Teacher Challenges In Implementing Cognitively Demanding Tasks In The Mathematics And Science Classrooms

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TEACHER CHALLENGES IN IMPLEMENTING COGNITIVELY DEMANDING TASKS IN THE MATHEMATICS AND SCIENCE CLASSROOMS

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Charles Ambler, Ph.D.
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Angelica Monarrez

2017
Dedication

A mis padres Rodolfo y Ana con mucho cariño por todo su apoyo.
TEACHER CHALLENGES IN IMPLEMENTING COGNITIVELY DEMANDING TASKS IN THE MATHEMATICS AND SCIENCE CLASSROOMS

by

ANGELICA MONARREZ RODRIGUEZ, M.S.

DISSERTATION

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Abstract

This mixed methods study examines secondary school mathematics and science teachers’ understanding of cognitive demand and the challenges in implementing tasks at different levels of cognitive demand. The conceptual framework for this study is grounded on the conception of cognitive demand proposed by Stein, Smith, Henningsen, and Silver (2000), which includes the following levels: memorization (level 1), procedures without connections (level 2), procedures with connections (level 3), and doing mathematics and science (level 4). The study attempts to address the following research questions: 1) To what extent are secondary mathematics and science teachers able to recognize, solve and construct tasks at different levels of cognitive demand? 2) Are there relationships among teachers’ ability to recognize, solve, construct, and implement tasks at different levels of cognitive demand? and 3) What are secondary mathematics and science teachers’ challenges in recognizing, solving, constructing, and implementing cognitively demanding tasks (CDTs)? CDTs are considered tasks at level 3 and 4. We used a cognitive demand survey to test teachers’ (N=58) ability to recognize, solve, and construct tasks at different levels of cognitive demand. We employed semi-structured interviews and classroom observations to examine a subset of teachers’ (n=13) challenges in implementing cognitively demanding tasks in mathematics and science classrooms. Correlation and inferential methods were used to analyze data in response to quantitative research questions whereas meaning coding technique was employed to analyze qualitative data. Main results suggest that teachers had challenges distinguishing between the levels of cognitive demand related to procedures with and without connections. Teachers also had challenges solving tasks at the highest levels of cognitive demand and constructing tasks at the levels of procedures with connections. From the correlation analysis, we found statistically significant associations between recognizing a task at level 2 with recognizing a task at level 3 as well as between recognizing a task at level 3 with recognizing a task at level 4. Analysis of the teachers’ interviews revealed challenges related to students’ knowledge, teachers’ knowledge, and external factors. The reported teachers’ challenges may
result in declining the cognitive demand level into procedures without connections. Implications for professional development are also discussed.
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Chapter 1: Introduction

Implementing mathematical tasks that require more than memorization is necessary to assure that students understand different mathematical concepts. Teachers have to be prepared to provide students with a cognitive array of mathematical tasks. Higher-order mathematical tasks are difficult to teach and solve but should be encouraged because students can be engaged in the process of mathematical thinking (Stein, Grover & Henningsen, 1996). That is why the use of tasks at higher levels of cognitive demand plays a major role in the mathematics classroom as well as the science classroom. To be able to increase cognitive demand in mathematics and science classrooms teachers themselves need to be engaged in recognizing, solving, and constructing such tasks. Moreover, the Principles and Standards for School Mathematics (NCTM, 2000) recommends that teachers actively engage in ongoing professional development, interact with other colleagues, and self-reflect about their teaching. The Next Generation Science Standards (2013) argue that students need to be exposed to science practices so that students are prepared for the modern workforce. In this study, the challenges middle school mathematics teachers face in implementing higher-order mathematical tasks based on the experiences in a professional development setting will be shared as well as their understanding of the different levels of cognitive demand (i.e. memorization, procedures without connections, procedures with connections, and doing mathematics).

While there are recommendations to implement tasks that go beyond memorization, studies show that in reality, teachers are still implementing tasks that only require memorization more often. In an international video analysis of mathematics teachers by the TIMSS Video Study, they found that 95% of the time spent in U.S. classrooms was spent performing
mathematical work (USDE-NCES, 2003). However, most of that time was spent doing routine exercises. Moreover, 67% of eight-grade mathematics problems per lesson were spent in low complexity problems with 27% of moderate complexity and only 6% of high complexity. Additionally, this study found that more than two-thirds of the time were spent using procedures while only 20% of the time was spent making connections (USDE-NCES, 2003). Thus, more efforts need to be put forward to a wider implementation of mathematical tasks that require higher cognitive demand since tasks that engage students to “make purposeful connections to meaning or relevant mathematical ideas lead to a different set of opportunities for student thinking” (Stein et al., 2000, p. 11). Those opportunities allow students to make personal connections with mathematics.

Also, the performance of U.S. students is not very strong. In the mathematics part of the TIMSS, U.S. students performed significantly lower on average than international students in 8th and 11th grades (USDE-NCES, 2003). Nationwide, according to the 2013 National Assessment of Educational Progress (NAEP), only 35% of eight-grade students demonstrated mathematical proficiency (NCES, 2014). While looking at the Hispanic population of the United States, only 21% of Hispanic eight-graders reached proficiency in 2013. In Texas, over half of the students fell below proficiency levels (NCES, 2014). In science, based on the results from the 2015 NAEP, students in 8th performed higher than in 2009 and 2011. However, they are way below the advanced level with the majority performing below the proficient level. In Texas, the percentage of students who performed at or above the NAEP Proficient level was 37% in 2015 (NCES, 2015) with only 26% of Hispanics reaching proficiency. Only 10% of students in the United States reached the advanced international benchmark (Martin, Mullis, Foy & Stanco, 2012).
If students are falling behind in mathematics and science proficiency, they will not have the proficiency to succeed in college. Even beyond college, people need to have a better understanding of mathematics in order to “make well-informed decisions by formulating conjectures and testing hypothesis” (Conley, 2010, p. 236). Hence, students need to be provided with opportunities to engage with cognitively demanding mathematics, tasks that require more thinking to be solved, in ways that will help them in their future.

1.1 Purpose

According to Stein, Smith, Henningsen, and Silver (2000) providing teachers with opportunities to discuss what makes a task challenging, how classroom events influence the tasks, and reflect on their practice is beneficial for teachers. This study provides teachers with those experiences and examines teachers’ knowledge and practices in understanding and implementing of cognitively demanding tasks (CDTs). CDTs are those tasks that require more thinking to solve. The purpose of this study was to determine how learning about the different levels of cognitive demand of mathematical tasks might influence implementation of such tasks and what are the challenges teachers face implementing CDT. Professional development does not commonly include learning about cognitive demand. This study analyzes the extent to which teachers (1) recognize tasks at different levels, (2) solve tasks at different levels, (3) construct tasks at different levels, and (4) challenges in implementing CDT in the classroom. Figure 1.1 is a representation of what this study is attempting to examine, particularly the relationship to recognition (R), solution (S), and construction (C) and ultimately the effect it has on implementation (I) of the CDT in mathematics/science classroom.
The purpose of this study is to examine secondary teachers’ understanding of CDT and the implementation of such tasks in their classrooms. Moreover, this study aims to examine the following research questions:

1. To what extent are secondary mathematics and science teachers able to recognize, solve and construct tasks at different levels of cognitive demand?

2. Are there relationships among teachers’ ability to recognize, solve, construct, and implement tasks at different levels of cognitive demand?
3. What are secondary mathematics and science teachers’ challenges in recognizing, solving, constructing, and implementing CDTs?

1.3 **SIGNIFICANCE OF STUDY**

In this study, teachers were able to discuss with peers about the levels of cognitive demand. Allowing them to discuss their ideas with other teachers might lead to a richer understanding of the different levels of cognitive demand. The significance of this study comes from the need to enable students to solve high-level tasks. This study provides data for teachers’ understanding of cognitive demand by documenting whether teachers can recognize, solve, and construct tasks at different levels of cognitive demand and the challenges in implementing them in the classroom.

Numerous studies have focused on different types of implementation of cognitively demanding tasks in the classrooms (Arbaugh & Brown, 2005; Boston, 2012; Boston & Smith, 2009; Boston & Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013; McDuffie & Mather, 2006; Patterson, Musselman & Rowlett, 2013; Stein, Grover & Henningsen, 1996; Stein & Kaufman, 2010; Thomas & Williams, 2008). Most of these studies were based on a standard-based reform that calls for higher levels of cognitive demand (Arbaugh & Brown, 2005; Bayazit, 2013; Boston, 2012; Boston & Smith, 2009; Boston & Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013; Norton & Kastberg, 2012; Porter, 2002; Porter, 2004; Stein, Engle, Smith & Hughes, 2008; Stein, Grover & Henningsen, 1996; Stein & Kaufman, 2010). Based on research, learning about cognitively demanding tasks in a professional development environment has helped teachers understand and implement those tasks in the classrooms (Boston, 2012; Boston & Smith, 2009; Boston & Smith, 2011; Stein, Grover & Henningsen, 1996; Stein & Kaufman, 2010). However, more research needs to be done to understand the factors that are
impeding teachers to implement cognitively demanding tasks. Finally, we need to understand better the effects of cognitively demanding tasks on students’ achievement and how it correlates with teachers’ understanding and learning of the different levels of cognitive demand.

This study is grounded on two aspects of teaching: learning opportunities (“how much thinking is called for in the classroom”) and teaching opportunities (“the kind of teacher knowledge needed to sustain students’ high-level thinking in the classroom”) (Tchoshanov, 2011, p. 145). In addition, this study will add to the current literature on the utilization of the cognitive demand framework (Henningsen & Stein, 1997; Stein & Kaufman, 2010; Stein, Grover, & Henningsen; 1996) to research about selecting and implementing high-levels of CDT. This study builds on research about learning of cognitive demand levels in a professional development environment for mathematics and science teachers. (Arbaugh & Brown, 2005; Arbaugh, Lanin, Jones, & Park-Rogers, 2006; Barriteau, 2013; Boston, 2006, 2012, 2013; Boston & Smith, 2009; Boston & Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013; Tekkumru-Kisa, 2013; Tekkumru-Kisa, Stein, 2015; Zohar, 2004). By understanding the challenges of teachers in implementing cognitively demanding tasks we can provide teachers with opportunities to overcome those challenges.

1.4 LIMITATIONS

This study examines secondary teachers’ understanding and implementation of cognitively demanding tasks through limited number of classroom observations. It is important to point out that few classroom observations may not accurately indicate the level of cognitive demand on all the tasks presented to the students. That is why there is an attempt to understand the teachers’ reasoning for implementing or not implementing cognitively demanding tasks by interviewing them. Also, external influences such as personal experiences, other professional
development experiences, leadership in their districts and school cannot be controlled for but may affect the results of this study. Finally, there is no intent to generalize the results of this study because of the small sample size in the quantitative part. Also in the qualitative part of the study generalizability is not of interest but rather an understanding of the phenomena for theory.

The organization of this dissertation is as follows. In Chapter 1, the purpose, three research questions, significance, and limitations of the study were explained. Chapter 2 starts by providing an overview of teachers’ knowledge, mathematics and science knowledge, and the importance of mathematics and science tasks. After that, the literature review of cognitive demand is divided into two parts: cognitive demand frameworks in mathematics and cognitive demand frameworks in science. In the cognitive demand in mathematics section, it was found that studies have focused on teachers and students. In the cognitive demand in science sections, it was found that studies have focused on teachers and curriculum. Chapter 3 provides the methodology utilized in this study. There was a pilot study and the main study. This study follows a mixed method design; a cognitive demand survey was given to all teachers then a smaller sample was chosen for interviews. This chapter also explained the data analysis and demographic information of teachers that participated in this study. Chapter 4 shows the results separated by pilot study and main study for each research question. Finally, Chapter 5 provides a discussion of all results, implications for theory, practice, and policy, future research, and a conclusion.
Chapter 2: Literature Review

Developing mathematical and science tasks that focus on higher-order thinking is critical for students’ understanding of mathematical and science concepts. *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) contain recommendations for teachers to challenge their students and assure that they have learned the concepts well. In addition, the Next Generation Science Standards (NGSS) also provide recommendations for science teachers to provide students with opportunities to engage in scientific and engineering practices by engaging in the practices of inquiry. According to the National Research Council (2012) “learning science and engineering involves the integration of the knowledge of scientific explanations (i.e., content knowledge) and the practices needed to engage in scientific inquiry and engineering” (p.11).

Teachers have to assess students’ understanding (Shepard, Hammerness, Darling-Hammond, Rust, Snowden, Gordon, Gutierrez, & Pacheco, 2005). Mathematical and science tasks, among other means, are used to assess student knowledge and understanding of content. Boston and Smith posit (2009), "different kinds of tasks lead to various types of instruction, which subsequently result in different opportunities for students' learning" (p.122). In addition, "In effective teaching, worthwhile mathematical tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually" (National Council of Teachers of Mathematics [NCTM], 2000, p. 18). “Engaging in the practices of science helps students understand how scientific knowledge develops; such direct involvement gives them an appreciation of the wide range of approaches that are used to investigate, model, and explain the world” (NGSS, 2013, p. 1). Implementing tasks at higher levels of cognitive demand plays a
major role in any mathematics and science classroom. Some evidence suggests that selecting
cognitively demanding tasks affects students learning (Boaler, 2002; Boaler & Staples, 2008;
Hiebert & Wearne, 1993; Stein & Lane, 1996).

In this chapter, a review of the literature relevant to four main areas significant to this
study is presented: teacher knowledge, knowledge of mathematics and science teachers,
mathematical and science tasks, and cognitive demand. In the first section, three knowledge
bases for teachers are identified and discussed (subject matter knowledge, pedagogical content
knowledge, and curricular knowledge). In the second section, three knowledge bases for
mathematics teachers are identified (mathematical knowledge for teaching, content knowledge
for teaching mathematics, and specialized content knowledge). In addition, five aspects of
knowledge related to science are identified (science subject matter, academic and research
knowledge, pedagogical content knowledge, professional knowledge, classroom knowledge).
Technology pedagogical content knowledge is also explained. In the third section, the role of
mathematical and science tasks in the classroom is presented. Finally, in the fourth section, the
cognitive demand construct is unpacked by presenting different cognitive demand frameworks in
mathematics and studies that have utilized those frameworks followed by an overview of
different cognitive demand frameworks in science with several studies as well.

2.1 TEACHER KNOWLEDGE

Shulman (1986) moved from research on teaching focused on classroom management,
organization, and lesson planning to research focusing on the actual content of the lessons taught,
the questions asked, and the explanations offered. He argued for a need to examine both
pedagogical knowledge and content knowledge and suggested a major component of teacher
expertise that encompasses those two: pedagogical content knowledge. This study draws on
Shulman’s (1986) categories of content knowledge: subject matter (content) knowledge, pedagogical content knowledge, and curricular knowledge. Content knowledge refers to knowledge of facts and concepts as well as the structures of the subject matter. Pedagogical content knowledge includes “the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations in a word, the ways of representing and formulating the subject that make it comprehensible to others” (Shulman, 1986, p. 9). Curricular knowledge refers to the knowledge of programs, resources, and instructional materials available. In the following section, I will present the knowledge needed specifically for mathematics teachers.

2.2 Teachers’ Mathematics and Science Knowledge

Based on Shulman’s (1986) categories of teacher knowledge, other scholars have specified the knowledge that mathematics and science teachers need. One aspect of mathematics teachers’ knowledge is mathematical knowledge for teaching (MKT) which is a combination of pedagogical content knowledge and subject matter knowledge (Hill, Ball, & Schilling, 2008). Some researchers have examined the relationship between MKT and teachers implementation of cognitively demanding tasks (Charalambous, 2010; Stein & Kaufman, 2010) as well as the relationship between content knowledge for teaching mathematics as part of MKT and the relationship between cognitively demanding tasks (Wilhelm, 2014). Charalambous (2010) found some evidence that suggests that there is a positive association between the teachers’ MKT and the level of cognitive demand at which tasks at enacted at their lessons. Wilhelm (2014) found that teachers’ mathematical knowledge for teaching and conceptions of teaching and learning mathematics were related to their enactment of cognitively demanding tasks. Other studies have focused on other aspects of mathematics teacher knowledge, such as content knowledge for
teaching mathematics and specialized content knowledge (Hill, 2007). Content knowledge for teaching mathematics includes common content knowledge (i.e. what a middle school student should learn) and specialized content knowledge (i.e. what an average well-educated adult should know) (Hill, 2007).

In science, one study examined pedagogical content knowledge and found that when science teachers try to teach unfamiliar concepts, they often pose low cognitive demand questions (Carlsen, 1993). Another study found teaching experience as a major source of PCK along with adequate subject-matter knowledge in a chemistry classroom (Vandriel, Verloop, & de Vos, 1998). Barnett and Hodson (2001) found that exemplary science teachers utilize four kinds of knowledge: academic and research knowledge (science content knowledge, knowledge about the nature of science, and knowledge about how and why students learn), pedagogical content knowledge (knowing how to set teaching goals, organize a sequence of lessons into a coherent course, conduct lessons), professional knowledge (knowledge of curriculum documents, the duties of teachers, union matters, information about school administration and procedures for communicating with parents), and classroom knowledge (knowledge that a teacher has to their classroom and students) (p. 438).

Another aspect of PCK that applies to both mathematics and science is technology PCK (TPCK). Nies (2005) argues that a teacher should develop knowledge of how their subject matter relates to technology. According to Mishra and Koehler (2006) “TPCK is the basis of good teaching with technology and requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face” (p. 1029).
2.3 MATHEMATICAL AND SCIENCE TASKS

In order to understand the relationship between classroom instruction and student learning Stein et al. (2000) examined the cognitive demands of instructional tasks “since instructional tasks form the basis of students’ opportunities to learn mathematics” (p. 3). Doyle (1983) defined the term academic tasks by “the answers students are required to produce and the routes that can be used to obtain these answers” (p. 161). The term task focuses on three essential aspects of students’ work: 1) the product students are required to create, 2) the operations needed to create the product, and 3) the resources that are given to students (Doyle, 1983). The focus of this study will be based on the theory that the tasks assigned to the classroom by the teachers and done by the students determine students learning (Doyle, 1988; Stein & Smith, 1998). A mathematical task is defined by Stein et al. (1996) as,

A classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea. An activity is not classified as a different or new task unless the underlying mathematical idea toward which the activity is oriented changes. Thus, a lesson is typically divided into two, three, or four tasks rather than into many more tasks of shorter duration. (p. 460)

According to the NGSS (2013), mathematical tasks are widely used in science to predict behavior, test hypothesis, and test the validity of predictions. Science tasks must adhere to the view by the National Research Council (2012) that “science is not just a body of knowledge that reflects current understandings of the world; it is also a set of practices used to establish, extend, and refine that knowledge. Both elements –knowledge, and practice– are essential (p. 26). Stein et al. (1996) developed the mathematics task framework (see figure 2.1) and showed the different phases that mathematical tasks go through in order to affect students’ learning. There are
different phases in how mathematical tasks are presented: the way they are represented in instructional materials, the way they are set up in the classroom, and the way they are implemented in order to affect student learning. Stein et al. (1996) also show the factors that influence these phases such as how teacher goals impact the setup phase and classroom norms influence the implementation phase.

Figure 2.1: Mathematical task framework (from Stein et al., 1996)

Finally, different kinds of tasks also affect students’ learning because they provide students a broad range of opportunities to learn and think (Boston, 2006; Stein & Smith, 1998). Thus, tasks that only require memorization provide students with one type of opportunity to learn while tasks that require more conceptual understanding provide students with a different kind of opportunity to think (Stein & Smith, 1998). For this reason, it is essential to examine tasks through the lens of cognitive demand. In the following section, I will explain the levels of cognitive demand and its relationship to mathematical tasks.
2.4 COGNITIVE DEMAND

Cognitive demand has been defined as the kind of thinking process required when solving a task (Smith and Stein, 1998). According to Smith and Stein (1998),

Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking. (p. 269)

Smith and Stein (1998) separated low-level from high-level cognitive demands where memorization and procedures without connection fall at the low-level and procedures with connections and doing mathematics are at the high-level (see Figure 2.2). As explained by figure 2.2 tasks at the level of memorization (level 1) involve reproducing previously learned facts, rules, formulae, and definitions as well as committing them to memory. Tasks at this level “cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure” (Smith & Stein, 1998). Usually, such tasks have no connection to the meaning of facts, rules, formulae, or definitions.

Procedures without connection (level 2) are algorithmic by nature and require a limited cognitive demand for completion. Moreover, such tasks do not ask for connection to the concepts or meaning that underlies the procedure. Procedures with connections (level 3) tasks focus students’ attention on an understanding of mathematical concepts and ideas. Such tasks usually are represented in multiple ways (e.g., numerical, graphical, visual, concrete, symbolic) and require making connections among representations. The highest level of cognitive demand according to Smith and Stein (1998) - doing mathematics and science (level 4) - requires non-routine, non-algorithmic thinking to explore mathematical concepts, processes, and relationships.
Doing mathematics tasks demands significant cognitive effort due to the unpredictable nature of the problem-solving process at this level.

<table>
<thead>
<tr>
<th>Low-Level Cognitive Demands</th>
<th>High-Level Cognitive Demands (CDTs)</th>
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<tbody>
<tr>
<td><strong>Memorization Tasks</strong></td>
<td><strong>Procedures With Connections Tasks</strong></td>
</tr>
</tbody>
</table>
| - Involve either producing previously learned facts, rules, formulae, or definitions or committing facts, rules, formulae, or definitions to memory.  
  - Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.  
  - Are not ambiguous- such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.  
  - Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced.  

**Procedures Without Connection Tasks**
- Are algorithmic. Use of the procedure is either specifically called for, or its use is evident based on prior instruction, experience, or placement of the task.  
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.  
- Have no connection to the concepts or meaning that underlie the procedure being used.  
- Are focused on producing correct answers rather than developing mathematical understanding.  
- Require no explanations or explanations that focus solely on describing the procedure that was used.

**Doing Mathematics Tasks**
- Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathways explicitly suggested by the task, task instructions, or a worked-out example).  
- Require students to explore and to understand the nature of mathematical concepts, processes, or relationships.  
- Demand self-monitoring or self-regulation of one's own cognitive processes.  
- Require students to access relevant knowledge in working through the task.  
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.  
- Require considerable cognitive effort and may involve some level of anxiety for the student due to unpredictable nature of the solution process required.

Figure 2.2: The Task Analysis Guide (from Stein et al., 2000)

The four levels of cognitive demand have been explained in this section and how they relate to mathematical tasks. In the following section, studies that have been done on the utilization of cognitively demanding tasks will be presented and examined. For the purposes of
this study, I refer to cognitively demanding tasks as tasks that are at level 3 and 4. As explained by Stein et al. (200) level 3 and 4 tasks are considered high-level cognitive demand. It is important to understand how researchers have studied the cognitive demand levels and mathematical tasks at different levels.

2.5 THE COGNITIVE DEMAND CONSTRUCT IN MATHEMATICS

Several authors have used the cognitive demand construct with different levels where each author may talk about processes, knowledge or both. Bloom’s (1956) taxonomy levels have also been used: knowledge, comprehension, application, analysis, synthesis, and evaluation. Bloom’s taxonomy has one cognitive domain that is related to mental skills, and each level is linked to the main verb that is attached to it. Andrew Webb (1997) Depth of Knowledge (DOK) has four levels: recall, basic application, strategic thinking, and extended thinking. Webb’s levels are nominative and “each level is dependent upon how deeply students understand the content in order to respond, not simply the verb used” (Hess, 2006, p. 4). For example, Level 1 can ask students to recall a simple or more complex concept. Andrew Porter (2002) has five levels: memorize, perform procedures, communicate understanding, solve nonroutine problems, and conjecture/generalize/prove. Stein et al. (2000) and they apply to mathematical tasks: memorization, procedures without connections, procedures with connections, and doing mathematics. Stein et al. levels are more related to processes where they see memorization of facts at the lowest level, performing procedures without connections at the second level connecting procedures at the third level, and making generalizations at the highest level.

Numerous studies support the implementation of cognitively demanding tasks based on current education reforms (Bayazit, 2013; Campbell, Davis & Adams, 2007; Porter, 2002; Porter, 2006; Resnick, 2006; Stein, Engle, Smith & Hughes, 2008). First, Porter (2002) in his
presidential address at the American Educational Research Association (AERA) provided tools for measuring the content of instruction and alignment. Figure 2.3 describes the two-dimensional matrix developed by Porter that describes mathematics content with topics as rows and the cognitive demand as columns (Porter, 2002, p. 4). In this figure, somebody can analyze the level of cognitive demand that the teacher is implementing based on a specific mathematical and science topic being presented. According to Porter (2002), “Knowing the content of instruction, educational materials, content standards, and professional development is key to monitoring the implementation and effects of education reform” (p. 3). He proposes this matrix for alignment of content instruction, but he argues that it can also be used for analysis of instructional materials. While the categories of cognitive demand are different from Stein, Smith, Henningsen, and Silver (2000), they provide a similar framework to examine cognitive demand in various concepts.

![Content Matrix](image)

Figure 2.3: Content Matrix (Porter, 2002, p. 4)

Later on, Porter (2006) related the content matrix to the intended, enacted, assessed, and learned curricula, although he does not discuss the learned curricula and focus mostly on the
enacted curricula. The intended curriculum is the one stated in the standards; the enacted curriculum is the taught by teachers for the students, the assessed curriculum is the one that’s tested (Porter, 2006). Thus the content matrix can be used to measure the intended, enacted and assessed curricula. For example, it can be utilized to measure textbooks (intended), measure classroom observations (enacted), and tests (assessed). Porter (2006) also argues that it can help measure vertical and horizontal alignment and explains in the following paragraph:

- Once the enacted, intended, and assessed curricula have been measured, questions can be asked about the extent to which content is similar across them. To the extent content is the same, they are said to be aligned. For example, one might ask to what extent a student achievement test is aligned with a state’s content standards. In fact, the No Child Left Behind Act of 2001 (NCLB) requires that each state aligns assessments to content standards. If the content assessed is exactly the same as the content represented in the standards, alignment is perfect. There are two ways in which alignment can be less than perfect: Content in the standards may not be assessed, and content assessed may not be in the standards. (p. 9)

As we can see, cognitively demanding tasks have to go beyond implementation in the classrooms and as Porter (2006) explains the need to be an alignment of all different pieces of the curriculum.

Campbell, Davis, and Adams (2007) provided a framework that will allow teachers with English language learners’ population to increase the cognitive demand in the classrooms. In regards to the use of cognitive demand tasks the authors claimed, “When teachers identify and make explicit the factors that increase cognitive demands, they can then assess students’ understandings in those areas and provide appropriate support in instruction.” (Campbell, Davis
& Adams, 2007, p. 6). Also, they argue that for students, the cognitive demand will depend on how long they have been exposed to a specific concept, thus for some students the cognitive demand may appear higher than for others based on their prior knowledge. This article was the only study that was aimed at English language learners’ issues. However, this study did not collect any data, and their only purpose was to present the framework.

In an article written by Stein, Engle, Smith and Hughes (2008) five practices were provided for teachers. The authors of this paper assert “teachers are often faced with a wide array of student responses to cognitively demanding tasks and must find ways to use them to guide the class toward deeper understandings of significant mathematics” (p. 314). Thus five practices are provided to facilitate mathematical discussions around cognitively demanding tasks.

The five practices are: (1) anticipating likely student responses to cognitively demanding mathematical tasks; (2) monitoring students’ responses to the tasks during the explore phase; (3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase; (4) purposefully sequencing the student responses that will be displayed; and (5) helping the class make mathematical connections between different students’ responses and between students’ responses and the key ideas. (p. 321)

What Stein et al. (2008) proposed is the integration of all the five practices as a whole-class discussion after students are presented with high levels of cognitive demand tasks. In science education, there is a similar five practices model (Cartier, Smith, Stein, and Ross, 2013). These practices were also designed to “advance the science understanding of the class as a whole during task-based discussions” (p. 28). The five practices are as follow:

(1) anticipating how students are likely to respond to a task; (2) monitoring what students actually do as they work on the task in pairs or mall groups; (3) selecting particular
students to present their work during the whole-class discussion; sequencing the student work or products that will be displayed in a specific order; and connecting different students’ responses and connecting the responses to key scientific ideas. (p. 28)

The language of both models is similar because they both focus on fostering productive discussions either in the mathematics classroom or in the science classroom.

Bayazit (2013) examined three elementary school mathematics textbooks used in Turkey. Grades 6, 7, and 8 are considered elementary in Turkey while in the United States they are considered middle school grades. He used the task analysis framework from the levels of cognitive demand to analyze the tasks at those textbooks. 174 tasks were identified from any content area from Grades 6 through 8 (Figure 2.4 from Bayazit, 2013). Around 75% of those tasks were related to high levels of cognitive demand while the 25% of the rest were related to low levels of cognitive demand. However, out of the 75% of those in the high levels, the vast majority (63.8%) were procedures with connections and the rest (11.5%) doing mathematics. Among the 25% of the low level, the majority (23%) were procedures with connections with a small percentage in the memorization level (1.7%) (Bayazit, 2013). We can claim that the majority of the tasks fall between the middle levels of cognitive demand. There were no other articles of this kind for American textbooks.
Finally, an editorial piece written by Resnick and Zurawsky (2006) for the American Educational Research Association (AERA) calls for the use of cognitive demand. Resnick and Zurawsky (2006) contends that it is important to raise cognitive demand levels to enhance career options for students. Raising the cognitive demand levels in the classroom is essential especially for minority students since it is necessary to prepare them for a better future in mathematics and science by giving them access to higher mathematics. Better-prepared teachers and high curriculum standards are needed (Resnick & Zurawsky, 2006). The authors concluded with the following for policy makers:

First, embrace high expectations for all students in mathematics. Informed civic engagement and a competitive, global economy demand higher levels of technical skill. Second, institute curriculum policies that broaden course-taking options for traditionally underserved students. This includes avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework. Third, raise cognitive demand in mathematics teaching and learning in both
elementary and secondary schools. Elevated thinking processes come into play when students focus on mathematical concepts and connections among those concepts. High cognitive demand is reinforced when teachers maintain the rigor of mathematical tasks, for example, by encouraging students to explain their problem-solving. (p. 3)

Cognitively demanding tasks implementation is a job that has to be done by teachers, policy makers, and administrators.

2.5.1 Teacher lens

The vast majority of these studies examined teachers’ understanding and implementation of cognitively demanding tasks in a professional development environment (Arbaugh & Brown, 2005; Arbaugh, Lanin, Jones, & Park-Rogers, 2006; Barritteau, 2013; Boston, 2006; Boston, 2012; Boston, 2013; Boston & Smith, 2009; Boston & Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013; McDuffie & Mather, 2006; Otten, 2012; Patterson, Musselman & Rowlett, 2013; Stein, Grover & Henningsen, 1996; Stein & Kaufman, 2010; Tchoshanov, Lesser & Salazar, 2008; Thomas & Williams, 2008) and three that were not in a professional development setting (Choppin, 2011; Silver, Mesa, Morris, Star & Benken, 2009; Tchoshanov, 2011). Stein, Grover and Henningsen, (1996) argue about the study of implementation of cognitively demanding tasks, “Of particular interest to mathematics reform are those instances in which tasks start out as cognitively demanding but, during the course of implementation, decline into somewhat less-demanding activities versus those cases in which tasks start out as cognitively demanding and remain so.” (p. 461-462). These studies focused on different aspects of the implementation. For instance, Jackson, Garrison, Wilson, Gybbons and Shahan, (2013) examined the setup and introduction stage, while McDuffie and Mather (2006) examined the instructional materials in this selection. One study used a different framework called Depth of
Knowledge (DOK) for mathematics, which also consists of four levels: recall and reproduction, skills and concept, strategic thinking, and extended (Webb, 2002), to check whether teachers can accurately recognize problems at each level (Patterson, Musselman & Rowlett, 2013). Webb’s (2002) depth-of-knowledge levels can be used in four different content areas: language arts, mathematics, science, and social studies.

Other studies focus on different aspects of the lesson. In a dissertation by Otten (2012) participation and cognitive demand were examined. In this study participatory demand was a significant predictor of standardized gain scores but no relationship between standardized gain scores and cognitive demand. In another dissertation, Barritteau (2013) concentrated on the classroom management and its effect on the setup and implementation phases of cognitively demanding tasks. In this dissertation, Barritteau (2013) found that teacher beliefs affect instruction and students opportunities to learn. In addition, the level was often decreased to off tasks behavior. However, when elements of classroom management were addressed in the lesson plan high levels of cognitive demand were maintained.

According to Arbaugh and Brown (2005), “Professional development can be described as an experience that promotes teacher change – in beliefs, in knowledge, and/or in practice” (p. 501). Some studies suggest that learning about cognitive demand levels in a professional development environment helps teachers analyzing the different tasks but advice that the professional development periods should be longer than one year (Boston, 2006; Thomas & Williams, 2008). Boston and Smith (2009) also found that after participating in a professional development workshop, teachers significantly increased the level of cognitive demand and maintained it throughout the study. In another study of a task-centric professional development workshop teachers were found to improve their ability to select and implement cognitively
challenging tasks (Boston & Smith, 2011). McDuffie and Mather (2006) followed one teacher and saw how her selection and lesson planning shifted during the professional development. However, they found that the tasks selected by the teacher were from lower-level cognitive demand tasks and that the text-based lessons were also mostly following procedures. Thus there is a need for an alignment among text-based tasks and teacher-made tasks.

Boston (2006) conducted an experimental control group study with 18 teachers participating in a professional development initiative and ten teachers who did not participate in the professional development initiative. She found that teachers who participated in the initiative were more likely to use high-level tasks than those who did not participate. In a similar study, Smith and Stein (2009) found that teachers had challenges in recognizing tasks at level three procedures with connections. Later on, Boston and Smith (2011) went further and examined the selection and examination of cognitively demanding tasks at three different points: before and after their participation and a follow-up two years later this time without a control group. Those teachers that exhibited improvements through the time were those who used tools and frameworks, were self-reflective and mentored pre-service teachers (Boston & Smith, 2011).

Boston (2012) used an instrument called Instructional Quality Assessment (IQA) Mathematics Toolkit to examine the quality of classroom instruction. This instrument was also used in other articles (Boston & Smith, 2009; Boston & Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013). Boston (2012) argues, “The IQA can also provide an assessment of teachers’ ability to maintain the cognitive demands of high-level tasks throughout implementation in a mathematics lesson.” (p. 93). In the qualitative analysis of classroom observations of videotapes, Boston (2012) found that after teachers participated in the Enhancing Secondary Mathematics Teacher Preparation Project (ESP) teachers were able to utilize
cognitively demanding tasks in their classrooms and were able to maintain the level. At the same time, the classroom observations revealed a lack of whole-group discussions.

Later, Boston (2013) conducted a mixed-methods study that suggested that the professional development workshop provided teachers with the opportunity to increase their knowledge of cognitive demands. Teachers also developed new ideas of mathematical tasks. In regards to the knowledge of cognitive demand, Boston (2013) analyzed whether teachers were able to recognize the different levels and found the following:

Teachers were successful at classifying “Doing Mathematics” tasks as a high level on both the pre- and post-workshop task sort. When teachers incorrectly classified a doing mathematics task as having low-level demands on the post-workshop task sort, their rationales indicated that: (1) the task did not require an explanation; (2) the task did not connect to a real-world context; or (3) the task did not require mathematical thinking. (p. 21)

Based in Boston (2013) results we can see that for teachers in her study a high level task requires an explanation and connection to a real-world context. In order to understand teachers’ reasoning to classify a task as level 4 (doing mathematics), it is essential to allow them to construct their own tasks.

Another important result from Boston (2013) is that teachers had difficulty categorizing tasks at level 3 (procedures with connections) in both the pre- and post-workshop test. Compared to how teachers categorized the doing mathematics tasks, teachers incorrectly categorized the procedures without connections three times as often. According to Boston (2013)

The predominant rationale teachers provided for classifying a procedures with connections task as low level was the presence of a stated or implied procedure or
“steps” for solving the task (i.e., procedures with connections were classified as “Procedures without Connections” tasks), overlooking the opportunities for developing mathematical connections and understanding embedded in the task. (p. 22)

While tasks at level 3 (procedures with connections) also require steps for solving the task, they go further and require more thinking that just doing all the steps.

Stein, Grover, and Henningsen (1996) used narrative summaries and observations to measure the teachers’ understanding and implementation of cognitively demanding tasks. They studied the level of cognitive demand, among other variables, in the task setup phase and the task implementation phase. There was some consistency between the features of the tasks during the setup phase and the implementation phase, but the actual levels of cognitive demand seemed to decline. In other words, the level seemed higher at the setup phase and declined at the implementation phase. Stein and Kaufman (2010) studied two topics in the standard-based mathematics curricula: Everyday Mathematics and Investigations. Their main focus was on the curriculum materials that teachers are assigned to use. Stein and Kaufman (2010) found that teachers were able to maintain high levels of cognitive demand tasks for problems dealing with investigations but not so much for problems dealing with everyday mathematics.

Similarly, Jackson, Garrison, Wilson, Gybbons and Shahan (2013) were interested in how teachers introduced tasks, but they also examined the concluding whole-class discussions after the task ended. This quantitative study includes 165 teachers from different districts by using an expanded version of the IQA instrument (see Boston, 2009) to assess the quality of instruction for 460 lessons. While only providing findings that are descriptive in nature, Jackson, Garrison, Wilson, Gybbons and Shahan (2013) argue, “Our findings suggest that the quality of
the setup appears to be related to students’ opportunities to learn in the concluding whole-class discussion.” (p. 677).

Arbaugh and Brown (2005) used written artifacts, but they also conducted interviews and evaluated at the mathematical tasks used in the classroom along with study group meetings. In this study, Arbaugh and Brown (2005) examined how a group of seven high school mathematics teachers selected mathematics tasks in geometry classes over a period of 8 months, with the purpose of increasing the use of cognitively demanding tasks. The authors concluded, “The teachers in this group showed more concern with the relationship that tasks have with the work of students at the end of the study than was the case at the beginning of the study.” (Arbaugh & Brown, 2005, p. 518).

Arbaugh et al. (2006) examined 26 secondary teachers that were using a problem-based mathematics textbook in one district. According to Arbaugh et al. (2006) a problem-based textbook is “designed to constitute a coherent set of materials that develop mathematical ideas through a problems-based approach... the instructional sequence is highly dependent on active student involvement in exploration and sense-making and less on the ‘Demonstrate and Practice’ instructional model” (p. 518). In this qualitative study of classroom observations, the authors found that instructional practices fell along a wide continuum of lesson implementation. On the interviews, they found that teachers’ beliefs towards students’ abilities play an important role in lesson implementation. They separated the tasks from their lessons in different groups in the low-lesson quality group half of the tasks were coded as requiring a high level of cognitive demand while on the high lesson quality group the cognitive demand was maintained high for most of the time. Arbaugh et al. (2006) argue that even when teachers are presented with resources like these textbooks might not result in the teachers using the textbooks as intended.
Teachers’ beliefs may lead to decreasing the cognitive demand required by the mathematical tasks (Arbaugh et al., 2006). They conclude that more research is needed since their intent was not to understand teachers’ motivations for their instructional decision-making.

One study examined the tasks adaptations and teacher attention to student thinking (Choppin, 2011). Five teachers that have adopted the Connected Mathematics Project (CMP) materials for five years and it was used as their primary curriculum were observed and interviewed. Choppin (2011) examined whether the attention above was related to “teachers’ tendencies to reduce, maintain, or even enhance the enacted complexity of tasks from the CMP materials (p. 192). The author concluded that two teachers made adaptations based on their observations of how students think and conjectures about the development of student thinking. These adaptations focused on their interpretation of student thinking. Three other teachers focused more on the evaluation of whether the answer was right or wrong. As a consequence, their adaptations were not informed by their observations, and the complexity of the task was reduced. Chopin (2011) argues, “The comprehensive inclusion of challenging tasks in the curriculum materials was crucial to teacher learning (p. 193).

Similarly to Chopin’s (2011) study, Silver, Mesa, Morris, Star & Benken (2009) didn’t focus on a professional development workshop. Silver et al. (2009) centered on the mathematical tasks provided by the portfolio entries of candidates of the certification by the National Board for Professional Teaching Standards in the area of Early Adolescence/Mathematics. Their main focus was to identify what mathematical learning opportunities were provided to students and how the teachers provided these opportunities. After analysis of the portfolios, the authors found that only half of the teachers submitted at least one task that was cognitively demanding. In addition, frequencies were calculated for tasks in Assessing Mathematics Understanding (AU)
and Developing Mathematical Understanding (DU) entries in which they found that 38% of the
tasks were high levels in the AU entries and only 30% of the tasks were high levels in the DU
entries. These results indicate that the teachers in this study were more worried about assessing
high-demand tasks than developing an understanding of high-demand tasks.

Thomas and Williams (2008) utilized the Mathematical Tasks Framework (MTF) and
culturally relevant pedagogy in their study of 30 secondary mathematics teachers. Thomas and
Williams (2008) posit “The MTF provides an adaptable approach for teachers to use in
examining the cognitive effort required of students in order to achieve success in completing an
assigned task.” (p. 112). Other studies have utilized this framework as well for examining the
cognitive demand needed in the tasks (Arbaugh & Brown, 2005; Boston & Smith, 2009; Boston
& Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013). Results from this study
show that in a professional development environment, teachers feel more confident in
recognizing the cognitive demand of a task but had some trouble classifying their task when
constructing it. They created five tasks and classified them as high level (doing mathematics or
procedures with connections) when in fact some were low level (procedures without
connections) (Thomas & Williams, 2008). Likewise, Patterson, Musselman, and Rowlett (2013)
found that the teachers’ tests created by them did not match with the Depth of Knowledge
(DOK) model. They argue,

As teachers create their assessments for learning and utilize depth of knowledge, it is
important that they ensure that mathematics curriculum flows well across coursework.
However, our findings indicate that teachers’ actual DOK levels of assessment and the
targeted DOK levels proposed by Webb’s model are disconnected. Additionally,
teachers’ perceived levels of DOK and their actual levels of DOK are disconnected. This
situation makes for an interesting problem in need of a solution if we want to positively impact the focus and cognitive level of educational experience for students across all grades and courses. (p. 43)

Another important finding was related to teacher content knowledge. Tchoshanov, Lesser, and Salazar (2008) found that there is a strong connection between teacher knowledge and student achievement. The authors created a teacher knowledge survey and administered it on a professional development workshop to 22 in-service teachers, and they analyzed whether the survey was related to the students’ TAKS scores. The teacher knowledge survey consisted of 33 multiple-choice problems that were addressing TAKS objectives and used three different levels of cognitive demand. Their Level 1 was called facts and procedures and included the first two levels from Stein (memorization and procedures without connections). Level 2 was called concepts and connections and included level 3 from Stein (procedures with connections) and Level 3 was called models and generalizations and included level 4 from Stein (doing mathematics). In this study, the authors focused on constructs such as facts, procedures, concept, models, and generalizations. On the first level, they considered memorizing facts and performing computations on the first level, which includes levels 1 and 2 from Stein. Their second level involves the justification of solutions and connection of concepts, which appeared on Level 3 from Stein. Finally, they included generalizations on their Level 3, which is included in Level 4 from Stein. On the teacher knowledge results by cognitive demand levels 75% of teachers correctly solved tasks at Level 1 while only 48% correctly only tasks at Level 2 and 52% correctly solved problems at Level 3. These results show that teachers had more difficulty at solving tasks at Level 2 (procedures with connections from Stein). In another mixed methods study by Tchoshanov (2011) with 102 teachers, there was also a positive correlation between
content knowledge and student achievement. Tchoshanov (2011) also found that those teachers whose mathematical knowledge is more connected and conceptual were also more conceptual on their teaching. Tchoshanov (2011) argues, “We believe that the major outcome of this study that contributes to the field of mathematics education research could be summarized in the following way: teacher content knowledge of concepts and connections is significantly associated with student achievement and lesson quality in middle grades mathematics” (p. 162). It is important to note that concepts and connections are the equivalents to Stein’s level 3 procedures with connections.

Overall, most studies observed the teachers in the classrooms. The majority of these studies were qualitative in nature except two studies with a mixed methods approach (Boston & Smith, 2009; Stein & Kaufman, 2010) and one with a quantitative approach (Jackson, Garrison, Wilson, Gybbons & Shahan, 2013).

### 2.5.2 Student lens

These articles aimed to examine students understanding of concepts when they are presented with tasks at different levels of cognitive demand (Alkhalifa, 2005; Gilbert, 2016; Henningsen, 1997; Kotsopoulos, Lee & Hide, 2012; Norton & Kastberg, 2012; Otten and Soria, 2014). Two of these studies were qualitative in nature and collected classroom observations and written tasks (Norton & Kastberg, 2012), and assessments (Alkhalifa, 2005). Three used a quantitative approach (Gilbert, 2016; Kotsopoulos, Lee & Hide, 2012; Wijaya, van den Heuvel-Panhuizen, Doorman, Robitzsch, 2014) and two used a mixed methods approach (Henningsen, 1997; Otten and Soria, 2014).

There was only one article that claimed that lower cognitive levels would provide better results with all materials. In this case, materials refer to the questions and assignments given to
the students. According to Alkhalifa (2005) when the cognitive level is high, and the materials are complex, then learning is delayed. However, it is important to point out that this study only focuses on one concept: mathematical series. Furthermore, they only used an instructional hypermedia system. They also used a different framework called Cognitive levels of thought that is based on Bloom’s Taxonomy: knowledge, comprehension, application, analysis, and evaluation.

There was one study that focused on preservice teachers’ understanding of cognitive demand paired with algebra II students by using letter writing among them (Norton & Kastberg, 2012). Preservice teachers had to evaluate their own tasks and then assess the tasks as implemented by the students. The letter writing method served as a way for preservice teachers to understand students’ mathematical thinking and aid them in their task design. Two preservice teachers were chosen for this study and presented them as case studies. Students demonstrated higher levels of cognitive demand thanks to the preservice teachers’ reconstruction of tasks and questioning. Another study with eighth-grade students found a discrepancy between the cognitive demand for homework and those tasks during classroom instruction (Kotsopoulos, Lee & Hide, 2012). Overall, two-thirds of the time the assigned homework had a higher or a lower cognitive demand level than the classroom instruction where the desirable outcome was some correlation between the level of classroom instruction and the homework. Kotsopoulos, Lee, and Hide (2012) discuss, “Classroom instruction and learning cannot be divorced from what occurs later, in the home, when students engage in mathematics homework” (p. 362).

According to Henningsen (1997), the purpose of her study was to identify, examine, and illustrate the ways in which classroom-based factors shape students’ engagement with high-level mathematical tasks. This study was part of the QUASAR project and found that teachers would
decline the level of cognitive demand by doing the following: removing the aspects of the task that made it high-level, shifts in focus from the understanding to the correctness or completeness of the answer, and inappropriate amount of time allotted.

Gilbert (2016) conducted a quantitative study of 479 primarily Latino middle school students. The author was interested in investigating the relationship between different aspects of student motivation and performance on mathematical tasks varying in levels of cognitive demand. The tasks were related to the Common Core State Standards for Mathematics (CCSS-M). Different aspects informed motivation: interest (enjoyment one gets from a task), utility (relevancy for the students’ future), efficacy (self-confidence in learning mathematics), achievement goal orientations (what achievement means to the student), and negative emotions (students’ emotional reactions to class experiences). All these aspects were measured after students completed a self-report motivation measure and mathematics assessment. Gilbert (2016) wrote, “The findings regarding performing a procedure suggest that higher interest and efficacy in mathematics, lower performance-avoidance goals, and fewer experiences of negative emotions related to progress toward less cognitively demanding facets of mathematical competence” (p. 654) while utility and mastery approach goals were more related to the more cognitively demanding parts.

Otten and Soria (2014) focused on the relationship between enactment of tasks at different levels of cognitive demand and the participatory demand and students’ performance. Instructional material and enacted curriculum were coded for cognitive demand. All written tasks of instructional material were considered high in terms of cognitive demand. In the enacted curriculum the three teachers participating in the study failed to maintain the cognitive demand level during the set-up phase and the look-back phase. Based on results from pre- to post-tests
students did not improve. Otten and Soria (2014) argue that maintaining a high level of cognitive demand and allowing students to express mathematical relations rather than computations might lead to better performance. Bieda (2010) also examined the enactment of tasks. In this case, she focused proof-related tasks, those that require students to justify and prove in the classroom. With observations of 7 middle school classrooms, Bieda (2010) found that students were not given enough opportunities to prove. Most students gave generalizations without justifications because teachers did not allow them to do so. Teachers responded in one of the three following ways “(a) Teachers gave no feedback to elicit a justification, (b) teachers sanctioned the conjecture as valid without justification, or (c) teachers asked other students to state whether or not they agreed with a student’s conjecture” (Bieda, 2010 p. 366). Findings from the interviews show that teachers thought there was not enough time for the class to work on problems or discuss their work.

Another study examined students’ difficulties in solving context-based mathematics tasks (Wijaya, van den Heuvel-Panhuizen, Doorman, Robitzsch, 2014). The following cognitive demand levels categorized the tasks pertinent to this study: reproduction, connection, and reflection defined by the Programme for International Student Assessment (PISA). In addition, they examined the students’ difficulties based on several stages of problem solving: comprehending a task, transforming the task into a mathematical problem, processing mathematical procedures, and interpreting or encoding the solution in terms of the real situation. In all three types of tasks, students made the most mistakes in the two stages of problem solving. Especially in the reflection tasks, the majority of errors were made in the transforming stage.
Similarly, as in the case of the field of mathematics education, several types of research have utilized the construct of cognitive demand in science education either to talk about processes, knowledge or both. Many authors have used Bloom’s Taxonomy (1956) to examine science classrooms. As mentioned above there are six levels of Bloom’s Taxonomy: knowledge, comprehension, application, analysis, synthesis, and evaluation. Others have used the two-dimensional matrix shown in Figure 2.3 from Porter (2002) where the two-dimension matrix contains topics as rows and cognitive demand in the columns. Also utilized in science is Webb’s Depth of Knowledge (1997) since the four levels: recall, basic application, strategic thinking, and extended thinking were created by Webb to examine the alignment of science and mathematics standards and assessments.

Edwards and Dall'Alba (1981) developed a scale of cognitive demand for analysis of printed secondary science materials. The authors defined cognitive demand as the demand placed on the cognitive abilities through the following dimensions: complexity, openness, implicitness, and level of abstraction. Complexity is related to the nature of each operation and the links between operations. Openness is related to “the degree to which a task relies on the generation of ideas” (Edwards & Dall'Alba, 1981, p. 159). Implicitness is related to the degree to which the student is required to go beyond the available data. Finally, the level of abstraction is related to “the extent to which a task deals with ideas rather than concrete objects or phenomena” (p. 159). After analyzing secondary science texts, they identified and based on the dimensions from above six groups of cognitive demand were identified. Similarly to another framework, the first group (the lowest) contains tasks that would require students to recall or follow a simple set of
laboratory instructions, while group 6 (the highest) contains tasks such as assessing the impact and evaluate.

In 2012, the National Research Council wrote that “science is not just a body of knowledge that reflects current understanding of the world; it is also a set of practices used to establish, extend, and refine that knowledge. Both elements—knowledge, and practice—are essential” (NRC, 2012, p. 26). This view of science is supported by the Next Generation Science Standards (NGSS). Cartier, Smith, Stein, and Ross (2013) talk about ways in which teachers can draw from different resources to select and modify tasks that are aligned with that view of science. Cartier et al. (2013) focused on three categories of tasks (1) experimentation, (2) data representation, analysis, and interpretation, and (3) explanation. Then each of these categories is separated into low cognitive demand and high cognitive demand based on tasks and teacher actions (Figure 2.5) to show how the cognitive demand can either be lowered or higher. According to Cartier et al. (2013) “A task that requires students to invest significant effort in making sense of the underlying science phenomena or concepts is a high cognitive demand task” (p. 10). In experimentation tasks students follow a detailed protocol (low-level) or makes the decision about what data to collect and how to collect it. In data representation, analysis, and interpretation tasks the authors explain how a task that asks students to represent and analyze data can is a low-level task if it does not prompt students to think about how best to represent the data. In explanation tasks, students can provide explanations without justification (low-level) or draw upon a variety of representational tools (high-level). Two of the authors that created the cognitive demand levels, Margaret S. Smith and Mary Kay Stein helped developed this framework for science tasks.

<table>
<thead>
<tr>
<th>Low Cognitive Demand</th>
<th>High Cognitive Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks</td>
<td>Teachers Actions</td>
</tr>
<tr>
<td></td>
<td>Experiment</td>
</tr>
<tr>
<td>----------------------</td>
<td>-------------------------------------</td>
</tr>
<tr>
<td><strong>Students-</strong></td>
<td>follow a highly specified procedure.</td>
</tr>
<tr>
<td></td>
<td>do not make choices about what data to collect or how to collect it.</td>
</tr>
<tr>
<td></td>
<td>are not engaged in being critical about the data collection procedure.</td>
</tr>
<tr>
<td><strong>The Teacher-</strong></td>
<td>does not help students understand that data collection is occurring in the service of answering a question.</td>
</tr>
<tr>
<td></td>
<td>introduces the experiment after she/he has already provided didactic information on the underlying concepts.</td>
</tr>
<tr>
<td><strong>Students-</strong></td>
<td>must make decisions about what data to collect and/or how to collect it.</td>
</tr>
<tr>
<td></td>
<td>compare/contrast or critique experimental protocols, considering issues such as reliability and “fit” between data gathered and the underlying question driving the experiment.</td>
</tr>
<tr>
<td></td>
<td>are not engaged in being critical about the data collection procedure.</td>
</tr>
<tr>
<td><strong>The Teacher-</strong></td>
<td>ensures that students understand how their data collection must help them achieve the goal of answering a particular question.</td>
</tr>
<tr>
<td></td>
<td>introduces the experiment after she/he has already provided didactic information on the underlying concepts.</td>
</tr>
<tr>
<td></td>
<td>accepts only very specific representation types of strategies. (i.e., multiple solutions or strategies are not possible).</td>
</tr>
</tbody>
</table>
In addition to the previous framework Tekkumru-Kisa, Stein and Schunn (2014) developed a new task analysis guide in science (TAGS). They took the task analysis guide from Stein et al. (2000) which was created for mathematical tasks and develop it for science tasks. According to the authors, “The primary purpose of the TAGS is to identify the level and kind of reasoning required of students in order to successfully engage with a task that focuses on science content and/or scientific practices.” (p. 663). This new framework has five levels: memorization tasks, tasks involving scripts, tasks involving guidance for understanding, and doing science tasks. The authors have considered two levels inside of tasks involving guidance for understanding. However, the TAGS has two dimensions: the integration of science content and practices (columns) and cognitive demand (rows) as shown in Figure 2.6. The lower levels of cognitive demand are presented in the bottom rows (memorization tasks and tasks involving scripts), and the higher level is presented in the top rows (tasks involving guidance for understanding, and doing science tasks).
According to Tekkumru-Kisa, Stein, and Schunn (2014) “Doing science tasks (level-5) are the most open or unstructured. These kinds of tasks require students to self-regulate their own cognitive process in order to monitor, and, if necessary, adjust their approach” (p. 666). Doing science is aligned with the level 4 task “doing mathematics” from the task analysis guide. Tasks that require guidance for understanding (level-3 and level-4) are also considered high-level, but since they often have suggested pathways to be solved, they fall lower than doing science tasks. “There is less ambiguity for students regarding what to do; nevertheless, the suggested pathways cannot be followed mindlessly. Rather, following them requires students to understand what they are doing and why” (p. 666). This task is similar to procedures with connections in the mathematics counterpart. The two lower-level tasks (level 1 and 2) focus more on getting the correct answer. For example, in tasks involving scripts (level 2), the instructions of the task clearly state what students are expected to do. In the framework for mathematical tasks, this level is called procedures without connections in which students are supposed to substitute values in a formula. Finally, Memorization tasks (level-1) require exact reproduction of previously seen materials (i.e., definitions, rules, formulas, and principles) and what is to be produced be clearly and directly stated. Similarly, the level 1 in the mathematics framework is also called memorization task and is also about students remembering formulas, rules, or definitions. The second dimension of the framework (columns) is a concern with scientific practices, science content, and integration of content and practices. Figure 2.6 shows that levels 4 and 5 are achieved with the integration of content and practices. The authors explain, The cells at the intersection of the cognitive demand level-5 and “scientific practices” and “science content” are grayed out because “working like a scientist” inherently constitutes engaging in scientific practices and science content at the same time. Therefore, it is not logically feasible to require students to think like a scientist but solely focus on science content or scientific practices. Integration also inherently involves higher cognitive demand when students are responsible for the integration (i.e., it is not scripted). (p. 675)
Other cells are grayed out based on the same logic. For example, in the memorization tasks, it is not possible to achieve the integration of content and practices.

<table>
<thead>
<tr>
<th>Cognitive Demand Level</th>
<th>Scientific Practices (e.g., argumentation and investigation)</th>
<th>Science Content (i.e., scientific body of knowledge)</th>
<th>Integration of Content and Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Doing Science Tasks</td>
<td></td>
<td>Doing Science (DS) Engaging in practices to make sense of content and recognize how scientific body of knowledge is developed</td>
</tr>
<tr>
<td>4</td>
<td>Tasks Involving Guidance for Understanding</td>
<td>Guided Practices (GP) Being guided for understanding practices</td>
<td>Guided Integration (GI) Guidance for working with practices tied to a particular content</td>
</tr>
<tr>
<td>3</td>
<td>Tasks Involving Scripts</td>
<td>Guided Content (GC) Being guided for understanding particular content</td>
<td>Scripted Integration (SI) Following a script to work on practices tied to content</td>
</tr>
<tr>
<td>2</td>
<td>Memorization Tasks</td>
<td>Scripted Practices (SP) Following a script to work on practices</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Memorization Tasks</td>
<td>Memorized Practices (MP) Reproducing definitions/ explanations of practices</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Memorized Content (MC) Reproducing definitions, formulas, or principles about particular content</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.6: The Task Analysis Guide in Science (from Tekkumru-Kisa, Stein & Schunn, 2014)

One article proposed a framework that addresses English language learners’ language proficiencies and cognitive abilities in science classrooms (Bautista, 2014). The framework has two dimensions, in one dimension are the English language proficiency levels and in the other dimension the cognitive levels. Five levels of English proficiency are explained: starting (L1), emerging (L2), developing (L3), expanding (L4), and bridging (L5). In L1 students rarely communicate in English. In L2 can engage in simple conversations. In L3 students can understand more complex language. In L4 students can adequately communicate in English. In L5 students have language skills that can be compared to a native English speaker. The second dimension is based on Bloom’s taxonomy: remembering, understanding, applying, analyzing, evaluating, and creating. According to Bautista (2014), “It is every science teacher’s responsibility to help ELLs accomplish higher-order thinking, regardless of their language
abilities. Teachers must commit to putting time and effort into modifying their lesson plans, so ELLs achieve the same goals as mainstream students” (p. 37).

Bautista (2014) provides several strategies and tasks for each cell of the two-dimension framework with the strategies for L1 and L2 together and the strategies for L3, L4, and L5 together. For example, a task for a student in the remembering level can be to provide a simple fill-in-the-blank definition of conductors and insulators with a word bank. Thus the students in this level of proficiency can be encouraged to orally define conductors and insulators with the help of a word bank. Students in the L3, L4, and L5 English proficiency level can be asked to verbally define and explain conductors or insulators by providing examples from their daily lives or surroundings. In the creating level students in the L1 and L2 English proficiency level “present their engineering design, product, results, and modification in the form of a poster that includes pictures taken during the investigation. They compare the pictures of the original and modified product on the poster by labeling the materials they used” while those in the L3, L4, and L5 “ELLs present their engineering design, product, results, and modification in the form of a poster that includes pictures taken during the investigation. L3, L4, and L5 students can use long paragraphs to explain their design process on the posters. They can provide an oral presentation of their work, explaining their rationale, evaluation of their product, and how they modified it as a result of their findings.” (Bautista, 2014, p.35)

2.6.1 Teacher lens

Some studies about cognitive demand and science education have focused on the teachers and implementation of cognitively demanding tasks at different states of the lessons. (Huntley, 1999; Nehring, Pabler, & Tiemann, 2017; Stiller, Hartmann, Mathesius, Straube, Tiemann, Nordmeier, Kruger, & Upmeier zu Belzen, 2016; Tekkumru-Kisa, 2013; Tekkumru-Kisa, Stein, 2015; Zohar, 2004; Zohar, Schwartzer, & Tamir, 1998). Some of these studies were done in a professional development environment (Tekkumru-Kisa, 2013; Tekkumru-Kisa, Stein, 2015;
Zohar, 2004) while others were done in the classrooms (Zohar, 2004; Zohar, Schwartzer, & Tamir, 1998). One study was done with preservice teachers (Stiller, Hartmann, Mathesius, Straube, Tiemann, Nordmeier, Kruger, & Upmeier zu Belzen, 2016). Three studies utilized a quantitative methodology (Stiller, Hartmann, Mathesius, Straube, Tiemann, Nordmeier, Kruger, & Upmeier zu Belzen, 2016; Zohar, 2004; Zohar, Schwartzer, & Tamir, 1998), four utilized a qualitative methodology (Huntley, 1999; Tekkumru-Kisa, 2013; Tekkumru-Kisa, Stein, 2015; Zohar, 2004), and one used a mixed-methods (Nehring, Pabler, & Tiemann, 2017).

Zohar, Schwartzer, and Tamir (1998) examined the application of higher-order thinking skills in biology classrooms in Israel. The authors were interested in the cognitive level of the questions asked by teachers in classes, homework, assignments and tests in junior high school and high school. They used the following levels: knowledge, comprehension (from Bloom's taxonomy), higher-order thinking (a combination of analysis, application, synthesis, and evaluation of Bloom’s levels), and low-level application (questions that required more than comprehension but less than application). This quantitative study found that in classroom discourse and test comprehension questions are more common in both high school biology and junior high school with about one-third of the total number of questions. Also, the use of application questions is very limited in both classrooms discourse. Another interesting finding is that the frequency of knowledge questions was found to be positively related to the total number of questions asked by teachers. The more questions they asked, the larger the frequency of knowledge questions was found.

Zohar (2004) conducted a qualitative study in a professional development workshop with Israeli junior and high school science teachers. Zohar (2004) used the term higher-order thinking where the recall of information is a lower level of cognitive demand and analyzing, synthesizing,
and evaluating as well as science inquiry skills are examples of higher-order cognitive demand. Zohar (2004) found that “the cognitive demands of tasks requiring higher order thinking are often lowered in order to reduce ambiguity while also turning the task into something more familiar” (p. 306). When a teacher was able to keep the higher-order thinking was because the teacher directed students to relevant scientific knowledge, did not provide the answer, and kept the students’ participation active. In those cases, the teachers reflected feeling satisfied (Zohar, 2004.) Zohar (2004) argue that when teachers lower the cognitive level of the activities, the intended purpose of the activity is lost as well,

While teaching various subject matters, a transmission-of-knowledge pedagogy may lead to rote or nonmeaningful learning and to the acquisition of inert knowledge (i.e., knowledge that cannot be applied to new situations and is not retained for long periods). However, when this pedagogy is used in the teaching of thinking, students’ opportunities to engage in active thinking are reduced because of the reduction in the cognitive requirements of the task. In such cases, although teachers may administer learning activities that were specifically designed to make students think, they may go through the activities without actually being engaged in any active thinking (p. 308.)

In her dissertation study, Tekkumru-Kisa (2013) talks about the enactment of cognitively demanding science tasks. She focused on the term teacher noticing by presenting teachers in professional development (PD) setting videos of classrooms that enacted high-level, cognitively demanding science tasks. She used an earlier version of the TAGS framework explained in the section above to define the enactment of cognitively demanding science tasks. Teacher noticing is related to identifying what is important in a teaching situation. The results of this study suggest that showing teachers videos cases that depict classroom interactions helped teachers understand
what they were seeing and connect it to the level outlined in the TAGS framework as well as the teacher’s role in the maintenance or decline of the cognitive demand. In another study by Tekkumru-Kisa and Stein (2015), they argue, “even though attention to students’ thinking is important, teachers’ being aware of their own teaching actions while teaching is also critical, particularly in classrooms that are using cognitively challenging tasks” (p. 128). When teachers reflect about others teaching and their own teachers they might develop an awareness of how to maintain the level of cognitive demand in their classrooms.

There was only one study that focused on the integration of mathematics and science in the classroom and the cognitive demand construct (Huntley, 1999). First, the author argues that defining the integration of mathematics and science is complicated since curriculum has focused on either mathematics with science, where science is used to establish problem context in a mathematics course or science with mathematics, where mathematics is used as a tool for solving scientific problems. Huntley (1999) was interested in classrooms in which both mathematics and science play the same role. In this case study of two classrooms where full mathematics and science instruction were intended the author found that in both cases the cognitive demand of the tasks was low. Teachers maintained the full authority of the classroom and only implemented a recall and follow procedures tasks. While this study only shows two classrooms that attempted to fully integrate mathematics and science it poses an important question not only of integration of mathematics and science and also the how to implement cognitively demanding tasks.

2.6.2 Curriculum lens

Some research about cognitive demand in science classrooms have also focused on the students and the curriculum (Bautista, 2014; Contino, 2013; Fortus, Dershimer, Krajcik, Marx, & Mamlok-Naaman, 2004; Kragten, Admiraal, & Rijlaarsdam, 2013; Iding, Klemm, Crosby, &
Speitel, 2002; Lee, Kim, & Yoon, 2015; Liang & Yuan, 2008; Liu & Fulmer, 2008; Vachliotis, Salta, Vasiliou, & Tzougraki; 2011). Eight of the studies utilized a quantitative methodology (Contino, 2013; Fortus, Dershimer, Krajcik, Marx, & Mamlok-Naaman, 2004; Kragten, Admiraal, & Rijlaarsdam, 2013; Iding, Klemm, Crosby, & Speitel, 2002; Lee, Kim, & Yoon, 2015; Liang & Yuan, 2008; Liu & Fulmer, 2008; Vachliotis, Salta, Vasiliou, & Tzougraki; 2011). The majority of the students were conducted in the United States with four carried out in other countries (Kragten, Admiraal, & Rijlaarsdam, 2013; Lee, Kim, & Yoon, 2015; Liang & Yuan, 2008; Vachliotis, Salta, Vasiliou, & Tzougraki; 2011). Results from those studies are presented below.

Two articles utilized Bloom’s taxonomy (remember, understand, apply, analyze, evaluate, and create) as their framework (Lee, Kim, & Yoon, 2015; Liang & Yuan, 2008). Lee, Kim, and Yoon (2014) examined the intellectual demands of the science curriculum in Korea and Singapore in primary science and Liang, and Yuan (2008) reviewed the alignment of the Chinese national physics curriculum guidelines and 12th-grade exit examinations. Liang and Yuan (2008) argue that Bloom’s taxonomy has been widely used in Chinese education and that is why they used it. Lee, Kim, & Yoon (2015) claim that Bloom's taxonomy contains verbs that explain the cognitive processes with more clarity.

After examining two most recent physics examinations in a province in China, Liang and Yuan (2008) found that both tests measures have more emphasis at the understanding cognitive level and they both over-represented the curriculum guidelines in the apply and analyze cognitive levels. When analyzing the curriculum and the examination they found that neither encourages creativity, critical thinking, and students’ abilities to conduct scientific inquiry. According to Liang and Yuan (2008),
On the one hand, the over-representation of test items with higher cognitive demands in the 12th-grade exit examinations has probably positively impacted classroom practices; that is, teachers generally pay more attention to the development of student understanding and routine/non-routine problem-solving in teaching. High school students are therefore more likely to develop a solid knowledge base in physics and other science disciplines upon graduation. However, on the other hand, the development of creative thinking, decision-making, and real-world problem-solving skills that are not reflected in the assessments is thus largely ignored in science learning and teaching (p. 1833.)

Likewise, Lee, Kim, and Yoon (2015) found that both curricula from Korea and Singapore focused on the lower levels of cognitive demand. Most of the learning objectives are the dimensions of conceptual and understand or below.

Two studies used Porter’s framework (two dimension content matrix) (Contino, 2013; Liu & Fulmer, 2008). One of the advantages of using the Porter framework is that “it adopts a common language to describe curriculum, instruction and assessment thus are appropriate for analyzing the alignment between any two of curriculum, instruction, and assessment” (p. 375.) In addition, this framework contains the Porter alignment index, a formula to find the alignment with a value between 0 and 1. Having an index number between 0 and one makes it a useful tool for quantitative analysis. Contino (2013) examined the alignment between curriculum and assessment in the earth science standards-based system in the New York State. In one of the state assessments (Regents Exam) Contino (2013) found the following: Remember (24%), Understand (34%), Apply (37%), Analyze (1%), Evaluate (5%), and Create (0%). “Over ninety percent of the questions occurred at lower order thinking skills (Remember, Understand, and Apply) and less than ten percent of the questions occurred at higher order thinking skills (Analyze and
Evaluate)” (Contino, 2013, p. 69). When examining the cognitive levels in the Core Curriculum, the major focus is on lower level cognitive skills such as understand and apply. The porter alignment was .35, which means slightly aligned.

Liu and Fulmer (2008) also conducted a quantitative study of alignment in the New York state. They were interested in the science curriculum and some New York state regent exams. They first argue that it is important to examine the alignment between the curriculum and state exams because often the alignment is not assumed and if they are unaligned they might produce inaccurate results. The investigation of cognitive levels in the alignment is because students must master a science concept at different cognitive levels (Liu & Fulmer, 2008). Results of the analysis suggest that there are clear differences in alignment between the levels of cognitive demand. In physics and chemistry, the curriculum emphasizes the levels of understand and apply. In physics tests, there is a shift in higher-order thinking, and in chemistry tests, there is a shift toward lower-order thinking skills. Some students might do well in the chemistry test since there is alignment between curriculum and state test but that does not mean that they are learning higher-order thinking skills.

Vachliotis, Salta, Vasiliou, and Tzougraki (2011) focused on the validation and reliability of systemic assessment questions (SAQs) for assessing students’ meaningful understanding of organic reactions in the 11th grade in high school in Greece, particularly diagrams. “Meaningful learning seems to be related to the achievement of higher-order cognitive skills. If someone wishes to foster and assess meaningful learning, they need to emphasize those cognitive processes that go beyond recall of knowledge” (Vachliotis, Salta, Vasiliou, and Tzougraki, 2011, p. 338). They aimed to study some SAQs questions, but most questions were only assessing recall of knowledge and only one was assumed to demand higher cognitive skills. Most
questions were fill in the blank and multiple choice. After statistical analysis, the authors find that the SAQs are suitable as an assessment tool for meaningful learning. However, they fail to mention how the assessment that contains only one question that was considered higher order thinking succeeds at assessing meaningful learning, which was one of the main arguments of the study.

Kragten, Admiraal, & Rijlaarsdam, (2013) were also interested in diagrams in a secondary science classroom. They conducted a quantitative study of 18 biology exams in Amsterdam by conducting a hierarchical regression analysis to examine which features of each task, student, and diagram were related to its difficulty. Bloom’s taxonomy was also used to classify the level of cognitive demand of a task. Other features studied in the model were diagram components, familiarity, and prior content knowledge. Kragten, Admiraal, & Rijlaarsdam, (2013) found that there is an interaction effect between tasks with high cognitive demand and prior content knowledge. These results suggest that prior content knowledge and cognitive demand should be studied together.

Iding et al. (2015) examined figures and tables in a constructivist science text. A constructivist text was defined as an inquiry-bases text that also required students to be actively involved. The text is a marine biology text in which “information is given in figures or tables, minimizing long sections of narrative paragraphs characteristic of traditional texts. Guided inquiry in the procedure or question sections scaffold students into using more open-ended exploratory learning” (Iding et al., 2015, p. 443). Figures and tables in one unit of the text were rated based on the following taxonomy: knowledge acquisition (forming an initial representation), knowledge application (reorganizing/assimilating information), and knowledge creation (creating something new). The majority of tables and figures focused on knowledge
acquisition although there was also a high percentage of tables and figures focusing on knowledge application. There were no knowledge creation tables and figures throughout the text. However, the authors argue that knowledge creation was not expected since the text is an introductory-level science test. Iding, et al. (2015) only focused on one unit because it was an introductory-level unit. Also, from the beginning, the authors assumed that they were not going to find any tables and figures at the knowledge creation level. In the conclusion section, the authors also argue that “What sets constructivist science texts apart is the large proportion of material devoted to knowledge application phase to give the students the background they need so they may advance to the knowledge creation phase” (Iding et al., 2015, p. 449). These results are conflicted with the initial definition of a constructivist text in which students are expected to be more exposed to more open-ended exploratory learning.

Fortus et al. (2004) researched the scientific knowledge of 92 students after being in a design-based science (DBS) classroom. In a design-based science classroom, students are presented with real-world problems that are ill-defined and may have more than one solution. This was a quantitative study with pre- and post-tests that were multiple choice questions that were low, medium, and high cognitive demand. 1 item to the test was a low-demand item, 2 items were medium-demand, and 2 were high-demand. The authors did not explain why those items were considered low, medium or high demand but rather used the cognitive demand items to suggest that the pre and post-tests contained items at each level. Fortus et al. (2004) concluded that there was a substantial science learning after being part of the DBS classroom. However, it was not explained how students performed when accounting for each level of cognitive demand on the tests.
A literature review of studies of cognitive demand in science has been presented. The majority of studies used Bloom’s taxonomy to classify the level of cognitive demand of a task or used Porter’s index to examine the alignment between curriculum and state exams. Others have used the framework of mathematics education and revised it to use it in a science classroom (i.e., TAGS framework). For the purpose of this study, the framework for mathematical tasks with the four levels (memorization, procedures without connection, procedures with connections, and doing mathematics) from Stein et al. (2000) will be used to examine recognition, solution, construction, and implementations of tasks at different levels of cognitive demand.
Chapter 3: Methodology

This study examines the extent to which secondary mathematics teachers can recognize, solve, and construct mathematical tasks at different levels of cognitive demand and the ways in which they impact their implementation of cognitively demanding tasks. Data was collected from a professional development project that was conducted from 2012 to 2016. This professional development project was a workshop that allowed teachers to learn about the different levels of cognitive demand among other topics that were not part of this study.

Table 3.1 shows the relationship and the data sources as well as the analysis methods and participants for each. The first question is related to whether teachers can solve, recognize and construct tasks at different levels of cognitive demand and it will be measured by the surveys with all participants from the professional development workshop and analyzed with statistical analysis. The second question is related to whether teachers can implement tasks at different levels of cognitive demand by observing the teachers in the workshop as well as in the classroom. This question will be analyzed by qualitative analysis. Finally, the third question is related to the challenges the teachers face when selecting, designing, and implementing cognitively demanding tasks. The data source of this research question will be the interviews with a purposefully selected sample of participants and will be analyzed quantitatively. In the following chapter, the setting, participants, data collection and data analysis plans will be explained in detail. In addition, Table 3.2 shows a timeline for the duration of this study were activities for data collection, data analysis, and writing are shown and the length of each one of them.

Table 3.1: Research questions and data sources

<table>
<thead>
<tr>
<th>Research questions</th>
<th>Data sources</th>
<th>Participants</th>
<th>Analysis methods</th>
</tr>
</thead>
</table>

51
1. To what extent are secondary mathematics and science teachers able to recognize, solve and construct tasks at different levels of cognitive demand?

2. Are there relationships among recognition, solution, construction, and implementation of tasks at different levels of cognitive demand?

3. What are secondary mathematics and science teachers’ challenges in recognizing, solving, constructing, and implementing CDTs?

Table 3.2: Timeline

<table>
<thead>
<tr>
<th>Activity</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data collection pilot study (survey)</td>
<td>January 2013</td>
</tr>
<tr>
<td>Data collection pilot study (microteaching observations)</td>
<td>Spring 2013</td>
</tr>
<tr>
<td>Data collection pilot study (interviews)</td>
<td>Fall 2013</td>
</tr>
<tr>
<td>Interview transcriptions</td>
<td>December 2014</td>
</tr>
<tr>
<td>Data collection main study (survey)</td>
<td>Summer 2014</td>
</tr>
<tr>
<td>Data collection main study (workshop observations)</td>
<td>Fall 2014 and Fall 2015</td>
</tr>
<tr>
<td>Data collection main study (interviews)</td>
<td>Fall 2015</td>
</tr>
<tr>
<td>Transcriptions</td>
<td>Fall 2015</td>
</tr>
<tr>
<td>Data analysis</td>
<td>December 2015-January 2016</td>
</tr>
<tr>
<td>Writing</td>
<td>February 2016-February 2017</td>
</tr>
<tr>
<td>Revisions</td>
<td>March-April 2017</td>
</tr>
</tbody>
</table>

3.1 Setting

This study focuses on secondary mathematics teachers participating in a larger professional development workshop at the University of Texas at El Paso, entitled Teacher Quality (TQ). The TQ grant was aimed to support training and retention of secondary mathematics teachers. The Texas Higher Education Coordinating Board is a federal initiative for using professional development to improve teaching and learning that funded the TQ grant. Two cohorts of this grant were part of this study. The first cohort met for two years, 2012-2013. The
second cohort met in 2014 and had been extended for the second year, which will start in Summer 2015. The University of Texas at El Paso is a Hispanic-serving university located at the Paso del Norte border region of Texas. This region has one of the lowest median incomes in the state of Texas (Texas Higher Education Coordinating Board, 2011). El Paso has a large population of Hispanics (80%), and 73% speak another language other than English at home (U.S. Census Bureau). All the activities presented in this study were part of the TQ grant. However, other activities are part of the grant but not part of this study.

3.2 PARTICIPANTS

This section describes the procedure for selection and recruitment of teachers to participate in this study.

3.2.1 Selection Criteria and Recruitment

The secondary mathematics teachers involved in the TQ grant were asked to take part in this study. There was no penalty from the grant if they decide not to participate in this study, they still got the learning experiences from this study and the grant. While selecting participants based on the willingness to participate may have some drawbacks such as a small sample of teachers participating in the study, teachers should not be forced and should feel comfortable to participate in this study. There were 20 participants in the pilot study and 38 participants for the main study. The selection criteria for teachers to take part in the TQ grant were the following: in-service math teachers currently teaching in Texas public schools and from a high-need local education agency campus. All the teachers participating in the TQ grant have met the criteria aforementioned. All the teachers that were accepted to participate in the grant were asked to fill out the survey as part of this study. The selection of all the teachers of the TQ grant to participate
in this study was beneficial because they teach at schools from different districts throughout the area. In addition, it provided a representative sample of secondary mathematics and science teachers in the area. Participants of the TQ grant were contacted directly during the professional development sessions. Those who agreed to participate in the study answered the first part of the study, which is the survey. A second criterion was used to select teachers to be interviewed. Teachers were rated based on their survey answers, the level of cognitive demand on the classroom, and the level of cognitive demand in the microteaching. The selection criteria for the interviews is the following: based on their average rating from the survey, cognitive demand level of the task in the classroom and cognitive demand level of the task of the microteaching, teachers with a high, medium and low rating will be selected for an interview. Teachers that were selected for interviews were either contacted directly during the professional development workshop or by email to schedule an individual interview.

3.2.2 Participants Information

3.2.2.1 Descriptive statistics of the pilot study

When teachers applied for the grant, they filled out a participant information form. First, each participant was given a code in order to assure anonymity. The pilot study consisted of 20 teachers from 7 different public schools in 4 different districts of the city. 6 teachers taught at a rural district while 14 taught at an urban district at the time of this study. The majority of the teachers were female (70%) (Table 3.4). The vast majority of the teachers reported their race as Hispanic (85%) and three teachers reported their race as white non-Hispanic (15%) (Table 3.5). They all reported teaching at a public school (Table 3.6), and the majority (70%) reported teaching in an urban school district (Table 3.7). Their teaching experience years ranged from half
a year to 12 years with a mean of 5.575 years and a standard deviation of 3.57 years (Table 3.8).

The majority of teachers in the pilot study were mathematics teachers from 5th grade to 8th grade (90%) while two (10%) were science teachers. Table 3.3 shows the information of each participant in the pilot study and Tables 3.4-3.8 show the descriptive statistics by gender, race, type of school, type of school district, and experience years.

Table 3.3: Participants' information in pilot study N=20

<table>
<thead>
<tr>
<th>Participant Code</th>
<th>Gender</th>
<th>Race</th>
<th>Type of school</th>
<th>Type of school district</th>
<th>Experience years</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>518-407</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>6</td>
<td>5th-grade math and science</td>
</tr>
<tr>
<td>518-521</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Rural</td>
<td>7</td>
<td>6th-grade math</td>
</tr>
<tr>
<td>518-510</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>0.5</td>
<td>6th-grade math</td>
</tr>
<tr>
<td>518-617</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>10</td>
<td>6th-grade math</td>
</tr>
<tr>
<td>518-490</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>3</td>
<td>6th-grade math</td>
</tr>
<tr>
<td>518-217</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Rural</td>
<td>1</td>
<td>6th-grade math</td>
</tr>
<tr>
<td>518-111</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>3</td>
<td>7th-grade math</td>
</tr>
<tr>
<td>518-240</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Rural</td>
<td>5</td>
<td>7th-grade math</td>
</tr>
<tr>
<td>518-619</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>1</td>
<td>7th-grade math</td>
</tr>
<tr>
<td>518-107</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Rural</td>
<td>5</td>
<td>6th-8th-grade math</td>
</tr>
<tr>
<td>518-317</td>
<td>Female</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>10</td>
<td>7th-8th-grade math</td>
</tr>
<tr>
<td>518-518</td>
<td>Female</td>
<td>Hispanic, White</td>
<td>Public</td>
<td>Rural</td>
<td>11</td>
<td>8th-grade math</td>
</tr>
<tr>
<td>518-462</td>
<td>Female</td>
<td>Hispanic, White</td>
<td>Public</td>
<td>Urban</td>
<td>5</td>
<td>7th-grade math</td>
</tr>
<tr>
<td>518-924</td>
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<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>2</td>
<td>7th-grade math</td>
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<tr>
<td>518-662</td>
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<td>Hispanic</td>
<td>Public</td>
<td>Rural</td>
<td>3</td>
<td>6th-grade math</td>
</tr>
<tr>
<td>518-983</td>
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<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
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<td>8th-grade math</td>
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<tr>
<td>518-425</td>
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<td>Hispanic</td>
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<td>8th-grade math</td>
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<td>518-815</td>
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<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
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<td>8th-grade math</td>
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<tr>
<td>518-429</td>
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<td>Hispanic, White</td>
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<td>Urban</td>
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<td>8th-grade math</td>
</tr>
<tr>
<td>518-120</td>
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<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>4</td>
<td>8th-grade math</td>
</tr>
</tbody>
</table>
Table 3.4: Descriptive statistics by gender of pilot study

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>14</td>
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<td>Male</td>
<td>6</td>
<td>30.0</td>
</tr>
<tr>
<td>Total</td>
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<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.5: Descriptive statistics by race/ethnicity in pilot study

<table>
<thead>
<tr>
<th>Race/Ethnicity</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hispanic</td>
<td>17</td>
<td>85.0</td>
</tr>
<tr>
<td>White, non-Hispanic</td>
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<td>15.0</td>
</tr>
<tr>
<td>Total</td>
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<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.6: Descriptive statistics by type of school in pilot study

<table>
<thead>
<tr>
<th>Type of school</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

Table 3.7: Descriptive statistics by type of school district in pilot study

<table>
<thead>
<tr>
<th>Type of school district</th>
<th>Frequency</th>
<th>Percent</th>
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</thead>
<tbody>
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<td>Rural</td>
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<td>30.0</td>
</tr>
<tr>
<td>Urban</td>
<td>14</td>
<td>70.0</td>
</tr>
<tr>
<td>Total</td>
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<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.8: Descriptive statistics of teaching experience in the pilot study

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching experience</td>
<td>20</td>
<td>.5</td>
<td>12.0</td>
<td>5.57</td>
<td>3.57</td>
</tr>
</tbody>
</table>

3.2.2.2 Descriptive statistics the main study

The main study consisted of 38 teachers from 13 different schools in 2 different districts. All these teachers taught in an urban district, and they were all public schools (Table 3.12-3.13). 25 of the teachers were female (65.8%), and 13 were male (34.2%) (Table 3.10). 28 teachers reported their race as Hispanics (73.7%) while eight teachers reported their race as white non-
Hispanic (21.1%) and two as African American (5.3%) (Table 3.11). Their teaching experience ranged from half a year to 28 years with a mean of 7.474 years and a standard deviation of 6.48 years (Table 3.14). 22 (58%) of the teachers in the pilot study were mathematics teachers, and 16 were (42%) were science teachers. Table 3.9 shows the participants’ information of the pilot study while Tables 3.10-3.14 show the descriptive statistics by gender, race, type of school, type of school district, and experience years.

Table 3.9 Participants' information in the main study N=38

<table>
<thead>
<tr>
<th>Participant Code</th>
<th>Gender</th>
<th>Race/Ethnicity</th>
<th>Type of school</th>
<th>Type of school district</th>
<th>Experience years</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>542-010</td>
<td>Female</td>
<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>12</td>
<td>6th-8th-grade math</td>
</tr>
<tr>
<td>542-023</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>11</td>
<td>8th-grade math</td>
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<td>542-122</td>
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<td>Urban</td>
<td>Public</td>
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<td>7th-8th-grade math</td>
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<tr>
<td>542-046</td>
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<td>7th-8th-grade math</td>
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<td>8th-grade math</td>
</tr>
<tr>
<td>542-004</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>15</td>
<td>9th-grade math</td>
</tr>
<tr>
<td>542-003</td>
<td>Female</td>
<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>2</td>
<td>9th-grade math</td>
</tr>
<tr>
<td>542-034</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>1.5</td>
<td>9th-grade math</td>
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<td>542-038</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>5</td>
<td>9th-grade math</td>
</tr>
<tr>
<td>542-028</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>8</td>
<td>9th-12th-grade math</td>
</tr>
<tr>
<td>542-113</td>
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<td>Hispanic</td>
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<td>biology, mathematics</td>
</tr>
<tr>
<td>542-142</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>9</td>
<td>biology, chemistry, science life, physical</td>
</tr>
<tr>
<td>542-134</td>
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<td>earth</td>
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<td>542-132</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
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<td>8th-grade science</td>
</tr>
<tr>
<td>542-119</td>
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<td>Urban</td>
<td>Public</td>
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<td>8th-grade science</td>
</tr>
<tr>
<td>542-140</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>0.5</td>
<td>9th-grade science</td>
</tr>
<tr>
<td>542-124</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
<td>3</td>
<td>9th-grade science</td>
</tr>
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<td>542-015</td>
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<td>Public</td>
<td>5</td>
<td>9th-grade math</td>
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<td>542-001</td>
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<td>Urban</td>
<td>Public</td>
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<td>Algebra I</td>
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<td>542-107</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
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<td>biology, chemistry</td>
</tr>
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<td>542-125</td>
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<td>Hispanic</td>
<td>Urban</td>
<td>Public</td>
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<td>9th-grade science</td>
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</table>
Table 3.10: Descriptive statistics by gender in the main study

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
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<td>25</td>
<td>65.8</td>
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<td>Male</td>
<td>13</td>
<td>34.2</td>
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<tr>
<td>Total</td>
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</table>

Table 3.11: Descriptive statistics by race/ethnicity in the main study

<table>
<thead>
<tr>
<th>Race/Ethnicity</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>African American</td>
<td>2</td>
<td>5.3</td>
</tr>
<tr>
<td>Hispanic</td>
<td>28</td>
<td>73.7</td>
</tr>
<tr>
<td>White non-Hispanic</td>
<td>8</td>
<td>21.1</td>
</tr>
<tr>
<td>Total</td>
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<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.12: Descriptive statistics by type of school in the main study

<table>
<thead>
<tr>
<th>Type of school</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public</td>
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<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.13: Descriptive statistics by type of school district in the main study

<table>
<thead>
<tr>
<th>Type of school district</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
</table>
Table 3.14: Descriptive statistics of teaching experience in the main study

<table>
<thead>
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<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching experience</td>
<td>38</td>
<td>.5</td>
<td>28.0</td>
<td>7.47</td>
<td>6.48</td>
</tr>
</tbody>
</table>

3.2.2.3 Descriptive statistics of pilot and main study combined

Overall 58 teachers from 19 different schools in 5 different districts participated in the study. All teachers taught at a public school with six teachers at a rural school and 52 at an urban school. The majority were females (67.2%) and 32.8% males. The vast majority reported their race/ethnicity as Hispanic (77.6%), 11 teachers reported their race/ethnicity as White, non-Hispanic (19%), and two as African American (3.4%). Teaching experience years ranged from half a year to 28 years with a mean of 6.819 and a standard deviation of 5.688. In total 40 (69%) teachers that participated in this study were mathematics teachers while 18 (31%) teachers were science teachers. Tables 3.15-3.19 show the descriptive statistics by gender, race, type of school, type of school district, and experience years.

Table 3.15: Descriptive statistics by gender in both studies

<table>
<thead>
<tr>
<th>Gender</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>39</td>
<td>67.2</td>
</tr>
<tr>
<td>Male</td>
<td>19</td>
<td>32.8</td>
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<tr>
<td>Total</td>
<td>58</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.16: Descriptive statistics by race/ethnicity in both studies

<table>
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<tr>
<th>Race/Ethnicity</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>African American</td>
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<td>3.4</td>
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<tr>
<td>Hispanic</td>
<td>45</td>
<td>77.6</td>
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<tr>
<td>White non-Hispanic</td>
<td>11</td>
<td>19.0</td>
</tr>
<tr>
<td>Total</td>
<td>58</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.17: Descriptive statistics by type of school in both studies

Urban 38 100.0
<table>
<thead>
<tr>
<th>Type of school</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public</td>
<td>58</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 3.18: Descriptive statistics by type of school district in both studies

<table>
<thead>
<tr>
<th>Type of school district</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural</td>
<td>6</td>
<td>10.3</td>
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<tr>
<td>Urban</td>
<td>52</td>
<td>89.7</td>
</tr>
<tr>
<td>Total</td>
<td>58</td>
<td>100.0</td>
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</tbody>
</table>

Table 3.19: Descriptive statistics of teaching experience in both studies

<table>
<thead>
<tr>
<th>Teaching experience</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>58</td>
<td>.5</td>
<td>28.0</td>
<td>6.82</td>
<td>5.69</td>
</tr>
</tbody>
</table>

3.3 DATA COLLECTION

This study follows a mixed method sequential (quan →QUAL) design (Johnson & Onwuegbuzie, 2004) in two stages. In this design, the capital letters in qualitative part denote high priority where the lower case letters denote lower priority for the quantitative part. Thus the design of this mixed-methods study has a higher priority on the qualitative data and lower priority on the quantitative data. One of the strengths of a mixed methods design is that it provides “stronger evidence for a conclusion through convergence and corroboration of findings” (Johnson & Onwuegbuzie, 2004, p. 21). A mixed methods research proposes a third research paradigm, aside from the quantitative and qualitative paradigms, in which both quantitative and qualitative research are relevant and useful by maximizing the strengths and minimizing the weaknesses of both (Johnson & Onwuegbuzie, 2004, p. 21). In this study, I draw from a mixed methodology by utilizing the strength of quantitative results from the survey, which will allow selecting those participants to be interviewed. In the interviews, I draw from qualitative methodology to understand why teachers implement (or not) cognitively demanding tasks and what are the challenges. In the first stage, teachers answered a survey, and in the
second stage, interviews were conducted. Data was collected from the following sources: a) cognitive demand survey, b) workshop observations (microteaching), c) classroom observations and d) individual interviews with a purposefully selected sub-sample of teacher-participants. Figure 3.1 shows the process of data collection for this study. As mentioned above, the quantitative data was collected first. Once there was an average score of the results from the surveys as well as the level of cognitive demand used in the microteaching sessions and classroom observations, teachers were selected to be interviewed. Each one of these components is explained in detail in the following sections.

![Research design of the study](image)

**3.3.1 Surveys**

The instrument used as a test is the cognitive demand survey (see Appendix A). This survey served to assess teachers understanding of tasks at different levels of cognitive demand and was administered after one session where teachers have learned about the four levels of
cognitive demand. The teachers learned about cognitive demand and the four different levels in one of the sessions. One of the main presenters of the TQ grant is in charge of the design and delivery of the learning activities, and he explained the topic of cognitive demand and the levels. Teachers answered the survey at the end of one of the workshop sessions. The answers were individual, and it took around 20 to 25 minutes to finish.

During one workshop session of the pilot study, teachers talked about the cognitive demand construct and discussed it. They also spoke of the different levels of cognitive demand and were shown figure 2.1. After that, they were asked to solve the cognitive demand survey (see Appendix A). The purpose of this survey was to assess whether teachers were able to recognize, solve, and construct tasks at different levels of cognitive demand. This survey contained four different tasks around the concept of similarity. For the recognition part, teachers had to rate each one of the tasks with a scale from 1-4 where 1 is the lowest cognitive demanding and 4 is the highest cognitive demand. The level 1 task asked teachers to write the definition of similar figures. In the level 2 task teachers were given two similar rectangles, rectangle A and rectangle B, the width of rectangle A is 6, and the width of rectangle B is 12. The length of rectangle B is 20. Teachers had to find the value of the width of rectangle A, labeled x. The level 3 task had two irregular shapes labeled figure A and figure B. The instructions were the following: Figure A was transformed by scaling 1:2 in a horizontal direction and 1:0.5 in a vertical direction. What is the ratio of the area of Figure B to the area of Figure A? Finally, in the level 4 task teachers were asked to derive the Pythagorean relationship $AC^2 + BC^2 = AB^2$ by using similarity. For the construction part, teachers were asked to develop tasks at each level of cognitive demand (memorization, procedures without connections, procedures with connections, and doing
mathematics) in the concept of area of a triangle. Also, they had to explain why they think the task they developed is a task at that particular level and then provide a solution for their task.

For the main study, before they were given an explanation about cognitive demand, teachers were asked to discuss in teams their understanding. Four teams were formed, and after their discussion, they were instructed to write their definitions in large easel pads to present to the rest of the group. Figure 3.2 shows the definition of team 1 in which they wrote “the amount (how much) of critical thinking required to problem solve.” Team 2 wrote something similar when they wrote “level of complex thinking required to solve a problem” but also added, “Be able to recall prior knowledge and apply to a given situation.” They stressed the word “apply” to signify the importance of application (Figure 3.3). Figure 3.4 shows the answer written by team 3 in which they first wrote bullet points about connecting prior knowledge to new information and how the brain scaffolds new information. They also wrote the following definition: “cognitive demand refers to thinking by connecting prior knowledge with new information through scaffolding.” Finally, figure 3.5 shows the definition given by team 4 which was provided by several bullets such as “how hard you should have to think,” finding multiple solutions to 1 problem,” and “visual to numerical to symbolic.” After the whole group had discussed all their definitions, the presenters talked about the cognitive demand construct and its four different levels by showing Figure 2.1. Similarly to the pilot study, teachers in the main study were given a survey. Since the main study consisted of mathematics and science teachers the topic of the survey for recognition and solution was rate and for construction was proportionality (see Appendix B). They were also asked to rate each task from 1-4 where 1 is the lowest cognitive demand and 4 is the highest cognitive demand. In the task for level 1, teachers had to write a formula for the rate. In the task for level 2, teachers had to solve the following problem: The
Rabbit runs 30 meters in 4 seconds. What is his rate? For the level 3 task teachers had to solve the following task: Rabbit and Turtle run a 60 meter “over and back” race from a starting point to a tree (30 m), then back to the starting point again. Rabbit’s speed over is 6 m/s and back is 4 m/s. Turtle’s speed both ways is 5 m/s. Who will win the race? Finally, for the level 4 task this is the problem that was given to them: Rabbit and Turtle run $d$ meter “over and back” race from a starting point to a tree ($d/2$), then back to the starting point again. Rabbit’s speed over is $r_1$ m/s and back is $r_2$ m/s. Turtle’s speed over is $r_3$ m/s and back $r_4$ m/s. Rabbit and Turtle have equal average speeds. Would Rabbit win the race? Specify conditions under which Rabbit could win.

In the construction part teachers had to develop a task at each level, then explain why they consider that task is at that specific level, and then solve it.

Figure 3.2: Cognitive demand definition by team 1
Figure 3.3: Cognitive demand definition by team 2

- Be able to recall prior knowledge and apply to a given situation
- Level of complex thinking required to solve a problem

Figure 3.4: Cognitive demand definition by team 3

- How we connect prior knowledge to new information
- How your brain scaffolds information

Cognitive demand refers to thinking by connecting prior knowledge to new information through scaffolding.
3.3.2 Microteaching sessions

As part of the professional development workshop in the TQ grant, teachers have to develop a lesson to be presented at the workshop called a microteaching. They had to submit this lesson as if it was for their classroom and it lasted around 45 minutes. These observations took place during the professional development workshop sessions when they presented the microteaching sessions. Of particular interest was the level of cognitive demand of the tasks presented during these sessions in order to know which level they choose to present to their peers. The observation field notes followed an ethnographic approach, in this approach the ethnographer participates and gets immersed in the setting by observing and developing relationships with the participants and creates written records of the observations (Emerson, Fretz, & Shaw, 2011). The microteaching sessions helped understand the implementation of cognitively demanding tasks. The tasks presented at the microteaching sessions were analyzed based on the four different levels of cognitive demand, that is, if a memorization task is
presented, then it would be rated as 1. If a task that is considered procedures without connections is presented, then it would be rated as 2 and so on. Hence, we would be able to determine the level of cognitive demand being used during these sessions. Several discussions were held with my advisor to reach a consensus on the level of cognitive demand used during the microteaching sessions since we were both present during these sessions.

3.3.3 Interviews

The semi-structured interviews served to give voice to teachers’ ability to recognize, solve, and design cognitively demanding tasks as well as their challenges in implementing CDT in middle school and secondary mathematics classroom. “Qualitative interviewing provides an open-ended, in-depth exploration of an aspect of life about which the interviewee has substantial experience, often combined with considerable insight” (Charmaz, 2002, p. 675). The purpose of a semi-structured interview is to be “sufficiently structured to address specific topics related to the phenomenon of study, while also leaving space for study participants to offer new meanings to the study focus” (Galleta, 2013, p. 24). The interview protocol was developed based on the research question, but it also allowed for the interviewer to expand based on the teacher's responses. Thus qualitative interviewing provided more insight into the teachers’ reasoning to implement cognitively demanding tasks as well as the challenges these teachers face. Also, qualitative interviews allowed answering the why question that quantitative inquiry often does not answer. One semi-structured interview with each participant selected was conducted because the protocol was developed to address the research questions. “The interviewer’s questions ask the participant to describe and reflect upon his or her experiences in ways that seldom occur in everyday life. The interviewer is there to listen, to observe with sensitivity, and to encourage the person to respond. Hence, in this conversation, the participant does most of the talking”
Participants were selected to be interviewed based on the following criteria: a) teachers that know the different levels of cognitive demand (i.e. a high rating on the survey) b) teachers that know the levels and apply it in a peer setting (i.e. the level utilized during the microteaching was somewhat high), and c) teachers that know and implement the levels in a peer setting and in a classroom setting (i.e. teachers that utilized a high level in the classroom setting). Teachers from each criterion were selected. The interviews conducted lasted approximately 45 minutes and were conducted in a private space at UTEP or any place of preference of the teachers. I transcribed all the interviews. All participants were told that if a follow-up interview was needed to clarify anything from the previous interview, they would be contacted. However, this was not necessary. Also, all personal information was kept confidential in the transcriptions and teachers were assigned a pseudonym to assure anonymity. For the interview protocol, see Appendix C.

3.3.4 Classroom observations

Classroom observations were conducted for the smaller sample of teachers who were interviewed. Similarly, as with the microteaching sessions, the level of the tasks implemented in the classroom was analyzed (if a memorization task is presented then it was rated as 1 if a task that is considered procedures without connections is presented then it was rated as 2 and so on). In order to examine the implementation of cognitively demanding tasks, one classroom session was analyzed. Observing the application of cognitively demanding tasks in two settings (peer setting and classroom setting) allowed cross-referencing the results. In addition, observing the actual classroom provides more detailed information than just a peer setting in which teachers may choose to implement a higher or lower cognitive demand task.
In this section, the methods for data collection have been explained. This study used a mixed methodology for data collection in two stages. In the first stage, all teachers participating in the professional development were asked to answer the cognitive demand survey. At the same time, observations were done during the professional development workshop sessions, especially the microteaching sessions. From the results on this stage, a smaller sample of teachers was selected to interview and observe in their classrooms. Once data was collected, it was analyzed as explained in the following section.

3.4 DATA ANALYSIS

All data sources were analyzed to answer the research questions. Survey responses were coded the following way: for the recognition section of the survey each task was graded to check whether the teachers correctly identified the level of the task, for the solution task the answers was carefully graded to check if they correctly solved the task, and for the construction task, the tasks were carefully analyzed to check whether each task constructed at the level required. For the solution and construction surveys a rating scale was used where 1 was given if there is no answer or answer is wrong, 2 if there is an incomplete answer, and 3 if the answer is correct. I rated their answer and then had several discussion meetings with my advisor in order to reach agreement on the ratings for solution and construction. There was one teacher that responded to all survey questions with the same answer. I met with her, and after seeing the questions again, she realized her mistake and corrected it. After the survey responses had been coded, statistical analyses were conducted. Descriptive statistics was used to understand the teachers’ responses and selection criteria for the interviews. In addition, correlation analysis was carried out to analyze whether there is a relationship between recognition, solution, construction, and
implementation. Independence tests were be conducted to test whether recognition, solution, and construction are related to implementation.

The semi-structured interviews and workshop observations were coded based on the grounded theory technique shown in Charmaz (2006). The grounded theory technique was utilized in this study to analyze the interviews and workshop observations since it allows collecting data and analyzing it in a way that the theory comes from the data itself. In the grounded theory technique, the researchers gather rich data and then they “evaluate the fit between their initial research interests and their emerging data” (Charmaz, 2006, p. 17). These data sources were coded to look for patterns to find categories and themes, “Coding is the pivotal link between collecting data and developing an emergent theory to explain these data” (Charmaz, 2006, p. 46). The grounded theory analysis technique is done in two phases: initial and focused coding. First, during the initial coding, the coding is close to the data and is coded with words that reflect action. Data was coded line by line because that way the researcher remains open to the data and see nuances in it (Charmaz, 2006, p. 50). One of the advantages of the initial coding is that in early stages of research it allows finding gaps and holes and then gather more data if needed. According to Charmaz (2006), in the focused coding, the codes are more directed, selective and conceptual and “it requires decisions about which initial codes make the most analytic sense to categorize your data incisively and completely” (p. 57). In this stage, the larger amount of data gets analyzed based on the codes that were selected in the initial coding. As in the survey ratings, several meetings were held with my advisor in order to reach agreement on the categories and themes. All interview transcriptions were analyzed using QSR International’s NVivo 11 qualitative data analysis Software. This software is an analytical tool that allows organizing all qualitative data while looking for themes.
In this chapter, the methods of participant recruitment, data collection, and data analysis have been explained in detail. In summary, this study took place in a professional development workshop entitled Teacher Quality. All teachers from this workshop were invited to participate in this study. However, it was not mandatory, and they could refuse or decide to stop participating at any time. Data sources include a survey, workshop observations, classroom observations and interviews. That is why this study follows a mixed methodology design in which the quantitative portion helped to select participants for the qualitative portion of the study. Interviews allowed examining the challenges in-depth and understanding teachers’ reasoning about the challenges of implementing cognitively demanding tasks. Classroom observations and microteaching sessions allowed identifying the levels of the tasks. The following chapter shows the results of the surveys, correlation analysis, and interviews.
Chapter 4: Results

This chapter presents all the results from this study. The research questions of this study served to organize the findings. First, the results of the surveys are presented separated by the pilot study and the main study. Then the correlation results are also separated by pilot study and the main study. Finally, based on the teachers’ answers and the third research question the results from the interviews are presented together.

4.1 Survey results

Research Question 1: To what extent are secondary mathematics and teachers able to recognize, solve and construct tasks at different levels of cognitive demand?

In this section, the results of the cognitive demand survey were analyzed in order to answer the first research question. This survey was administered to both studies, and it was developed to assess whether teachers were able to recognize, solve, and construct tasks at different levels of cognitive demand. The pilot study served as a trial to the cognitive demand survey, and some changes were made to address any issues during the first round of survey implementation. Each result is shown next, separated by the pilot and the main study.

4.1.1 Pilot study

In the recognition part of the survey, teachers were given four different tasks, and they were required to rate the level of each task from 1-4 (1 the lowest cognitive demand and 4 the highest cognitive demand). There was one problem from each level, but they were not placed in order (i.e. the first task was not level 1). In addition to providing a rating for the task, teachers had to explain their reasoning for their rating. Table 4.1 shows the percentages of correct
answers in the cognitive demand level survey of the pilot study. It shows the percentages by levels of cognitive demand. For each part of the survey, recognition, solution, and construction, there were 20 teachers. In the recognition part, teachers had more challenges in level 3 (procedures with connections). In the solution part, teachers had more challenges in level 4 (doing mathematics). In the construction part, teachers had more difficulty constructing tasks at level 3 (procedures without connections). Next, each percentage will be explained in detail by providing some examples.

Table 4.1: Percentages of correct answers in the cognitive demand level survey of pilot study

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recognition</strong></td>
<td>85%</td>
<td>70%</td>
<td>50%</td>
<td>70%</td>
</tr>
<tr>
<td><strong>Solution</strong></td>
<td>100%</td>
<td>95%</td>
<td>55%</td>
<td>0%</td>
</tr>
<tr>
<td><strong>Construction</strong></td>
<td>95%</td>
<td>90%</td>
<td>40%</td>
<td>70%</td>
</tr>
</tbody>
</table>

The level 1 task asked teachers to write the definition of similar figures. 85% of teachers correctly identified the task at level 1. Those who correctly identified the task as a level 1 wrote: “only need to memorize a definition,” “recall, stating a definition,” and “just a definition.” Others who incorrectly identified this task as a level 1 task wrote: “Task is demanding, recall on knowledge some thought process is required to develop the definition” (this participant rated it as level 2), another participant wrote: “they need to remember that the angles and sides must correspond from one figure to another.”

70% were able to identify the mathematical task at level 2. This mathematical task required teachers to find a missing side of a rectangle when given the sides of a similar triangle. Those who correctly identified this task as level 2 explained: “it is not requiring a recall or definition, you need to have some concept of similar figures proportions, etc.,” “it is a two because they are required to follow procedure,” and “all they had to do was write a proportion
and solve for x.” Among those who incorrectly identified this task, one participant wrote, “Only need to memorize a process” (rated it as 1). Another participant explained, “The figures are similar, so I set up a proportion” (this participant rated it as level 3).

Only half were able to identify the task at level 3. This task had two irregular figures (A and B). Figure B was drawn by transforming figure B by scaling 1:2 in a horizontal direction and 1:0.5 in a vertical direction. The question asked for teachers to find the ratio of the area of figure B to the area of figure A. In the rating explanation those teachers who correctly identified the task at level 3 wrote: “requires some procedures and effort making connections with math”, “they are using math with connections”, and “cognitive demand is high because of the need to understand how area is affected when scaled.” Explanations given by those who incorrectly identified this task are: “just had to do the procedure” (rated as level 2), “having to explain how compensating one for the other is equal to 1” (rated as level 4), and “it requires mathematical thinking to come to a solution” (rated as level 4).

70% were able to identify the task at level 4 correctly. This task required teachers to derive the Pythagorean theorem based on a picture of a triangle. Some of the explanations by those who were able to identify this task at level 4 were: “it takes mathematical thinking to come to solution,” “you have having to derive a formula from a picture you are doing mathematics with understanding,” and “this is very cognitively demanding problem because you must be able to access all of your previous information.” Some teachers did not provide an explanation. The rest of the teachers rated it as level 3 and wrote “procedures with connections is needed,” “Connecting geometry to math, there are many rules to connect.”

In the solution part of the survey, teachers had to solve the same tasks from the recognition part. All teachers were able to solve the task at level 1 correctly. The level 1 task
called for the definition of similar figures teachers responded with answers such as the following: “2 figures were congruent, corresponding angles and corresponding sides are proportional”, “2 shapes that have congruent angles and proportional side lengths”.

95% correctly solved the task at level 2. In figure 4.1, the teacher wrote the proportions and then solved for x. As we can see from the picture, the teacher first created the proportion 6/12 then x/20 and set them up equal to each other and then crossed them as if showing that is doing a method of the rule of 4. The teacher then showed the long division and finally crossed the right answer: 10. By setting up proportions, the teacher solved this task correctly, and it also shows how this is a procedure without connections task.

Figure 4.1: Example of a solution with a proportion

In figure 4.2, the teacher set up the proportions to find the scale factor by showing that 6/12 is equal to 1/2 and then divided the side proportional to the unknown side by the scale factor. In other words, this teacher divided 20 by 2. In addition, this teacher added the same long division as the previous example. Finally, the solution in this case also contains a written explanation: “the solution of x should be 10 because the simplified ration is 1:2 which would make x=10.” One participant that didn’t solve this task correctly made a mistake with the arithmetic operations.
55% of teachers correctly solved the task at level 3. When solving this task, some teachers wrote, “The ratio is 1:1 because the same ratio that was cut on length was added on the sides,” “the first scaling doubles the area because the doubling was done to one dimension. The second scaling reduces the area by half because it is done in one dimension. Also, both the enlargement and the doubling counteract each other.” Other teachers, as it is shown in Figure 4.3 opted out to putting a numerical value to each side, then performing the scaling that was said in the task, and then obtained the area to get to the conclusion that the area will have a ratio of 1:1. This teacher also drew two rectangles most likely because the figure in this task was irregular, so the teacher had to convert it to something that is more familiar.

Here are some examples of teachers that didn’t correctly solve this task “can’t explain it without understanding the ratio,” “the area of figure B will be four times as big as figure A,” the rest did not finish the task or didn’t write anything.
None of the teachers of the pilot study were able to solve the task at level 4. There were some attempts, but they were not able to finish the task. Some of these teachers started to write some similarities. However, they used the wrong similarities and couldn’t finish the problem. For example, Figure 4.4 shows how this teacher tried to write several similarities set them equal to each other but couldn’t reach a conclusion. One teacher wrote, “don’t remember.” Another teacher drew squares on each side of the right triangle and wrote “see picture” without providing any explanation. 13 out of the 20 teachers decided to leave the task blank.

![Figure 4.4 Example of an incorrect solution](image)

In the construction, task teachers were given a topic, and then they had to construct their tasks at each one of the levels. Also, they were also required to provide a solution to the task they developed as well as an explanation of why they think that task is at that level. 95% of teachers constructed a task at level 1. The vast majority of tasks that were created by teachers were about writing a formula or a definition. Some teachers explained the reasoning for their problem: “students only need to memorize the formula,” “because it can be recalled without meaning or understanding,” and “student does not need to apply to any solution.” One participant created a task that would require solving for a variable. That task was considered to be more procedural than a memorization task.

90% of teachers were able to construct a task at level 2 correctly. Some teachers created a task that would require finding the area of a triangle when given the values of the base and the
height. Some explanations were: “you are giving students the dimensions, all they are doing is substituting and solving,” “they need to know the process.” Others wrote a word problem that was procedural and required finding the area of a triangle. These teachers wrote: “they are just following the formula without much thinking involved,” “task is procedural without any connections to real-world applications.”

Slightly less than half (40%) of the teachers constructed a task at level 3. One participant created a task to find the area of a triangle by using similar triangles and explained that it was a level 3 task because they need to “have experience of similar figures and apply it to find area.” Another participant created the following task: Sam needs to find the height of the triangle inscribed inside a rectangle. She knows the area of the triangle is 3 units². Using the information given and the picture finds the height. That participant explained, “The student must see the relationship between the rectangle and the triangle. For example, the task in figure 4.5 shows a triangle and then provides its height, and hypotenuse and the student would have to find the area. The teacher explained that it was a procedure with connections “because it involves multi step, you need to understand a couple of concepts to fulfill the assignment.” When the teacher provided the answer it shows that the student would need to remember the Pythagorean theorem and then the formula for the area of a triangle to get the result.

Figure 4.5 Example of construction of a level 3 task

Those teachers that did not construct a level 3 task constructed a task that was more procedural since it only required to substitute values in a formula. In the example, in figure 4.6
the teacher created a very similar task as the one in figure 4.5, but in this case, the values of the height and base are given and labeled. The teacher wrote as an explanation “students are able to see where do units correspond to.” The answer was given next to it, which clearly shows that it only requires substituting values in the formula and computing the results. Those types of tasks were considered to be level 2 tasks. There was one participant that left this part unanswered.

![Image](image.png)

Figure 4.6 Example of construction of a level 2 task that was intended as a level 3

70% of teachers created a task at level 4. The majority of teachers that created a task at level 4 created a task that said: derive the formula for the area of a triangle. One participant explained, “It requires you to develop a method of finding the area of a triangle.” Another participant wrote. “There must be a deep understanding of concepts and many connections.” 5 participant constructed a task that was a level 2 because it was considered to be procedural. Those tasks required the substitution of values in a formula to solve for one variable. One participant left this part unanswered.

The pilot study served as a way to examine how the teachers understood the cognitive demand survey. In this pilot study, teachers did not learn about the four levels of cognitive demand before administering the recognition and solution part of the survey. It was decided that for the main study some examples and a brief explanation of the four levels of cognitive demand were given to the teachers. Also, before the construction task was administered there was an explanation of the four levels of cognitive demand, which included the following example for the
level 4 task: derive a formula. As a result, many teachers used the same example for the
construction part of the survey. For the main study that example was not used during the
explanation.

Table 4.2 is a summary of the strategies used by teachers in the solution and construction
part of the survey. To solve the task at level 1, the majority of teachers (75%) wrote a definition
of the angles and sides. To solve the task at level 2, the majority used either a scale factor (50%)
or a proportion (45%). To solve the task at level 3 the majority of teachers that were able to solve
it used numbers (30%). Nobody was able to correctly solve the task at level 4. The vast majority
(95%) constructed a task about defining a formula in the construction of a task at level 1. In
constructing a task at level 2, the teachers either wrote a task about using a formula (75%) or a
word problem (15%). Teachers that were able to construct a task at level 3 (40%) created a task
wrote a task about using concepts of similarity and area. The majority of the ones that did not
create a task at level 3 created a task at level 2 (55%). To construct a task at level 4, they created
a task that required students to derive a formula (70%). The ones that did not construct a task at
level 4 constructed a task at level 2 (25%).

Table 4.2: Summary of strategies used by teachers in the solution and construction part of the
survey

<table>
<thead>
<tr>
<th>Solution level 1</th>
<th>Strategy</th>
<th>% of teachers</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles and sides</td>
<td>75%</td>
<td>“Corresponding congruent angles and proportional corresponding sides.”</td>
<td></td>
</tr>
<tr>
<td>Shape and sides</td>
<td>20%</td>
<td>“Same shape different sides.”</td>
<td></td>
</tr>
<tr>
<td>No answer</td>
<td>5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution level 2</th>
<th>Strategy</th>
<th>% of teachers</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale factor</td>
<td>50%</td>
<td>“The scale factor is (\frac{1}{2}).”</td>
<td></td>
</tr>
<tr>
<td>Proportion</td>
<td>45%</td>
<td>12/6=20/10</td>
<td></td>
</tr>
<tr>
<td>No answer</td>
<td>5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution level 3</th>
<th>Strategy</th>
<th>% of teachers</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used numbers</td>
<td>30%</td>
<td></td>
<td>“The ratio of the area from figure A to figure B is 1:1.”</td>
</tr>
<tr>
<td>No computation</td>
<td>5%</td>
<td></td>
<td></td>
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80
### Table 4.1.2

<table>
<thead>
<tr>
<th>Level</th>
<th>Task Description</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 4</td>
<td>Didn’t finish</td>
<td>65%</td>
</tr>
<tr>
<td></td>
<td>No answer</td>
<td>35%</td>
</tr>
<tr>
<td>Construction 1</td>
<td>Formula</td>
<td>95%</td>
</tr>
<tr>
<td></td>
<td>Find the area of a triangle.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Level 2</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td>“What is the formula for the area of a triangle.”</td>
<td></td>
</tr>
<tr>
<td>Construction 2</td>
<td>Formula</td>
<td>75%</td>
</tr>
<tr>
<td></td>
<td>Find area</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Word problem</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>“John is finding the area of a triangle with a base length of 2 and a height of 3.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No answer</td>
<td>10%</td>
</tr>
<tr>
<td>Construction 3</td>
<td>Similarity</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td>“Triangles A and B are similar, find the area of triangle A.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Level 2</td>
<td>55%</td>
</tr>
<tr>
<td></td>
<td>No answer</td>
<td>5%</td>
</tr>
<tr>
<td>Construction 4</td>
<td>Derive formula</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>“Derive the formula of area of a triangle.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Level 2</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>“Apply the Pythagorean theorem to calculate the diagonal distance across a garden whose dimensions are 40 ft. by 15 ft.”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No answer</td>
<td>5%</td>
</tr>
</tbody>
</table>

### 4.1.2 Main study

Similarly to the pilot study, the teachers in the main study were given the cognitive demand survey. This survey had some changes based on the results of the pilot study. In addition, since the main study included both mathematics and science teachers the topics were modified to rate for recognition and solution and proportionality for construction. The cognitive demand survey was given after a brief explanation of the cognitive demand levels. A change had to be made after the first collection of the survey. The task at level one first asked teachers the following: if \(d\)-distance, \(t\)-time, and \(r\)-rate \(r=rate\), find a relationship for rate. This question resulted in confusion for teachers since by reading the word relationship they considered this task at a higher level than was intended. This question was then changed to ask for “what is a formula
for rate?” and the whole survey was administered a second time. Table 4.3 shows the percentages of correct answers in the cognitive demand level survey of the main study. It shows the percentages by levels of cognitive demand. For recognition and solution part of the survey, 38 teachers responded the survey. 35 teachers responded the construction part. In the recognition part teachers had more challenges in level 2 (procedures without connections). In the solution part, teachers had more challenges in level 4 (doing mathematics). In the construction part, teachers had more difficulty constructing tasks at level 4 (doing mathematics). Next, each percentage will be explained in detail by providing some examples.

Table 4.3: Percentages of correct answers in the cognitive demand level survey of the main study

<table>
<thead>
<tr>
<th></th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognition N=38</td>
<td>76%</td>
<td>63%</td>
<td>74%</td>
<td>82%</td>
</tr>
<tr>
<td>Solution N=38</td>
<td>100%</td>
<td>89%</td>
<td>53%</td>
<td>0%</td>
</tr>
<tr>
<td>Construction N=35</td>
<td>74%</td>
<td>97%</td>
<td>14%</td>
<td>2%</td>
</tr>
</tbody>
</table>

For the recognition part, they were required to rate the level of the task and then provide an explanation. 76% of teachers were able to recognize the task at level 1 correctly. This question required teachers to write a formula for the rate. In the explanation, those who correctly rated this task as level 1 wrote: “they just need to recall the info,” “this problem just requires recitation,” and “memorize the formula.” Some of the explanations given by those who correctly recognized this task as a level 1 are: “making a formula” (rated as level 2), “asks to apply prior knowledge to answer” (rated as level 3), “need to have some knowledge of formula,” “proportion,” “they just need to set up and solve.”

The task at level 2 was about finding the rate of a rabbit that runs 30 meters in 4 seconds. 63% were able to recognize the task at level 2. On the one hand, teachers who recognized this task as a level 2 wrote: “basic operation,” and “it takes somewhat of an understanding of rate and
knowing that it is based on a measure of two units.” On the other hand, those who rated this task as something other than level 2 wrote: “substitute the values” (rated as 1), “basic calculation” (rated as 1), “this process is one of memorization, there is no rigor in trying to solve it” (rated as 1).

In the level 3 task teachers were given a problem in which a rabbit and a turtle ran a 60 meter over and back race from a starting point to a tree (30 m) and then back to the starting point. 74% successfully recognized the task at level 3. In the explanations, some teachers wrote: “because they do give us some numerical values to work with,” “because they do give us some numeral values to work with,” and “higher level of cognitive demand to set up equation and solution.” Those who rated this task differently explained: “This problem requires computing times which simple equations manipulation and plugging in” (rated as 2), “have to apply concept” (rated as 4), “have to know that it’s not just rate but time” (rated as 4), and “only based on the calculations to be set up (rated as 2).

Level 4 task was more abstract as compared to the other three tasks. They were required to specify under which conditions the rabbit could win given that the rabbit’s speed over is \( r_1 \) m/s and back is \( r_2 \) m/s and the turtle speed over is \( r_3 \) m/s and back is \( r_4 \) m/s. In addition, they have equal average speeds. 82% correctly recognized the task at level 4. One participant wrote, “no numerical values, higher critical thinking.” Similarly, another participant wrote “highest cognitive demand because there is no value, so students have to use variables only. One participant that rated this task as a level 2 wrote: “high cognitive demand to set up equation and solution.” The rest of teachers who rated this task incorrectly didn’t provide any explanations.

All teachers from the main study were able to solve the task at level 1. In this part, teachers were asked to give the formula for rate or the definition of similar figures. All teachers
correctly solved by answering $r = \frac{d}{t}$. A vast majority (89%) of teachers successfully solved the task at level 2. In order to get the rate teachers solve this task in different ways. Figure 4.3 shows how a participant used long division to get the result. This example contains first a proportion $30\text{m}/4\text{sec}=7.5 \text{ m/s}$. In order to get the answer, the teacher used long division.

\[
\begin{array}{c}
30\text{m} \\
4\text{sec}
\end{array}
= 7.5 \text{ m/s}

\begin{array}{c}
4 \underline{13}0 \\
-28
\end{array}

\begin{array}{c}
20
\end{array}

\]

Figure 4.7: Example of a solution with a long division

Others simplified the proportion as it is presented in figure 4.4. In this case, the teacher wrote the formula first rate(speed) = distance/time, then substituted the values $30\text{m}/4$. After that, the formula was simplified as $15/2$ with a result of $7.5\text{m/sec}$. It is important to note that in this case, the teacher did not have the units on the denominator, it only shows a number 4. However, in the end, the results do show both the right answer and the correct units which are meters over seconds.

\[
\frac{\text{Rate (Speed)}}{\text{Time}} = \frac{30\text{m}}{4} = \frac{15}{2} = 7.5 \text{ m/sec}
\]

Figure 4.8: Example of a solution by simplifying a proportion

Others just left the proportion as $30/4$. The ones who did not solve it correctly made mistakes with the arithmetic (see figure 4.5). Figure 4.5 shows how this teacher started this problem correctly by writing first the corresponding units, meters over second; the second part is
also correct when the proportion is set up 30/4. However, the answer fails at the end since 30 divided by 2 is indeed 15 but 4 divided by 2 is not 1.

\[
\frac{\text{meters}}{\text{second}} = \frac{30}{4} = \frac{15}{1}
\]

15 meters per second

Figure 4.9: Example of an incorrect solution

Slightly more than half of the teachers (53%) solved the task correctly at level 3. Some teachers solved this task by computing the rate for the rabbit going back and forth separately and added them, then the rate for the turtle and found out that the turtle had won the race. In figure 4.6 it can be seen that this participant computed each rate separately. In this case, it shows 30/6 equal 5 and 30/4 equal 7.5. Even though this teacher does not show the addition, it shows that the rabbit rate is 12.5. The second set of operations show 30/5 equals to 6 and again 30/5 equals to 6. Again, without showing the addition, the teacher writes turtle 12, circles it and concluded that the turtle wins the race. Also, the teacher writes “turtle wins because he has a constant rate of change.” This approach was used by all of those who correctly solved this task.

Figure 4.10: Example of a solution by computing the rates separately

Those who did not solve it correctly either wrote that the rabbit would win the race by doing the same procedure as explained above but wrote that the rabbit would win since it will
make the race in 12.5 seconds instead of the turtle who made it in 12 seconds. Some used the incorrect operations and therefore didn’t solve this task successfully as it is shown in figure 4.7. In this example, the teacher only writes 6 m and 4 m equal to 5 m plus 5 m. The teacher fails to solve this task correctly because the rate was not computed. The written explanation is also incorrect because it says that the rabbit won the race but based on the computation above there will be a tie.

Figure 4.11: Example of an incorrect solution by adding the rates

Another answer was that there would be a tie because they just got the average speed.

Figure 4.8 shows an example of a teacher that concluded that the rabbit and the turtle tie the race. In this example, the teacher found the average of both speeds and made them equal to each other. From the beginning, this method is assuming that they will be equal to each other. With this method, both sides are equal to 5. The teacher explains that it is a tie because “the average speed is the same.” The rest did not finish the task or left the task blank.

Figure 4.12: Example of an incorrect solution by computing the average rates
None of the teachers were able to solve the task at level 4. In this task, they had to specify under which conditions the rabbit would win when running to a tree and then back to the starting point, thus the whole distance is divided by two. The speed going back for the rabbit is \( r_1 \) m/s and back is \( r_2 \) m/s and the turtle speed over is \( r_3 \) m/s and back is \( r_4 \) m/s. One participant wrote, “The condition would be by knowing the rate,” Another one wrote, “the rabbit would win if he has a constant rate of change,” Others wrote, “Rabbit would win if the turtle had a slower rate coming and a faster rate going. Others left it blank or incomplete. One participant only wrote, “The problem is complex.”

74% of teachers constructed a task at level 1. Most of their tasks were about remembering a formula, writing a definition or setting up a proportion. One participant that created a task about explaining what a ratio is and a proportion wrote, “It is the concept from where students start the understanding of proportionality.” Another participant created the following task: “Name the equation for direct variation” and explained that it was a memorization task because “they just need to memorize the equation.” Those who did not construct a level 1 task constructed a level 2 task because it was more procedural than memorization. For example, one participant wrote, \( x/10=39/130 \), solve for \( x \). That participant then explained, “This is a memorization task because they just have to recall the procedure in order to find the value of \( x \).”

97% created a task at level 2. One participant constructed a task with a proportion and one missing value and then wrote: “must find \( x \) using cross multiplication, then division, very procedural no connection.” Others created a similar task but with a word problem such as the following, “4 laps equals a mile, how many miles are there in 20 laps.” There was only one participant that did not construct a level 2 task because this task was left unfinished.
Only 14% of teachers were able to create a task at level 3. One participant constructed the following task “what is the maximum area of a rectangle if the perimeter is 20” and explained that it was a level 3 task because “it requires to use prior knowledge of area and perimeter.” Another participant wrote, “Use fraction bars to show how proportions are ‘fractional parts’” and explained that “it shows the students the process and the reasoning” The following task was rated a level 2 task when the participant intended to write a level 3 task. “Find the amount needed to double or triple a recipe.” Another example of a level 2 task that was intended as a level 3 task is the following, “a can of pineapple costs $2.00 how many cans are you able to buy with $10.00.” Figure 4.13 also shows another example of a task that is intended as a level 3 but is only procedural. Even though it is using similar figures, there is only one value missing, so all it needs is to set up a proportion and solve for x. The reasoning was that it “connects real world problem to solving proportions. Finally, only five teachers left this part unanswered.

![Figure 4.13 Example of construction of a level 2 task intended as level 3](image)

Figure 4.13 Example of construction of a level 2 task intended as level 3

Only 2% successfully created a task at level 4. One participant wrote the following task: “is it possible the area and perimeter to be the same?” and said that it is a level 4 task because it requires using higher order thinking. 15 teachers created a task that was considered to be a level 2 such as “apples are sold at 2 lb. for $5. How many lbs. of apples can be bought with $25?” while 2 teachers left this question blank.
Table 4.4 shows a summary of strategies used by teachers in the solution and construction part of the survey. To solve the task at level 1, the majority of teachers (97%) wrote the formula. To solve the task at level 2, the majority of teachers used long division (10%), used a proportion (13%), just wrote the answer (53%), or provided a written response (13%). The ones that did not solve it had an arithmetic mistake (8%) or provided no answer (3%). To solve the task at level 3, the majority of teachers computed the rates separately (53%). The ones that did not solve it correctly either used a wrong operation (19%) said that it was a tie (8%), or did not provide an answer (10%). The majority (61%) provided no answer for the solution of the task at level 4. Teachers that constructed a task at level 1 created a task about writing a definition (23%) or a formula (28%). The vast majority of teachers constructed a task at level 2 about solving for x (88%). In the construction of a task at level 3, the vast majority wrote a task at level 2 instead (71%). Similarly, in the construction of a task at level 4, the majority created a task at level 2 (66%).

Table 4.4: Summary of strategies used by teachers in the solution and construction part of the survey

<table>
<thead>
<tr>
<th>Strategy</th>
<th>% of teachers</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution level 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formula</td>
<td>97%</td>
<td>(r=d/t)</td>
</tr>
<tr>
<td>Written explanation</td>
<td>3%</td>
<td>“Distance is equal to rate * time therefore if I solve for r I obtain the formula for rate (r=d/t).”</td>
</tr>
<tr>
<td>Solution level 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long division</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Proportion</td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td>Just answer</td>
<td>53%</td>
<td>(30/4)</td>
</tr>
<tr>
<td>Written response, no operation</td>
<td>13%</td>
<td>“divide total distance by time 7.5.”</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>8%</td>
<td>(30\text{m}/4\text{sec}, 15\text{m/s})</td>
</tr>
<tr>
<td></td>
<td>Solution level 3</td>
<td>Solution level 4</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Mistake</td>
<td>No answer</td>
<td>Separate operations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>53%</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Solution</td>
<td>No answer</td>
<td>Rabbit speed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>39%</td>
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### 4.2 Correlation Results

Research Question 2: Are there relationships among teachers’ ability to recognize, solve, construct, and implement tasks at different levels of cognitive demand?

In this section, the correlation analysis from the survey responses is shown. A series of correlation analysis were conducted for the pilot study and the main study to address the second question of this study. Each result is shown next, separated by the pilot and the main study.
I was interested in examining if there was a relationship between the teacher’s ability to recognize, solve, construct, and implement tasks at different levels of cognitive demand. Recognition, solution, and construction were based on the teacher’s responses from the cognitive demand survey, and the implementation part was based on the level of cognitive demand used when the class was observed. In this case, a chi-square test as a correlational probe was conducted. According to Huck (2004), this type of test is used when “the researcher is interested in whether a nonchange relationship exists between two nominal variables” (p. 468). The data was arranged into contingency tables, and then a chi-square test was used to determine whether there was a statistically significant relationship between two variables in each case.

The following two tables (Tables 4.5 and 4.6) show the values of the chi-square statistics \( \chi^2 \) and the p-value. At an alpha level of .05, some variables were significant. From the pilot study, recognizing a task at level 2 was related to the recognition at level 3 \( (\chi^2=3.81, \text{ p-value= .051}) \). In other words, those who correctly identified the task at level 2 were more likely to recognize the task at level 3. Another two variables that were significantly related were recognition at level 3 and recognition at level 4 \( (\chi^2=3.81, \text{ p-value= .051}) \). The majority of those who had challenges recognizing the task at level 3 also had challenges recognizing the task at level 4. Recognizing a task at level 3 was also related to solving a task at level 3 \( (\chi^2=5.05, \text{ p-value= .025}) \). Thus, teachers who correctly recognized the task at level 3 were more likely to solve a task at level 3. Recognizing a task at level 4 was also related to being able to construct a task at level 2. Constructing a task at level 2 was significantly related to implementation at level 3 \( (\chi^2=4.13, \text{ p-value= .042}) \). However, this correlation was negative, in other words, those who were able to construct a task at level 2 did not implement a task at level 3. Constructing a task at level 3 was related to implementing a task at level 2 \( (\chi^2=6.71, \text{ p-value= .010}) \). Thus the ones that
were able to construct a task at level 3 had fewer challenges implementing a task at level 2. Finally, implementing a task at level 2 was significantly related to implementing a task at level 3 ($\chi^2=4.61$, p-value= .032). This relationship was also negative. Therefore if they implemented a task at level 2, they had more challenges implementing a task at level 3.

For the main study recognition of a task at level 1 was related to recognition of a task at level 2 ($\chi^2=5.29$, p-value= .021). Then, teachers from the main study that correctly recognized a task at level 1 were more likely to recognize a task at level 2. Recognizing a task at level 3 was related to recognition of a task at level 4 ($\chi^2=4.20$, p-value= .040) and constructing a task at level 3 ($\chi^2=7.56$, p-value= .006). The majority of the teachers that had challenges recognizing a task at level 3 also had challenges recognizing a task at level 4. The correlation between recognizing a task at level 3 and constructing a task at level 3 was negative. Thus those who were able to recognize a task at level 3 had difficulty constructing a task at level 3. Being able to recognize a task at level 4 was also related to the solution of a task at level 2 ($\chi^2=9.52$, p-value= .002) and construction of a task at level 2 ($\chi^2=4.12$, p-value= .042). Teachers that were able to recognize the task at level 4 were more likely to solve a task at level 2 and construct a task at level 2. Teachers that solved a task at level 2 were more likely to construct a task at level 2 ($\chi^2=7.98$, p-value= .005). The majority of those who incorrectly solved a task at level 2 also had challenges constructing a task at level 2.

Table 4.5: Correlations of the pilot study

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>C1</th>
<th>C2</th>
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<th>C4</th>
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<td>.019</td>
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</table>

92
Table 4.6: Correlations of the main study

<table>
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<th>R_2</th>
<th>R_3</th>
<th>R_4</th>
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</tr>
<tr>
<td>S_2</td>
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<tr>
<td>S_3</td>
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</tbody>
</table>

R_4 .019 .726 3.81 1
.891 .394 .051
S_1 .098 .060 .050 .060 1
.754 .806 .823 .806
S_2 .186 .451 1.05 .451 .263 1
.666 .502 .305 .502 .608
S_3 .194 1.63 5.05 1.62 .051 .861 1
S_4 .098 .060 .050 .060 .133 .263 .051 1
.754 .806 .823 .806 .909 .608 .821
C_1 .186 .451 1.05 .451 .263 .05 .128 .263 1
.666 .502 .305 .502 .608 .814 .257 .608
C_2 2.13 .952 2.22 5.18 .139 .117 .022 .139 .117 1
.144 .329 .136 .023 .709 .732 .881 .709 .732
C_3 2.35 2.54 .833 1.94 .052 .702 .135 .052 .702 1.48 1
.125 .111 .361 .163 .819 .402 .714 .819 .402 .224
C_4 .019 .045 .952 1.63 .060 .451 .087 .060 2.46 .423 1.94 1
.891 .831 .329 .201 .806 .502 .769 .806 .117 .515 .163
C_4 4.80 .952 1.25 .952 .079 .263 .051 .078 .263 .556 3.33 .060 1
.028 .329 .264 .329 .778 .608 .822 .780 .608 .456 .068 .807
C_2 1.51 1.63 .000 .726 .079 .451 1.63 .078 2.46 .952 6.71 .045 2.14 1
.219 .201 1.000 .394 .778 .502 .202 .780 .117 .329 .010 .831 .143
C_1 5.04 1.27 2.20 .848 .055 .567 .020 .055 .567 4.13 .037 .010 2.69 4.61 1
.948 .260 .639 .357 .814 .452 .888 .814 .452 .042 .848 .919 .101 .032
C_4 .623 1.51 3.53 1.51 .098 5.96 2.89 .098 .186 .392 2.35 .019 .882 1.51 1.90 1
.430 .219 .060 .219 .754 .015 .089 .754 .666 .531 .125 .891 .348 .219 .168
Research Question 3: What are secondary mathematics and science teachers’ challenges in recognizing, solving, constructing, and implementing CDTs?

In this section, the results related to the third research question are shown. First, it shows all the information of the teachers that were interviewed. Then, the results are separated into categories based on the teachers’ narratives.

In order to address this research question, 13 teachers were interviewed. Five teachers were interviewed from the pilot study, and eight teachers were interviewed for the main study. Table 4.5 shows the demographic information of those teachers that were interviewed. The first five are from the pilot study (Mathew, Gina, Marco, Damian, and Anna). The following eight teachers were part of the main study (Isabel, Dylan, Jessica, Megan, Monica, Mayra, Cesar, and Derek). The vast majority of the teachers that were interviewed were Hispanic (77%) while only three teachers (23%) reported their race as white non-Hispanic. All were teaching at a public school in an urban district at the time of the study. The experience years among those who were...
interviewed ranged from half a year to twelve years. Two of them were teaching a 9th-grade science course while the rest were teaching a secondary mathematics course ranging from 7th grade to 9th-grade mathematics. One teacher was teaching 5th-grade mathematics and science.

Table 4.5: Demographic information of interviewed teachers

<table>
<thead>
<tr>
<th>Participant pseudonym</th>
<th>Race</th>
<th>Type of school</th>
<th>Type of school district</th>
<th>Experience years</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathew</td>
<td>White, non-Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>4</td>
<td>8th-grade math</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5th-grade math and</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>science</td>
</tr>
<tr>
<td>Gina</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>6</td>
<td>8th-grade math</td>
</tr>
<tr>
<td>Marco</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>5</td>
<td>8th-grade math</td>
</tr>
<tr>
<td>Damian</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>12</td>
<td>8th-grade math</td>
</tr>
<tr>
<td>Anna</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>1</td>
<td>7th-grade math</td>
</tr>
<tr>
<td>Isabel</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>11</td>
<td>8th-grade math</td>
</tr>
<tr>
<td>Dylan</td>
<td>White non-Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>13</td>
<td>8th-grade math</td>
</tr>
<tr>
<td>Jessica</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>3</td>
<td>9th-grade science</td>
</tr>
<tr>
<td>Megan</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>1.5</td>
<td>9th-grade math</td>
</tr>
<tr>
<td>Monica</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>5</td>
<td>9th-grade math</td>
</tr>
<tr>
<td></td>
<td>White non-Hispanic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mayra</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>3</td>
<td>9th-grade science</td>
</tr>
<tr>
<td>Cesar</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>1</td>
<td>9th-grade math</td>
</tr>
<tr>
<td>Derek</td>
<td>Hispanic</td>
<td>Public</td>
<td>Urban</td>
<td>0.5</td>
<td>9th-grade math</td>
</tr>
</tbody>
</table>

Table 4.6 has the answers of the recognition part of the survey for the interviewed teachers. Three teachers correctly recognize 1 task out of the 4 given; one of them correctly identified the task at level 1 while the other two correctly identified the task at level 4. Four teachers correctly identified 2 of the tasks in the recognition part of the survey where all four of these teachers correctly identified the task at level 1 and 4. One participant correctly identified three of the tasks with only incorrectly identifying the task at level 1 (he wrote level 2 for this task). Five teachers correctly recognize all the four tasks. Thus based on these responses five teachers were considered high in the recognition survey (Damian, Anna, Jessica, Megan, and
Monica) because they responded correctly to all the tasks. Five teachers were considered medium (Mathew, Marco, Isabel, Dylan, and Mayra) because they either got two or three correct answers. Three teachers were considered low (Gina, Cesar, and Derek) because they only got one correct answer in the recognition part of the survey.

Table 4.6: Recognition answers of interviewed teachers

<table>
<thead>
<tr>
<th>Teacher</th>
<th>L-1</th>
<th>L-2</th>
<th>L-3</th>
<th>L-4</th>
<th># of correct answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathew</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Gina</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Marco</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Damian</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Anna</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Isabel</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Dylan</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Jessica</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Megan</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Monica</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Mayra</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Cesar</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Derek</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.7 has the rate of the solution part of the cognitive demand survey. Their solutions were rated from 1-3 where 1 was no solution or incorrect, 2 was partially correct, and 3 was correct. One participant earned only 1 task at a rate of 3 with one rate of 2 and two rates of 1. Eight teachers earned two tasks with a rate of 3 with 6 teachers getting those rates of 3 in the level 1 and level 2 tasks, 1 participant got those rates in the level 1 and level 3 tasks, and 1 participant getting those rates in the level 2 and level 4 tasks. Four teachers earned three tasks with a rate of 3 with two of them getting a rate of 1 to the level 4 task and the other two getting a rate of 2 to the level 4 task. None of the teachers that were interviewed got all the responses right on the solution part. Based on their responses four teachers were rated as high (Marco, Anna, Mayra, and Derek) because they correctly solved three of the tasks. Eight teachers were
considered medium (Mathew, Gina, Damian, Isabel, Dylan, Jessica, Monica, and Cesar). Only one teacher was considered low in this part of the survey (Megan).

Table 4.7: Solution results of interviewed teachers

<table>
<thead>
<tr>
<th></th>
<th>L-1</th>
<th>L-2</th>
<th>L-3</th>
<th>L-4</th>
<th># of correct answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathew</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Gina</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Marco</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Damian</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Anna</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Isabel</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Dylan</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Jessica</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Megan</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Monica</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Mayra</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Cesar</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Derek</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4.8 is showing the rates of the construction part of the cognitive demand survey. Only one participant got one rate of 3 to the level 2 task. Seven teachers got two rates of 3. Six of them got them in the level 1 and level 2 task and 1 in the level 2 and level 4 task. Four teachers got three rates of 3. Two of those teachers got it for tasks at levels 1, 2, and 3 while the other two got them for the tasks at levels 1, 2, and 4. One teacher got four rates of 3. One teacher was considered high because he was able to construct tasks at each level (Mathew). Eleven teachers were considered medium (Gina, Marco, Damian, Anna, Isabel, Dylan, Jessica, Monica, Mayra, Cesar, and Derek) because they successfully constructed two or three tasks. One teacher was considered low (Megan) because she was only able to construct one task (procedures without connections) successfully.

Table 4.8: Construction results of interviewed teachers
<table>
<thead>
<tr>
<th></th>
<th>L-1</th>
<th>L-2</th>
<th>L-3</th>
<th>L-4</th>
<th># of correct answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathew</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Gina</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Marco</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Damian</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Anna</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Isabel</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Dylan</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Jessica</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Megan</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Monica</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Mayra</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Cesar</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Derek</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Finally, in Table 4.9 the level of cognitive demand from the observations is shown. In the classroom, three teachers implemented tasks at level 1; one participant implemented tasks at level 2, six teachers at level 3, and three teachers at level 4. In the microteaching, one teacher was considered high (Anna) because she presented a lesson that contained a task at level 4. Eleven teachers (Mathew, Gina, Marco, Damian, Isabel, Dylan, Jessica, Megan, Monica, Cesar, and Derek) presented a lesson with a level 2 or level 3 task. One teacher presented a lesson with a level 1 task (Mayra). In the classroom part, three teachers (Damian, Anna, and Dylan) presented a lesson with a task at level 4. Seven teachers (Mathew, Isabel, Jessica, Megan, Monica, Cesar, and Derek) presented a lesson with a task at level 2 or level 3. Three teachers (Gina, Marco, and Mayra) presented a lesson with a task at level 1. After getting an average of all survey answers as well as the cognitive demand in the implementation of the microteaching and classroom, teachers that were interviewed were categorized either high, medium, or low. Overall, Gina, Megan, Mayra, and Cesar had the lowest scores from the surveys and the observations. Damian and Anna obtained the highest scores. Overall, Mathew, Marco, Isabel, Dylan, Jessica, Monica, and Derek were in the middle.
Table 4.9: Implementation level

<table>
<thead>
<tr>
<th>Level Microteaching</th>
<th>Level Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gina</td>
<td>1 Mayra</td>
</tr>
<tr>
<td>Gina</td>
<td>2 Jessica</td>
</tr>
<tr>
<td>Gina</td>
<td>3 Gina</td>
</tr>
<tr>
<td>Gina</td>
<td>4 Gina</td>
</tr>
<tr>
<td>Monica</td>
<td>3 Megan</td>
</tr>
<tr>
<td>Monica</td>
<td>3 Megan</td>
</tr>
<tr>
<td>Damian</td>
<td>3 Megan</td>
</tr>
<tr>
<td>Dylan</td>
<td>3 Megan</td>
</tr>
<tr>
<td>Anna</td>
<td>4 Anna</td>
</tr>
<tr>
<td>Derek</td>
<td>3 Megan</td>
</tr>
</tbody>
</table>

During the interviews, we talked about their difficulties in recognizing, solving, constructing, and implementing mathematical tasks at different levels of cognitive demand. There were several instances in which the teachers talked about cognitively demanding tasks. Teachers stated some challenges when thinking about their lesson plans. The results from the interviews are presented for both cohorts combined. The challenges for teachers in both cohorts were similar, so that is why they will be presented combined. Once the data from the interviews was analyzed, different codes emerged. The following are some of the codes that emerged from the data: low expectations of students, lack of knowledge of students, student needs, English language learners, special education students, teachers’ knowledge, lack of knowledge of teachers, challenges, time, outside forces. Different themes emerged when talking about the challenges in implementing cognitively demanding tasks. Thus the codes mentioned above were then turned into categories based on the challenges each code expressed.

All interviews were coded line by line and overall more than thirty codes emerge from the data. The codes were chosen based on the main research question of the study in addition to the number of teachers that mentioned the particular code and the frequency of those references. For example, 8 teachers referred to the lack of mathematical knowledge by students’ code, 10
teachers mentioned the lowering level of cognitive demand code, 7 teachers mentioned the teacher knowledge code, 8 teachers mentioned the outside factors, 7 teachers mentioned the English language learners, and 5 teachers mentioned the student needs a code. These codes are those in which the teachers expressed challenges in implementing cognitively demanding tasks. The different codes were analyzed and then put into the categories depending on the specific topics that teachers mentioned. The main categories for this section are: Challenges related to students such as students’ knowledge and English language learners, challenges related to teachers’ knowledge, and outside challenges such as time and testing. These categories are further explained in the following section. Excerpts have been taken from the thirteen teachers’ interviews: Mathew, Gina, Marco, Damian, Anna, Isabel, Dylan, Jessica, Megan, Monica, Mayra, Cesar, and Derek.

4.3.1 Challenges related to students

This section is related to two ways that teachers talked about their students as part of the challenges of implementing cognitively demanding tasks (levels 3 and 4). Teachers mentioned challenges related to students in two ways: about their knowledge and about having English language learners in their classroom. Each of those is explained further in the following subsections with examples from the interviews.

4.3.1.1 Challenges related to students’ knowledge

This section is about the challenges related to students’ knowledge based on the teachers’ responses in the interviews. Themes about students’ knowledge appeared more when teachers were talking about construction and implementation of cognitively demanding tasks. Some examples of these themes were: lowering the level of cognitive demand, lack of mathematical
knowledge by students, English language learners. When talking about their ability in constructing tasks ten teachers mentioned having difficulty creating tasks at higher levels and then mentioned their students as the reason. Likewise, when discussing the implementation of cognitive demand tasks seven teachers mentioned being unable to use any task at level 4 while three mentioned that the higher level they utilized in the classroom is level 2. These teachers stated the students’ level as an important aspect when selecting those tasks. They sometimes lower the level of the tasks because they feel that students are not ready and that their mathematical ability is not enough to succeed in solving those types of problems.

Mathew, an 8th-grade mathematics teacher, said: “You can find wonderful tasks, but if they are too far above the level of your students the usefulness of the lesson would be lost.” It seems like he was afraid of losing those students that are not ready. In the same interview, he mentioned a student that dropped out of one of his classes because she felt that she could not handle the class even though Mathew felt that she could have been able to succeed that class. It is experiences like these that may draw teachers’ decisions to lower down the level of instruction. We can see in this following excerpt his decision to reduce the level of his tasks:

Mathew: I want to be level 3 all the time, that’s what I want. I can’t always be there tough because I mean, even now you have some students they want to always be led. They don’t want to make that, they don’t want to always make that leap themselves, so you have to start at the level 2, level 1, and build up, but I’d like to be able to start at level 3 and have them start drawing all their past knowledge and make the connections and develop, and then start pushing level 4 I think I’d be so cool to work at a level 4 all the time, but I’m not there yet.

Interviewer: why not? Why do you think that?
Mathew: because when I try to push a lot of level 4, I still lose students.

Even though Mathew talks about the experience of losing students when the cognitive demand is higher, he was consistent with what he said about trying to keep the lessons at Level 3 since during classroom observation his lesson consisted of tasks at Level 3, but in the microteaching part, his task was at level 2. He wasn’t the only one worried about “losing students” since Mayra, a 9th grade science teacher also said something related “how do I get to that point without completely losing them…how do I do that again without losing them, without getting them so confused to like I don’t get this because a lot of the kids they will shut down if they don’t understand.” Mayra was reflecting on how to use the higher level of cognitive demand (doing mathematics, doing science). She was asking themselves how to help those students that may not understand tasks at higher levels. It seems that it was hard for her to answer those questions and was often thinking about her students’ knowledge.

Instruction is often based on students’ knowledge and what the teachers assume their students’ knowledge is. Teachers make decisions upon presenting students with different topics and tasks. These decisions affect the level of cognitive demand those tasks would have. For example, in the following excerpt from Gina, a 5th-grade mathematics and science teacher, we can see how she bases her decision on her students’ mathematical knowledge: “The way I interpret the different levels of cognitive demand depends on the student themselves you know…what foundation they lack in order for me to kind of distinguish if their cognitive demand needs to be more challenged.” We can notice that she stresses on the foundation that students are lacking. Thus she bases her problems based on that, thinking that some students would lack the mathematical knowledge necessary to solve those problems. Overall, Gina’s rating from the survey and the classroom and microteaching sessions was low. In her classroom, she presented a
task at level 1. She did not mention how she assesses her students’ understanding, but she was aware of her students’ prior knowledge and how it could be related to whether they would be able to solve tasks at higher levels of cognitive demand.

Also, Gina who teaches 5th-grade mathematics and science, further mentioned how every student comes at different levels and how the cognitive demand is based on each student’s knowledge. “What I might consider be a high-level type of questioning or problem solving it differs on each student like if I were to consider it high some other student I think would have difficulty didn’t have difficulty at all,” she said. In this excerpt, we can see how she is reflecting on how some tasks would be harder for some students than for others. She continues talking about her students’ prior knowledge. Mayra, a 9th-grade Biology teacher, also talks about students’ prior knowledge she says that

Mayra: some of the kids it’s hard to get to that higher level, you can push them, and they will struggle, they struggle so much to get that title even going all procedural sometimes they’ll struggle with that which I’ve noticed, and it’s like ok, what’s going on here, it really does just depend on the students themselves and how well like their background is science and how well they understand the topic that we already covered

Similarly, Marco an 8th-grade mathematics teacher thinks about the level of the whole class: “and of course we try to find tasks that would be challenging for the class but some periods… the challenging level may not be as high as some of the tasks for other classes, and basically, that’s how we kind of determine what kind of class”. In the microteaching session, Marco presented a task at level 3 while in the classroom he presented a task at level 1. In another part of the interview, Gina said that most teachers do repetitive problems that can be solved by mechanical steps. “They’re being taught to do steps to solve problems, and that’s I think that’s
what they get comfortable with.” She was thinking about how sometimes is difficult to implement higher levels of cognitively demanding tasks. She feels that since students are used to solving problems at low levels of cognitive demand, then they would struggle with other problems. Monica, a 9th grade mathematics teacher that presented a task at level 3 both in the microteaching session and in the classroom, had a similar sentiment towards implementing higher levels of cognitive demand in her classroom. She feels that her students are too used to using the calculator even for simple multiplications “sometimes they have to put one times zero, or one times three in the calculator.” When asked about her confidence in implementing the four levels she answered,

Monica: In my classroom, I wish I could tell you that I feel a lot confident but I know that my kids won’t get to that part and then some, I know that they’re here and you know what you never know, let your kids, but if you were in my classroom and just knowing my kids they do not rise up to the challenge, they won’t, unfortunately they won’t…I think my kiddos would go as far as the second level.

In this excerpt, Monica was first explaining that she does not feel confident using higher levels and then starts talking about how her students will not “rise up to the challenge” and even said that she thinks that her students will only go to the second level. Even though Monica does not say how exactly she assesses her students understanding, she says that based on her observations she knows that they will not be able to solve those problems.

When talking about the level of cognitive demands used in the classroom one teacher, Damian, and 8th-grade mathematics teacher, said that he starts with level 1 and then builds up to a higher level. His reasoning was based on his students’ knowledge:
Damian: I always start with level 1, and that’s been another form of debate you know it’s like sometimes they say that to start in a level 4 let them work their way down, and it’s hard for me to accept that kids can start at a level 4 I think that they need that level 1 and then the building up to that level 4 so yeah when we’re introducing something new um we usually start with level 1 and then we build and build and build.

This excerpt shows how Damian does not feel like students can start at a high level but he said that he builds up to high levels. During the class observation, Damian presented his class with tasks that were high level (Level 4) and overall, Damian was considered high based on his responses to the survey, microteaching, and classroom observations. Even when he was not completely confident about his students’ ability on solving cognitively demanding tasks, he was still trying to get them to that level. In contrast to Damian, another teacher, Anna who’s a 7th-grade mathematics teacher, said that she believes that students can work at high levels of cognitive demand:

Anna: I like to keep instruction at level three I push myself to try to incorporate as much level 4 as I can, but I feel pretty comfortable with level three I think a lot of what we do in class right now is at that level, maybe between a 2 and a 3 but more a three I try to keep it there. I think that in the long run you know it’s what helps them understand the material; I don’t want them to just remember steps or just remember what they need to do. I want them to be able to understand this so they can apply the math to any question to any you know any type of question that they may face. For example, the STAAR test that we just took yesterday I don’t know what’s on there, but I know the concepts that they need to be familiar with so I feel like if I can keep the instruction at a 3 or a 4 it would help them apply those concepts into whatever they face, whatever they have to do.
Throughout the whole interview, Anna said that she feels convinced that students can solve tasks at high levels of cognitive demands. She said that she often gets surprised to see that her students can solve those problems. Equally to Damian, Anna’s lesson during the classroom observation was at a high level (Level 4), and she was only considered high overall. It is important to understand why teachers like Damian and Anna are able to implement tasks at higher levels even though sometimes they may not be sure whether their students would be able to solve those problems. Dylan, an 8th-grade mathematics teacher, also thought that students should be able to solve problems at higher levels of cognitive demand. However, he is often cautious about not challenging his students too much. He said, “you don’t want to make [the problems] unreachable because if you just give them problems that are too far advanced and they never solve them, then they get discouraged … it’s a fine line you’ve got to give them something to get their confidence up but then always through something in there to challenge them so where they don’t get too discouraged.” It is essential for teachers to think about their students’ knowledge and ability to solve cognitively demanding tasks before implementing them while also finding ways to challenge them.

Another theme that emerged from the data related to the students is the students’ motivation. Cesar, a 9th-grade mathematics teacher, said that he mostly implements tasks at level 2. “The biggest, the absolute biggest challenge is student motivation, a lot of these kids it’s hard enough just to get them to do the first two levels just here is the steps, apply it here, you have them try to think and they just they won’t.” For Mayra, a 9th-grade science teacher, getting students to be interested in mathematical problems is also a challenge: “I know a lot of the kids they struggle with that kind of thing. What? You want us to do what? But how do we, what? So getting them to the highest level, it’s a challenge.” Both Mayra and Cesar were considered low
based on all responses and observations. Another teacher, Monica, a 9th-grade mathematics teacher said that she often tries to “entertain them” to get them interested:

Monica: It's just challenging just to get the kids interested and sometimes yeah we have because sometimes you have to entertain them so ok I entertain them, I got them but when it comes to math there they go, and I was like no, no come back, come back we already discussed this, and I even make stories or something for them to just engage them and you still have those faces that they are here but they are not actually here, and that’s kind of hard.

Jessica, a 9th-grade science teacher, told me at the beginning of the interview that she has been using a new curriculum in her school that focuses more on project-based learning. Overall, she was categorized as medium based on the surveys and the level of the tasks of the observations. When asked about constructing tasks at different levels she said,

Jessica: Well I think yeah the fourth one because sometimes you have to spend, the kid, you don’t know what they are going to come in with first so sometimes you need to spend a lot of time with the memorization and then the making the connections or doing without connections and then connections and then just trying to get them to see how that is in the big picture and connects with the real world, sometimes it’s a struggle

Interviewer: Why do you think it’s a struggle with the kids?

Jessica: I don’t think they’ve always been used to being forced to critically think and so they kind of get comfortable with the “I just need to know what I need to know,” and then “Give me the right answer but I don’t want to explain why ” so just getting them comfortable I guess with sharing what they think can sometimes be hard so just making them feel comfortable with
Even though the question was about constructing tasks at different levels, she started talking about the students. First, she said that she has to spend much time during memorization then says that students are not used to thinking critically. Later in the interview, she continued by adding that students in high school only want to do the minimum. She added that since using the project-based learning curriculum used in her class, she has been feeling more comfortable using all levels of cognitive demand and hoped that by the end of the year she would be able to implement them more effectively. In the following subsection, the challenges related to English language learners are explained.

4.3.1.2 Challenges related to English language learners

Several teachers explained that they have English language learners (ELLs) in their classrooms. This section is a subsection of challenges related to students’ knowledge because even though is not about the students’ mathematical knowledge some teachers view students’ proficiency in English as a challenge in the mathematical classroom. They reported this as one of the challenges to implementing higher levels of cognitive demand because of their language. Also, some teachers also mentioned that having special education students might prevent them to use more cognitively demanding tasks. In the following excerpt from an interview we can see how Derek, a 9\textsuperscript{th}-grade mathematics teacher, feels that ELLs and special education students pose a challenge because they take longer to process information:

A: So what would you say would be the biggest challenge and you say the highest levels which are, let say the last two, three and four are the most cognitively demanding, right? What are the biggest challenges in trying to implement those in your classroom?
Derek: The biggest challenges? Probably with students that are ELL or the special ed. students it’s harder to come across those concepts because it takes them a little bit longer to process the information and make the connection and trying to make it more meaningful to the students. That’s probably another hard way to connect mathematics to students because I know a lot of students don’t like math so connecting the concept that we are learning to students is probably the harder thing to do.

When dealing with special education students, Derek says that he approaches them and shows them the steps to solve the problems and then he asks them to solve the problems.

Dylan, an 8th-grade mathematics teacher, is another teacher that referenced English language learners when talking about the challenges. In regards to ELLs he says that he relies on other teachers to help him translate since he doesn’t speak Spanish but he adds that “in class I know I lose them all the time, the ELLs because I talk English, and they barely understand it, let alone I don’t know if they understand it, so yeah I think that’s the hardest part I think it’s communicating.” He mentioned that the hardest part is when implementing word problems since students have issues understanding some of the words but that procedural problems seem to be easier for ELLs to work on. When asked about the factors when thinking about which tasks to implement in his classroom his response was “I always think about my ELLs how I am going to modify it, how I’m going to simplify the terms for the rest of the class.” Both Dylan and Derek are monolingual English speakers. Thus their lack of a second language might determine their view about English language learners.

Some teachers acknowledge having Spanish speakers ELLs in their classroom but don’t see it as a challenge since they also speak Spanish and feel that they have the tools to help them. Similarly, Isabel, an 8th-grade mathematics teacher, says that in her classroom there is a large
population of ELLs and says that they “have to modify based on their language because we have
a lot of kids coming from Mexico, so we have to just to do it.” However, she did not talk about
modifying for ELL students as a challenge but more as something she has been doing since she
started teaching. She has been giving students the translation of the vocabulary and being a
Spanish speaker herself she mentions that she knows the terminology in Spanish and she often
uses it with them.

4.3.2 Challenges related to teachers’ knowledge

This section is about the challenges related to teachers’ knowledge based on the
conversations from the interviews. Themes about teachers’ knowledge appeared more when
teachers were talking about the solution, recognition, and implementation of cognitively
demanding tasks. Some examples of these themes were: level 4, differences between levels, lack
of mathematical knowledge by teachers. In the implementation of cognitively demanding tasks
teachers were thinking about their mathematical knowledge. Some teachers feel that they needed
to know the solutions to implement them in the classrooms. Thus implementing tasks at the
highest level (Level 4-doing mathematics) was seen as a challenge. Others talked about being
confused about the differences of between the four levels, in particular between Level 2
(procedures without connections) and Level 3 (procedures with connections). Ten teachers
mentioned having difficulty recognizing levels between 2 and three while two teachers said they
have issues recognizing between levels 3 and 4. Also, ten teachers expressed having difficulty
solving tasks at level 4.

In connection to their mathematical knowledge, Damian, an 8th-grade mathematics
teacher, stated: “I would not say I’m confident using level 4 because again I don’t think that… I
don’t have all the answers as to where things came from”. Clearly, he does not feel comfortable selecting tasks at a higher level because he does not feel prepared to solve those problems by himself. He kept saying, “again I would say probably level 4 simply because the level 4 that I am aware of now because of the workshop the ones that I've been exposed to then those I have no problem using but stuff that I don’t know where things came from then there is no way that I would even attempt to explain or teach because I wouldn’t know what I'm talking about”. He battles on his decision to implement higher level tasks because he does not want to present students with a problem that he does not understand or doesn’t know how to solve. In this study, the level of Marco’s microteaching lesson was 3, and he reduced that lesson for his classroom lesson. When Marco whom teachers 8th-grade mathematics, was asked why he reduced the level in his classroom, he mentioned that his classroom had special education students and he did not believe that they were able to solve those problems.

During the interview and in one of the professional development workshop sessions Damian who teaches 8th-grade mathematics indicated that he was grateful that this workshop was based on reinforcing their mathematical knowledge and pedagogical content knowledge since his experience at other professional development sessions that he has attended have focused more on pedagogical knowledge. Interestingly, Damian kept saying the quote “where things come from” because the way he understands Level 4 (doing mathematics) is that it is about making generalizations without any other tools. Even though he struggles with his self-confidence about his mathematical knowledge, he has been able to implement highly cognitively demanding tasks for his classrooms as was mentioned before. Megan, a 9th-grade mathematics teacher, was another teacher that wanted to feel confident that she could solve the highest level problems before she could implement them in her classroom. Overall, Megan was considered low based on
all her responses and observations, for example in the solution part she was rated as low because she was able to solve only one task (procedures without connections). She said, “I need to be able to say, develop something that will get them to think at the fourth level and feel confident that if I am able to do it, they are able to do it, or I can teach it with that strength, but since I am still kind of, I don’t feel I have that backing, like everything else is ok”.

During one session of the professional development workshop, teachers worked in teams to solve a task at Level 4 (doing mathematics) and one teacher mentioned about this activity to reflect on her mathematical knowledge as a teacher. The following excerpt shows how Gina, a 5th-grade mathematics, and science teacher felt when solving this problem:

Gina: There’s some things that were mentioned in my group, but then I really couldn’t explain it but my the person that was in my group knew where I was going with it and could explain it better, so it kind of makes me feel good that I was on the right path. I had the idea of. The overall idea and which made led me to think, you know, I do have the possibility of getting to that level. But I was, I did notice that I was not, it was like if I was observing my own students and I was one of those students. That my cognitive thinking to solve this problem wasn’t as high as the other person in my group and vice versa. Like the other people in our group, you could notice the different levels. But together it did confirm the thinking that we all had but one could explain it better because that particular person in our group had more experience.

Solving the task at the highest level as well as working in teams made her think about her mathematical knowledge. It is fascinating to see how at the beginning of the activity she could not explain her reasoning but her teammates helped her, and she felt better because she was on “the right path.” It also empowered her to think that she can reach that level, a level in which she
could be able to solve those highly cognitive demanding tasks. She said, “I was thinking in my head well I do have some knowledge I’m just missing more information.” It is also fascinating how she reflected on how her students might feel when solving these types of problems as we can see from this part taken from the excerpt: “it was like if I was observing my own students and I was one of those students that my cognitive thinking to solve this problem wasn’t as high as the other person in my group”. When she was asked about doing a similar activity with a high-level task in teams, she said that she would do that and that she tries to allow individual work as well as group work.

The teachers revealed that there are similarities between some levels and how those similarities have made it difficult for them to recognize, solve and ultimately implement those tasks. For example, Damian (8th-grade mathematics teacher) mentioned, “level 3 is a little bit harder to construct … it is similar to level 2 but it’s, it takes it further and is not as simple as here’s a formula get an answer you know it’s a step further and it they are a little bit more challenging to come up with those.” Damian makes an interesting reflection when he points out that he is not completely sure on how to differentiate between tasks at Level 2 (procedures without connections) and Level 3 (procedures with connections). Teachers learned and were presented with examples at each level. However, it is evident that there is still confusion between the differences. The following excerpt from Marco’s interview shows this confusion even further:

Marco: I’m not sure. And I know I struggle with that throughout the year. I’m not sure because it’s for me sometimes the demand goes from the second one all the way to the fourth. I think you kind of, you kind of reach that fourth one when you’re trying to come up with that method, so I would say that the third one is somewhere in between. I’m not
sure what it is but its somewhere in between… yeah, I would say that the level 3 is probably. That’s the one that goes doing math, and that’s the one that I’m not sure, and I’ve never really understood well so I would say the third stage the third level from low to high would be the one that I have most trouble recognizing.

In this example, Marco, an 8th-grade mathematics teacher, is talking about the differences between levels. He says that for him the cognitive demand on the task goes from Level 2 to Level 4. He can recognize that Level 4 is when students have to develop their method, but his understanding of Level 3 is still not clear. Teachers should be able to recognize these levels in order to provide students with a broad array of tasks that goes beyond substituting numbers on a formula (Level 2).

4.3.3 Challenges related to external factors

This section is about the challenges related to external factors based on responses from the interviews. Themes about students’ knowledge appeared more when teachers were talking about the implementation of cognitively demanding tasks. Some examples of these themes were: outside forces, planning, modify. Those themes were then separated into two categories: time constraint challenges and curriculum factor challenges.

4.3.3.1 Time constraint

Teachers are often concerned about the way they spend their time in the classroom. They have to think about the time they would devote to each lesson. Moreover, time is a major factor because they feel pressured to finish the curriculum and get students ready for the standardized test. Thus, time was mentioned as a significant factor in implementing tasks at different levels of cognitive demand. For example, Mathew, an 8th-grade mathematics teacher said, “We have a set
amount of time to do everything.” Mathew also talked about not wanting to lose students which can be related to the lack of time since he might felt that he would not have time to bring those students back to the lesson if they are feeling lost. Also, when I asked Damian, an 8th-grade mathematics teacher who had also said that he does not want to implement tasks at higher levels if he is not sure about the answers, about what factors he thinks when designing a lesson plan he said, “Definitely time … it is kind of tough to do certain things especially the things that we are so restricted with time.” Damian explained that his classes are only forty-seven minutes long and during that time he needs to do a bell ringer, which sometimes might take longer than expected and then do his lesson. Also, he further explains that is very hard to do project-based lesson because they would take some time. He said, “The expectations that are placed upon us … so all these things that are expected of us teachers take up time”. These expectations from the school and others sources (i.e. district, state) may sometimes take time away from the teachers.

Some teachers talked about being pressured about finishing all the material on time. For example, Monica who teaches 9th grade mathematics expressed how she feels because she needs to keep going with the material and some of her students are not ready. She said, “you need to be here, you need to be here, you need to be here at this certain time, and you need to be teaching systems of equations, you need to be, you’re like oh my god my kids are not up to that right now, and you can only do so much in class.” She added that she feels pressured by the school administration to finish everything on time. She mentioned, “Right now we are getting pressured because of the test you know so we could only show them ok you only have two days to show them that the concept of quadratic formula, so only one day and so sometimes I think the lesson that we can actually project is short changed because we don’t have the time to do it.” The day the interview was conducted she told me that she should be doing paperwork, she said
Monica: we are too busy calling parents. Like right now I should be either grading or calling parents for kids that are failing. They are going to get on my behind because well ‘how come you didn’t tell the parent that they were failing.’ I was like I don’t have time, and then well how do I know they are failing if I don’t grade. You know, so it’s like ok, and you know, so it’s just hard, especially when we are being attacked with all these, the PLCs is something that we are supposed to be doing, and it has to do with class, but then we are filling out paperwork, filling out. I don’t know what else they want us to do

Monica’s excerpts exemplify how she feels about doing other things that are not related to teaching but that the school is asking her to do. She wishes she had more time to finish all the things that the school is expecting her to do.

Finally, Isabel mentioned that the “constraint of time” is an important factor for her. She also told me that she has many students that are performing at a 3rd-grade level even though she teaches 8th-grade mathematics. During the interview, she said that even though she teaches 8th-grade algebra, most of her students are performing at a 3rd or 4th-grade level. In the following excerpt, she explains that the constraint of time is an important factor,

Isabel: we have the constraint of time with the strict curriculum, that we have to finish by such a day so what I do every time, I start like a standards. I always start with my activity right away because if I start with a concept and I see the kids understand, I rather move on because I know that’s going to give me time to review at the end…I kind of already block myself of doing is the very high one because I’m afraid that I start so high and then I don’t finish the curriculum goal on time.

She mentions that she blocks herself and doesn’t even think about the highest level because otherwise, she would not have time to finish the curriculum on time. Also, if she sees that
students understand the topic, she moves on. She also has to move on even though her students sometimes need more help since they are at a lower level.

4.3.3.2 Curriculum factor

Teachers also think about the Texas Essential Knowledge and Skills (TEKS) which are the state standards (Texas Education Agency, 2015) and the concepts they need to teach before thinking about the kind of tasks they will add to their lessons. For example, at the beginning of the interview, Anna, a 7th-grade mathematics teacher, commented, “I have to start with the TEKS in mind I have to make sure that whatever task I have selected follows the TEKS.” Similarly, Gina a 5th-grade mathematics and science teacher that was considered low in the overall rating, said that she must consider the concepts she has to teach: “Before I select any task in the classroom I have to consider what concepts I’m going to introduce or delivery.” Later on, Anna talked about the challenges when selecting. Let us consider the following excerpt in which Anna claims that the main challenge is time:

Anna: again the TEKS. Cause you have to. They tell us what it is we need to teach. Also, the task does it lend itself to working in teams, working with groups, the length of time it would take to complete the task. And basically does it meet the objective that we have, so those are the main things. I look at when selecting and considering tasks

Interviewer: what would you say would be your main challenge?

Anna: from those factors?

Interviewer: uhh uhh
Anna: the time I think the time factor, you know cause I can plan a lesson and allow two days and maybe the kids are not where they need to be on the second day and you know it’s for I think for any teacher time is always the biggest obstacle.

First of all, this excerpt shows all the factors that she has to think about before considering her tasks, the TEKS, the time, and the objective. When she was asked about the main challenge, she claimed that time is the main factor and that for all teachers time is also the main factor. In this case, she talked about time spent on lessons based on her students and whether the students are where she is expecting them to be at the end of the lesson. In that sense, this part of the theme can be related to the students’ knowledge. We should also remember that Anna was considered high overall, most of her responses in the recognition, solution, and construction parts of the survey were high. In addition, she was also able to implement tasks at level 4. Thus, if teachers had enough time, they might be able to implement more cognitively demanding tasks.

Similarly, when asked about the biggest challenge, Megan, a 9th-grade mathematics teacher, had a similar answer explaining that she first thinks about her students’ level and the time she has for instruction,

Megan: Certain things I consider. What level they are at that point and how I can take them, and then time, because we have to be moving, and moving, and moving. So we are starting exponential, the rules of exponents, so I’m thinking if I spend too much time on each one example, example, no, review then, let’s get to work. Let’s figure something out and let’s come up with some kind of activity.

The way that Megan talks about everything she has to do shows that she often stresses about how much time she for each activity. In the interview, she started speaking faster when she mentioned: “we have to be moving, and moving, and moving.” If teachers do not have time to
review with students, it will be very hard to assess whether students are understanding. Therefore, it might be why teachers do not want to implement CDTs in their classrooms. Even though the categories from the teachers’ narratives have been explained separately, we can see that some of them are related. For example, if teachers do not have time and think that students are not ready then implementing cognitively demanding tasks becomes a challenge for them. Curriculum factors and time constraints can also be related.

In this chapter, the results were presented based on each of the research questions. For the first question, we found that teachers had challenges distinguishing between the middle levels. Teachers also had challenges solving tasks at the highest levels of cognitive demand and constructing tasks at the middle levels. From the correlation analysis, we found some correlations in the pilot study (recognizing a task at level 2 was related to recognizing a task at level 3) and others in the main study (recognizing a task at level 3 was related to recognizing a task at level 4). From the interviews, we found that both science and mathematics teachers talked about the following challenges: challenges related to students’ knowledge as well as challenges related to English language learners, challenges related to teachers’ knowledge, and challenges related to external factors (time constraints and curriculum factors). A discussion and conclusion of these results are presented in the next chapter.
Chapter 5: Discussion and Conclusion

In this chapter, a discussion of the results from this study will be presented. First, a section describes lessons learned from the pilot study that was implemented in the main study. Next, the importance of this study and its implication to the field of mathematics education will be explained. Then, an interpretation of the results in regards to teachers’ recognition, solution, and construction of cognitive demand levels will be given followed by an explanation of the findings from the qualitative data. Followed by the significance of the study, recommendations, and its limitations. Finally, a section on future research will be presented.

5.1 DISCUSSION OF THE RESULTS

Research Question 1: To what extent are secondary mathematics and science teachers able to recognize, solve and construct tasks at different levels of cognitive demand?

One of the purposes of this study was to examine teachers’ understanding of cognitive demand, more importantly, the recognition, solution, and construction of tasks at different levels of cognitive demand as well as the challenging in implementing those tasks. The main data to examine the way teachers understand the recognition, solution, and construction of cognitive demand was the cognitive demand survey. In the recognition part of the survey, the results show that teachers had more issues recognizing tasks at level 3 in the pilot study and more issues recognizing tasks at level 2 in the main study. This result is further supplemented by the teachers’ interviews in which they mention having issues differentiating between the middle levels. The following excerpt from Damian’s interview show how he was having trouble differentiating between levels 2 and 3. “[Level 3] is similar to level 2 but it, it takes it further and is not as simple as here’s a formula get an answer you know it’s, it’s a step further.” Also, this
result expands results from Smith and Stein (2009) in which they found that the teachers in their professional development workshop also had challenges in recognizing tasks at level 3.

In regards to the solution part of the survey, the results show that teachers had issues solving tasks at the highest levels of cognitive demand (levels 3 and 4) for both studies. This result suggests that mathematical knowledge is an important factor in solving tasks at highest levels of cognitive demand. Anna explained that for her the highest level is more abstract and thus more difficult for her to solve, “for me the level 4 is more difficult because since that’s more of a … I guess the abstract and the making the connections I feel more comfortable and better at solving problems where things are more concrete, and things are more I do not know I guess they follow a certain sequence.” It is important to understand whether teachers can solve cognitively demanding tasks before expecting them to implement them in the classroom.

In the construction part of the survey, teachers had the opportunity to create their mathematical tasks at the four different levels of cognitive demand. Teachers had more difficulty constructing tasks at level 3 in the pilot study while teachers from the main study had more difficulty constructing tasks at level 3 and 4. As mentioned above teachers in the main study did not get the example about “deriving a formula” for constructing a task at level 4 before the cognitive demand survey was administered. The previous explanation may have led to a low rate in this part since teachers from the pilot studies opted to construct a task exactly like the one explained above. Teachers in the main study chose to create tasks at level 4 that were more procedural. This result was also supported by some of the interviews, for example in the following excerpt Anna explains how she can create tasks at level 2 but for level 3 she will have to find a problem, but she will have challenges creating it herself:
“I think it’s definitely easier to come up with problems at the first two levels, level 3 I’m ok with that. Like I’m comfortable enough where I can find problems. And maybe even sometimes create them at that level. But level four, just because I don’t know, sometimes it seems like I want to be sure that they get it. And if students struggle with the concept a little bit, as a teacher, I feel like I want to give them all the tools they need and all the guidance, instead of just kind of letting them work.”

In a study by Thomas and Williams (2008) teachers also had difficulty classifying their own tasks and often classified a task as a level 3 when it was a level 2 task. Many teachers stated that they prefer to modify the tasks rather than creating their own so that might be another reason why teachers would have more difficulty creating their own tasks.

Research Question 2: Are there relationships among teachers’ ability to recognize, solve, construct, and implement tasks at different levels of cognitive demand?

For this research question, there was a statistical analysis of the teachers’ responses to the surveys to examine whether there was a relationship between the recognition, solution, construction, and implementations of tasks at different levels of cognitive demand. The results of the statistical analysis indicate that in the pilot study there is a significant correlation between recognizing a task at level 2 and recognizing a task at level 3. Level 2 task is procedures without connections while level 3 task is procedures with connections. In the section above, it was explained that some teachers had difficulty differentiating those levels. This result shows that those who clearly understood level 2 were more likely to understand level 3 and thus recognize the differences between the levels. In a similar way, there was a statistical relationship between recognizing a task at level 3 and recognizing a task at level 4.
Between recognizing and solving there was only one level that was significant. Recognizing a task at level 3 was related to solving a task at level 3. Perhaps those who had a high understanding of how a task at level 3 should look like are those who were more likely to solve it. There was no evidence to suggest that recognizing tasks at the four levels was related to constructing tasks at any level. Regarding implementation, only implementing tasks at level 1 was related to recognizing a task at level 1. One possible answer for this is that the majority of teachers were able to implement tasks at level 1 and the percentage of implementation decreased as the level increased.

Between solving and constructing there is no evidence to suggest that there is any relationship among them. Constructing a task at level 2 was negatively correlated with implementing a task at level 3. By looking at the results, we see that those who correctly constructed tasks at level 2 did not implement tasks at level 3. An explanation for this is that not many teachers implemented tasks at level 3. However, there was no correlation between constructing a task at level 2 and any of the other levels of implementation. Constructing a task at level 3 was related to implementing a task at level 2. The majority of those who had challenges constructing a task at level 3 also had challenges implementing a task at level 2. Finally, those who were able to implement a task at level 2 had challenges implementing a task at level 3.
Figure 5.1: Recognition, Solution, Construction, and Implementation framework supported by teachers’ narratives
Research Question 3: What are secondary mathematics and science teachers’ challenges in recognizing, solving, constructing, and implementing CDTs?

The main purpose of this study was to understand the challenges teachers face in implementing cognitively demanding tasks in the classroom. The results provided by the qualitative part of this study show how teachers think about different factors before creating a lesson plan and implementing it in the classroom. We engaged secondary mathematics and science teachers in learning about the cognitive demand levels. The following themes emerged from the interview data as the main challenges in the implementation of cognitively demanding tasks: challenges related to students’ knowledge, challenges related to teacher’s knowledge, and challenges related to outside factors.

Figure 5.1 shows individual quotes of the way teachers talked about each of the components of this study: recognition, solution, construction, and implementations of tasks at different levels of cognitive demand. These individual quotes exemplify how teachers talked about each one of the parts in this study. The themes from the interviews are related to how teachers responded to the components mentioned above. In the recognition part, we can see that teachers talked about the difficulty between the middle levels and themselves, they used phrases such as “the third level… I have most trouble recognizing” or “depending how it’s asked it can be the second and third.” In the solution part, they talked more about their mathematical knowledge. For example, they used phrases like “I know I can do one and two.” Teacher’s use of “I” shows that they are thinking about themselves and whether they can recognize and solve the tasks. In the construction part, they mentioned their students’ knowledge as a challenge. They use phrases such as “I want to make sure that they get it” and “I'm not that creative.” In this case, teachers use either “I” to refer to themselves or “they” to refer to the students. In the
implementation part, they talked about the students’ knowledge, their mathematical knowledge, and external factors as challenges. In the implementation part, they use phrases such as “I wouldn’t say I’m confident using level 4,” “I’m afraid that if I start so high, and then I don’t finish the curriculum goal on time,” and “these kids trying to keep them on a path to get everything done they usually need the straight forward.” In this case, we can see how they talk about themselves (“I”), the students (“these kids”), and curriculum and time.

In the first theme, teachers talked about the students’ knowledge as the main factor to implement any cognitively demanding task. Some of the teachers mentioned lowering the level of their courses. Arbaugh et al. (2006) found that many of the teachers talked about the students’ lack of basic skills in mathematics and expressed that deficit in knowledge as a reason for not allowing students to use the textbook as intended and consequently lowering the level. McDuffie and Mather (2006) claim that teachers’ beliefs about students’ learning are critical. Teacher beliefs about students’ knowledge might lead to lowering the level of cognitive demand if teachers think that students are not ready for tasks that are more cognitively demanding. Also, the results from this study in which teachers had difficulty recognizing between levels 2 and 3 are consistent with quantitative findings from Boston (2013) in which teachers had difficulty categorizing tasks in the middle levels. While teachers in the main study had less difficulty recognizing the middle levels they still had challenges solving and constructing tasks at level 3. When trying to construct tasks at level 3, teachers often constructed tasks at level 2 that were procedural. More research needs to be done about the differences between levels 2 and 3 to examine how to adapt procedural tasks to a task that requires more connections (Boston, 2013). Thus, engaging teachers on discussing the differences between a task that is procedural and a task that has procedures with connections should be a part of professional development.
One of the teachers reflected on how the cognitive demand on each problem depends on each student’s knowledge. Similarly, Boston (2013) argues “tasks that have high-level demands as written may not result in high-level thinking and reasoning as implemented in the classroom if they are not appropriately aligned with students’ prior knowledge and experiences (i.e., student have too much or too little exposure to similar tasks or the underlying mathematical ideas)” (p. 13). From the results of this study, some teachers said that they had lowered the level of the cognitive demand of their tasks because they feel that students are not ready. Even though, neither mentioned how they assess their students’ knowledge before implementing any tasks we can consider that their assumptions are based on their teaching experience, informal observations, and past assessments. During the teacher quality workshop, teachers were encouraged to solve different types of problems. Stein, Engle, Smith and Hughes (2008) claim, “if [teachers] put themselves in the position of their students while doing the task, they can anticipate some of the strategies that students with different degrees of mathematical sophistication are likely to produce and consider ways that students might misinterpret problems or get confused along the way” (p. 323). Solving problems at different levels of cognitive demand allowed the teachers not only to expand their knowledge but also to put themselves in the students’ position when solving problems. According to Cartier et al. (2013) “It is important for all students to have opportunities to learn science by participating in tasks that require them to think hard about the ideas and phenomena they are encountering” (p. 10).

Another important theme that emerged from the interview data was the teachers’ confidence or lack thereof in their mathematical knowledge. According to Stein and Kaufman (2010), “As a teacher prepares for the lesson, a limited understanding of the mathematics involved may lead him or her to fail to recognize the mathematical integrity of the task, thereby
altering it in ways that (unintentionally) change (and often reduce) the level of cognitive demand of the task” (p. 9). Some teachers might need extra help to develop their mathematical content knowledge before they try to utilize the highest levels of cognitive demand in their classrooms. In the workshop, the presenters focused largely on providing teachers with the opportunity to challenge them and solve mathematical problems that were more cognitively demanding. However, the focus of this study was not to examine the effects on the workshop and their mathematical content knowledge.

It has been shown that often time is a primary factor that prevents certain problems to be implemented, especially those at high levels of cognitive demand (Henningsen & Stein, 1997). Bieda (2010) interviewed some middle school teachers and when asked about their decision not to implement high-level tasks teachers said that they were concerned about not having enough time to finish all material in the lesson. In this study, teachers mentioned time constraints as a challenge because of all the other duties they are expected to fulfill and not enough time. Anna was a teacher that excelled in all parts of the survey as well as the implementation of cognitively demanding tasks. She talked about how she believes that her students were ready to face tasks that require more thinking. At the same time, she mentions time as a challenge. Teachers like Anna should be helped to reach their full potential, and other teachers should be encouraged so they too, can implement higher levels of cognitive demand without stressing about the time.

Another important aspect that emerged from this study is the way teachers think about the construct of cognitive demand. Stein et al. (2000) talk about cognitive demand as part of the mathematical task. They also discuss how the level of the cognitive demand of the task can be declined based on different factors of the task during task enactment. Results from this study suggest that for teachers, the challenge goes beyond the mathematical task. Teachers talked
about the students, content knowledge, and external factors as main challenges. They did not mention specific details about the task but rather about the intersection of all the challenges with the mathematical task. Figure 5.2 is a visual representation of the intersection between the mathematical/science task and the teacher, which includes the challenges related to teachers (i.e. mathematical knowledge). Another intersection is between the mathematical/science task and the students, which include challenges, related to students (i.e. students’ knowledge and English language learners). The other intersection is between teacher and student, which include challenges, related to external factors (i.e. time and curriculum).

Figure 5.2: Challenges in relation to the task, teacher, and student

5.2 IMPLICATIONS FOR RESEARCH, PRACTICE, AND POLICY

In this study, we invited teachers to learn about the concept of cognitive demand and the four levels by Stein et al. (2000). Other studies have examined teachers’ recognition and selection of the levels of cognitive demand (Boston, 2006; Boston and Smith 2011; Smith and Stein, 2009). However, this study further expands research on cognitive demand by allowing
teachers to construct their tasks at different levels on a given topic. In addition, we also encouraged teachers to solve mathematical tasks at the various levels of cognitive demand. This study contributes to research in the field of mathematics education by understanding whether teachers can solve and construct their tasks. Silver et al. (2009) found that teachers performed better in assessing mathematics understanding than developing mathematical understanding. The framework shown in figure 5.1 shows that research about cognitive demand should be done thinking about the different aspects presented in this study: recognition, solution, construction, and implementation. In addition, from figure 5.2 shows the intersection between the challenges teachers face with implementing cognitively demanding tasks. Thus research should also focus on more than the cognitive demand of the task but also other aspects such as the teachers, the students, and external factors.

A key aspect of this study was that teachers attended the professional development workshop in which they were encouraged to learn about the levels of cognitive demand and reflect on their teaching practices. Arbaugh and Brown (2005) also believe that engaging teachers in a professional development experience and learning about the levels of cognitive demand allows teachers to think about “the relationship between mathematical tasks and the work of students in their classes” (p. 525). This study shows that professional development based on mathematical content knowledge and cognitive demand is beneficial for secondary teachers. For example, professional development that focuses on demonstrating how teachers can create tasks at the four levels of cognitive demand for different topics. This type of professional development will not only enhance their mathematical content knowledge but also their confidence in implementing cognitively demanding tasks in their classrooms.
Furthermore, this study is significant because we encouraged teachers to talk about the challenges in implementing cognitively demanding tasks. By allowing teachers to reflect on their understanding of cognitive demand and the challenges in their implementation we get a better idea of how teachers feel. Henningsen (1997) concluded that teachers decline the level of cognitive demand by removing the aspects that made it high level. This study unpacks why teachers might decide not to implement tasks at highest levels of cognitive demand. Cartier et al. (2013) and Stein et al. (2008) provide the five practices to orchestrate task-based discussion in mathematics and science. Utilizing the five practices might help teachers implementing tasks at higher levels and maintaining the level. However, one of the main findings from this study was that teachers viewed having English language learners in their classroom as a challenge. The teachers that participated in this study worked in a school in which the majority of their students have different levels of English language proficiency. Therefore, they need to be prepared to teach these students to enhance their mathematical knowledge while also understanding their needs as ELLs. School principals should provide secondary mathematics teachers with training and resources to work with ELLs. More preparation is needed for mathematics teachers to understand how they can help ELL students learn mathematics instead of just assuming that they will not be able to solve cognitively demanding tasks because they do not understand English well.

5.3 Future research

Teachers should be engaged in learning more mathematical content; they need more practice not only recognizing but also solving and constructing different tasks. When teachers construct their own tasks, they are challenged to their knowledge as well as their students’
knowledge. The examination of qualitative data such as interviews and observations suggests that teachers consider many factors before implementing different tasks, especially those tasks that are cognitively demanding (i.e. tasks at level 3 and 4). The analysis of interviews allowed us to go deeper in the study of the implementation of cognitively demanding tasks by enabling us to understand what teachers think and why they make certain decisions when thinking about tasks. However, more research needs to be done to understand the factors that are impeding teachers to implement cognitively demanding tasks. Challenging teachers to solve, construct, and implement tasks at different levels of cognitive demand, especially at high levels, should be a major role of professional development workshops.

Another possible study could be about the implementation of cognitively demanding tasks in a primarily English language learner’s classroom. English language learner students might be at a disadvantage since they are learning the language and the concepts at the same time. However, Bautista (2014) argues, “It is every science teacher’s responsibility to help ELLs accomplish higher-order thinking, regardless of their language abilities” (p. 37). In addition, teachers in a primarily ELL classroom may lower the cognitive demand level thinking that students are not ready. According to a survey by the National Center for Education Statistics (2002), 42% of teachers have ELLs in their classrooms, but only 12.5% reported that they are prepared to work with them. Teachers feeling unprepared to work with ELLs is why more research needs to be done to help teachers that are in that position create better opportunities to learn for all kinds of students. Some teachers in this study expressed that one of the challenges of implementing cognitively demanding tasks is having ELLs in their classrooms. However, the ability to work with ELLs was not a focus of this study, and that theme emerged from the interview data. There is a need to explore further mathematics and science teachers’ beliefs and
challenges towards ELLs and the use of cognitively demanding tasks in their classroom. Issues of equity must be further examined and addressed if teachers are deliberately implementing tasks at lower levels of cognitive demand in their classrooms to ELL students. ELL students should be getting the same opportunities to solve tasks at higher levels and the same preparation to be able to succeed in college.

Another part of this study suggests that more research needs to be done about teachers’ construction of tasks at different levels of cognitive demand. Researching teachers’ construction of tasks at different levels has not been widely examined. Most research has focused on teacher’s selection of tasks (Boston, 2006; Boston, 2012; Boston, 2013; Boston & Smith, 2009; Boston & Smith, 2011; Jackson, Garrison, Wilson, Gybbons & Shahan, 2013; Stein, Grover & Henningsen, 1996; Stein & Kaufman, 2010). Also, there were no studies in the literature review in which teachers were encouraged to solve tasks at different levels of cognitive demand. Teachers in this study had challenges with constructing and solving tasks at different levels, and they also suggested that they prefer to gather tasks from other sources and sometimes modify them. A study that further explores the effects of allowing teachers to construct and solve tasks at different levels of cognitive demand has the potential to benefit not only researchers but also other teachers.

5.4 CONCLUSION

There is a need for teacher engagement in solving and constructing mathematical tasks at different levels of cognitive demand. As presented above, teachers are challenged by solving and constructing cognitively demanding tasks at levels 3 (procedures with connections) and 4 (doing mathematics) while recognizing the tasks at different levels was somewhat less challenging for
them. That is why it is critical to involve teachers in a discussion on solving and constructing tasks at higher levels as well as in discourse on differences between tasks at different levels. According to Stein et al. (2000), there are many implications for teaching practice when solving and constructing tasks at different levels of cognitive demand. One of the most important phases mentioned by Stein et al. is the implementation phase where teachers can lower the demand of the tasks. During implementation, teachers should reflect on their own practice. In addition, Stein et al. (2000), contends that teachers have mentioned numerous advantages by using the mathematical tasks framework by either reflecting with other teachers or by themselves. In this regard, Stein et al. (2000) assert “encouraging teachers to raise their interpretations of classroom actions to this more general level can allow them to see specific classroom events as ‘cases of’ something larger, more coherent, more meaningful, and, perhaps, more memorable (p. 38). One teacher talked about his understanding of the different levels of cognitive demand “I feel confident that I would be able to recognize them I do…I think that now I have a good understanding of what these look like and what kind of problems and tasks they are”. Teachers’ ability to solve is highly correlated with the ability to construct mathematical tasks at different levels of cognitive demand. Moreover, teachers should be encouraged to solve and construct challenging problems to promote the use of cognitively demanding tasks in their classrooms.

We proposed a methodological perspective that suggests a holistic view of the connections between recognition, solution, construction, and implementation of tasks at different levels of cognitive demand were examined. This framework contributes to the field of mathematics education as a way to study the implementation of cognitive demand levels as a whole. Figure 5.1 shows a few excerpts from the interviews from each part of this study. These excerpts exemplify how the teachers talk about each of the components of this study.
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Appendix A: Cognitive demand survey-Pilot study

COGNITIVE DEMAND SURVEY

Task-1. Use the figure below to answer the following question.

Figures A and B are similar. What is the length x of Figure A?

1.1. Explain your solution below:

1.2. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-4.

Rating: ______

Explain your rating below:
Task-2. What is the definition of similar figures?

2.1. Write down your definition below:

2.2. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-2.

Rating: _____

Explain your rating below:

Task-3. Triangle $\triangle ABC$ below has a right angle $\angle ACB$ and height $CD \perp AB$.

![Diagram]

Use the picture above to derive the Pythagorean relationship $AC^2 + BC^2 = AB^2$.

3.1. Explain your solution below:

3.2. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-3.

Rating: _____

Explain your rating below:

Task-4. Use the diagram below to answer the question that follows.
Figure A was transformed by scaling 1: 2 in a horizontal direction and 1: 0.5 in a vertical direction. What is the ratio of the area of Figure B to the area of Figure A?

4.1. Explain your solution below:

4.2. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-4.

Rating: _____

Explain your rating below:
5. Develop tasks at different levels of cognitive demand for the following topic – Area of a triangle.

5.1. Memorization Task:

Explain why the task you developed is a memorization task.

Provide a solution for your memorization task below:

5.2. Task using Procedures without Connections:

Explain why the task you developed is a task using procedures without connections.

Provide a solution for your procedures without connections task below:
5.3. Task using Procedures with Connections:

Explain why the task you developed is a task using procedures with connections.

Provide a solution for your procedures with connections task below:

5.4. Task on Doing Mathematics:

Explain why the task you developed involves a doing mathematics task.

Provide a solution for your doing mathematics task below:
Appendix B: Cognitive demand Survey - Main study

COGNITIVE DEMAND SURVEY - 1

Task-1. The Rabbit runs 30 meters in 4 seconds. What is his rate?

1.3. Explain your solution below:

1.4. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-1.

Rating: ______. Explain your rating below:

Task-2. If \( d \) – distance, \( t \) – time, and \( r \) – rate, what is a formula for rate?

2.1. Write down your response below:

2.3. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-2.

Rating: ______. Explain your rating below:
Task-3. Rabbit and Turtle run $d$ meter “over and back” race from a starting point to a tree ($d/2$), then back to the starting point again. Rabbit’s speed over is $r_1$ m/s and back is $r_2$ m/s. Turtle’s speed over is $r_3$ m/s and back $r_4$ m/s. Rabbit and Turtle have equal average speeds. Would Rabbit win the race? Specify conditions under which Rabbit could win.

3.3. Explain your solution below:

3.4. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-3.

Rating: _____. Explain your rating below:

Task-4. Rabbit and Turtle run a 60 meter “over and back” race from a starting point to a tree (30 m), then back to the starting point again. Rabbit’s speed over is 6 m/s and back is 4 m/s. Turtle’s speed both ways is 5 m/s. Who will win the race?

5.5. Explain your solution below:

5.6. Using a scale 1-4 (1 – lowest cognitive demand, 4 – highest cognitive demand), rate the task-4.

Rating: _____. Explain your rating below:  __ __ __ __ __
Develop tasks at different levels of cognitive demand for the following topic – **Proportionality**.

1. *Memorization Task*:

   Explain why do you think that the task you developed is a memorization task?

   Provide a solution for your own task below:

2. *Task using Procedures without Connections*:

   Explain why do you think that the task you developed is a task using procedures without connections?

   Provide a solution for your own task below:
3. Task using Procedures with Connections:

Explain why do you think that the task you developed is a task using procedures with connections?

Provide a solution for your own task below:

4. Task on Doing Mathematics/Science:

Explain why do you think that the task you developed is a doing mathematics/science task?

Provide a solution for your own task below:
Appendix C: Interview protocol

1. Could you begin by explaining to me your lesson planning process from conception, implementation and to completion?

2. When planning your classes, do you select, modify or design your own tasks? Why or why not?

3. What is your understanding of the different levels of cognitive demand? Could you give some examples?

4. How confident do you feel recognizing the different levels of cognitive demand?

5. At which level do you have most difficulty recognizing tasks? Why?

6. How confident do you feel solving problems with different levels of cognitive demand?

7. At which level do you have most difficulty solving tasks? Why?

8. How confident do you feel constructing the different levels of cognitive demand?

9. At which level do you have most difficulty constructing tasks? Why?

10. How confident do you feel using different levels of cognitive demand in your classroom?

11. Which level of cognitive demand do you feel most comfortable with in your classroom? Why?

12. Which level of cognitive demand do you feel most uncomfortable with in your classroom? Why?

13. What are the main factors you consider in selecting tasks at a certain level of cognitive demand to implement in your classroom?

14. Is there anything else you would like to tell me about your experiences with CDTs?
Vita

Angelica Monarrez Rodriguez was born in Juarez, Chihuahua, Mexico. She has a bachelors degree in mathematical science with a minor in education from the University of Texas at El Paso. During her time as an undergraduate, she worked as a tutor in mathematics as well as a peer leader for Pre-Calculus and Calculus courses. Soon after graduation, she was admitted in the Masters program in Statistics at The University of Texas at El Paso. As a graduate student, she was a research assistant on the implementation of supplemental instruction for Pre-Calculus at El Paso Community College. She also worked as a teaching assistant on her last year as a master student for statistics courses. Her jobs have helped her realize the strong passion she has for research. In 2012 she was accepted as a doctoral student in the Teaching, Learning, and Culture with a specialization in mathematics education. During this time she was worked as a research assistant for several projects funded by the U.S. Department of Education and National Science Foundation. She has presented her research at several prestigious conferences such as the Psychology of Mathematics Education-North American Chapter (PME-NA), the annual meeting of the American Educational Research Association (AERA), and the American Educational Studies Association (AESA).

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