot t_j - \underline{y}_j \cdot t_i}{t_j - t_i}.$$

III. Resulting Algorithm and Its Analysis

Resulting algorithm. Once we computed the values $\underline{X}_i = \max(0, |x_i| - \Delta_i)$, $\overline{X}_i = |x_i| + \Delta_i$, $\underline{y}_i = \ln(\underline{X}_i)$, and $\overline{y}_i = \ln(\overline{X}_i)$, we can compute

$$\underline{k} = \max\left(0, \max_{i>j} \frac{\underline{y}_i - \overline{y}_j}{t_j - t_i}\right); \quad \overline{k} = \min_{i<j} \frac{\overline{y}_j - \underline{y}_i}{t_i - t_j};$$

$$\underline{z} = \max_{i>j} \frac{\underline{y}_j \cdot t_i - \overline{y}_i \cdot t_j}{t_i - t_j}; \quad \overline{z} = \min_{i<j} \frac{\overline{y}_i \cdot t_j - \underline{y}_j \cdot t_i}{t_j - t_i};$$

then we compute $\underline{A} = \exp(\underline{z})$ and $\overline{A} = \exp(\overline{z})$.

Comment. These formulas are based on the assumption that we have correctly estimated the bounds Δ_i on the measurement errors and thus, the desired values A and k satisfy the inequalities $|x_i| - \Delta_i \le A \cdot \exp(-k \cdot t_i) \le |x_i| + \Delta_i$. If we underestimate these bounds, the actual values A and k may not satisfy these inequalities; moreover, in this case, it may be possible that no values A and k satisfy all these inequalities, i.e., that there are no solutions for k and/or A at all. This means that the corresponding intervals $[\underline{k}, \overline{k}]$ and/or $[\underline{A}, \overline{A}]$ are empty, i.e., that $\underline{k} > \overline{k}$ and/or $\underline{A} > \overline{A}$.

So, if we apply the above algorithm and get $\underline{k} > \overline{k}$ and/or $\underline{A} > \overline{A}$, this means that we have underestimated the measurement errors.

Computation time of the new algorithm. To compute each of the four bounds, we find n^2 ratios corresponding to $n \cdot n = n^2$ possible pairs of indices i and j. Then, we take n^2 steps to find the smallest and the largest of these ratios. Thus, computations take time $O(n^2)$.

For large n, this is much smaller than the time $O(n^{3.5})$ that is used by the linear programming algorithm.

Possibility of parallelization. If we have an unlimited number of processors, then we can use n^2 processors to compute all the ratios in parallel – i.e., in time of one computational step.

In general, if we have N numbers $r_1, \ldots, r_N$, we need $\log_2(N)$ time to find the largest and the smallest of these values; see, e.g., [2], [6]. Indeed:

- On the first step, the 1st processor computes the maximum $r'_1 = \max(r_1, r_2)$, the second processor computes the maximum $r'_2 = \max(r_3, r_4)$, then we have $r'_3 = \max(r_5, r_6)$, $r'_4 = \max(r_7, r_8)$, etc.
- At the second step, the 1st computer computes the maximum

  $$r''_1 = \max(r'_1, r'_2) = \max(r_1, r_2, r_3, r_4),$$

  while the second computes the maximum

  $$r''_2 = \max(r'_3, r'_4) = \max(r_5, r_6, r_7, r_8),$$

  etc.
- At the third step, the first computer computes the value

  $$\max(r''_1, r''_2) = \max(r_1, r_2, \ldots, r_8).$$

- ...
- Similarly, at a step s, the first computer computes the maximum

  $$\max(r_1, r_2, \ldots, r_{2^s}).$$

- ...

When $2^s = N$, i.e., when $s = \log_2(N)$, then we compute the desired maximum, so this computation indeed takes $\log_2(N)$ steps.

In our case, we have $N \le n^2$ numbers, so computing the maximum takes

$$\log_2(N) \le \log_2(n^2) = 2 \cdot \log_2(n) = O(\log_2(n))$$

steps. Thus, the new algorithm is indeed highly parallelizable. To be more precise, it belongs to the class NC of all the problems that can be solved in polylog time (i.e., in time bounded by a polynomial of $\log_2(n)$) on a polynomial number of processors.

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