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Why the Presence of Point-Wise ("Punctate") Calcifications or Linear Configurations of Calcifications Makes Breast Cancer More Probable: A Geometric Explanation

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Why the Presence of Point-Wise ("Punctate") Calcifications or Linear Configurations of Calcifications Makes Breast Cancer More Probable: A Geometric Explanation

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Abstract

When a specialist analyzes a mammogram for signs of possible breast cancer, he or she pays special attention to point-wise and linear-shaped calcifications and point-wise and linear configurations of calcification – since empirically, such calcifications and combinations of calcifications are indeed most frequently associated with cancer. In this paper, we provide a geometric explanation for this empirical phenomenon.

1 Signs of Breast Cancer

Early diagnostics of breast cancer is important. Breast cancer is curable if detected early. One of the most frequently used methods of detecting breast cancer is regular mammography. When a mammogram shows calcifications, a specialist determines whether these calcifications are indicative of the cancer. If yes, additional tests are performed (such as biopsy) that enable the doctors to diagnose cancer and start the treatment.

How specialists diagnose breast cancer. Calcifications are classified as suspicious based on the size of the classification and on the geometric shape of their configuration; see, e.g., [3, 4]. Specifically, in terms of size:

- small, point-wise ("punctuate") calcifications are highly suspicious, while
- larger-size calcifications are considered benign.

In terms of configurations:

- calcification that group together in a small area (point) or that form a straight line are suspicious, while

- calcifications that are evenly distributed in a reasonable-size region – or even throughout the whole breast – are considered benign.

Open problem and what we do in this paper. To the best of our knowledge, there is no convincing theoretical explanation of the above empirical facts. In this paper, we show that geometric analysis can provide such an explanation.

2 From Physical Analysis of the Problem to Its Geometric Description

What are we looking for: physical description. The main feature of cancer is its growth: if not stopped, it can cover larger and larger areas, and through metastasis spread to the whole body, often causing the death of a patient.

So, to detect cancer, we look for configurations that suddenly appeared and that can potentially spread through the whole body.

How to translate this physical description into geometric terms: calcifications mean symmetry violation. The original healthy tissue is homogeneous, i.e., locally, it is invariant with respect to shifts and rotations. Also, in the original homogeneous distribution, there is no fixed size, i.e., the original configuration is also (locally) invariant with respect to *scalings* $\vec{x} \to \lambda \cdot \vec{x}$.

When calcifications appear, this symmetry is violated. Indeed, in this case, there is a calcification at some location, but there is no calcification at some other locations, and these other locations can be obtained from the calcification location by a shift. Thus, the resulting configuration is no longer invariant with respect to all the shifts.

Which symmetry violations are physically preferable. According to physics, while it is possible to have a state transition that goes from a highly symmetry state to a state with no symmetries, such transitions are highly improbable.

The more symmetries are violated, the less probable is the corresponding transition. From this viewpoint, the most probable transitions are the ones that retain the most symmetries; see, e.g., [1].

Resulting symmetry groups. The original configuration is invariant with respect to all shifts, rotations, scalings, and their composition. These *symmetries* – i.e., transformations that preserve the configuration – form a *group*. We will denote this group by G_0.

To describe a general element from this group, we need to describe four parameters: two to describe shift, one to describe rotation, and one to describe scaling. In this sense, this group is 4-dimensional.

After spontaneous symmetry violation, only some symmetries remain. Let us denote the remaining group of symmetries by G.

The above argument shows that we should select the largest proper subgroups of the original group G_0, i.e., the proper subgroups with the largest possible dimension.

How resulting geometric shapes are related to symmetry groups. If we have a configuration with a calcification at some point p, and this configuration is invariant with respect to some group G, then, for each transformation $g \in G$, the point $g(p)$ should also be a calcification. Thus, the set of the calcifications contains the whole orbit $G(p) \stackrel{\text{def}}{=} \{g(p) : g \in G\}$.

So, the shape of each calcification, and the shape of each configuration of calcifications, should consist of orbits of subgroups of the group G. Let us use these conclusions to come up with the resulting geometric shapes.

3 Geometric Analysis of the Problem

Analogy with shapes of celestial bodies. A similar symmetry-violation argument can be used to describe the shapes of most frequent celestial bodies; see, e.g., [2].

Possible shapes of orbits. The paper [2] provides a full description of all possible orbits of subgroups of the group G_0.

Specifically, a 2-D orbit is the whole plane, and the generic form of a 1-D orbit is a logarithmic spiral, which has the form $r = r_0 \cdot \exp(k \cdot \varphi)$ in polar coordinates (r, φ). The degenerate forms of the logarithmic spiral are: a circle (corresponding to $k = 0$), a half-line (corresponding to $k \to \infty$), a straight line (the limit of shifted half-lines), and a point (corresponding to $r_0 = 0$).

Analyzing different shapes. Let us check which of these orbits has the largest possible symmetry group.

- A logarithmic spiral has a 1-D symmetry group: its elements are compositions of rotation by φ_0 and scaling by $\exp(k \cdot \varphi_0)$.
- A circle has a 1-D symmetry group: all rotations.
- A half-line has a 1-D symmetry group: all scalings.
- A line has a 2-D symmetry group: scalings and shifts along this line.
- A point also has a 2-D symmetry group: all rotations around this point and all scalings.

Thus, the largest possible dimension of orbit is 2, and 2-D symmetry groups correspond to straight lines and points.

First conclusion: geometric analysis explains why linear and point-wise configurations are suspicious. In search for early cancers, we are interested in emerging calcifications and emerging combinations of calcifications.

Our analysis has shown that most of such calcifications and combinations of calcifications should allow the largest possible symmetry groups, and that the resulting geometric shapes are a straight line and a point.

This explains why point-wise calcification are marked as suspicious, and why point-wise and linear combinations of calcifications are suspicious.

Point-wise and linear shapes are suspicious, but are they dangerous? As we have mentioned earlier, the main danger of cancer is that tends to spread to the whole body. From the geometric viewpoint, are the point-wise and linear shapes indeed dangerous?

To answer this question, let us go back to physics. If we have a symmetric configuration, then, while symmetry violations are possible, still the most probable dynamics is the one that preserves the original symmetry [1].

So, if each shape grows, it will turn into a larger shape with the same symmetry. What is this larger shape?

A point is invariant with respect to all scalings (which, in coordinates centered at the original point have the form $\vec{x} \to \lambda \cdot \vec{x}$) and all rotations. If it grows, it will include one different point (r_0, φ_0) with $r_0 = 0$. Since the resulting shape is invariant with respect to all these symmetries, it will therefore include all possible points (r, φ): indeed, each such point can be obtained from (r_0, φ_0) if we scale it by a factor $\lambda = r/r_0$ and then rotate by the angle $\varphi - \varphi_0$. From this viewpoint, a point-wise shape is indeed dangerous: once it grows, it tends to take over the whole body.

A similar conclusion can be made about a straight line. Indeed, we can always select coordinates so that this line is an X-axis, i.e., consists of all the points $(x, 0)$. If it grows, it will contains a point (x_0, y_0) outside this line, i.e., a point for which $y_0 \neq 0$. Then, by an appropriate scaling and shift, we can get all the points (x, y) – at least all the points for which y has the same sign as y_0: we take a scaling by $\lambda = y/y_0$ and then shift by $x - \lambda \cdot x_0$. Thus, the linear shape is also indeed dangerous.

Second conclusion: geometric analysis explains why linear and point-wise configurations are dangerous. The above analysis shows that both point and linear shapes are not only typical shapes of emerging configurations, they are also *dangerous* – in the sense that their grows tends to take over the whole body.

This is one more reason to consider these shapes to be suspicious.

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