4-2017

Which Material Design Is Possible Under Additive Manufacturing: A Fuzzy Approach

Francisco Zapata
University of Texas at El Paso, fazg74@gmail.com

Olga Kosheleva
University of Texas at El Paso, olgak@utep.edu

Vladik Kreinovich
University of Texas at El Paso, vladik@utep.edu

Comments:
Technical Report: UTEP-CS-17-09a
To appear in *Proceedings of the Joint 17th Congress of International Fuzzy Systems Association and 9th International Conference on Soft Computing and Intelligent Systems*, Otsu, Japan, June 27-30, 2017.

Which Material Design Is Possible Under Additive Manufacturing: A Fuzzy Approach

Francisco Zapata
Department of Industrial, Manufacturing, and Systems Engineering
University of Texas at El Paso
500 W. University
El Paso, Texas 79968, USA
fazg74@gmail.com

Olga Kosheleva and Vladik Kreinovich
University of Texas at El Paso
500 W. University
El Paso, Texas 79968, USA
olgak@utep.edu, vladik@utep.edu

***Abstract*—Additive manufacturing – also known as 3-D printing – is a very promising new way to generate complex material designs. However, even with the modern advanced techniques, some designs are too complex to be implemented. There exist an empirical formula that describe when the design is implementable. In this paper, we use fuzzy ideas to provide a theoretical justification for this empirical formula.**

I. Formulation of the Problem

Additive manufacturing is important. Additive manufacturing – also known as 3-D printing – is a very promising new way to generate complex material designs; see, e.g., [2], [4].

Additive manufacturing allows us to generate objects layer-by-layer, and thus, come up with very complex objects.

Limitations of additive manufacturing. While additive manufacturing has many successes, it is not a panacea.

Current equipment for additive manufacturing only reproduces the desired design with a certain accuracy.

- For simpler shapes, small deviations from the desired configuration do not affect their functionality.
- However, for more complex, more detailed objects, with multiple small elements, even a small change in the configuration can ruin the result.

For example, one of the biomedical application of additive manufacturing is the design of tiny blood vessels to be placed inside a patient's body. An ideal blood vessel should have uniform width. If, due to implementation inaccuracy, we have a vessel with widely varying width, this will severely impede the blood flow through this vessel.

For such complex objects, most additive manufacturing equipments do not guarantee the desired accuracy. To manufacture such complex objects, it is necessary to come up with more accurate additive manufacturing equipment. Such more accurate more sophisticated equipment has indeed been invented; however, the main limitation of such state-of-the-art equipment is that it remains very expensive.

And, of course, some designs are still too complex to be implemented even by the most up-to-date equipment; so, even more accurate equipment is needed.

978-1-5090-4917-2/17$31.00

It is desirable to estimate the complexity of a given design. While some 3-D printers are already reasonably cheap, the up-to-date additive manufacturing equipment is not cheap. It is therefore desirable to avoid losing money on trying to implement impossible designs.

On the other hand, we do not want to use the corresponding equipment to implement designs which are too simple for this equipment – and which therefore can be implemented by much less sophisticated (and cheaper) equipment.

It is therefore desirable to be able to estimate the complexity of the design before the manufacturing, so that we would be able to see whether a given equipment can implement this design – and if it can, whether this same design can be implemented by a cheaper equipment.

How complexity is estimated now: an empirical formula. The state-of-the-art empirical formula describing the complexity of a design estimates this complexity is described in this section; for a detailed description, see, e.g., [7]. This formulas is based on the division of the design into several sections i.

A section which has only one type of material in it is relatively easy to manufacture. The difficulty comes from the sections where several different materials have to be in close vicinity.

If we make sections sufficiently small, then in each section, we have at most two different materials – so there is no need to consider sections with three or more materials.

Let us denote the fraction of this section which is filled with one of these materials by $v_i \in [0, 1]$. Then, the other material fills the remaining fraction $1 - v_i$. In this case, the empirical formula for the complexity C is

$$C = \sum_i v_i^\eta \cdot (1 - v_i)^\eta,$$

where η is an empirical parameter.

Open problem. The problem with empirical formulas is that when a formula does not have a theoretical justification, it is less reliable, because it is not clear

- whether this dependence indeed follows from first principles — and thus, can be applied to other situations as well,

- or is somewhat accidental, based on a relatively small sample, and probably will not hold in other cases.

What we do in this paper. In this paper, we provide a theoretical justification for this formula. In this justification, we use fuzzy logic ideas [5], [6], [8].

II. Commonsense Analysis of the Problem and Its Fuzzy-Based Formalization

Idea. As we have mentioned, a section i with only one material – i.e., when we have a large fraction of one material and practically no presence of the other material – is relatively easy to manufacture. The complexity comes when we have a significant amount of one material *and* the significant amount of the other material.

From the idea to the numerical description of the section's complexity. In general the section's complexity c depends on the parameter v_i. In precise terms, this means that $c = c(v_i)$ for an appropriate function $c(v_i)$.

- When we have only one material, i.e., when $v_i = 0$ or $v_i = 1$, there is no complexity, so we should have
$$c(0) = c(1) = 0.$$
- When both materials are present, there is complexity, so we should have $c(v_i) > 0$ for $v_i \in (0, 1)$.

Many physical ideas are based on the fact that every analytical function can be expanded in Taylor series, and, as a good approximation, we can take the sum of the first few terms in this expansion; see, e.g., [3]. As a first approximation, it makes sense to consider as few terms as possible.

Let us apply this idea to the unknown function $c(v_i)$: let us consider its Taylor series

$$c(v_i) = c_0 + c_1 \cdot v_i + c_2 \cdot v_i^2 + c_3 \cdot v_i^3 + \ldots$$

Let us find the smallest number of terms for which the resulting approximate function

$$c(v_i) = c_0 + c_1 \cdot v_i + \ldots + c_k \cdot v_i^k$$

satisfies the properties $c(0) = c(1) = 0$ and $c(v_0) > 0$ for $v_i \in (0, 1)$.

Let us start with the simplest possible approximation, in which we keep only the 0-th order (constant) term in the expansion:

$$c(v_i) = c_0.$$

In this case, from $c(0) = c(1) = 0$, we conclude that $c_0 = 0$ and thus, $c(v_i) = c_0 = 0$ for all $v_i \in (0, 1)$ – contrary to our assumption that for such v_i, we have $c(v_i) > 0$.

Similarly, in the next approximation

$$c(v_i) = c_0 + c_1 \cdot v_i,$$

from the conditions that $c(0) = 0$ and $c(1) = 0$, we conclude that $c_0 = c_1 = 0$ and thus, that $c(v_i) = 0$ for all $v_i \in (0, 1)$.

Thus, to satisfy both properties, we need to consider at least quadratic terms. Let us first consider quadratic terms, i.e.,

$$c(v_i) = c_0 + c_1 \cdot v_i + c_2 \cdot v_i^2.$$

In this case, the condition $c(0) = 0$ implies that $c_0 = 0$, and thus, that

$$c(v_i) = c_1 \cdot v_i + c_2 \cdot v_i^2.$$

Now, the condition that $c(1) = 0$ implies that $c_1 + c_2 = 0$, so $c_2 = -c_1$ and thus,

$$c(v_i) = c_1 \cdot (v_i - v_i^2).$$

The requirement that $c(v_i) > 0$ or all $v_i \in (0, 1)$ implies that $c_1 > 0$.

This formula can be further simplified if we take into account that all we are interested in is relative complexity of different designs. From this viewpoint, the absolute value of the complexity is irrelevant, what is important is relative complexity of different designs. In particular, nothing will change if we simply divide all the complexity values by the same constant.

If we take c_1 as this value, then the formula for the complexity of a section take a simpler form

$$c(v_i) = v_i - v_i^2 = v_i \cdot (1 - v_i).$$

This formula has a clear fuzzy interpretation. One can easily see that this formula has a clear fuzzy interpretation. Indeed, it is reasonable to take the proportion v_i of one of the materials as the degree to which this material is present in this section. Similarly, the proportion $1 - v_i$ of the other material can be taken as the degree to which the other material is present.

As we have mentioned, the section is complex only if both materials are present in this section, i.e., when the first material is present *and* the second material is present. Thus, to get the degree of complexity of a section, it is reasonable to apply an appropriate "and"-operation (t-norm) $f_\&(a, b)$ to the degrees v_i and $1 = v_i$ to which both materials are present:

$$c = f_\&(v_i, 1 - v_i).$$

In particular, if we use the algebraic product $f_\&(a, b) = a \cdot b$, one of the simplest and the most widely used "and"-operations, we get $c = v_i \cdot (1 - v_i)$, which is exactly the formula that we got by considering the Taylor series.

The fact that the fuzzy-logic formalization of the common sense ideas leads to the same formula as the more mathematical Taylor series approach make us more confident that this formula is true.

An additional commonsense property of the above formula. We describe the two-material situation by describing the proportion v_i of one of the materials in the section. Alternatively, we could describe the same situation by describing the proportion $1 - v_i$ of the other material in this section.

Both values v_i and $1 - v_i$ thus describe the exact same physical situation. Therefore, it is reasonable to require that the complexity $c(v_i)$ should not change if we simply re-name

the materials without changing anything in the section itself. So, we must have

$$c(v_i) = c(1 - v_i)$$

for all v_i.

It is easy to see that the above formula satisfies this commonsense property.

How accurate is this quadratic approximation? Every time we approximate a function by the sum of the first few terms in the Taylor expansion, a natural question is: how accurate is the resulting approximation?

A natural answer to this question can be obtained if we allow one more term in this expansion:

- If addition of one more term drastically changes the situation, then our original approximation is rather crude, and we should not trust the results of using this approximation too much.
- On the other hand, if the addition of one more term does not change the result, this means that the original approximation was reasonably accurate.

From this viewpoint, let us see what happens if we add a cubic to our expansion, i.e., if we consider the expressions of the type

$$c(v_i) = c_0 + c_1 \cdot v_i + c_2 \cdot v_i^2 + c_3 \cdot v_i^3.$$

It make sense to require that the resulting function satisfies all the commonsense properties:

- the two properties that we considered earlier: $c(0) = c(1) = 0$ and $c(v_i) > 0$ for $v_i \in (0, 1)$, and
- the new commonsense property, that $c(v_i) = c(1 - v_i)$ for all v_i.

The two polynomials $c(v_i)$ and $c(1 - v_i)$ coincide for all v_i if and only al their coefficients are equal. In particular, the coefficients at v_i^3 in both polynomials must be equal. However, if we substitute $1 - v_i$ into the above formula, we get

$$c(v_i) = c_0 + c_1 \cdot (1 - v_i) + c_2 \cdot (1 - v_i)^2 + c_3 \cdot (1 - v_i)^3.$$

Thus,

$$c(v_i) = c_0 + c_1 \cdot (1 - v_i) +$$

$$c_2 \cdot (1 - 2v_i + v_i^2) + c_3 \cdot (1 - 3v_i + 3v_i^2 - v_i^3).$$

If we open parentheses, we see that the only resulting term proportional to v_i^3 is the term $-c_3 \cdot v_i^3$, with the coefficient $-c_3$.

So, this coefficient must be equal to the similar coefficient c_3 in the expansion of $c(v_i)$. We have $-c_3 = c_3$, hence $c_3 = 0$. Thus, the expression $c(v_i)$ must be quadratic – and we already know that in this case, this complexity expression is proportional to $v_i \cdot (1 - v_i)$.

Here, the addition of the next order term did not change the formula – so we can conclude that this approximation is reasonably accurate.

III. Combining Complexity of Sections into a Single Complexity Value

Formulation of the problem. Now that we know the value of the complexity

$$C_i = c(v_i) = v_i \cdot (1 - v_i)$$

of each section i, we need to combine these complexities into a single value that describes the complexity of the overall design.

What are the reasonable properties of the combination function? Let us first start with the case of two sections 1 and 2. Let $f(C_1, C_2)$ be an overall complexity of a 2-section design in which the first section has complexity C_1 and the second section has complexity C_2.

If one of the sections does not have any complexity $C_1 = 0$, then the overall complexity is simply equal to the complexity of the second section $f(0, C_2) = C_2$.

The combination result should not depend on the order in which we consider these two sections, so we should have $f(C_1, C_2) = f(C_2, C_1)$. In other words, the operation $f(C_1, C_2)$ should be commutative.

Let us now consider a 3-section design, with sections of complexities C_1, C_2, and C_3. Then, we can compute the overall complexity of this design as follows:

- first, we compute the complexity C_{12} of the combination of the first two sections, as $C_{12} = f(C_1, C_2)$;
- then, we apply the same combination function f to combine the complexity $C_{12} = f(C_1, C_2)$ of the first two sections an the complexity C_3 of the third section.

As a result, we get the value

$$f(C_{12}, C_3) = f(f(C_1, C_2), C_3).$$

Alternatively, we can first combine the complexities of the second and third sections into a single complexity $C_{23} = f(C_2, C_3)$, and then combine the resulting complexity C_{23} with the complexity of the first section, resulting in

$$f(C_1, C_{23}) = f(C_1, f(C_2, C_3)).$$

It is reasonable to require that the resulting estimates coincide, i.e., that

$$f(f(C_1, C_2), C_3) = f(C_1, f(C_2, C_3)).$$

In other words, the operation $f(C_1, C_2)$ must be associative.

The function $f(C_1, C_2)$ must also be increasing with respect to both C_1 and C_2: indeed, if increase the complexity of one of the sections, the overall complexity increases.

It is also reasonable to require that small changes in complexities C_i should lead to small changes in the overall complexity $f(C_1, C_2)$, i.e., in mathematical terms, that the function $f(C_1, C_2)$ is continuous.

What are the functions satisfying these properties? The above properties are similar to the properties of an "or"-operation (t-conorm) in fuzzy logic: namely, they correspond

to the properties of an *Archimedean* t-conorms $f_\vee(a,b)$, for which, for $a<1$, we always have $f_\vee(a,a)>a$.

Thus, we can use the known result about the classification of such t-conorms [5], [6] and conclude that the operation $f(C_1,C_2)$ has the form

$$f(C_1,C_2)=g^{-1}(g(C_1)+g(C_2))$$

for some strictly increasing function $g(C)$ for which $g(0)=0$; here, g^{-1} denotes the inverse function.

In this case, the relation $C=f(C_1,C_2)$ that describes the complexity C of a 2-section design in terms of the complexities C_1 and C_2 of the sections take an equivalent form

$$C=g^{-1}(g(C_1)+g(C_2)).$$

By applying the function $g(x)$ to both sides, we can get a simplified expression for this relation:

$$g(C)=g(C_1)+g(C_2).$$

Another reasonable property: scale-invariance. As we have mentioned earlier, complexity – like many other quantities – is defined modulo a measuring unit. If we change the measuring unit to a new one which is λ times smaller, then all the numerical values of complexity get multiplied by λ. This is similar to the fact that if for measuring length, we replace the original meter with a 100 times smaller unit – centimeter – all numerical values get multiplied by a factor of 100.

It is reasonable to require that the result of a combination operation should not depend on this re-scaling, i.e., that for every C_1, C_2, C, and λ, if we have

$$g(C)=g(C_1)+g(C_2)$$

then we should also have

$$g(\lambda\cdot C)=g(\lambda\cdot C_1)+g(\lambda\cdot C_2).$$

Let us analyze which functions $g(x)$ satisfy this scale-invariance property.

Which functions $g(x)$ are scale-invariant: analysis of the problem. The above property implies that if we change both C_1 and C_2 to $C_1'=C_1+\Delta C_1$ and $C_2'=C_2+\Delta C_2$ so that the sum $g(C_1)+g(C_2)$ does not change, i.e., that

$$g(C_1')+g(C_2')=g(C_1)+g(C_2),$$

then we should also have

$$g(\lambda\cdot C_1')+g(\lambda\cdot C_2')=g(\lambda\cdot C_1)+g(\lambda\cdot C_2).$$

For a small ΔC_1, we have

$$g(C_1')=g(C_1+\Delta C_1)=$$
$$g(C_1)+g'(C_1)\cdot\Delta C_1+o(\Delta C_1),$$

where $g'(x)$ denotes the derivative of the function $g(x)$. Similarly, we have

$$g(C_2')=g(C_2)+g'(C_2)\cdot\Delta C_2+o(\Delta C_2).$$

Thus, the condition that

$$g(C_1')+g(C_2')=g(C_1)+g(C_2)$$

implies that

$$g'(C_1)\cdot\Delta C_1+g'(C_2)\cdot\Delta C_2+o(\Delta C_i)=0,$$

i.e., that

$$\Delta C_2=-\frac{g'(C_1)}{g'(C_2)}+o(\Delta C_i).$$

Similarly, we have

$$g(\lambda\cdot C_1')=g(\lambda\cdot C_1+\lambda\cdot\Delta C_1)=$$
$$g(\lambda\cdot C_1)+g'(\lambda\cdot C_1)\cdot\lambda\cdot\Delta C_1+o(\Delta C_1)$$

and

$$g(\lambda\cdot C_2')=g(\lambda\cdot C_2+\lambda\cdot\Delta C_2)=$$
$$g(\lambda\cdot C_2)+g'(\lambda\cdot C_2)\cdot\lambda\cdot\Delta C_2+o(\Delta C_2).$$

So, the condition

$$g(\lambda\cdot C_1')+g(\lambda\cdot C_2')=g(\lambda\cdot C_1)+g(\lambda\cdot C_2)$$

takes the form

$$g'(\lambda\cdot C_1)\cdot\lambda\cdot\Delta C_1+g'(\lambda\cdot C_2)\cdot\lambda\cdot\Delta C_2+o(\Delta C_i)=0.$$

Substituting the above expression for ΔC_2 in terms of ΔC_1 into this formula, we conclude that

$$g'(\lambda\cdot C_1)\cdot\lambda\cdot\Delta C_1-g'(\lambda\cdot C_2)\cdot\frac{g'(C_1)}{g'(C_2)}\cdot\lambda\cdot\Delta C_1+$$
$$o(\Delta C_i)=0.$$

Dividing both sides of this equality by $\lambda\cdot\Delta C_1$, we conclude that

$$g'(\lambda\cdot C_1)-g'(\lambda\cdot C_2)\cdot\frac{g'(C_1)}{g'(C_2)}+o(1)=0,$$

i.e., in the limit when $\Delta C_i\to 0$, that

$$g'(\lambda\cdot C_1)=g'(\lambda\cdot C_2)\cdot\frac{g'(C_1)}{g'(C_2)}.$$

Moving all the terms containing C_1 to one side of this equality and all the terms containing C_2 to another side, we conclude that

$$\frac{g'(\lambda\cdot C_1)}{g'(C_1)}=\frac{g'(\lambda\cdot C_2)}{g'(C_2)}.$$

This is true for all possible values C_1 and C_2, which means that the ratio $r\stackrel{\text{def}}{=}\dfrac{g'(\lambda\cdot C)}{g'(C)}$ does not depend on C at all, it depends only in λ:

$$\frac{g'(\lambda\cdot C)}{g'(C)}=r(\lambda),$$

thus $g'(\lambda\cdot C)=r(\lambda)\cdot g'(C)$.

For every two values λ_1 and λ_2, we can apply this formula directly for $\lambda=\lambda_1\cdot\lambda_2$, getting

$$g'(\lambda\cdot C)=r(\lambda)\cdot g'(C)=r(\lambda_1\cdot\lambda_2)\cdot g'(C).$$

Alternatively, we could apply it first for λ_2, getting

$$g'(\lambda_2 \cdot C) = r(\lambda_2) \cdot g'(C),$$

and then for λ_1, getting

$$g'(\lambda \cdot C) = g'(\lambda_1 \cdot (\lambda_2 \cdot C)) = r(\lambda_1) \cdot g'(\lambda_2 \cdot C) =$$

$$r(\lambda_1) \cdot (r(\lambda_2) \cdot g'(C)) = (r(\lambda_1) \cdot r(\lambda_2)) \cdot g'(C).$$

By comparing the above two expressions for $g'(\lambda \cdot C)$, we conclude that

$$r(\lambda_1 \cdot \lambda_2) \cdot g'(C) = (r(\lambda_1) \cdot r(\lambda_2)) \cdot g'(C),$$

i.e., that

$$r(\lambda_1 \cdot \lambda_2) = r(\lambda_1) \cdot r(\lambda_2).$$

It is known (see, e.g., [1]) that every monotonic function $r(\lambda)$ satisfying the above equality has the form $r(\lambda) = \lambda^q$ for some real number q. Thus, the equality $g'(\lambda \cdot C) = r(\lambda) \cdot g'(C)$ takes the form $g'(\lambda \cdot C) = \lambda^q \cdot g'(C)$. In particular, for every real value x, if we take $\lambda = x$ and $C = 1$, we get $g'(x) = c \cdot x^q$, where we denoted $c \stackrel{\text{def}}{=} g'(1)$.

By integrating this formula, we can get a formula for $g(x)$. The integration formula depends on where $q = -1$ or $q \neq -1$.

For $q = -1$, we get $g(x) = c \cdot \ln(x) + C_0$, where C_0 is the integration constant, which contradicts to our requirement that $g(0) = 0$. So, this case is impossible, and $q \neq -1$.

For $q \neq -1$, we get $g(x) = \dfrac{c}{q+1} \cdot x^{q+1} + C_0$. In this case, the requirement that $g(0) = 0$ implies that $q > -1$ and $C_0 = 0$. Thus, $g(x) = \text{const} \cdot x^\eta$, for $\eta \stackrel{\text{def}}{=} q + 1$.

Thus, we arrive at the following conclusion.

Resulting formula for computing the overall complexity of the design. If $C_1, \ldots, C_n$ are complexities of the individual sections, then we can compute the overall complexity C of the design by using the formula

$$g(C) = \sum_{i=1}^{n} g(C_i).$$

For the above scale-invariant function $g(x) = \text{const} \cdot x^\eta$, this implies that

$$\text{const} \cdot C^\eta = \sum_{i=1}^{n} \text{const} \cdot C_i^\eta.$$

This formula can be simplified if we divide both sides by the same multiplicative constant:

$$C^\eta = \sum_{i=1}^{n} C_i^\eta.$$

We already know that $C_i = v_i \cdot (1 - v_i)$, so we conclude that

$$C^\eta = \sum_{i=1}^{n} (v_i \cdot (1 - v_i))^\eta = \sum_{i=1}^{n} v_i^\eta \cdot (1 - v_i)^\eta,$$

i.e., that

$$C = \left(\sum_{i=1}^{n} v_i^\eta \cdot (1 - v_i)^\eta \right)^{1/\eta}.$$

This formula can be further simplified if we recall that our objective is not so much to come up with a numerical value of the design, but rather to understand which designs are possible under given technology and which are not, by comparing the resulting complexity value with a threshold value describing the state-of-the-art technology.

From this viewpoint, it does not matter whether we use the original scale for the complexity or use any monotonic non-linear re-scaling – as long as the ordering between different values of design complexity remain the same.

Thus, instead of the original difficult-to-describe complexity value C, it makes sense to consider simpler-to-describe re-scaled value $\widetilde{C} = C^\eta$ for which

$$\widetilde{C} = \sum_{i=1}^{n} v_i^\eta \cdot (1 - v_i)^\eta.$$

Conclusion. This is exactly the empirical formula describing the complexity of different designs.

Thus, we have indeed provided a theoretical explanation for this empirical formula – an explanation that uses both fuzzy ideas and fuzzy results.

Acknowledgments

This work was supported in part by the National Science Foundation grants HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and DUE-0926721, and by an award "UTEP and Prudential Actuarial Science Academy and Pipeline Initiative" from Prudential Foundation.

The authors are thankful to the anonymous referees for valuable suggestions.

References

[1] J. Aczél, *Lectures on Functional Equations and Their Applications*, Dover, New York, 2006.
[2] A. Bandyopadhyay and S. Bose (eds.), *Additive Manufacturing*, CRC Press, Boca Raton, Florida, 2015.
[3] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Addison Wesley, Boston, Massachusetts, 2005.
[4] I. Gibson, D. Rosen, and B. Stucker, *Additive Manufacturing Technologies: 3D Printing, Rapid Prototyping, and Direct Digital Manufacturing*, Springer Verlag, New York, 2015.
[5] G. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic*, Prentice Hall, Upper Saddle River, New Jersey, 1995.
[6] H. T. Nguyen and E. A. Walker, *A First Course in Fuzzy Logic*, Chapman and Hall/CRC, Boca Raton, Florida, 2006.
[7] C. Spadaccini, "Topology optimization and reverse homegenization in design of thermoelastic properties", *Proceedings of the ASME 2015 International Mechanical Engineering Congress & Exposition IMECE'2015*, Houston, Texas, November 13–19, 2015.
[8] L. A. Zadeh, "Fuzzy sets", *Information and Control*, 1965, Vol. 8, pp. 338–353.