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Why Right-Brain Cultures Are More Flexible: A Possible Explanation of Yu. Manin's Observation

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Abstract

Yuri Manin, a renowned mathematician, observed that it is much easier for a person raised in a right-brain culture to adjust to the left-brain environment than vice versa. In this paper, we provide a possible explanation for this phenomenon.

1 Manin's Observation: Formulation of the Problem

Manin's observation: formulation of the problem. In his talk [?], Yuri Manin, a renowned mathematician, made several observations about cultures that place more emphasis on left-brain, logical, discrete reasoning (e.g., traditional Western cultures) and cultures that place more emphasis on right-brain, intuitive, continuous reasoning (e.g., traditional Chinese culture and other Oriental cultures).

One of his observations is that it is much easier for a person raised in the traditional right-brain culture to adopt to a left-brain one than vice versa. For example:

- many students and researchers who are originally from China blossom in the Western academic institutions, but
- for a Western researcher it is much more difficult to fully adjust to a Chinese academic institution.

What we plan to do. In this paper, we provide a possible mathematical explanation for this phenomenon.

2 Formalization of the Problem

Need for a simplified model. To understand the corresponding adjustment phenomena, let us consider the simplest possible mathematical model for such adjustment. For that, we need to formalize what we mean by right-brain (continuous) and left-brain (discrete) cultures, and what we mean by adjustment.

Right-brain (continuous) and left-brain (discrete) cultures: a simplified description. In mathematics, the most natural notion of continuity is the continuity of a function. From this viewpoint, we will model:

- knowledge representations corresponding to right-brain (continuous) cultures by continuous functions, and
- knowledge representations corresponding to left-brain (discrete) cultures by discrete (piece-wise constant) functions.

We need to select finite-parametric families of functions. At any given moment of time, we can only store finitely many parameters, so we have to restrict ourselves to finite-parametric families of functions. How do we select these families?

Functional dependencies $y = f(x)$ are ubiquitous in nature. For example, for a given body:

- the force y is a function of acceleration x (second Newton's law),
- the voltage y is a function of the current x (Ohm's law),
- the acceleration y caused by the Sun's gravity is a function of the distance x ,

etc. In all these cases, we can observe the values $y_i = f(x_i)$ of the desired function $f(x)$ corresponding to different inputs $x_1 < \dots < x_n$:

- forces corresponding to different accelerations,
- voltages corresponding to different values of the current, etc.

It is therefore reasonable to consider functions generated by this information, i.e., by finitely many values $x_1, \dots, x_n$ and the corresponding values $y_1, \dots, y_n$.

Resulting families. In the continuous case, for every two tuples $(x_1, \dots, x_n)$ and $(y_1, \dots, y_n)$, we need to consider a continuous function $f(x)$ for which $f(x_i) = y_i$. The simplest such function is a piece-wise linear function that connects the corresponding points, i.e., a function for which:

- $f(x) = y_1$ for $x \leq x_1$,
- $f(x) = y_n$ for $x \geq x_n$, and

- $f(x) = y_i + \frac{x - x_i}{x_{i+1} - x_i}$ when $x_i \leq x \leq x_{i+1}$.

In the discrete case, the simplest possible idea is to use a piece-wise constant function whose value $f(x)$ at each point x coincides with the value y_i at the nearest point x_i . In this case,

- $f(x) = y_1$ for $x < \frac{x_1 + x_2}{2}$,
- $f(x) = y_n$ for $x \geq \frac{x_{n-1} + x_n}{2}$, and
- $f(x) = y_i$ for $\frac{x_{i-1} + x_i}{2} \leq x < \frac{x_i + x_{i+1}}{2}$.

What does adjustment mean in these terms? Since we are talking about functions, a natural meaning of adjustment is approximation:

- for a right-brain (continuous) person to adjust to the left-brain (discrete) culture means that we try to approximate a discrete function by continuous ones;
- similarly, for a left-brain (discrete) person to adjust to the right-brain (continuous) culture means that we try to approximate a continuous function by discrete ones.

As a numerical measure of approximation quality, we can use the Least Squares difference – the one that is mostly used in data processing. In our case, this means that, in effect, as a measure of difference between two functions $f(x)$ and $g(x)$ we consider the quantity $\int (f(x) - g(x))^2 dx$. For this integral to be finite, we need to limit ourselves to a finite interval $[L, U]$.

Now, we are ready to formulate the problem in precise terms.

3 Definitions and the Main Result

Definition 1. Let $L < U$ be real numbers, and let n be a positive integer.

- By an n -parametric continuous function, we mean a function that, for some tuples $L \leq x_1 < x_2 < \dots < x_n \leq U$ and $(y_1, \dots, y_n)$, has the form

- $f(x) = y_1$ for $x \leq x_1$,
- $f(x) = y_n$ for $x \geq x_n$, and
- $f(x) = y_i + \frac{x - x_i}{x_{i+1} - x_i}$ when $x_i \leq x \leq x_{i+1}$.

- By an n -parametric discrete function, we mean a function that, for some tuples $L \leq x_1 < x_2 < \dots < x_n \leq U$ and $(y_1, \dots, y_n)$, has the form

- $f(x) = y_1$ for $x < \frac{x_1 + x_2}{2}$,

- $f(x) = y_n$ for $x \geq \frac{x_{n-1} + x_n}{2}$, and
- $f(x) = y_i$ for $\frac{x_{i-1} + x_i}{2} \leq x < \frac{x_i + x_{i+1}}{2}$.

Definition 2. By a distance between two functions $f(x)$ and $g(x)$, we mean the value $d(f, g) \stackrel{\text{def}}{=} \int_L^U (f(x) - g(x))^2 dx$. For every real number $\varepsilon > 0$, we say that f and g are ε -close if $d(f, g) \leq \varepsilon$.

Proposition 1. For every n and for every $\varepsilon > 0$, for every n -parametric continuous function $f(x)$, there is a $2n$ -parametric discrete function $g(x)$ which is ε -close to $f(x)$.

Proposition 2. For every n and $N > n$, for every non-constant n -parametric continuous function $f(x)$, there exists a real value $\varepsilon > 0$ such that no N -parametric discrete function is ε -close to $f(x)$.

Conclusion. So, while a discrete function can be approximated by continuous functions with any possible accuracy, there is always a limit with which a discrete function can approximate a continuous one. In other words, it is much easier for continuous functions to approximate discrete ones than vice versa, it is much easier for a continuous function to adjust to a discrete family than vice versa. This is exactly what we tried to explain.

4 Proofs

Proof of Proposition 1. Let $f(x)$ be a continuous function corresponding to the values $x_1 < \dots < x_n$ and $y_1, \dots, y_n$. To create an approximating discrete function, we approximate each jump by a fast-increasing linear function.

Specifically, let us select a small $\delta > 0$, and consider $2(n-1)$ values

$$x_1^- < x_1^+ < x_2^- < x_2^+ < \dots < x_{n-1}^- < x_{n-1}^+,$$

where $x_i^- = \frac{x_i + x_{i+1}}{2} - \delta$ and $x_i^+ = \frac{x_i + x_{i+1}}{2} + \delta$. As the corresponding y -values, we take $y_i^- = y_i$ and $y_i^+ = y_{i+1}$.

One can check that the resulting $2n$ -parametric continuous function $g(x)$ differs from the original discrete function $f(x)$ only on $n-1$ intervals $[x_i^-, x_i^+]$ of length 2δ each – and thus, of total length $2(n-1) \cdot \delta$. On these intervals, the values of both functions $f(x)$ and $g(x)$ are bounded from below from $y \stackrel{\text{def}}{=} \min(y_1, \dots, y_n)$ and from above by $Y \stackrel{\text{def}}{=} \max(y_1, \dots, y_n)$. Thus,

$$|f(x) - g(x)| \leq Y - y,$$

hence $(f(x) - g(x))^2 \leq (Y - y)^2$, and therefore, $d(f, g) \leq (Y - y)^2 \cdot 2(n-1) \cdot \delta$.

So, when $\delta \rightarrow 0$, we get $d(f, g) \rightarrow 0$. The statement is proven.

Proof of Proposition 2. Since the n -parametric continuous function $f(x)$ is not a constant, there exists an i for which $y_i \neq y_{i+1}$. On the corresponding interval $[x_i, x_{i+1}]$, the function $f(x)$ is linear, with a non-zero slope $k \stackrel{\text{def}}{=} \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$.

Any N -parametric discrete function $g(x)$ is based on N pairs (x'_i, y'_i) that divide the whole interval $[L, U]$ into N subintervals on each of which the function $g(x)$ is constant. In particular, some of these intervals cover the whole interval $[x_i, x_{i+1}]$. Thus, the interval $[x_i, x_{i+1}]$ is divided into $\leq N$ parts of each of which $g(x)$ is a constant. The sum of the lengths of these parts is $x_{i+1} - x_i$, thus at least one of these parts must have the length ℓ which is at least $h \stackrel{\text{def}}{=} \frac{x_{i+1} - x_i}{N}$ – else, if all their lengths are smaller, their sum would be smaller than $x_{i+1} - x_i$.

On this part of length ℓ , a linear function $f(x)$ with slope k is approximated by a constant. The distance $f(f, g) = \int_L^U (f(x) - g(x))^2 dx$ between $f(x)$ and $g(x)$ is greater than or equal to the integral $\int (f(x) - g(x))^2 dx$ limited to this part. One can easily check that for a linear function, the smallest ℓ^2 -distance from a constant is when this constant is equal to a midpoint between the two extreme points of the linear function. On this part, the corresponding integral is thus equal to

$$\int_{-\ell/2}^{\ell/2} (k \cdot x)^2 dx = 2 \int_0^{\ell/2} k^2 \cdot x^2 dx = 2k^2 \cdot \frac{1}{3} \cdot \left(\frac{\ell}{2}\right)^3.$$

Since the distance $d(f, g)$ is larger than or equal to this integral and $\ell \geq h$, we have

$$d(f, g) \geq 2k^2 \cdot \frac{1}{3} \cdot \left(\frac{h}{2}\right)^3.$$

Any positive real number which is smaller than the right-hand side of this inequality can thus be taken as the desired value $\varepsilon > 0$. The proposition is proven.

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