Relationship Between Cognitive Types Of Teacher Content Knowledge And Knowing-To Act: A Mixed Methods Study Of Mexican Borderland Middle School Teachers

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Recommended Citation

Cruz Quinones, Maria De Los Angeles, 'Relationship Between Cognitive Types Of Teacher Content Knowledge And Knowing-To Act: A Mixed Methods Study Of Mexican Borderland Middle School Teachers' (2014). Open Access Theses & Dissertations. 1223.

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RELATIONSHIP BETWEEN COGNITIVE TYPES OF TEACHER CONTENT KNOWLEDGE AND KNOWING-TO ACT: A MIXED METHODS STUDY OF MEXICAN BORDERLAND MIDDLE SCHOOL TEACHERS

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Dedication

I dedicate my dissertation to my father who supported me throughout my journey of studies for the PhD and who recently passed away. He taught me to be a hard worker and to never give up. This dissertation is also lovingly dedicated to my family. A special feeling of gratitude for my loving mother, Maria Dolores, whose words of encouragement and push for tenacity rings in my ears. I dedicate this work and give special thanks to my husband Luis Armando for his understanding and support of this accomplishment. Finally, I am grateful for the unending support from my sisters Reyna, Rosario and Dolores who never left my side and are very special.
RELATIONSHIP BETWEEN COGNITIVE TYPES OF TEACHER CONTENT KNOWLEDGE AND KNOWING-TO ACT: A MIXED METHODS STUDY OF MEXICAN BORDERLAND MIDDLE SCHOOL TEACHERS

by

MARIA DE LOS ANGELES CRUZ QUINONES, B.S., M.Ed.

DISSERTATION

Presented to the Faculty of the Graduate School of
The University of Texas at El Paso
in Partial Fulfillment
of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education
THE UNIVERSITY OF TEXAS AT EL PASO
December 2014
Acknowledgements

I would like to gratefully and sincerely thank my Chair, Dr. Mourat Tchoshanov, for his guidance, understanding, and patience during my graduate studies at the University of Texas at El Paso. I would also like to thank Dr. Ana Macias for her assistance, mentorship, and most importantly, for her friendship. She always was there with the best disposition to help me to accomplish this important milestone in my life. I would like to thank all the individuals who became involved in the process that I went through in order to achieve this accomplishment.

I would like to thank the participants of my study because this study would not have been possible without their participation. I would also like to thank Ricardo Vizcarra and the rest of the teachers who helped me to verify the translation of one of the instruments used in the study.

Finally, and most importantly, I would like to thank my loving family for their tolerance, support, understanding, and unending encouragement for the completion of my PhD studies.
Abstract

This study analyzed middle school mathematics teachers’ content knowledge and its relationship with teachers’ “knowing-to act” ability. Understanding what kinds of knowledge has a direct influence on teaching practices and student learning is critical in order to improve teacher education programs and professional development. An Explanatory sequential mixed methods design was used in the study. It involved collecting quantitative data and explaining the quantitative results with in-depth qualitative data. In the quantitative phase of the study, two surveys were administered to N=70 middle school mathematics teachers in the Mexican borderland to assess whether their mathematical content knowledge was related to their “knowing-to act”. The correlational analysis of these surveys showed no statistically significant correlation between overall mathematical teacher content knowledge (total score on TCKS) and the “knowing-to act” ability (KtAS). However, a statistically significant correlation between the specific cognitive type of teacher knowledge - models and generalizations - and the “knowing-to act” was reported. The qualitative phase provided a deeper understanding of the quantitative results: the exploration of the “knowing-to act” enacted during mathematics instruction with four middle school mathematics teachers from the quantitative sample was conducted using a specifically designed classroom observation protocol. The analysis of the observation together with the results of the KtAS provided revealing differences among teacher’s actions observed and the teacher’s responses on the survey. Overall, the analysis of the qualitative data reflected findings from the quantitative phase of the study. Two main findings were reported in the study: (a) the lack of correlation between the mathematical teachers content knowledge and their “knowing-to act” during teaching mathematics, which was reflected by the data collected from the case studies; (b) a statistically significant correlation between knowledge of models and generalizations (T3), which added to the discussion that teachers who performed higher on the cognitive type 3 items of the TCKS were able to know how to act at the moment more frequently than teachers with a limited T3.
This research provided in-service teachers and other participants in the education field with awareness about the active knowledge that is needed to enact the teachers’ knowing-to act in teacher preparation programs in Mexico that can be used to support teachers and students in the United States. Further studies are needed in which the association and exploration of other kinds of knowledge for teaching mathematics and students learning can be analyzed. For instance, research on “knowing-to act” in the United States or other countries can also be worthy of a study; how would teachers act in KtA situations during their mathematics instruction in the USA, Canada, or Russia? In addition, this study allows comparisons among Mexico and countries where data is already collected in regards to teacher knowledge in the area of Mathematics, such as Russia, the U.S., Latin American countries, and other countries that participated in the TEDS-M Study 2012.
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Chapter 1: Introduction

1.1 Overview

This study examined the mathematical teacher content knowledge of Mexican middle school teachers in Mexico. A second type of teacher knowledge, “knowing-to act”, was also analyzed in order to determine if any relationship existed between mathematical teacher content knowledge and “knowing-to act”. This study provided an understanding and description of this relationship. A mixed methods study was developed in which an explanatory sequential research design was used. During the quantitative phase, two surveys were administered, one that measured the mathematical teacher content knowledge, and the second one explored the teachers’ “knowing-to act”. Then, looking for a deeper understanding of this relationship, a qualitative phase was implemented. And interview and class observation data were collected in order to enrich the interpretation of information provided in the quantitative phase.

This chapter introduces several important aspects of this research. The rationale for conducting this study will be presented in the research problem section. In addition, the purpose of the study and the description of the specific research site will be provided. Also, reasons for the importance and relevance of conducting this study in Mexico at the middle school level will be discussed. An introduction to the conceptual framework and philosophical foundations that were used as lenses for conducting this study will be presented. Additionally, a definition of terms and necessary background information will be provided.

1.2 Research Problem

Teacher knowledge is a critical topic for teaching, learning and culture. The process of teaching and learning is a decisive aspect to shape culture. Therefore, to be focused on components of this process such as teacher knowledge is relevant for the educational field. Since the knowledge possessed by teachers has an impact on students’ learning, it is an educational issue worthy of study. In the past twenty-five years, a growing number of studies focusing on teacher knowledge have been conducted (e.g. Shulman, 1986; Taylor, 2011; Tchoshanov, 2011).
However, teacher knowledge is very broad, and it includes different kinds of knowledge. The criticism and classification of the different types of knowledge that a teacher should possess in order to teach mathematics effectively become relevant for teachers, teacher education, scholars, and policy makers. These classifications and conceptualizations of teacher knowledge will allow pre-service and in-service teachers to become aware of the knowledge that they need to possess as the knowledge base for teaching mathematics. Based on research focusing on this worthy issue, policy makers and teacher education programs can make decisions about how teachers should be prepared in order to help students to learn mathematics.

In the field of mathematics education, scholars have addressed only some kinds of knowledge and their components, as it is discussed in chapter two (e.g., An, Kulm, & Wu, 2004; Davis, & Simmt, 2006; Tchoshanov, 2011). Some of the categorizations of teacher knowledge in mathematics are mathematical teacher content knowledge (e.g. Tchoshanov, 2011), pedagogical content knowledge (e.g. An et al., 2004), knowledge of curriculum (e.g. Ball, Thames, and Phelps, 2008; Shulman, 1986), and “knowing-to act” (e.g. Mason, 1998), among others. The complex nature of the mathematical knowledge for teaching challenges investigators to research and define with precision each kind of teacher knowledge. In addition, interactions among these kinds of knowledge are crucial as a part of the knowledge base for teaching mathematics and, as such, some scholars have recognized and studied it (e.g. An et al., 2004; Koehler and Mishra, 2009). Therefore, further research is needed about the nature of the interaction among specific kinds of teacher knowledge. Additionally, knowing what kinds of knowledge have a direct influence on teaching practices could be used to enhance teacher education programs and teaching practices. This study aimed to provide insights that will provide awareness to teacher education programs and policy makers to make important decisions in regards to what teachers need to know for teaching mathematics in an effective manner.

Based on the need for research about interactions among different kinds of mathematical teacher knowledge, two types of teacher knowledge were explored in this study: mathematical teacher content knowledge and “knowing-to act”. Mathematical teacher content knowledge is the
first type of knowledge that was studied. There is an assumption in the mathematics education field that to some extent, the higher the level of mathematics studied by teachers the higher pedagogical skills they develop (Sorto, Marshall, Luschei and Carnoy, 2009). Research on the area of teacher knowledge (e.g. Sorto et al., 2009) shows that this assumption is not always correct, and this study added to this discussion. Sorto and colleagues (2009) showed that mathematics teachers in Panama and Costa Rica at the seventh grade are proficient in the content, but they exhibited a deficient teaching of mathematics. As reported in the previous study, a weak mathematical teacher content knowledge primarily affects mathematical instruction, which can in turn cause students have poor opportunities to learn (Tchoshanov, 2011), and student frustration and a negative attitude towards mathematics (Sorto et al., 2009). Moreover, in mathematics it is very important that teachers be able to make connections among mathematical concepts in order to help students to make sense of mathematics.

The second type of knowledge that was studied is “knowing-to act”. “Knowing-to act” is the process in which “knowledge [that] enables people to act creatively rather than merely react to stimuli with trained or habituated behavior” (Mason and Spence, 1999, p.136). According to Mason and Spence (1999), there is an absence of “knowing-to act” that leads mathematics teachers not to be able to respond creatively in the moment even when they possess mathematical and pedagogical content knowledge. Consequently, this absence may limit the learning opportunities that teachers can offer. Research on this kind of knowing may help teachers develop the active knowledge needed to respond creatively in the moment.

Around the world, there is interest in researching teacher preparation programs to provide educational policy makers relevant information such as which types of teacher knowledge have a stronger impact on student learning (Tchoshanov, 2011). Several international studies focused on teacher preparation in countries such as Russia, USA, Singapore, Finland, etc. (e.g. TEDS-M Study, 2012). Research shows that most teacher preparation programs in the United States focus mainly on content. Fewer programs emphasize pedagogical knowledge, fewer still attempt to include content and pedagogy, but these programs have courses that focus primarily on one area
or another (Davis & Simmt, 2006). American teacher education programs often use a coarse distinction between mathematical knowledge and instructional knowledge. Mathematics courses are offered by the mathematics department, and the instructional courses are offered by education departments (Davis & Simmt, 2006; Sorto et al., 2009). In most American universities, teacher preparation programs have few courses that integrate the content and the pedagogy needed to teach that subject matter (Davis & Simmt, 2006). This problem was also pointed in Grossman (2008), he mentioned that teacher preparation programs do not provide enough preparation to teachers in order to be ready to teach in mathematics classrooms.

In Mexico, this phenomenon is also identified. For example, an analysis of the degree plan for mathematics middle school teacher preparation program (DGESPE, 2010) at “the normal school” (La normal superior), which is the main institution for teacher preparation in this country, shows that mathematics teacher preparation in Mexico fits with the patterns found in the United States’ teacher preparation programs (Davis & Simmt, 2006) as will be shown in the next chapter. It is critical for teachers to be prepared with content, pedagogy, and the blend of both content and pedagogy, among other important aspects for teaching mathematics (Shulman, 1987; Ball et al., 2008). Teacher preparation programs need to provide the support that teachers need in order to help their students to make sense of the topics, instead of only providing teachers with tools to present sequences of instructions.

1.3 **PURPOSE OF THE STUDY**

The intent of this study was to measure middle school mathematical teachers’ content knowledge and its relationship with teachers’ “knowing-to act” ability. An explanatory sequential mixed methods design was used, and it involved collecting quantitative data first and then explaining the quantitative results with in-depth qualitative data. In the quantitative phase of the study, two surveys were administered in order to collect data from 70 mathematics teachers in middle schools in the Mexican borderland to assess whether their mathematical content knowledge was related to their “knowing-to act”. The qualitative phase was conducted as a
follow up to the quantitative phase to help explain the quantitative results. In this explanatory follow-up, the plan was to explore the knowing-to enacted during mathematics instruction.

The research was conducted in a Mexican city located on the border with the United States. Since I am Mexican and I live in the Mexican borderlands, it placed me in a privileged condition to conduct research from the cross-cultural community perspectives that have shaped me. My education -from elementary school to a Master’s degree- was earned in Mexican public schools and universities. Thus, I was familiar with the social context of public education in Mexico. My bachelor’s degree is in computer science engineering and my Master is in mathematics education. Thus, my education focused on mathematics that allowed me to be a mathematics teacher in one University of that country. Therefore, my background enhanced my ability to do research in my own country.

Educational research in Mexico has been limited due to the lack of research funding (Reyes, 2013). In Mexico, there are 1804 educational researchers; this number is too small in comparison with the large Mexican educational system and the educational issues that have existed for several decades (The Organization for Economic Co-operation and Development-Center for Educational Research and Innovation, Revision Nacional de Investigación y Desarrollo Educativo, Reporte de los examinadores sobre Mexico, 2004). An attempt to compare the number of educational researchers in Mexico with the number of researchers in the area of education in the U.S. was done. However, information about it was not found.

Furthermore, the educational researcher preparation is very limited, only 14 PhD programs and 51 master’s programs are recognized by the National advising board of graduate studies CONACYT in Mexico (CONACYT, 2013). The absence of institutional conditions that allow the conduct of rigorous research also contributes to the limited opportunities for conducting educational research in Mexico (Barriga, 1998). In addition, an existing gap of research at the elementary and middle school levels that analyzes specific mathematical knowledge and instructional skills required for teaching effectively has been identified (National Mathematics Advisory Panel, Report of the Task Group on Teachers and Teacher Education,
2008b). Of great relevance is to point out that in Mexico, research on the middle school level is critical because this level has had the greater dropout rate from the basic education for several years. Basic education in Mexico consists of preschool, elementary and middle school (Reimers, 2001; Blasco, 2003, Secretaría de Educación Pública, 2010).

The target population of this study is mathematics middle school Mexican teachers currently teaching in Mexico on the border with United States. A relevant feature of the participants that were part of the research is that they did not belong to any “minority group” such as the perceptions that in the United States people have towards Mexican people (Orellana, 2001). The participants are Mexican origin teachers who lived in Mexico. They did not see themselves as “minorities” as most Mexican people are often considered by the American society (Oropesa & Landale, 2009) and specifically in the educational research. Therefore, becoming aware of the socio-cultural context where these participants lived and taught was not the same context typically known by people and educational researchers in the United States is critical for the readers.

This study added to current knowledge that may help to restructure mathematics teacher education programs, and hopefully impact the teaching and learning process in Mexican middle schools. According to the Ministry of Public Education in Mexico (Secretaría de Educación Pública, 2004), teaching and learning mathematics in middle schools focus on three purposes: to develop mathematical skills, promote positive attitudes towards mathematics, and acquisition of mathematical knowledge. As it will be demonstrated in Chapter 2, few studies in Mexico focus on teacher content knowledge and its relationship with teaching practices at the middle school level. Based on these needs and the limitations of educational research in Mexico at the middle school level in regards of mathematics, doing research in Mexico is critical and essential. In addition, conducting research in Mexico will allow for comparison studies with other countries, such as the United States or Russia, where studies of teacher content knowledge have been conducted (e.g. Tchoshanov, 2011).
The broader educational purpose of this study focused on understanding how mathematical teacher content knowledge influences teachers’ instructional decisions to act in particular ways. I used the quantitative component of this study to look critically at the interrelationship of the mathematical teacher’s content knowledge and the teachers’ “knowing-to act” at the moment of instruction. The qualitative component explored in depth the findings of the quantitative component, specifically the relationship between these two types of teacher knowledge, in order to provide an understanding and description of the teachers’ “knowing-to act” influenced by the mathematical teacher content knowledge.

1.4 CONCEPTUAL FRAMEWORK

This paper analyzed the mathematical teacher content knowledge and teachers’ “knowing-to act” (Mason and Spence, 1999) possessed by Mexican in-service teachers in classrooms at the middle school level in Mexico. This research drew on the Shulman’s teacher knowledge model (Shulman, 1986, 1987). Shulman (1986) distinguished three categories of teacher knowledge: a) content knowledge; b) pedagogical content knowledge; and c) curricular knowledge. He defined content knowledge as “the amount of organization of knowledge per se in the mind of the teacher” (Shulman, 1986, p.9). This category of knowledge includes knowledge of facts, connections, models, generalizations and how the understanding of this content knowledge is structured and generated. This study focused on this category of content knowledge, specifically on the mathematics content that teachers need for teaching effectively. Several scholars have focused on this category in mathematics (e.g. Ball, Thames, & Phelps, 2008; Rowland, 2008). However, few studies have examined the cognitive types of teacher content knowledge (e.g. Tchoshanov, Lesser, & Salazar, 2008) and its relationship with student achievement (e.g. Tchoshanov, 2011). This research examined the three different cognitive types of mathematical teacher content knowledge: cognitive type 1 (which refers to the teacher content knowledge of facts and procedure); cognitive type 2 (the knowledge of concepts and connections); and cognitive type 3 (knowledge of models and generalizations).
Following Mason and Spence (1999), in this research “knowing-to act” refers to “active knowledge which is present in the moment when it is required.” (p.135). They mentioned that this construct depends on the structure of attention in the moment, in other words, “knowing-to act” depends on what one is aware of (Mason, & Spence, 1999). Also a distinction was made about the different forms of knowing that are the central focus of institutionalized education. These forms of knowing are: factual knowledge, which is the knowing-that; knowing-how, which refers to knowing the technique and skills; and knowing-why, to be able to have a story to “account for phenomena and actions” (Mason & Spence, 1999, p.137). These three forms of knowing constitute the knowing-about as we can observe in Figure 1.1

![Figure 1.1: Interaction of Knowing-to and Knowing-about (Mason & Spence, 1999, p. 145)](image)

The nature of the interaction of these three types of knowing is complex as it could be observed in the Figure 1. We can say that some knowing-how is directly influenced by knowing-that, “knowing-to act” depends on knowing-how, and knowing-why involves knowing-that and knowing-how (Mason & Spence, 1998). More details about this interaction are described in chapter two.

1.5 **PHILOSOPHICAL ASSUMPTIONS**

This study was conducted considering multiple philosophical positions. Three philosophical foundations supported the study. One philosophical foundation was post-positivism, another was constructivism, and the last one was pragmatism. The first two
philosophical orientations as recommended in Creswell and Plano (2011) were considered throughout the implementation of the first three phases. This study employed a mixed methods approach in which both quantitative and qualitative approaches were involved. Thus, pragmatism was considered in the last phase of the study.

Post-positivist philosophies of knowledge generally argue that researchers “make claims for knowledge based on (1) determinism or cause-and-effect thinking; (2) reductionism by narrowing and focusing on select variables to interrelate; (3) detailed observations and measures of variables; and (4) the testing of theories that are continually refined” (Creswell & Plano, 2011, p.40). This perspective was taken to develop instruments, measure variables, and evaluate statistical results in the first phase of the study. Then, to move to the qualitative phase, the shift to the use of the lens of the constructivism perspective was applied.

The constructivist worldview is commonly associated with qualitative research. Constructivism is made up by “the understanding or meaning of phenomena, formed through participants and their subjective views” (Creswell & Plano, 2011, p.40). The overall philosophical foundations changed in this study according to the research design used, and it switches from post-positivism to constructivism.

The philosophical foundation of pragmatism finds the reality as both singular and multiple. Pragmatism focuses “on the consequences of the research, on the primary importance of the question asked rather than the methods, and on the use of multiple methods of data collection to inform the problems under study” (Creswell & Plano, 2011, p.41). Pragmatism is used because this is a mixed methods study which included collecting data from surveys, observations and interviews as this philosophical assumption focused. Therefore, pragmatism was used as an umbrella foundation because of the sequential implementation of two strands where constructivism is used in the qualitative strand and post-positivism in the quantitative strand.
1.6 **Research Questions**

In order to accomplish the purpose of the study, the following questions guided the research:

1. To what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ knowing-to-act?
2. How do the teachers act in the KtA situations occurring in mathematics classroom?
3. How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?
4. To what extent do the qualitative findings explain the quantitative results of the study?

The first research question is quantitative in nature. Therefore, statistical methods were implemented to get a response to it. The nature of the second and third research question is qualitative. The implementation of qualitative methods was done. And the four research question is mixed methods in nature. In order to answer this question, the interpretation of the integration of the results of the first research question with the findings of the research question 2 and 3 was conducted.

1.7 **Significance of the Study**

The insights of this investigation provided institutions of higher education in Mexico and around the world with valuable information to contribute to the development of teacher education pre-service programs, specifically in the area of secondary school mathematics. Scholars argue conducting research on components, categories, features, and indicators of the knowledge base for teaching will generate a body of knowledge that could support teacher preparation programs (Even, 1990; Shulman, 1999; Pinto & Gonzalez, 2008). Since this study addressed the gap of research focused on teachers in mathematics education in Mexico, this study has a greater contribution to the field of mathematics education. In addition, this study
sought to contribute to the literature review on types of teacher knowledge studying mathematics teachers from perspectives not considered in Mexico (Pinto & Gonzalez, 2008), such as “knowing-to act”.

This study allowed future comparisons among Mexico and countries in which data is already collected in regards of teacher knowledge in the area of Mathematics, such as Russia, U.S., Latin American countries, and other countries that participated in the TEDS-M Study 2012. These comparisons will let policy makers, teachers, and scholars to learn about other teacher education programs, and situate the Mexican teacher preparation internationally, in order to be able to make improvements on the preparation of Mexican mathematics middle school teachers.

Additionally, the understanding and description presented in this study provides awareness of the importance of accentuating the “knowing-to act” in teacher preparation programs. The findings of the study allow in-service teachers and other participants in the education field to be aware of the active knowledge that is needed in order to enact the teachers’ “knowing-to act” during mathematics instruction.

The United States is made up of a growing number of students who have migrated from Mexico (Pew Research Center, 2009). It is projected that the Hispanic school-age population will increase by 166% by 2050 (to 28 million from 11 million in 2006), of which 69% of the Hispanic school-age population are of Mexican origin (Fry & Gonzales, 2008). Most of these students are incorporated in U.S. public schools at different grade levels in the entire country (Crosnoe, 2006). This study aimed to provide a newer perspective in regards to the realities of Mexican public schools in mathematics courses at the middle school level. Additionally, this study may help U.S. teachers understand Mexican students’ learning styles in order to offer learning environments in which students feel comfortable to construct their knowledge.

1.8 **The Role of the Researcher**

During my studies from elementary school to high school in Mexico, I had positive experiences regarding my mathematics classes. As a young person, I participated in several
mathematics competitions that took place in Ciudad Juarez. When I decided to go to college I chose a bachelor’s degree in engineering in computer science because that degree has numerous mathematics courses. Throughout my undergraduate studies I became aware of the need to find different ways to help students to understand mathematical concepts. Then, I realized that I had the facility to understand topics faster than my classmates, and I helped them to understand and grasp meaning. After finishing my engineering degree, I started to teach mathematics courses. I saw the same situation as when I was a student; many students faced difficulties grasping certain mathematical topics. At the moment, I felt that I wanted to help my students, but I could not do it because I did not have the preparation and knowledge base for teaching. Therefore, I decided to pursue a master’s degree in mathematics education in order to develop tools, knowledge and skills to support my students to construct their own knowledge. At that time, I observed the absence of research focused on many aspects of teaching mathematics. Thus, I started a Ph.D. program at The University of Texas at El Paso. During my graduate preparation, I discovered that there is a dearth of studies focused on how teachers know-to act in the moment during their lesson plans. Therefore, I became interested in the mathematics teacher knowledge.

I was born in a large border city in Mexico. Since 1973 its population has increased 400% (Morales, Rodriguez, and Sanchez, 2013). This growth of population is mainly due to the need for workers required by industrial companies in this region of the country (IMIP, 2003), and the accessibility of migration to the United States. The industrial manufacturing and automobile are the main sources on which the economy of the city is based (Monarrez, 2012). Since 2006 in the whole country, there is a war against the drugs trade. Several Mexican cities have suffered a huge impact on their functioning that has affected their citizens (Monarrez, 2012; Morales et al., 2013). The border cities are some of the areas most affected by the war and crime since their location next to the U.S. (Monarrez, 2012). During the last 6 years, thousands of homicides have occurred across the city, even in front of school buildings. This is the situation that people, students, and teachers live day-to-day in the city that was the site of this research.
1.9 Definition of Terms

Glasser and Smith (2008) pointed out that providing clear meanings for conceptual terms is crucial for the effective communication of educational research to the readers. A definition of terms will be provided to assist readers.

Teaching

This term is used in this study as Ball and her colleagues define as “everything that teachers must do to support the learning of their students” (Ball et al., 2008, p.395).

Teacher content knowledge

Teacher content knowledge in the sense of the study is defined, Shulman, as “the amount of organization of knowledge per se in the mind of the teacher” (Shulman, 1986, p.9).

Mathematical Teacher content knowledge

This term is used as Shulman (1986) defined content knowledge. However, this study focused on mathematics. Thus, mathematical teacher content knowledge includes knowledge of facts, procedures, connections, models, and generalizations.

Cognitive type of teacher content knowledge

In this study, the term cognitive type of teacher content knowledge means the specific type of mathematics knowledge and reasoning processes required by teachers in order to accomplish successfully an activity or task. In other words, we refer to cognitive types of mathematical teacher content knowledge as the knowledge of facts and procedures, knowledge of concepts and connections, and/or knowledge of models and generalizations (Tchoshanov, 2011)

Cognitive type 1

The term cognitive type 1 refers to knowledge of facts and procedures. This cognitive type 1 involves the knowledge required to perform routine procedures such as basic mathematical facts, rules, and algorithms (Tchoshanov, 2011).
Cognitive type 2

This term refers to the knowledge of concepts and connections. This cognitive type of mathematical teacher content knowledge involves conceptual understanding that allows making useful connections between mathematical topics, concepts, and procedures (Tchoshanov, 2011).

Cognitive type 3

Cognitive type 3 means knowledge of models and generalizations. This type of knowledge allows generating and testing conjectures, making generalizations, and providing theorems, among other aspects of mathematics. (Tchoshanov, 2011).

Knowing-to act

This study uses this term based on Mason (1998): “the kind of knowledge which enables people to act freshly and creatively.” (p.245). In other words, “knowing-to is active knowledge which is present in the moment it is required.” (Mason & Spence, 1999, p.135). “Knowing-to act” (KtA) is the knowledge that is recalled and used by teachers in the moment required. This type of knowing depends on the situations or context during mathematics instruction, and the level of awareness possessed by teachers.

Knowing-about

Knowing-about is constituted by knowing-that, knowing-how, and knowing-why. This type of knowledge is accumulated and trained to establish a habituated behavior. In other words, it involves factual knowledge, knowledge of techniques and skills to perform acts, and “having stories to account for phenomena and actions” (Mason & Spence, 1999, p.137).

KtA situations

These situations challenge the “knowing-to act” of the teachers and are commonly found during mathematics instruction. These situations are called “knowing-to Act” situations (KtA situations).
**Pedagogical content knowledge**

The term pedagogical content knowledge is used in the study as Shulman (1986) defined it. Shulman points out that this kind of knowledge not only considers the subject matter knowledge, but it is more focused on the aspects for teaching the subject matter such as different ways of representations of ideas, analogies, illustrations, explanations and demonstrations. Pedagogical content knowledge also includes the understanding of students’ perceptions and misconceptions of what makes a topic to be easy or complex to learn.

**1.10 Organization of the Study**

This study is organized in five chapters: introduction, literature review and conceptual framework, methodology, results and findings, and discussions and conclusions. The second chapter will review the literature about teacher knowledge, specifically mathematical teacher content knowledge and “knowing-to act”. A section of the second chapter will introduce the secondary education in Mexico. The literature review documented the existing lack of research in Mexico on mathematical teacher content knowledge and its relation to “knowing-to act” in the moment. The conceptual framework will also be discussed in the second chapter. The methodology will be presented in the third chapter, including sections describing the context, participants, each phase of the research design, instruments, data analysis and the methods to ensure the validity and trustworthiness of the study. Chapter four will discuss the results of each phase that answer the research questions. Finally, chapter five will provide interpretations and discussions about the research process and findings as well as limitations, implications for practice, and suggestions for further research.

**1.11 Summary**

In this chapter, several important aspects of the research were uncovered. The critical needs for conducting this study are clearly exposed in the research problem section. Research is needed about the interaction among specific kinds of teacher knowledge such as mathematical teacher content knowledge and “knowing-to act”. To know what kinds of knowledge have a
direct influence on teaching practices is critical in order to enhance teacher education programs and teaching practices. Thus, the purpose of the study was to measure middle school mathematical teachers’ content knowledge and its relationship with teachers’ “knowing-to act” at the moment and the understanding of this relationship. In order to achieve this purpose, these were the research questions of this study: 1) to what extent is the cognitive type of teachers’ knowledge associated with teachers’ knowing-to act? 2) How do the teachers act in KtA situations occurring in mathematics classroom? 3) How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey? And 4) to what extent do the qualitative findings explain the quantitative results of the study?

This research provided in-service teachers and other participants in the education field with awareness about the active knowledge that is needed to enact the teachers’ “knowing-to act: in teacher preparation programs in Mexico that can be used to support teachers and students in the United States. Additionally, a definition of terms list and necessary background information were provided.
Chapter 2: Literature Review and Conceptual Framework

2.1 Overview

In this chapter, two main components of my research will be presented: the literature review and conceptual framework. The literature review section will describe the context of education in Mexico in order to provide a better understanding of this study. The historical antecedents and the current situation of education in Mexico will be discussed. Lacunas in research on mathematical teacher knowledge at the middle school level in this country will be identified.

The second central component is the conceptual framework. This component describes the main studies that have placed an emphasis on teacher knowledge (e.g. Shulman, 1986, 1987; Ball, Thames, & Phelps, 2008). Conceptualizations of this field of inquiry will be discussed, such as content knowledge, pedagogical content knowledge, and “knowing-to act”. A comprehensive analysis of these conceptualizations and their relevance to teaching and learning mathematics will be also presented.

2.2 Literature Review

The literature review introduces the public educational system in Mexico. Historical antecedents of this system are also presented. In this study, I focused on middle school teachers, thus, relevant features of middle schools in Mexico are discussed, such as the curriculum, objectives, etc. Of great relevance is to introduce the mathematics teacher preparation at the middle school level in Mexico due to the fact that this research studied mathematics middle school teachers in this country. Also discussed is the current situation that teachers are going through in Mexico due to the new reform where several changes are directly affecting teachers. This chapter also reviews educational research in Mexico where numerous gaps were identified.

2.3 Historical Antecedents of Contemporary Public Education in Mexico

In 1917, the president of Mexico Venustiano Carranza introduced the concept of democratization into Mexico’s educational establishment, and it was stated in the Mexican
Constitution. The Constitution clearly stated that education was offered free and mandatory for all students only at the elementary school level (primaria, grades 1-6). Justice and independence was promoted in the Mexican constitution of 1917. The institutions of higher education were given autonomy to govern by themselves with the expectation that they contribute by providing educational research to enhance teaching and learning. However, states were not organized and struggled to follow what the constitution dictated, thus, it prolonged the need for a systematic examination of educational regulations in Mexico.

A relevant event occurred in 1921. The president’s University of Mexico City Jose Vanconcelos created the Ministry of Public Education (Secretaría de Educación Pública-SEP). From that time, the SEP has been in charge of overseeing and operating educational institutions at the national level for the different content areas. Its two main responsibilities are the development of a national curriculum and textbook adoptions (Secretaría de Educación Pública, 2010).

A significant event that would transform teacher education occurred in 1943; the creation of institutions that have had the responsibility of preparing teachers, “Normal Superior Schools”. However, initially these institutions prepared teachers only for elementary schools, due to the fact that mandatory education in Mexico included elementary school only. Nowadays, Normal Superior schools also prepare teachers for middle schools. Prior to 1943, middle school teachers only earned a bachelor’s degree in the area that they were teaching. In other words, there was no formal preparation for middle school teachers prior 1943.

Concurrently, the creation of a national teachers’ union (Sindicato Nacional de Trabajadores de la Educacion-SNTE) occurred to enhance the education in Mexico. SNTE advocates teachers’ rights and supervises working conditions (SNTE, 2011). In fact, Mexican law states that all teachers of the public education system must belong to this union (SNTE, 2011). Currently, school administrators, retired teachers, and part-time teachers also belong to the SNTE. The SNTE works collaboratively with the SEP on several employment issues such as the establishment of teacher salaries, schedules, management, and yearly pay increases.
In 1988, an amendment to the Mexican Constitution was promulgated by Mexico’s president Carlos Salinas de Gortari. This amendment consisted of the addition of secondary education to the basic education that is mandatory for all citizens. Furthermore, the establishment of admission assessments for students at the high school level and higher education was mandated.

In 2006, a standardized test was implemented in Mexico. It is called National Assessment of Academic Achievement in School Centers Test (Evaluación Nacional del Logro Académico en Centros Escolares-ENLACE). ENLACE standardized test is administered to elementary and middle school students throughout the country. This test measures mainly the content areas of mathematics and Spanish. However, this test does not have any influence on students’ grades. The test is administered every year in public and private schools. Students from 3rd grade to 6th grade are required to complete the test. Since 2009, ENLACE has been applied to the three grades included in middle school. Prior to 2009, the test was applied only to 9th grade students.

2.4 CURRENT SITUATION AND NEW REFORM

In Mexico, at this time, an educational reform is taking place. The motivation to promulgate this reform comes from a strong analysis and discussions about education (SEP, 2013a). This process involved educational authorities, SNTE managers, parents, teachers, and school administrators. Mexican government, and the union congress allowed the promulgation of this reform (SEP, 2013a) as well as agreements among the main political parties. This reform made several amendments to the Mexican Constitution in regards to basic education. Below are the main points that were modified:

- The constitution states that education must be of quality and with equity. Teachers’ role and permanent teacher professional development are considered in order to achieve an education of quality and with equity.
- The establishment of the teacher professional service to prepare, train, and assess teachers, administrators, among other aspects, is stated in the Constitution.
• The National System of Educational Assessment (Sistema Nacional de Evaluación Educativa-SNEE) was created to ensure the quality of the educational services provided in Mexico at the basic level. The SNEE has the responsibility of: providing indicators of what is working adequately, what it is not working well and the attempts to improve it, and identifying teachers and administrators’ needs to improve, develop their strengths and overcome their weaknesses.

The Constitution states that the National Institute for the Evaluation of Education (Instituto Nacional para la Evaluación de la Educación INEE) must evaluate the quality of education yearly. Thus, the INEE assesses educational authorities, programs, study plans, methods, and educational resources. In addition, the evaluation of schools, teachers, administrators and supervisors of private schools will be also implemented. Finally, this reform states that the only way to become a teacher of pre-school, or elementary school, or middle school is through the successful participation in the National Competition of Teaching Positions (Concurso Nacional de Plazas).

This reform has resulted in several meetings against it. This reform is different from the previous ones due to the fact that this is the first time that teachers will be evaluated. Principals and supervisors will be also evaluated. These evaluations have never been done before through exams, observations, student achievement, and other explicit method. In addition, this new reform is taking out some of the leverage that the teachers’ union SNTE had as on the hiring of teachers. SNTE will not be able anymore to decide who will be hired (SEP, 2013a). As well as, the supervision of the teachers’ work will also be done by the new institute (INEE). Teachers, or principals, or supervisors that do not perform well in the evaluation after the third attempt will be asked to retire (if they are almost ready to do it) or to get another position in which teaching is not involved, in the case of teachers.

Due to the changes involved in this new reform, resistance movements in the entire country are taking place. However, there are numerous misunderstandings of this new reform that contribute to this resistance such as: teachers will be fired immediately if they do not
perform well in the evaluation; teachers’ rights will be removed; schools will be privatized; among others (SEP, 2013a). These movements have caused physical confrontations among teachers, and government officials. Moreover, thousands of students have been affected by these movements due to the cancellation of classes. While teachers are meeting in public places to protest, students are missing classes. Although these movements are happening, the reform was already approved and began its implementation.

We can observe that teachers are going through a difficult situation in Mexico because of the many changes in the Constitution which affects directly the teacher's role and performance, and teachers are facing new standards in regards to teacher knowledge. These are the current circumstances that teachers who were participating in this study were living through in Mexico.

2.5 Educational System in Mexico

In Mexico, public education is divided in different levels: (a) pre-school; (b) elementary school (primaria), which includes grades from 1 to 6; (c) middle school (secundaria), which is composed of grades 7-9; and (d) high school (preparatoria), which includes from 10th grade to 12th grade, and higher education. Basic education in this country is composed of the educational levels that are compulsory for all Mexican citizens. Nowadays, mandatory education includes pre-school, elementary and middle school levels. In other words, in Mexico, Mexicans must study from 1st grade to 9th grade plus one year of pre-school (Sandoval, Aviles, Carbajal, Tovar, Villegas, & Montaño, 2005). Figure 2.1 illustrates a representation of the public educational system in Mexico.
The structure of the Mexican educational system is established by the law of general education (La Ley General de Educación), which states that it includes three kinds of education: basic, middle superior, and superior education (Secretaría de Educación Pública, 2010). As mentioned above, basic education includes pre-school, elementary and middle school. The completion of pre-school is a requirement to be admitted into elementary school, and graduation
from elementary school is required to attend middle school. The recognition of these milestones is done through a diploma (certificado).

The media superior education is composed of the high school level (preparatoria). Most often high school is composed of three grades, however, there are private schools or adult education programs that offer it also in two or four years for students that cannot be incorporated into regular high schools. The completion of this level is indispensable to be admitted to institutions of superior education. Superior education is offered by universities, colleges, institutes, Normal schools, and Technological schools (tecnológicos). In these institutions, students can earn associate degrees, bachelor’s degrees, master’s degrees, and doctorates.

2.6 MIDDLE SCHOOLS IN MEXICO

Officially, Mexican middle schools (secundarias) were created by law in 1915. However, serious attention was not given to middle schools until 1923 (Levinson, 2001). Prior to this year, middle school education was a part of college preparatory studies. Mexico had been influenced by the European tradition of having middle school education combined with professional studies (Levinson, 2001). In 1923, the subsecretary of education in Mexico, Bernardo Gastelum, reorganized the college preparatory studies by distinguishing a level of education that was an extension of elementary education; secondary education, which is now provided in middle schools. The central goals of middle school at the time were:

1) carry forth the task of correcting defects and sponsoring the general development of students begun in the primaria, 2) strengthen in each student the sense of solidarity with others, 3) create habits of unity (cohesion) and social cooperation, and 4) offer all students a great variety of activities, exercises, and teachings so that each one might discover a vocation and be able to dedicate him/herself to cultivating it. (Meneses, 1986, p. 408)

Middle schools were oriented toward urban and mostly professional classes (Levinson, 2001). The National University administered the middle school as part of its preparatory studies.
until 1925. The Office of Secondary Education was created in 1928, which was in charge of middle schools. At that time, middle school was focused on teaching methods and principles appropriated for adolescents. Students who attended middle school were able to choose technical or industrial workshops offered for preparation for work or study. The goal of these early middle schools was to provide students a specialized curriculum that offered ways to explore vocational options. However, this ultimate goal changed in the 1930’s when middle schools were focused on preparing workers and rural teachers.

Since the official creation of middle schools in Mexico, different educational philosophies such as the classical European traditions, foreign educational philosophies (American tradition), pedagogical mold of the German secondary schools, and the recognition of middle school as an extension of the elementary school had influenced the structure and goals of Mexican middle schools. In the 1940’s, after this influence, the middle school had its own agenda and structure. The number of middle schools began to increase to meet the needs of the middle-class aspirations (Levinson, 2001). Middle schools provided a crucial means of socioeconomic mobility for the working class. Mexican middle schools prepared workers for the new age of industrialization that was prevailing in the country. From 1950 to 1970, a rise of middle school enrollment of 1000 percent occurred. At that time, the restatement of the goal of middle schools was done: middle school “education is designed especially for the ‘integral formation’ of adolescents” (Meneses, 1986, p. 411).

Currently, there are four types or subsystems of public middle schools in Mexico: federal, state, technical, and TV middle schools (telesecundaria). In the past, these specific types were created for different purposes. For instance, federal middle schools were mostly built in cities with large populations (Levinson, 2001). Concerning the technical middle schools, graduated students acknowledged appropriate skills for immediately incorporating them into the regional workforce or continuing to high school or vocational studies (Levinson, 2001). Moreover, technical middle schools were built in rural areas or small towns due to the fact that in larger cities only federal middles schools were built. However, some of the purposes for the
classification of these schools have been removed. At this time, technical and federal schools are also built in urban communities in which larger populations exist. Moreover, the federal, state, and technical middle schools offer workshops (talleres) that prepare students to be incorporated after the completion of this education into the workforce. The TV middle school (telesecundaria) was the last type of middle school added to the educational system. It was created to support vulnerable groups of people (existing mostly in small rural areas) to overcome the difficulties of attending schools (SEP, 2012). In TV middle school, the instruction was provided via televised recording in classrooms.

Most of the middle schools in Mexican cities have two shifts (turnos): morning and afternoon. Commonly, some teachers work on the two shifts. However, the function of these schools is as two separate schools in the same building; one school in the morning and one school in the afternoon. In fact, the name of the middle school in the morning is different than the name of the afternoon middle school. Also, they mostly have different principals, students, teachers and personnel.

Middle school attendance has increased drastically since its creation. In 1925, there was a middle school enrollment of 12,435 students in Mexico (SEP, 2013b). By that time, the nations’ population was more than 14 million people (Instituto Nacional de Estadística y Geografía INEGI, 2013). The students’ enrollment represented less than one percent of the entire population. According to the 1960’s census, Mexico had a population of 34,923,129 (INEGI, 2013). The middle school enrollment of that time had reached 234,980 students in the whole country, which was less than one percent of the entire population (SEP, 2013b) as observed in Table 2.1. There were 81,249,645 of residents in Mexico in 1990 (INEGI, 2013). In this year, the enrollment reached 4,190,190 students at the middle school level (SEP, 2013b) which represented the 5.15% of the whole population. Currently, Mexico has a population of 112,336,538 (INEGI, 2013). According to the SEP (2013b) in 2010, the middle school enrollment rate is 6,137,546 students which represented 5.46% of the Mexican population. As we can
observe the attendance at the middle school level has changed radically through the passage of the time as observed in Table 2.1.

<table>
<thead>
<tr>
<th>Year</th>
<th>Middle School Student enrollment</th>
<th>Mexico’s population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1925</td>
<td>12,435</td>
<td>14,334,780</td>
</tr>
<tr>
<td>1960</td>
<td>234,980</td>
<td>34,923,129</td>
</tr>
<tr>
<td>1990</td>
<td>4,190,190</td>
<td>81,249,645</td>
</tr>
<tr>
<td>2010</td>
<td>6,137,546</td>
<td>112,336,538</td>
</tr>
</tbody>
</table>

2.7 **Mathematics Curriculum in Middle Schools in Mexico**

Mexican middle schools have three goals for teaching mathematics: developing skills, promoting positive attitudes, and learning mathematics (Secretaría de Educación Pública, 2004). With regards to developing skills, mathematics teachers should help students to develop skills that allow them to complete mathematics operations, communication skills, and discovery skills to learn mathematics and solve mathematical problems.

Teachers should promote positive attitudes towards mathematics. Therefore, they must promote collaborative work, respect, research, persistence, high self-esteem, and autonomy in their mathematics classrooms (Secretaría de Educación Pública, 2004). Additionally, learning mathematics at the middle school level implies understanding concepts and topics included in the following five mathematics areas: 1) arithmetic; 2) algebra; 3) geometry (trigonometry is added in 9th grade); 4) data representation and management; and 5) introduction to probability (Secretaría de Educación Pública, 2004).

At the secondary (secundaria) or middle school level, mathematics is taught as a single subject in each of the three grades, typically comprising five hours of instruction per week. Mathematics I is offered at the seventh grade level, while Mathematics II is imparted in eighth grade, and Mathematics III in ninth grade. Mathematics I encompasses linear equations, geometric properties for 2-D and 3-D figures, as well as a general introduction to probability. Mathematics II incorporates linear functions, introduction to statistics, and geometry, building on the concepts learned in Mathematics I. In the last year of middle school, in Mathematics III,
students are exposed to more in-depth geometry through analysis of triangle congruence, properties of polygons, and trigonometry. This curriculum is very similar to the middle school mathematics curriculum in the United States as we can observe in Table 2 a comparison of middle school mathematics curriculum of Mexico and the U.S.

Table 2.2: Middle School Mathematics Curriculum in Mexico and USA: Comparison Chart.

<table>
<thead>
<tr>
<th>Mexico Middle School (Secundaria)</th>
<th>United States: Texas Middle School</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7th Grade: Mathematics I</strong></td>
<td><strong>6th and 7th Grade: Mathematics</strong></td>
</tr>
<tr>
<td>Set of natural numbers, fractions and rational numbers; expressions using variables; linear equations; introduction to probability; geometric figures and their properties and proportionality.</td>
<td>Rational numbers; addition, subtraction, multiplication, and division of decimals, fractions, and integers; proportional relationships, expressions and equations, geometry and spatial reasoning; measurement; probability and statistics.</td>
</tr>
<tr>
<td><strong>8th Grade: Mathematics II</strong></td>
<td><strong>8th Grade: Mathematics</strong></td>
</tr>
<tr>
<td>Set of real numbers; algebraic expressions, square root; linear functions; proportionality; arithmetic and geometric progressions; elementary probability theory; statistics and geometric figures and their properties.</td>
<td>Number operations and quantitative reasoning, proportional and non-proportional linear relationships; transformational geometry; measurement; Pythagorean Theorem; probability and statistics.</td>
</tr>
<tr>
<td><strong>9th Grade: Mathematics III</strong></td>
<td>OR</td>
</tr>
<tr>
<td>Triangles and their congruence; quadrilaterals; polygons; areas of polygons; circles and inscribed (circumscribed) polygons; elementary trigonometric; quadratic function; Pythagorean Theorem; quadratic equations and probability theory and statistics.</td>
<td>Algebra I: Functional relationships; properties and attributes of functions; patterns and generalizations; solving equations and inequalities; linear functions; systems of linear functions; quadratic functions; laws of exponents; inverse variation; exponential growth and decay.</td>
</tr>
</tbody>
</table>

### 2.8 Mathematics Teacher Preparation at the Middle School Level

The role of teachers is very important in the process of teaching and learning. According to the Ministry of Public Education (2004), middle school mathematics teacher work includes: transferring information to their students; assessing the academic achievement of their students; analyzing situations related to content, organizing sequences that help the evolution of students’ procedures, posing problems, sharing solving strategies, and assessing different aspects of the didactic process.

Mexican teacher preparation is different depending on the institution that offers it. Differences among institutions, including private organizations, are related to length of time,
course format, and structure of the programs. Nonetheless, two main institutions are widely recognized by the Mexican government as institutions highly qualified and aligned to the SEP interests to prepare prospective teachers. The Normal Superior is one of these main institutions. The second is the National Pedagogical University (Universidad Pedagógica Nacional- UPN). As part of their preparation in such institutions, prospective teachers must complete a four-year, content-specific program of study, which includes teaching in actual classrooms with students assigned to them.

In order to have more chances to be considered for a teaching position in a public school, teachers should graduate from one or both of the institutions mentioned above. These institutions offer teacher preparation for middle school teachers. However, prospective teachers who attend the National Pedagogical University can teach 1st through 9th grade at public institutions, as well as instruct at private or public organizations such as community centers, hospitals, or factories.

It is estimated that 50% of middle school teachers acquired their preparation from the Normal Superior School or Pedagogical University and the other 50% hold other types of formal education, such as a bachelor’s degree in engineering, counting, biology, administration, and other areas (Sandoval, 2009).

Prior to 2009, Normal schools offered several qualifying courses which together with a high school diploma enabled prospective teachers to get a part time teaching position at the middle school level, either at a public or private institution. Once this qualification was met, a candidate could be hired as a part-time teacher, under the condition that they had to attend a Normal school teacher preparation program during weekends or summer semesters. This would allow them to obtain a permanent contract for a full-time position, once they completed the program.

The teacher preparation program is offered by the Normal Superior schools in two modalities: traditional and mixed modality (modalidad mixta). The traditional modality refers to attend the program over a four-year period. This modality allows high school graduates to be admitted and requires them to complete a social service prerequisite of 600 hours of public service.
On the other hand, the mixed modality program has a length of time of five years, and the social service prerequisite is not required. Particularly, this modality only enrolls current in-service part-time teachers. Graduated teachers from the Normal Superior School are able to teach only the course(s) in which they chose to specialize.

We can summarize the main components of the middle school teacher preparation at the Normal Superior schools in the following way: (a) coursework which includes 37 courses; (b) pre-service activities intended to support instructional practice by having pre-service teachers work in schools preparing lessons, observing classroom instruction, and analyzing student interactions; (c) practice teaching which takes place during the last two semesters when pre-service teachers co-teach up to three courses at a middle school; and (d) social service that involves 600 hours of voluntary, unpaid teaching.

2.9 **MIDDLE SCHOOL MATHEMATICS TEACHER PREPARATION CURRICULUM**

The secondary mathematics teacher preparation curriculum is designed by taking in consideration the nature of becoming a secondary school teacher. Therefore, the preparation at the Normal Superior School for middle school prospective teachers embraces three fields: General training, Common training, and Specific training (Dirección General de Educación Superior para Profesionales de la Educación-DGESPE, 2010).

General training represents 16% of the program and includes basic preparation that a teacher of any subject area is required to have. This kind of training focuses on the importance of understanding the Mexican educational system. Courses included in this field are Philosophical, legal and organized bases of the Mexican educational system, Education in the Historic Development of Mexico I & II, Study Strategies and Communication I & II, and Basic Education Problems and Policies.

Common training is unique to secondary school teacher preparation. This training contributes to the program with 35% of whole preparation, with courses such as Teenager Development I General Aspects, and three courses about teenager development but with different foci, Teaching
in Middle School I & II, Purposes and Contents of Basic Education I (elementary school, primaria) & II (middle school, secundaria), and Elective I & 2.

The third field is the specific training that is the content knowledge and pedagogical knowledge required to teach the subject chosen. This component of the program represents 49% of the total academic activity in the teacher preparation. For middle school mathematics teachers, this includes content-specific courses focused on mathematics such as Algebraic thinking, Numbers and their Relations, Geometrical Figures, Introduction to Teaching Mathematics, Cognitive Processes and Conceptual Change in Mathematics and Science. Additionally, four of the courses (Observation and Teaching Practice, one of Teaching Planning and Learning Evaluation) are also focused on mathematics.

The following table (Table 3) shows a sample degree plan obtained from a Normal superior School in the traditional modality. The coursework for each semester for a four year period is presented in Table 3. To be a mathematics teacher at the middle school level, prospective teachers must complete this teacher preparation program.

Table 2.3: Normal Superior Institution Degree’s Plan (DGESPE, 2010).

<table>
<thead>
<tr>
<th>First Semester</th>
<th>Second Semester</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Philosophical, legal and organized bases of Mexican educational system (4hrs/7credits)</td>
<td>- Education in the historic development of Mexico I (4hrs/7credits)</td>
</tr>
<tr>
<td>- Study strategies and communication I (6hrs/10.5 credits)</td>
<td>- Study strategies and communication II (4hrs/7credits)</td>
</tr>
<tr>
<td>- Basic education problems and policies (6hrs/10.5 credits)</td>
<td>- Teaching in middle schools (secundaria). Basic aspects I (4hrs/7credits)</td>
</tr>
<tr>
<td>- Purposes and contents of basic education I (4hrs/7credits)</td>
<td>- Introduction to teaching mathematics (4hrs/7credits)</td>
</tr>
<tr>
<td>- Teenager development I. General aspects (6hrs/10.5 credits)</td>
<td>- Purposes and contents of basic education II (lower secondary school) (4hrs/7credits)</td>
</tr>
<tr>
<td>- School and social context (6hrs/105 credits)</td>
<td>- Teenager development II. Growing and sexuality (6hrs/10.5 credits)</td>
</tr>
<tr>
<td></td>
<td>- Scholar process observation (6hrs/10.5 credits)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Third Semester</th>
<th>Fourth Semester</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Education in the historic development of Mexico II (4hrs/7credits)</td>
<td>- History of pedagogy and selected topics seminar I (4hrs/7credits)</td>
</tr>
<tr>
<td>- Algebraic thinking (4hrs/7credits)</td>
<td>- Geometrical shapes and solids (4hrs/7credits)</td>
</tr>
<tr>
<td>- Teaching in middle schools (secundaria). Basic aspects II (4hrs/7credits)</td>
<td>- Functions and the Cartesian plane (4hrs/7credits)</td>
</tr>
<tr>
<td>- Numbers and relations (4hrs/7credits)</td>
<td>- Change and variation (4hrs/7credits)</td>
</tr>
<tr>
<td>- Oral and written expression in teaching and learning (4hrs/7credits)</td>
<td>- Teaching planning and learning assessment (4hrs/7credits)</td>
</tr>
<tr>
<td>- Teenager development III. Identity and social</td>
<td>- Teenager development IV. Cognitive process (6hrs/10.5 credits)</td>
</tr>
</tbody>
</table>
Normal Schools administer an exam to assess the students’ performance, knowledge and skills. This exam is called the General Knowledge Exam and it is administered to students in their sixth and eight semesters. It is a standardized National test. The test consists of 300 multiple choice questions. This test is not considered for grading prospective teachers in their courses. It does not have any influence in the evaluation system used by teacher educators. This exam provides information in regards to student’s performance to assess if they meet the school’s expectations according to the plan of studies. It is only considered as a diagnostic test.

Currently, in Mexico, there is a competition to get a teaching position at the elementary and middle school level in any public school. It is called national competition of positions (Concurso Nacional de Plazas). This competition provides two teaching position categories in which graduated teachers and in-service teachers can apply. The categories are: category of permanent contract and the teaching position category per hours. In-service teachers who hold a part-time position, and want to apply for a permanent contract category are able to be part of the competition. This competition is national. Its main purpose is to recruit the best candidates to be teachers in the public education system. This competition consists of an exam that includes 110 multiple choice questions.
2.10 **Educational Research in Mexico**

In Mexico, educational research is limited for several reasons such as: the lack of research funding to conduct educational research (Reyes, 2013); the small number of educational researchers in the country (The Organization for Economic Co-operation and Development-Center for Educational Research and Innovation, Revision Nacional de Investigación y Desarrollo Educativo. Reporte de los examinadores sobre Mexico, 2004); the limited educational researcher preparation due to few graduate programs offered in the country (Consejo Nacional de Ciencia y Tecnología, 2013); and others.

For this study, a review of the literature on studies conducted in Mexico about mathematical teacher knowledge was conducted. This review shows that there is a deficiency of research in the field of inquiry of teacher knowledge in this country (Pinto & Gonzalez, 2008; Mochon & Andrade, 2009).

In Mexico, there are few studies identified that focus on mathematics teachers (Castañeda, Rosas, & Molina, 2011; Dueñas, 2009). Castañeda and colleagues (2011) analyzed a collection of video recordings of mathematical instruction at the secondary level. They focused on the mathematics discourse used by teachers to formulate generalizations, synthesis and summaries of classroom activities. Dueñas (2009) studied the social construction of mathematics middle school teachers. In Dueñas’s study, the author intended to provide an understanding of the relationship between the development of skills, attitudes and mathematical content knowledge, and the historical processes that have shaped mathematics teachers.

Understanding the mathematical knowledge for teaching possessed by Mexican prospective teachers at the end of their teacher preparation was one of the purposes of the Mochon and Hernandez’s study (2011). The authors offered a special final course at the Normal Superior School in Mexico City for these teachers. The results of this study indicated that the recognition of the mathematical teacher knowledge was merely instrumental. However, changes on prospective teachers’ views towards mathematics and teaching mathematics occurred.
There is a dearth of studies that have focused particularly on mathematical teacher knowledge in Mexico (e.g. Inzunsa & Guzman, 2011). Inzunsa and Guzman (2011) analyzed the teachers’ understanding of concepts about probability. They administered a questionnaire to 80 teachers of technical middle schools in Mexico. This questionnaire is composed of 17 items about probability concepts that they are teaching in their classrooms. The findings indicate that probability is a difficult area for middle school teachers. Teachers have a weak reasoning and lack of the use of diagrams was also identified. However, this study only focuses on a specific area of mathematics, probability. Further research on mathematical teacher knowledge based on the middle school curriculum taught by the participating teachers is needed.

The Mochon and Andrade’s study (2009) intended to explore the mathematical knowledge for teaching through the implementation of four instruments: classroom observation, an open questionnaire, a closed questionnaire, and interviews. Findings reported that elementary teachers have some shortages and conceptual difficulties in regards to their content knowledge. However, the instruments were administered to elementary school teachers, and only covered a few mathematics topics such as operations with natural numbers, mental calculation and estimation, fractions and decimals, proportionally and units conversion.

We can observe the existent lack of attention on teachers such as teacher knowledge, practices, beliefs, etc. (Pinto & Gonzalez, 2008). However, research on students’ learning and difficulties of particular mathematical topics is plentiful. This is an interesting finding of this literature review of the study. Although teachers are a fundamental part of the process of teaching and learning mathematics, and their preparation is crucial to teach effectively (Inzunsa & Guzman, 2011; Mochon & Hernandez, 2011), scholars in Mexico have not focused on them to conduct research (Pinto & Gonzalez, 2008).

In addition, an existing lacuna of research at the elementary and middle school levels that analyzes specific mathematical knowledge and instructional skills required for teaching effectively has been identified (National Mathematics Advisory Panel, Report of the Task Group on Teachers and Teacher Education, 2008b). Further research on the middle school level in
Mexico is needed due to the fact that this educational level has had the highest dropout rate in basic education for a long time (Dueñas, 2009; Reimers, 2001; Blasco, 2003, Instituto Nacional de Evaluación Educativa, 2010).

Conducting research on teacher content knowledge is essential due to the given fact that this kind of knowledge increases the ability for teachers to develop classroom activities, manage students’ interventions, help to design teaching strategies linked with the content, and be able to delve deeper in the “why” and “so what” of the content knowledge (Pinto & Gonzalez, 2008). In addition, there is a significant impact that teacher content knowledge has on student’s achievement (Tchoshanov, 2011; Mochon & Andrade, 2009). Teachers who possess weak content knowledge are limited in developing many of these abilities (Lopez, 1999; Pinto & Gonzalez, 2008). I was cognizant of the relevance and importance that teacher knowledge has in the teaching and learning process, thus, I investigated the conceptual frameworks that were critical to be considered to conduct research in this field of inquiry.

2.11 Conceptual Framework

The aim of this section is to present the conceptual framework on teacher knowledge that was used as a lens for this study. It is very important to consider that teaching is more than presenting sequences of instruction in classrooms (Mason, 1998). Teaching as Ball and her colleagues define such as “everything that teachers must do to support the learning of their students” (Ball et al., 2008, p.395).

2.12 Shulman Model

A perspective on teacher knowledge of different content areas was provided in the mid and late 1980’s by Shulman (1986) in his pioneer work. Most of the studies related to teacher knowledge (e.g. Ball, Thames & Phelps, 2008; An, Kulm and Wu, 2004) are based on the categories of teacher knowledge presented in the Shulman Model (1986, 1987). Shulman (1986) identifies the need to create a theoretical framework in which several questions could be addressed such as: “What are the domains and categories of content knowledge in the minds of
teachers? How are content knowledge and general pedagogical knowledge related? In which forms are the domains and categories of knowledge represented in the minds of teachers? What are promising ways of enhancing acquisition and development of such knowledge?” (p.9).

Shulman (1986) mentions that content knowledge is the knowledge that grows in the minds of teachers, placing emphasis on the content. Three categories were distinguished of content knowledge:

I. Subject matter content knowledge

II. Pedagogical content knowledge

III. Curricular knowledge

The first category is the subject matter content knowledge CK. This category involves knowledge that goes beyond just facts or concepts; it includes the understanding of structures of the subject matter. As Schwab (1978) mentions, CK includes the knowledge of substantive and syntactic structures. The substantive structures are the organization of fundamental concepts and principles to incorporate its facts in various manners. The syntactic structures are the diverse ways in which something is validated or invalidated.

Pedagogical content knowledge (PCK) is the next category. Shulman (1986) points out that this category of knowledge does not exclude the subject matter knowledge, it is considered in this knowledge. However, PCK is focused on the aspects for teaching the subject matter such as different ways of representation of ideas, analogies, illustrations, explanations and demonstrations. Pedagogical content knowledge also includes the understanding of students’ perceptions and misconceptions of what lead a topic to be easy or complex to learn.

Curricular knowledge is the third category of content knowledge. Shulman (1986) states “The curriculum is represented by the full range of programs designed for the teaching of particular subjects and topics at, a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances.” (p.10). Therefore the understanding about curriculum, curriculum materials and
alternative materials is critical to be known by teachers. Shulman’s framework (1986) of knowledge base for teaching was developed considering middle school teachers and different teaching content areas: English, biology, mathematics, and social studies.

In 1987, Shulman made some modifications to his model of categories of teacher knowledge. He reorganized and added more categories. Summarizing, these are the categories of Shulman model (1987): a) content knowledge, b) general pedagogical knowledge, c) curriculum knowledge, d) pedagogical content knowledge, e) knowledge of learners, f) knowledge of educational contexts, and g) knowledge of educational ends. Since these categories of knowledge are crucial to be in the mind of teachers, they were the main focus of teacher education programs of the late1980’s.

2.13 **Domains of Mathematical Knowledge for Teaching**

In the Study of Ball, Thames and Phelps (2008) a practice-based theory of content knowledge for teaching was developed. This model is built on Shulman’s model (1986). These scholars proposed a model that makes subdivisions of two categories that were considered by Shulman in his initial model (See Figure 3). Those are subject matter knowledge and pedagogical content knowledge, and also they consider that curricular knowledge Shulman’s category must be included inside of pedagogical content knowledge. Ball and colleagues (2008) studied these categories focused only on mathematics content.

Under the subject matter knowledge category, three domains were identified (Ball et al., 2008). Those are *Common content knowledge (CCK)*, *Horizon content knowledge* and *specialized content knowledge (SCK)*. Regarding the domain of common content knowledge, Ball and her colleagues described it as “the mathematical knowledge and skill used in settings other than teaching“(Ball et al., 2008, p.399). In other words, CCK is the knowledge and skills can held and used by somebody even this person is not a teacher in wherever place, such as in the market, office, etc. however, it does not mean that everyone possess this knowledge. It is not
exclusive for teaching. For example, a cashier in a restaurant should know adding and subtracting numbers and money amounts.

Horizon content knowledge is the other domain of subject matter knowledge category. It refers to the knowledge of the relationship between mathematical topics included in the curriculum, and the mathematical topics of the next grades. It also comprises the usefulness of making connections with later mathematical concepts. For instance, knowledge of adding, teachers know that this topic is fundamental to be learned before learning multiplication operations which is a topic taught in the following grades.

The last domain of this category is the specialized content knowledge (SCK). It implies the mathematical knowledge and skill unique for teaching. For instance, the skills and knowledge to identify patterns in students’ mistakes, knowing which strategies work better in general, “knowledge beyond that being taught to students. Understanding different interpretations of the operations in ways that students need not explicitly distinguish; it requires appreciating the difference between “take-away” and “comparison” models of subtraction and between “measurement” and “partition” models of division.” (Ball et al., 2008, p.400).

Regarding the other category of this model, pedagogical content knowledge includes three domains; knowledge of content and students (KCS), knowledge of content and teaching (KCT), and Knowledge of content and curriculum.
Figure 2.2: Domains of Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403).

Knowledge of content and students (KCS) is one domain of pedagogical content knowledge category. It is conceptualized as the interception of knowledge about the students and the knowledge of mathematics. Hill and colleagues defined KCS as “the knowledge that relate to both subject matter knowledge and PCK” (Hill, Ball, & Schilling, 2008, p. 377). For example, teachers need to know which methods are more favorable for students in order to experience success in their tasks, which teaching’s ways allow students to find an activity easy or difficult.

Knowledge of content and teaching (KCT) refers to the combination between knowledge of the mathematics and knowledge about teaching. In other words, KCT is the knowledge to design didactic activities which involves mathematical knowledge and pedagogical knowledge. KCT allows identifying which pedagogical strategies are more appropriated to approach a specific topic. Issues that students confront using certain strategy are part of this knowledge. The last domain is the Knowledge of content and curriculum. Ball and her colleagues (2008) did not define this category in their study. However, implicitly it could be deduced that knowledge
of curriculum means knowing about mathematics and the mathematical topics that are included in the curriculum and the materials available for its instruction.

2.14 **Content Knowledge**

Shulman’s (1986) categories of teacher knowledge consider the category of teacher content knowledge defined as “the amount and organization of knowledge per se in the mind of teachers” (p. 9). Focusing on mathematics, it includes the knowledge “that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems” (Hill et al., 2008, p.377-378). According to Bransford, Brown, and Cocking (2000) a solid domain of this category of knowledge requires three features: 1) “a deep foundation of factual knowledge”, 2) understanding of the “facts and ideas in the context of a conceptual framework”, and 3) organization of the knowledge “in ways that facilitate retrieval and application” (p. 16). Leinhardt and Smith (1985) state that teacher subject-matter knowledge is the knowledge of “concepts, algorithmic operations, and the connections among different algorithmic procedures…” (p. 247). Also, Fennema and Loef (1992) identified similar features of the knowledge needed for teaching mathematics which implies “…teacher knowledge of the concepts, procedures, and problem solving processes…” (p. 162). It also includes the organization of mathematical topics and the connections between them. Several studies have also focused on mathematical content knowledge (e.g. Hill, Ball, & Rowan, 2005; Ball et al., 2008). Tchoshanov, et al., (2008) pointed out that the cognitive demand is a function of the cognitive type of teacher content knowledge. Hence, three cognitive types of teacher content knowledge were identified: knowledge of facts and procedures, knowledge of connections and concepts, and knowledge of generalizations and models (Tchoshanov, 2011).
In past years, the indicators of teacher content knowledge considered were obtained from professional development. Also, research completed by the US Task Group on teachers and teacher education of the National Advisory Panel (2008b) shows that the manner in which teacher knowledge has been measured is through teacher certification, mathematics coursework, and a content knowledge test for teachers.

Nowadays, new ways to measure teacher knowledge are emerging. In Sorto and colleagues’ study (2009), the mathematics content knowledge and pedagogical content knowledge were measured using a questionnaire. This questionnaire was administered to 385 elementary and middle school teachers in Panama and Costa Rica. Items were posed in two formats: multiple choice question, and fill in the blank questions. This questionnaire measures
teacher content knowledge in the areas of basic operations, multiplication-division, fractions, geometry, measurement and statistics.

Furthermore, an instrument called Teacher content knowledge survey TCKS (Tchoshanov, 2011) was developed to assess the content knowledge of mathematics teachers at the middle school level. The data analysis could be done by a categorization of cognitive types of mathematical content knowledge included in this instrument. The main purpose of Tchoshanov’s (2011) was to create and use this survey in order to know whether and how the cognitive types of teachers’ content knowledge are associated with student achievement, and correlated with teaching practice. Tchoshanov developed a theoretical framework (see Figure 2.3) based on several scholars’ studies that he identified through his critical literature review such as the Shulman model. However, an emphasis is placed on the content knowledge category.

2.14.1 Cognitive Types of Teacher Content Knowledge

These cognitive types of teacher content knowledge contribute to measure the specific type of mathematical knowledge that a teacher has (Tchoshanov, 2011). Cognitive type 1 is the teacher knowledge of facts and procedures. Cognitive type 2 knowledge refers to teachers’ knowledge of concepts and connections. And cognitive type 3 knowledge includes the knowledge of teachers regarding models and generalizations.

Memorization and application of basic mathematical facts, rules, and algorithms to solve routine problems is required for cognitive type 1 knowledge. For example, “if a teacher is able to recall a rule for fraction division or to solve simple fraction division problem such as $1 \frac{3}{4} \div \frac{1}{2} =$, then we say that she has procedural knowledge of fraction division.” (Tchoshanov, 2011, p. 142). This cognitive type of knowledge (Type 1) has been also analyzed in several studies (e.g., Skemp, 1978; Stein, Smith, Henningsen, & Silver, 2000).

The quantity and quality of connections between mathematical procedures and ideas are part of the mathematical conceptual understanding in which the cognitive type 2 is focused (Tchoshanov, 2011). For example, a teacher is asked to solve a problem in multiple ways, in
order to do it the teacher needs to possess conceptual knowledge and not only procedural
knowledge about the mathematical concept.

Type 3 knowledge focuses on the theoretical part. This type of knowledge is about
models and generalizations. It includes conjecturing, generalizing, proving theorems, etc. For
example, “Is the following statement: \( \frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd} \) (a, b, c, and d are positive integers) ever true?”
(Tchoshanov, 2011, p.143). Cognitive type 3 and its relationship between generalizations and
representations has also been explored (Doerfler, 1991; Presmeg, 1997). It is important to clarify
that these types of knowledge do not follow any order; it means that it is not needed to possess
the previous type of knowledge to have any type of knowledge. A teacher could possess just one
type of knowledge, or two out of three, or the three cognitive types of knowledge.

2.15 Pedagogical Content Knowledge

Pedagogical content knowledge PCK, is the blend between content knowledge and
pedagogical knowledge. It means to know how to teach mathematics considering more than only
knowing about the subject matter or teaching strategies. PCK implies both PK and CK. They are
required in a well combination in order that teachers be able to deliver the content knowledge in
a proper manner to engage student to learn mathematics.

In An, Kulm and Wu’s study (2004), there is a theoretical framework developed
specifically on pedagogical content knowledge. It is based on Shulman’s model (1987) due to the
fact that they include the PCK and CK categories of Shulman model (1986, 1987). An and
colleagues (2004) define pedagogical content knowledge as “the knowledge of effective teaching
which includes three components, knowledge of content, knowledge of curriculum and
knowledge of teaching.”(p.146-147).
The first component is knowledge of content which means to have a solid knowledge about mathematics topics. The second one is knowledge of curriculum which implies to know the curriculum, what kind of curriculum materials, goals, and textbooks are better for teaching that class. The last component is knowledge of teaching, which means to know the pedagogical part; designing lesson plans, preparing instruction, methods of delivery instruction, and knowing the learner characteristics. An and colleagues (2004) stated that the central point of pedagogical content knowledge is knowledge of teaching. They considered that these three components are crucial to teach effectively. As in Figure 2.4, the relationship among the three components is presented. Also, Knowledge of teaching can be improved by the knowledge of the content (mathematics) and curriculum.

Numerous studies used the conceptualization of knowledge for teaching instead of teacher knowledge (Ball et al., 2008). In Hill, Ball, & Rowan’s study (2005) a definition for
mathematical knowledge for teaching is provided. This study defines mathematical knowledge for teaching as “the mathematical knowledge used to carry out the work of teaching mathematics.” (Hill et al., 2005, p.373). It implies explanations of concepts or terms, interpretations of students’ answers, analyses of textbooks with respect to the way they cover and present certain topics, management of diverse representations and examples of mathematical concepts, and proofs. In sum, this definition includes knowledge of mathematics, knowledge of teaching, and the intersection of both kinds of knowledge in order to be able to teach mathematical knowledge.

2.16 PEDAGOGICAL KNOWLEDGE

Pedagogical Knowledge PK means to know pedagogical strategies and methods for teaching. In addition, teachers need to update this knowledge in regards of effective tools for teaching such as technological tools, didactics and other fundamental tools that a teacher must have in order to engage their students. According to Fennema and Loef (1992), pedagogical knowledge includes teacher knowledge of pedagogical strategies, methods of teaching, classroom management, engagement techniques, and designing lesson plans. Also, Koehler, and Mishra (2009) recognized similar elements of the pedagogical knowledge such as the teacher’s knowledge about pedagogical strategies, methods, lessons, methods of assessment and all related with teaching.

2.17 KNOWLEDGE OF LEARNERS

Another kind of knowledge that has received little attention is the knowledge of learners identified by Shulman (1987). Learners play a crucial role in teaching and learning. Therefore, to learn to identify how students think, how pedagogical instruments can assist students to learn better, students’ weaknesses and strengths are critical aspects for teaching any subject. Also knowing about the students’ cultural backgrounds is of great relevance in order to provide comfortable learning environments. For example, classroom activities where students feel identified such as a mathematics word problem about a real situation that students go through
commonly or relates to their culture. According to Pinto and Gonzalez (2008) knowledge of learners also includes knowledge of student’s motivations, expectations and interests, ways of learning, preconceptions, conceptions, and difficulties towards specific areas of mathematics and topics.

Fennema and Loef (1992) categorize the knowledge of learners’ cognitions as a part of the knowledge base for teaching mathematics. It means the teachers’ understanding of how students’ thinking and learning is constructed regarding specific mathematics content. Also, the recognition of the difficulties and successes that students face using certain approaches to mathematical content is considered. Ball and colleagues (2008) called this kind of knowledge as knowledge of content and students (KCS). This kind of knowledge, KCS, is composed by two kinds of knowledge, one is content knowledge and the other one is knowledge of learners. Thus, the combination of “knowing about students and knowing about mathematics” (Ball et al., 2008, p.401) creates the KCS.

2.18 RELATIONSHIP BETWEEN TEACHER KNOWLEDGE AND BELIEFS

Teachers’ beliefs are considered part of the teacher knowledge. The beliefs definitely affect the way of teaching, and the perceptions towards students, content, school, and educational experiences, that teachers have. Fennema and Loef (1992) argue that teacher knowledge cannot be studied in isolation from teachers’ beliefs. According to Fennema and Loef (1992), “The context is the structure that defines the components of knowledge and beliefs that come into play.”(p. 162).

Fennema and Loef (1992) developed a model of teacher knowledge in which the context of the classroom is considered. It is composed of the following components: knowledge of the content of mathematics, knowledge of pedagogy, knowledge of students’ cognition, and teachers’ beliefs. This model displays the interaction and dynamism that exists in teacher knowledge (see Figure 2.5). The first component of this model is the content of mathematics which includes knowledge of concepts, procedures, and problem solving as already was
mentioned on page 6. It also includes the organization of mathematical topics. Knowledge of connections between mathematical concepts is essential in order to have a coherent organization. The pedagogical knowledge component is also included in this model of teacher knowledge. This component was previously defined on page 13. Knowledge of learners’ cognitions in mathematics is another component in this model. It focuses on knowing the ways of how students learn particular content (Fennema & Loef, 1992). Fennema and Loef (1992) conceptualize this kind of knowledge as a part of the interaction among content knowledge and pedagogical knowledge.

![Diagram](image)

**Figure 2.5:** Teachers’ Knowledge: Developing in Context (Fennema & Loef, 1992, p. 162).

Teachers’ beliefs are also a component of this model. The intersection of these three components: knowledge of mathematics, knowledge of learners’ cognitions, and pedagogical knowledge is associated with teachers’ beliefs. This association is shaped by the context specific knowledge. It is also important to remark that several aspects of teachers defined by the context emerge through teaching, such as knowledge and beliefs.
2.19 **Technological Pedagogical Content Knowledge**

Several studies have been focused on other kinds of knowledge that should be possessed by mathematics teachers (e.g. Koehler et al., 2009). According to the theoretical framework for teacher knowledge developed by Koehler and Mishra (2009), the technology integration is critical for teaching. This framework entitled Technological Pedagogical Content Knowledge (TPACK) includes some of the categories that are previously mentioned above. TPACK is based on three main categories of knowledge, which are content, pedagogy and technology. Interactions between these categories contribute to teach effectively. Since technology integration is more frequent in classrooms, TPACK is emerging today. It provides a better environment for learning and teaching.

The next diagram displays the interaction among the three main categories (Figure 2.6). Technological pedagogical content knowledge (TPACK) is the heart, the intersection of the three main categories.

![Figure 2.6: TPACK Framework and its Knowledge Categories (Koehler et al., 2009).](image-url)
In this framework of knowledge base for teaching, several categories that involved the technology component were added such as technology knowledge, technological content knowledge, technological pedagogical knowledge, and the technological pedagogical content knowledge.

Technology knowledge refers to the technological information, knowledge about software, hardware, calculators, and all technological instruments that can be utilized in teaching and learning. Technological content knowledge is the interaction between technology and content knowledge. Technological pedagogical knowledge is regarding the use of technology in order to develop and design didactic activities and methods for teaching.

The technological pedagogical content knowledge “TPACK is an understanding that emerges from interactions among content, pedagogy, and technology knowledge” (Koehler et al., 2009,). “TPACK recognizes the central role of content and pedagogy in uses of educational technology” (Bull, & Bell, 2009). A growing number of studies that focused on this kind of knowledge is evident (Lesser & Groth, 2008; Sorto & Lesser, 2009). Sorto and Lesser (2009) created several items to measure technological pedagogical content knowledge but focused on statistics for middle school teachers.

2.20 KNOWING-TO AND KNOWING-ABOUT: KNOWING THAT, KNOWING-HOW, KNOWING WHY

Mason and Spence (1999) present a framework in which several forms of knowing are taught in schools such as “knowing-that”, “knowing-how”, and “knowing-why”. These three forms of knowing compose the “knowing-about”. Particularly, “Knowing-that” refers to the factual, to know about facts, topics, among others. “Knowing-how” means to know how to do something, techniques or skills utilized in accomplishing a particular task. “Knowing-why” is about having an argument in order to structure actions and from which to reconstruct actions (Mason et al., 1999). Jong and Fegurson-Hessler (1996) categorize these kinds of knowing as: situational knowledge, conceptual knowledge, procedural knowledge, and strategic knowledge.
Mason and Spence (1999) recognize that “knowing-about” is immersed in the Shulman’s categories (1987). However, they considered that more than these categories are necessary to enable a teacher to act at the moment required. “Knowing-about” is considered as static knowledge meaning knowledge possessed by a person, but it does not mean that she/he is able to act creatively in a particular situation. Mason (1998) refers to the term of “knowing-to act” (KtA) as “the kind of knowledge which enables people to act freshly and creatively” (p. 245). “Knowing-to” act involves active knowledge “knowledge that enables people to act creatively rather than merely react to stimuli with trained or habituated behavior” (Mason et al., 1999, p.136).

According to Skemp (1979b), “Knowing-to act” is to be able to use the knowledge or technique in a novel situation. This type of knowing implies more than possessing the abilities or knowledge. The essence of “knowing-to act” is the use or call of the knowledge when required. In Rowland, Huckstep and Thwaites (2005) a categorization of the kinds of knowledge needed to teach mathematics is provided. This framework is called “knowledge quartet”. One of its components is contingency, which is similar to “knowing-to act”. This component is concerned about how the teachers perform when students ask questions in a particular way. Contingency encompasses two aspects, the first of which is an ability of the teacher to respond to children’s ideas. This ability requires judgment and deliberation of children’s thinking. For instance, teachers need to do or say something in order to keep their students engaged. The other aspect is in regards to a deviation of the agenda. Several times teachers should deviate from their agenda in order to help students to construct their knowledge instead of continuing teaching without allowing students to grasp the foundations. This component takes place when unusual situations (cannot be predicted) are happening in classrooms. “The teacher’s intended actions-can be planned, but the students’ responses cannot.”(Rowland et al., 2005, p.263).

According to Mason and Spence (1999) the absence of “knowing-to act” blocks students and teachers to respond creatively in the moment, thus, there is a need for distinguishing “knowing-to act” from “knowing-about” and their elements. Awareness provides the occurrence
of real possibilities to “knowing-to act”. Gatteno (1970, 1987) defines awareness “to refer to that which enables powers that have been integrated into one’s functioning to be employed” (as cited in Mason, 1998, p.254). Therefore, the educating of awareness could be done through the focus on attention. The emphasis of teacher-educators in attention to develop their own awareness in order to be able to help others to become aware of their awareness in turn is critical. There is a study that shows how mathematics and science pre-service teachers struggled to act in the mathematical moment (Wilhelm, Sherrod and Walters, 2008). In this study, a project-based framework was designed to provide opportunities to pre-service teacher to act in the moment and experience mathematics and science in different ways that they had not seen. Videotaped classroom interactions in which pre-service teachers attended the course, journals, and project artifacts for mastering content serve to determine how well pre-service teachers functioned in the mathematical moment. These methods and procedures did not take place in classrooms where pre-service teachers were teaching. Therefore, studies that focus on how teachers act during their instruction in mathematical classroom is critical to promote critical thinking and lead students to learn. Having research on teachers’ “knowing-to act” could lead to find ways to improve this kind of knowing in pre-service teachers. Considering “knowing-to act” as a fundamental part of teacher knowledge, professional development activities could be implemented using activities and projects. Professional development such as this will promote and help in-service teachers to develop this kind of knowledge.

According to Mason and Spence (1999) there are three approaches to what “knowing-to act” in the moment means. The first one is the experiential approach which emerges as a group of situations elicited from teaching and learning mathematics. The second approach is more theoretical but also based on experiences. And the last one is the theoretical approach which is intended to clarify the meaning of knowing-to act during teaching and doing mathematics.

In the experiential approach, the “knowing-to act” is emerged from the generation of awareness in specific incidents during teaching and doing mathematics. The following description of an incident can help to clarify this approach. For instance, when one student asked
if the solution of one problem done by her or himself is correct, it is a situation arisen during teaching mathematics. This incident can promote the critical thinking of the student if the teacher does not answer yes or no immediately. For instance, several questions that help students to grasp the meaning could be posed in regards to his/her question. However, if the teacher says yes or no immediately after the student is asked, a limitation of the reasoning of the student about other possible procedures or about the correctness of his/her own way used to solve the problem takes place. The manner in which the teacher acts in this situation depends on the level of awareness and attention placed in this situation, and also of distinct types of knowledge.

The second approach is related to the association mentioned above. In other words, it is how the situations or incidents during the class and the knowledge base for teaching mathematics are related. This approach is based on the Ryle’s work (1949), in which different kinds of knowledge are analyzed. These kinds of knowledge are also considered part of the teacher knowledge by numerous studies (e.g. Shulman, 1986; Tchoshanov, 2011). These are that distinctions made by Ryle (1949). “Knowing-that” refers to the factual knowledge. “Knowing-how” means to know the procedure to perform acts, and “knowing-why means to have stories to account for phenomena and actions” (as cited in Mason et al., 1999, p. 137). Regarding the “knowing-to act”, it is seen as instantaneous; it is in the moment in which one idea comes to your mind.

The following process is the relationship among the different types of knowing: when “knowing-to act” takes place in a moment, the “knowing-how” is in charge of the development of the fresh idea; “knowing-that” builds the foundation, “the base energy upon which all else depends and on which actions depends; “knowing-why” provides an overview and sense of direction that supports connection and link making and assist reconstruction and modification if difficulties arise en route” (Mason et al., 1999, p. 146). “Knowing-how” provides the procedures, manners of doing things, in order to change and transform the situation, “and providing the various knowing with fresh situations upon which to operate” (Mason et al., 1999, p. 147-148). This process occurs in a certain context and environment. The level of awareness of the teacher
combined with specific elements of the situation that help to promote or come up with experience are what lead to the production of “knowing-to act” in the moment.

The third approach is the theoretical approach. It is through three distinctions which offer a way of thinking about “knowing-to act” in the moment. 1) “Responsive-knowing which produces background experience through which to respond in future” (Mason et al., 1999, p. 148). 2) “Active-knowing in which you can actively initiate use of knowing” (Mason et al., 1999, p. 148). 3) “Reflective or meta-knowing” it means that “you are aware of knowing and can describe and reflect upon that knowing” (Mason et al., 1999, p. 148).

2.21 CONCLUSIONS

Teacher knowledge is a focal point of study. There are several aspects of teacher knowledge that undoubtedly have an impact on student learning. Therefore, analyzing teacher knowledge will contribute to the field of mathematics education, and provide tools to prepare mathematics teachers with a stronger knowledge that has an influence on students’ gaining knowledge.

There are several contributions found in the literature review. However, there are still many gaps to address regarding teacher knowledge and its impact on teaching and learning. For example, studies have conceptualized or explored a specific kind of teacher knowledge, but the influence on students’ achievement is not primarily addressed nor is the relationship among other types of knowledge. A main contribution of Tchoshanov’s study is “teacher content knowledge of concepts and connections is significantly associated with student achievement and lesson quality in middle grades mathematics” (Tchoshanov, 2011, p 162). In addition, throughout this study, the importance of the pedagogical knowledge has in teaching mathematics is identified. Teachers are not able to teach effectively if they do not know pedagogical strategies, methods, or simply how to deliver content. Teacher content knowledge plays an important role in teaching mathematics even in the elementary grades as Hill, and colleagues (2005) showed in their findings.
Different kinds of the knowledge base for teaching have been considered by several studies in mathematics education (e.g. Shulman, 1986; Ball et al., 2008; Tchoshanov, 2011). Further studies are needed in which the association and exploration of these kinds of knowledge and students learning can be described. It is critical to understand what influences teachers’ instructional decisions to act in particular ways during specific moments throughout teaching mathematics in their classrooms. Therefore, studies that research how the “knowing-to act” of teachers during their instruction in the mathematical classroom matters in producing student learning are essential.

The big picture of the field of inquiry on teacher knowledge was discussed in this chapter. However, this study was based on Shulman’s study (1986) in regards to the content knowledge category. Tchoshanov’s study (2011) about cognitive types of content knowledge was used as foundation for this research on examining mathematical teacher content knowledge of middle school teachers in Mexico. In addition, “knowing-to act” conceptualizations provided by Mason and Spence (1999) served as lenses to analyze the Mexican middle school teachers’ “knowing-to act” in Mexico.

### 2.22 SUMMARY

This chapter included essential information in regards to the literature review and theoretical framework of this research. A comprehensive description of the educational system in Mexico was provided. Also insights into the curriculum of mathematics middle school as well as the curriculum of the mathematics teacher preparation at the middle school level in Mexico were described. The current circumstances of education in Mexico, historical antecedents of education in that country, and research that is focused on mathematics teachers were presented. The second part of this chapter provided an extensive view of this field of inquiry of teacher knowledge in mathematics. Descriptions of different types of teacher knowledge as well as relevant empirical studies on those types were discussed. The methodology and the research procedures that were implemented in the study will be discussed in the following chapter.
Chapter 3: Methodology

3.1 Overview

This chapter will present the methodology that was implemented in this mixed methods study. The chapter begins with the purpose statement for the study. Next, the description of the research design will be discussed. This section is followed by the sampling techniques that were utilized as well as the context of the study. After that, the study will be discussed in four phases: quantitative phase, interim phase, qualitative phase and interpretation phase. Each phase will describe the methods, data sources, data analysis, and procedures that were implemented.

3.2 Mixed Methods Purpose Statement

The purpose of this study was to measure the cognitive type of middle mathematics school teachers’ content knowledge and its relationship with teachers’ “knowing-to act” ability. An explanatory sequential mixed methods design was used, and it involved collecting quantitative data first and then explaining the quantitative results with in-depth qualitative data.

In the first, quantitative phase of the study, two surveys were administered in order to collect data from 70 mathematics teachers in middle schools in the Mexican borderland to assess their content knowledge and “knowing-to act”. Moreover, a correlational analysis was conducted to examine if teachers’ mathematical content knowledge was related to their “knowing-to act”. The second, qualitative phase was conducted as a follow up to the quantitative phase to help explain the quantitative results. In this explanatory follow-up, the exploration done was of teachers’ actions performed during mathematics instruction in their classroom by four mathematics middle school teachers purposefully selected from the sample of the quantitative phase.

In this study, the implication that teachers’ content knowledge might have an impact on how teachers act in the classroom is critical for the field of mathematics education. The analysis of the teacher content knowledge was done utilizing the different cognitive types: cognitive type 1 (knowledge of facts and procedures); cognitive type 2 (knowledge of concepts and connections); and cognitive type 3 (knowledge of models and generalizations). Knowledge of
the relationship between teacher content knowledge and “knowing-to act” may help teacher education programs to implement activities or courses that allow teachers to develop their “knowing-to act” based on their content knowledge. Therefore, it is essential to know what types of teacher content knowledge are possessed in order to make possible to determine to what extent a teacher’s “knowing-to act” is impacted by her/his cognitive types of content knowledge.

This mixed methods study provided further understanding of the relationship between teacher content knowledge and “knowing-to act”. As Chapter 2 showed, few studies have used a mixed methods approach to analyze teacher content knowledge and “knowing-to act” (KtA). Therefore, this study contributed to the mixed methods community promoting the use of this approach. Based on this study, teacher education programs will be able to make adjustments in certain aspects of teacher knowledge during their preparation. Moreover, this study provided awareness to teachers about the importance that teacher mathematical knowledge has in order to know how to act in the teaching moments that challenge them. And teachers will be more cognizant about engaging in professional development that helps them to expand their content knowledge.

3.3 Research Questions

The following questions guided the research:

1. To what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ “knowing-to act”?

2. How do the teachers act in the KtA situations occurring in mathematics classrooms?

3. How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?

4. To what extent do the qualitative findings explain the quantitative results of the study?
3.4 **Research Design**

In connection with the literature review and the research questions, a mixed methods study was conducted. A mixed methods study is more than only the implementation of quantitative and qualitative methods; it has its own philosophical assumptions as quantitative and qualitative does.

Mixed methods research is a research design with philosophical assumptions as well as methods of inquiry. As a methodology, it involves philosophical assumptions that guide the direction of the collection and analysis and the mixture of qualitative and quantitative approaches in many phases of the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. Its central premise is that the use of quantitative and qualitative approaches, in combination, provides a better understanding of research problems than either approach alone (Creswell & Plano, 2007, p.5).

In this study, a mixed method research design was used for several reasons: (a) this type of approach allows the researcher to mix the quantitative and qualitative methods to offset the weaknesses of each method to draw on the strengths of both (Bryman, 2006); (b) implementing both quantitative and qualitative methods provided a comprehensive account of the relationship between mathematical content knowledge and “knowing-to act” of middle school teachers in Mexico, through the elaboration, explanation, clarification, and illustration of the quantitative results with the qualitative findings (Bryman, 2006; Greene, Caraceli, and Graham, 1989); (c) the diverse nature of the research questions of the study indicated the mixed methods approach was the most appropriate to conduct the study; (d) the mixed methods research approach allowed the qualitative phase to extend the breadth of the relationship between the two specific kinds of teacher knowledge studied (mathematical teacher content knowledge and “knowing-to act”) that were analyzed in the quantitative phase; and (e) conducting a mixed method study provides credibility due to the fact of the integrity of findings (Bryman, 2006).
A mixed methods sequential explanatory design was utilized in the study. It is a design in which “the researcher begins by conducting a quantitative phase and follows up on specific results with a second phase. The second phase is implemented for the purposes of explaining the initial results in more depth.” (Creswell & Plano, 2011, p.82). Thus, the mixed methods sequential explanatory design is composed of two phases: quantitative followed by qualitative (Creswell, Plano, Gutmann, & Hanson, 2003). In the first phase, quantitative methods were conducted. And then in the second phase, qualitative methods were used. During the first phase the administration of surveys and their analysis was done. The second phase was built on the first phase; therefore, an interim phase was inserted before the second phase. In the interim phase the considerations and decisions needed to start the qualitative phase were made. A fourth phase was added where the interpretation of the integration of the findings of the qualitative phase that helped to explain the results of the quantitative analysis was conducted. The rationale for this design was that quantitative analysis and results could be better explained and illustrated with the implementation of qualitative methods. In addition, studies in which both methods are used provide credibility to the findings (Bryman, 2006, Creswell & Plano, 2011).

This design implied the sequential implementation of a quantitative phase followed by an interim phase that preceded the qualitative phase and finalized with the interpretation phase, as mentioned. According to Creswell and Plano (2011), this research design of explanatory sequential mixed methods study implies multiple philosophical assumptions: post-positivism, constructivism, and pragmatism. These philosophical assumptions supported a specific phase of the study. The quantitative phase is supported by the post-positivism. The qualitative phase is supported by the constructivism assumptions. In the quantitative phase, numerical data was collected through the administration of two surveys: teacher content knowledge survey (TCKS) and knowing-to act survey (KtAS). Then, the analysis of the answers on these surveys was done. During the interim phase, based on the results of the analysis of the surveys, a case selection (N=4) was decided upon as well as the development of an observation protocol. After that, in the qualitative phase, classroom observations and interviews were conducted. Then, the following
implementation of a fourth phase was done. This phase is called the integration phase. In this phase the pragmatism assumption was considered. Using the results of the analysis of the qualitative data, the interpretation and explanation of the obtained results in the quantitative phase was provided in the interpretation phase (see Figure 3.1).
3.5 **Context of the Study**

Mexico, the country where the study was conducted is a third world country. It is located next to the U.S. Mexico is largely dependent on the economy of the United States. Currently, there is a problem of crime in the whole country. Corruption, poverty, and the illegal drug trade, among other issues are part of the social conditions that people live in Mexico. Over 112 million people live in Mexico (INEGI, 2013). However, there are 53.3 million people living in poverty (Consejo Nacional de Evaluación de la Política de Desarrollo Social-CONEVAL, 2013). In regards to education, Mexico has a deficiency in education of 32.5 million people (40.2%) who have not completed the basic education from the population of adults over 15 years old (INEA, 2010). More than 6 million students are enrolled at the middle school level in the country, including private schools (SEP, 2010).

This study was conducted in a Mexican city located on the border with the United States. This city has particular features such as cultural diversity, and ‘floating’ population. People from Central America countries and other states of Mexico live in that city. Its geographic location is appropriated by people trying to reach their American dream and cross the border into the U.S. Most people live in this city until they move to the United States side; therefore, the population of the city is called a “floating” population. However, this city has a large population. According to the census done by the INEGI (2013), there was a population of about 1,332,131 people in the city in 2010.

As mentioned in Chapter 1, the population of this border city has increased 400% since 1973 (Morales, Rodriguez, and Sanchez, 2013). This growth is mainly due to the need for workers required by industrial companies in this region of the country (IMIP, 2003), and the accessibility of migration to the United States. Industrial and automobile manufacturing are the main sources on which the official economy of the city is based (Monarrez, 2012). In 2006 in the whole country, there was a war against the drugs trade. Also, a war among drug cartels started in Mexico. The border cities are some of the areas most affected by the war and crime since their
location next to the U.S. (Monarrez, 2012). During the last 6 years, thousands of homicides have occurred across the city, even in front of school buildings. Beside homicides, extortions and robberies have overwhelmed this city. This crime together with the migration to the U.S. is the situation that everybody lives including students, and teachers in the city that is the site for this study.

In Chapter 2, we saw that in Mexico there are different types of public middle schools: federal, state, technical, and TV middle schools. In this study, we focused on two types of middle schools, A and B, because of the complexity of accessibility, the logistic procedures that are involved in recruiting participating teachers from different types of middle schools, and the lack of interest to participate in the research. In addition, TV middle schools provide the instruction via televised recording in classrooms. Thus, this last type of middle schools is not considered in this study. There were 26 middle schools in this city located in the Mexican borderland with United States that were the sources of the teachers recruited as participants. These middle schools are located throughout the whole city, including areas in where families with extreme poverty live. Students’ populations, students’ social status, and school’s infrastructure are varied. The student enrollment in the schools ranges from 262 to 1063 students (Sistema Nacional de Información Estadística Educativa-SNIEE, 2010). The number of students per classroom ranges from 23 to 55 (SNIEE, 2010) as observed in Table 3.1. Also, the dropout rate varies from 0% to 22.14%, and the course failure rates range from 0.38% to 54.21% of students (for any course not only math).

Table 3.1: Demographic Information of Participant Middle Schools (SNIEE, 2010).

<table>
<thead>
<tr>
<th>ID School</th>
<th>Students’ Enrollment</th>
<th>Students per Classroom</th>
<th>Dropout Rate</th>
<th>Course Failure Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1063</td>
<td>51</td>
<td>5.3%</td>
<td>23.33%</td>
</tr>
<tr>
<td>B</td>
<td>732</td>
<td>44</td>
<td>7.77%</td>
<td>28.18%</td>
</tr>
<tr>
<td>C</td>
<td>809</td>
<td>39</td>
<td>3.62%</td>
<td>30.04%</td>
</tr>
<tr>
<td>D</td>
<td>1004</td>
<td>51</td>
<td>7.27%</td>
<td>18.08%</td>
</tr>
<tr>
<td>E</td>
<td>807</td>
<td>43</td>
<td>0.43%</td>
<td>25.11%</td>
</tr>
</tbody>
</table>
3.6 Sampling

Sampling selection of participants was done using the sequential mixed methods sampling technique (Tashakkori & Teddlie, 2003), it means the sample for the quantitative phase was the frame for the qualitative phase, and the techniques for participant selections were implemented sequentially: (a) seventy mathematics middle school teachers were selected and invited to fill out two surveys. The criterion for selection of these teachers focused on what they were teaching at the time of the study; all participants were teaching at least one mathematics course at middle school level. A survey (TCKS) was administered to these (70) teachers to measure their content knowledge. A second survey (KtAS) was about the teachers’ actions enacted during teaching mathematics at particular moments. (b) Based on the results of these two surveys, the researcher selected 4 mathematics’ teachers who performed in a specified way in the
surveys, which will be clarified in the interim phase. After that, teachers were observed and video recorded in their mathematics classrooms. These four teachers were also interviewed individually.

3.7 Quantitative Phase

3.7.1 Participants

The sample for this phase consisted of 70 Mexican teachers. As mentioned in previous chapters, in Mexico there are four different types of middle schools. However, three out of the four types were suitable for this study because those types of schools have middle schools where the instruction was provided in classrooms by teachers (Types A, B, and C). The sampling strategy used was: all schools from type A middle schools were first visited. Also several schools from type B middle school were visited because the complex access to the schools and logistic procedures did not allow visiting more schools of this type. And in the type C middle schools, all mathematics teachers attended a mathematics academy; this attendance was mandatory. Teachers were invited to be part of the study during the academy, and they were allowed to answer the surveys of this study. However, teachers from this last type of middle schools did not want to be part of this research. Teachers from each middle school visited from the type A and B middle schools mentioned above were asked to be part of this research. The researcher did not know who and how many teachers were going to participate from each school because that information was not provided. According to Tashakkori and Teddlie (2003), the convenience sampling strategy implies the selection of the sample that is “…both easily accessible and willing to participate in the study…” (p.170). This definition of sampling matched the process for sampling implemented in this study. The participating teachers were teaching mathematics at the middle school level at the time of the study. The teacher sample was drawn from 26 different public middle schools which belong to two out of the three types of public

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1The kinds of the participating middle schools in the study are not revealed in order to keep the confidentiality of the participants because it might lead to disclosure of their identity. In addition, it might cause political conflicts because of the current situation in regards to the new educational reform approved.
middle schools in Mexico. Seventy teachers filled out two surveys: one was the teacher content knowledge survey (TCKS), and the other one was the knowing-to act survey (KtAS). These surveys assessed the content knowledge of the teachers and their “knowing-to act”.

3.7.2 Quantitative Research Design

In this phase, the quantitative data was collected. The data sources were the two instruments mentioned above. The administration of both surveys was simultaneous. In other words, teachers were provided with both surveys at the same time. Mathematics middle school teachers were allowed to complete both surveys in two hours. One survey is the Teacher Content Knowledge Survey (TCKS) and the second one is the Knowing-to Act Survey (KtAS). The administration of the surveys was face-to-face: the researcher and the participating teachers. The administration of these surveys took place during the school year 2013-2014. First, teachers filled out several questions with their demographic information (see Appendix F). One of the instruments used, the teacher content knowledge survey (TCKS) instrument, was designed by Tchoshanov (2011) to measure the cognitive types of content knowledge. TCKS was translated to Spanish for the Mexican teachers. The TCKS was administered and answered in 90 minutes which was the maximum time allowed. With the administration of the TCKS, an analysis about teacher content knowledge of Mexican Mathematics secondary school’s teachers (middle school) came possible.

The knowing-to act survey (KtAS) instrument was developed by the researcher and two committee members. One of the two members has expertise in mathematics, mathematics education, and instrument development. The other member of the committee has expertise in research methods, statistics, statistics education, and instrument development. The KtAS survey assessed the “knowing-to act” ability of the mathematics middle school teachers. The analysis of the data collected from each survey was done separately. In connection with the literature review and research questions for this study, a correlation analysis was designed and implemented to
seek for significant correlations among the two kinds of teacher knowledge and its components: mathematical teacher content knowledge (Type 1, 2 and 3) and teacher “knowing-to act”.

This research design was implemented in order to answer the first research question of the study: to what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ knowing-to act?

3.7.3 Instruments

Two survey instruments were used in this study. One instrument was the teacher content knowledge survey. The other one was the knowing-to act survey. A teacher Content Knowledge Survey (TCKS) is the instrument designed to assess the teacher content knowledge based on the three cognitive types identified: Type 1 knowledge of facts and procedures, Type 2 knowledge of connections and concepts, and Type 3 knowledge of models and generalizations (Tchoshanov, 2011). This knowledge is categorized in the three cognitive types mentioned above. The survey contained items that measured each of the cognitive types. Therefore, the mathematical teacher content knowledge was examined through the items that measured the cognitive types of content knowledge.

This survey consists of 33 multiple choice-items about relevant topics for secondary grades teachers’ knowledge: Number sense, Algebra, Geometry and Measurement, Probability and statistics. The team who developed the instrument consisted of interdisciplinary faculty with expertise in the following domains: mathematics, mathematics education, statistics and statistics education and represented many institutions such as university, community college and local schools. The main steps were the selection of items for the survey, the classification of items by cognitive type, and the suitable positioning of an item in a specific cognitive type (one item can measure one or more cognitive types). The instrument was field-tested during 2005-2006 (Tchoshanov, Lesser, & Salazar, 2008). The alpha coefficient technique (Cronbach, 1951) was utilized to evaluate the reliability of the teacher content knowledge survey instrument. “The
value of the coefficient of .839 suggests that the items comprising the TCKS are internally consistent (standard error $= .59$).” (Tchoshanov, 2011, p.148).

The sum of the three cognitive types of knowledge items is 33, which is the total of the items of the TCKS. There are 10 items that measure the cognitive type 1. For instance, the item number 7 shown in Illustration 3.1 is an item that measures the knowledge of facts and procedures. The cognitive type 2 is measured by 13 items. The item number 8 in Illustration 3.1 is an example of an item that measures the cognitive type 2 due to the fact than in order to answer it correctly, the teacher needs to have conceptual understanding and not only procedural knowledge. The rest of items (10) are measuring the cognitive type 3. For example, in Illustration 3.1, the item number 9 measures the knowledge of models and generalizations. The teacher content knowledge survey does not have any division per cognitive type. All the items are located randomly throughout the survey. Appendix G includes the complete TCKS.
Illustration 3.1: Examples of TCKS Items.

As Illustration 3.1 shows the items are multiple choice. Four choices are provided for each item. Only one choice is the correct answer. Items that were not answered on the TCKS are considered incorrect. Several studies (e.g. Sorto et al., 2009) reported that items unanswered are considered incorrect because persons do not answer the items because of the lack of knowledge. Also, the time used to answer an item increased when there is a lack of knowledge. This leads to more unanswered items on surveys or tests. Teachers were given a score of 1 per each correctly
answered item. The score of 0 was given to each item answered incorrectly. During the survey, teachers were allowed to use a calculator. Teachers answered the surveys individually. The researcher was supervising teachers while they were answering the surveys. Teachers could obtain a maximum raw score of 33 on the TCKS.

In regards to the knowing-to act survey (KtAS), it is an instrument designed by the researcher of this study and two members of her doctoral committee. The survey evaluates the ways in how teachers recall their active knowledge to act in the moment when it is required. In other words, it measures the actions and decisions that a teacher takes during mathematics instruction. The evaluation of these actions were considered as the most desirable according to an expert analysis and the literature review of the best teaching practices of mathematics. The expert analysis was accomplished with a team of faculty professors in the area of mathematics, statistics, and mathematics education. There were four professors who were part of this expert analysis. This team analyzed and determined which choices are the most desirable in indicating the level of preparation to teach mathematics. The review of the best teaching practices in mathematics shows which primary teaching practices help to increase student learning, and which ones lead to decrease student engagement and learning (Zemelman, Daniels, and Hyde, 2005). For instance, teaching practices that increase student learning are on the KtAS survey are questioning and making conjectures, content integration, cooperative group work, and others. On the other hand, the teaching practices that lead to decrease learning of mathematical knowledge are: teaching by telling, repetitive written practice, single answers and single methods to find answers, among others. Following the expert analysis and the recommendations on teaching mathematics (Zemelman, Daniels, and Hyde, 2005), the assessment of the “knowing-to act” of mathematics middle school teachers was conducted through the KtAS.

The KtAS survey is composed of 11 items. Each item describes a classroom situation. The nature of the situation varies. Even though these situations were included because they are commonly found during mathematics instruction, they challenge the “knowing-to act” of the teachers. These situations are called “knowing-to Act” situations (KtA situations). The selection
of these particular situations was done considering previous research where the implication of the challenge of the “knowing-to act” was already identified (e.g. Mason & Spence, 1998; Rowland et al., 2005). For instance, Mason and Spence (1998) recognized that when a student asks if the answer or steps are correct, the “knowing-to act” of the teacher can be enacted. As the previous situation described, the situations included in this survey are already identified as an opportunity where the teacher’s “knowing-to act” arises and promote the critical thinking of students. The odd and even split-half reliability coefficient (Crocker & Algina, 1986) was utilized to evaluate the reliability of the knowing-to act survey instrument. The value of the coefficient of .58 suggests that the items comprising the KtAS are internally consistent.

There are four categories in which the classroom situations provided in the KtAS survey are classified: student misconceptions (Shulman, 1986; Kulm, Capraro, Capraro, Burghardt, & Ford, 2001; An et al., 2004); student difficulties (Fennema & Loef, 1992; Ball et al., 2008); situations that are a challenge for the teacher (Mason & Spence, 1998; Rowland et al., 2005); and emerging situations (Mason & Spence, 1998; Rowland et al. 2005). Each item presents five choices to be ranked. Each choice is a possible action taken by teachers. They should rank the five options listed below in each of the situations presented according to their most likely action to do first from 1 to 5 in which the number 1 indicates the action that they would do first in that situation and so forth. The following figure shows an example of one item of this survey (Illustration 3.2):

```
1. When a student solves a particular mathematical problem and asks if the answer or steps are correct, which of the following would you most likely do? Rank in order from 1 to 5 (1 - you most likely do first, 5 - you least likely do)

   — Tell him/her whether his/her response is correct.
   — Ask another student to confirm if it is correct or not.
   — Ask the student to read the corresponding section of the textbook.
   — Ask the whole group to share what they think.
   — Ask the student to explain his/her solution.

Explain the choice you most likely do first below:
```
Illustration 3.2: Example of KtAS Item and Ranking Format.

Teachers were asked to respond to the KtAS survey within the time allowed of 30 minutes. If teachers selected the choice ranked as number 1 by the expert analysis done for this survey as their number 1 choice also, they would also obtain a 1 in that item. If the teachers selected another choice, they would get a 0 in that item. Then, the total score of the KtAS survey is provided by the sum of all the items. This survey is provided in Appendix E.

3.7.4 Variables

Based on the literature review, the following independent variables were considered in the analysis of this study: cognitive type 1 (T1), cognitive type 2 (T2), cognitive type 3 (T3), and the total scores of teacher content knowledge survey (TCKS). The dependent variable is the “knowing-to act” (KtAS) which means the total scores on the survey. Cognitive types are measured through the number of correct answers of the items for each type of knowledge in the teacher content knowledge survey. The number of total scores of the TCKS is calculated by adding all the correctly answered items in the survey. The “knowing-to act” was examined through the scores obtained in the knowing-to act survey (KtAS).

3.7.5 Data analysis

In this phase, the description of the variables was done through the use of basic statistics (descriptive) (see Chapter 4). Data analysis was performed using correlational analysis (Pearson coefficient) to determine how the cognitive types and teacher content knowledge total scores are related to the teacher’s “knowing-to act”. The decision to select this parametric technique is based on the first research question of this study. All the procedures of this analysis were done using the data analysis tools of Microsoft Excel software.

3.8 INTERIM PHASE

As a mixed methods sequential explanatory design, the qualitative phase was built on the quantitative phase. Therefore, based on the results of the previous phase (quantitative), the
decisions about the sampling and the development of one instrument for collecting qualitative data was made at this interim stage of the research process. Considering the possible options for connected data analysis, strategies such as the selection of participants whose scores met certain criteria were considered.

3.8.1 Point of Interface for Mixing

The primary point of interface for mixing for the explanatory sequential design was at this point. This point of interface for mixing was the connection between both quantitative and qualitative strands. This mixing occurred after analyzing the quantitative results and deciding which results were going to be used to guide the qualitative phase. Therefore, in the interim phase, important decisions were made for conducting the qualitative phase. Specifically, the case selection was conducted, and the observation protocol was developed.

3.8.2 Case Selection

A purposive sampling strategy, also known as criterion sampling, for the selection of participants was utilized based on the quantitative results of the two surveys: TCKS and KtAS. According to Miles and Huberman (1994), criterion sampling is a type of sampling which “includes all cases that meet some criterion, useful for quality assurance” (p.28). Table 3.2 shows the criteria that used to make the case selection: a teacher who has a high score on the TCKS and a high score on the KtAS was one of the four teachers. Another teacher who got a low score on the TCKS, and a high score on the KtAS was also selected as well as a teacher whose performance on TCKS was high, but on the KtAS was low. Lastly, another teacher whose performance on both surveys TCKS and KtAS was low was selected.

Table 3.2: Criteria for Case Selection Based on the Quantitative Phase.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>TCKS</th>
<th>KtAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher 1</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Teacher 2</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Teacher 3</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>Teacher 4</td>
<td>Low</td>
<td>Low</td>
</tr>
</tbody>
</table>
3.8.3 Protocol Development

The development of the protocol for classroom observation was conducted in this phase. Based on the results of the quantitative phase. The observation protocol was designed to answer the second question: how do the teachers act in the KtA situations occurring in the mathematics classroom?

The observation protocol consisted of the same classroom situations that were presented in the KtAS. There are 11 situations in this survey. As described earlier in this chapter, there is a categorization of these situations: (a) student misconceptions; (b) student difficulties; (c) challenge for the teacher; and (d) emerging situations. The classroom situations were clustered according to their nature in the observation protocol. For example, in the category of student misconceptions, there are three classroom situations. The first one is when a student solves a particular mathematical problem and asks if the answer or steps are correct. Another situation is when a student provides a nonsensical solution to a given problem. And the third situation under this category is when a student makes a mistake. In the category of student difficulties, there are four situations. Also, there are four situations in the category of challenge for the teacher. According to the categorization of the situations, they were organized and located on the survey. As mentioned in definition of terms in Chapter 1, the classroom situations included in the KtAS survey and in the observation protocol are referred to as KtA situations.

After each classroom situation in the KtAS survey, some space was left below each situation in order for the researcher to be able to write down notes about the phenomena observed. The difference between the observation protocol and the KtAS is that in the KtAS there are five possible actions to select that can be performed by teachers. Teachers in the KtAS ranked those actions from 1 to 5 as their most likely to do first. In the observation protocol, there are no choices to be ranked. There is a blank space to capture the teacher’s actions in those situations. Moreover, there is also more space on the back of the protocol to take notes about
emerging situations where the “knowing-to act” appears. After the development of this protocol, the research moved to the next phase, the qualitative phase.

3.9 Qualitative Phase

This phase involves qualitative research. This phase of the study sought to understand the practices (actions) enacted by mathematics middle school teachers in Mexico during mathematics instruction. Qualitative research places a holistic emphasis (Jackson & Verberg, 2007) on the phenomena under study. It means to conduct the research of mathematics middle school teachers in natural settings where their “knowing-to act” is enacted. As Marshall and Rossman (2011) state a particular setting influences the human actions performed in it. Therefore, to study the participating teachers’ actions during instruction in the mathematics classroom was crucial in order to understand and describe the “knowing-to act” of mathematics middle school teachers.

In Mexico, middle schools work in a different manner than in American middle schools. Therefore, to be in the environment where the mathematics teachers teach every day allows us to understand what a Mexican teacher does, and how the process of teaching and learning takes place in Mexican middle schools and specifically in the mathematics classrooms. Thus, the “knowing-to act” of the teachers was captured through face-to-face interactions with the participant and the natural setting where the actions were enacted.

In this phase of the research, the objective was not making generalizations as a quantitative study does. Quantitative research has the aim to measure and analyze causal and correlational relationships among phenomena within a value-free framework with the purpose of generalization (Denzin & Lincoln, 1994) as we saw it in the first phase of the study. Since each individual has his or her own thoughts, feelings, values, and beliefs that directly influence what he or she is and does, a generalization was not generated in this qualitative phase of the study.

Since the purpose of this qualitative phase was to understand and explore the teachers’ performance during mathematics instruction in which their “knowing-to act” was involved, the
observation and analysis of teachers’ “knowing-to act” was ensured. Quantitative research is based on a post-positivist approach as mentioned in Chapter 1. In post-positivist philosophies, statistical methods are used, such as hypothesis testing, regression model, correlation, etc., (Gravetter & Wallnau, 2013), as implemented in the quantitative phase. However, in the qualitative phase, we were not focused on proving any hypothesis or making a correlation between variables. This phase sought to understand an issue in order to provide insights about it and build a base to comprehend something else (Stake, 2008). To count just how many actions teachers take during mathematical instruction to bring active knowledge does not help to describe and understand those actions. Therefore, there was no better approach to conduct this phase of the study than conducting it from a qualitative research paradigm such as constructivism as mentioned in Chapter 1.

3.9.1 Design of the Qualitative Phase

As the literature review showed, there is a dearth of studies that focus on teachers’ “knowing-to act” in situations that require active knowledge. Of great relevance for the field of mathematics education is to develop awareness among middle school teachers of mathematics about their actions or “knowing-to act” during mathematics instruction because this knowledge can increase or limit the students’ opportunities to learn mathematical concepts. Therefore, an exploration of the “knowing-to act” of four mathematics middle school teachers was conducted in order to identify and classify those teachers’ actions. This qualitative phase documented actions that four mathematics middle school teachers employed while teaching their mathematics courses. The research questions addressed in this phase are:

2. How do the teachers act in the KtA situations occurring in mathematics classroom?

3. How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?
3.9.2 Case Study Method

The research questions addressed in this phase are exploratory in nature. A case study method was selected. According to Yin (2003), “The case study as a research strategy comprises an all-encompassing method- covering the logic of the design, data collection techniques and specific approaches to data analysis” (p.12). In other words, a case study is a comprehensive research strategy (Stoecker, 1991) which allows the explanation and exploration of the classroom actions and performance by the mathematics middle school teachers during mathematics teaching.

In the design for this qualitative phase, a multiple-case study method was utilized in order to identify teachers’ actions during instruction of four middle school teachers engaged in their classes, as the literature review showed that no published studies address this. A multiple case study is also called a collective case study (Stake, 2008). It is utilized when several cases are studied jointly help to understand a phenomenon, population, or general condition (Stake, 2008). The multiple-case study was designed to represent the actions performed during instruction by mathematics middle school teachers. Thus, this study contributed to the field by relating the actions employed by four Mexican middle school teachers in their mathematics classrooms.

The design of this study is an instrumental case study. According to Stake (2008), it means the study of a case such as a person, or a specific group, or organization to present insights into a particular issue. Using this research design, the case helped to get a better understanding of something else. For instance, the study of four mathematics middle school teachers selected to be observed as they teach. These observations helped to provide a comprehensive understanding of the knowledge that calls the actions utilized by teachers to teach mathematics in moments that require active knowledge. In other words, the multiple “case study is of secondary interest, it plays a supportive role,… the choice of case is made to advance understanding of that other interest” (Stake, 2005, p.445). Therefore, exploring the actions enacted during instruction of each of the four mathematics teachers allowed the researcher to
understand the complexities of Mexican middle school teachers’ actions during mathematics instructions.

3.9.3 Participants

In this phase of the study, the sampling strategy for the selection of participants that was utilized is purposive sampling, as mentioned in the interim phase. From the 70 teachers that participated in the quantitative phase, 4 teachers were selected to participate in the qualitative phase (see Figure 3.2). Since this is an instrumental case study, the selection of four mathematics middle school teachers was decided in order to capture the teachers’ “knowing-to act” during mathematical instruction. These teachers met the criteria previously mentioned about the scores obtained in two surveys that were administered in the first phase of the study as reported in Chapter 4.

![Figure 3.2: Qualitative Sample](image)

The four middle school teachers in mathematics participating in this phase of the study worked in the settings described earlier in this chapter. All participants were teaching at least one mathematics course at middle school level at the time of the study. Cross-case analysis was used as a part of the criterion for selecting teachers. It consisted of the understanding of the “knowing-to act” of teachers whose performances in two surveys was done in a contrasting manner as observed in Table 6 with teachers 2 and 3. Based on results of the two surveys, the researcher
selected 4 mathematics’ teachers to be interviewed and their teaching observed and video recorded (three classes).

3.9.4 Data Sources

To explore the “knowing-to act” of the four mathematics middle school teachers during mathematics instruction, I employed multiple data sources. Individual sources of data for each of the four middle school teachers consisted of field notes that I took during and after the classroom observations. I was using video recordings to document quotes that teachers spoke when teaching mathematics at middle school level. Additional sources of data were the individual interviews with each of the four mathematics teachers. These interviews were transcribed and video recorded in order to capture as many details as possible. Also, I took field notes based on these individual interviews.

Classroom Observations

The classroom observations took place at the end of the school year 2013-2014. I video recorded the classes that I observed to try to cover every detail possible. I observed and video recorded a total of 12 classes, three for each participating mathematics teacher. The three classroom observations for each teacher were conducted within a week’s period during April or May. There was an exception where one of the teachers was observed twice before the spring break in Mexico, and was observed the following week after the break, thus the observations of this teacher were conducted within a three week period. I observed the four teachers teaching different mathematical topics. There was enough time to write and expand field notes after each observation. Each class lasted 45 minutes; thus, in total I observed and video recorded 9 hours. Also, I took handwritten notes during the observations. Data from this source helped me to answer the second research question of the study.

Field notes

I typed up and expanded my field notes (handwritten notes) within 24 hours of each observation in order to recall as much as possible. The field notes were descriptive and
reflective. They included classroom situations, teacher’s actions, ideas, quotes, impressions and speculation about what occurs during instruction related to the teacher’s actions. Data from this source helped me to answer research questions number 2 and 3.

**Point of interface for Mixing**

After the classroom observations and the field notes were expanded, the analysis for the observations was performed. The triangulation between the teacher’s actions observed and the KtAS survey was conducted. In this triangulation, several teacher’s actions were identified that differed from what teachers responded previously on the KtAS survey. In this point of interface for mixing, the decision for connecting the findings of the observations with the results of the KtAS survey was made. This connection was based on this triangulation. This point of interface for mixing allowed the design of the interview questions for the qualitative phase. Video recordings were selected to be shown during the interviews with the teachers and questions about the difference between what they responded on the KtAS survey and what they did during the observations were prepared.

**Interviews**

Individual interviews with four mathematics middle school teachers were conducted. The questions for the interview were designed to answer the third research question: How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their acts are not aligned with their responses on the Knowing-to-Act survey? I conducted semi structured interviews in which I prepared the questions to be asked in the interview after watching each video recording. I allowed the participating teachers to describe their practices in order to be able to identify their actions and intentions. These interviews were video recorded. The alignment of this data source with the research questions was done through the third question. The design of the interview questions was based on the research question, the teacher’s actions observed, and the responses on the KtAS during the quantitative phase.
There were a total of four interviews. I used the same format of semi structured interview for each middle school teacher. The interview consisted of showing video recordings of the participating teachers during their teaching. Seven video recordings were shown to two out of the four teachers; ten video recordings were presented to another teacher; eleven video recordings were shown to the other participating teacher. The number of video recordings shown depended on the participant. In some participant’s classes, certain KtA situations were observed only once. However, other participants’ classes exhibited the same situation several times.

Each interview lasted 60 minutes. The amount of time was chosen to offer teachers enough time to answer and reflect on each question, and to leave open the conversation in the interview to allow the emergence of important aspects or details that could help me to understand the “knowing-to act” of teachers during mathematics instruction.

The format of the interview was: watch the video clip, and ask the three following questions after the video recording:

1) Describe what you saw in the video clip
2) What was your thinking to act this way in the situation captured in the video clip?
3) Your response in the KtAS was the following (show the participant her/his response in the survey). It is different from the action performed in the video clip. Explain why.

With these questions, teachers were allowed to describe the situation that occurred in the video recording. After that, the participating teachers were asked to justify their actions enacted in that situation. Then, the teachers were provided with their responses on the KtAS survey and asked about why they acted in a different way from what they responded on the survey that would do. Teachers provided their reasons about this difference between the actions observed and the responses on the KtAS survey. This process was repeated every time that a different video recording of their teaching was shown. Teachers’ responses in the interviews helped me to identify and understand the “knowing-to act” of this teacher.
3.9.5 Data Collection and Analysis

As mentioned above, several data collection methods were used to gather data for the qualitative phase: classroom observations, video-recordings of the classroom observations, field notes, and semi-structured interviews. The data collection process for this qualitative phase started with the classroom observations and video recordings of the classroom observations that I observed as Table 3.3 shows. Then, I took field notes during the classroom observations, and I expanded these notes after the observation. After that, the analysis of these data was performed to design the interview questions. Also, I checked the responses of these four teachers on the KtAS survey. The responses on the KtAS survey together with the analysis of the classroom observations allowed the development of the interview questions. Then, the semi structured interview with each of the four mathematics middle school teachers was conducted and videotaped. Also field notes and transcriptions were done from these interviews. Table 3.3 summarizes the different data sources for the qualitative phase and the corresponding collection dates.

Table 3.3: Qualitative Data Sources.

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Dates (2014)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classroom observations</td>
<td>April 9, 29, May 2, 7, 8, 12, 13</td>
</tr>
<tr>
<td>Video recording of the observations</td>
<td>April 9, 29, May 2, 7, 8, 12, 13</td>
</tr>
<tr>
<td>Field notes of the observations</td>
<td>April 10, 30, May 3, 8, 9, 13, 14</td>
</tr>
<tr>
<td>Semi-structured interviews</td>
<td>Jun 26-30</td>
</tr>
<tr>
<td>Video recording of the interviews</td>
<td>Jun 26-30</td>
</tr>
<tr>
<td>Field notes and transcriptions of the interviews</td>
<td>Jun 26-30</td>
</tr>
</tbody>
</table>

Data analysis began after the class observation, when the field notes were expanded. Next, the analysis of the videotapes was done in order to describe and reflect on details that could not be captured by the researcher during the class observation. After that, a coding process was developed to identify the teachers’ actions. Then, I analyzed the raw field notes and expanded field notes. I coded this data. A code was assigned to each specific teacher’s action. As mentioned previously, I refer to action as what the mathematics middle school teacher says and does. Teacher’s actions such as the recognition of the situation in which “knowing-to act” should
occur, the action performed to promote critical thinking, the action performed that limits the opportunities for students to learn. The codes were developed based on these actions due to the fact that the literature review did not identify codes previously defined in early studies. A triangulation process was used among the data collected with different sources in order to get a more complete appreciation of the teachers’ actions during mathematics instruction.

After this coding process, frequencies of these codes were obtained. Then, this analysis of the classroom observations and the KtAS survey responses allowed the design of the interview questions. Data from the interviews were analyzed. First, each interview was video recorded. Next, interviews were transcribed. Then, via triangulation of the multiple methods utilized to collect data, the analysis was conducted. This analysis allowed me to create certain themes of the middle school teachers’ reasons for acting in certain ways during mathematic instruction and responding differently on the KtAS survey.

3.10 Integration Phase

After the classroom observation analysis, the semi-structured interviews were designed for each teacher. Then, the analysis of these interviews was conducted. Several findings obtained from this analysis were integrated with the quantitative results. This integration of results and findings provided the meta-inferences for this study and the answer to the fourth research question:

- To what extent do the qualitative findings explain the quantitative results of the study?

This phase also includes the interpretation of the integration of the results from the previous phases. The discussion of this integration phase will be provided in Chapter 5.

3.11 Discussion on Language

Language is a key aspect discussed throughout this research. The native language of the participants of the study was Spanish. Thus, the first language issue was that the study is written in English. Several translations of TCKS and KtAS surveys, the field notes of the classroom
observations as well as the interview questions and transcriptions of the interviews were done. The translator plays an important role in the research: “The researcher gains access to the ideas and experiences of the participants through the translator, and it is through the translator that the research participants’ voices are heard” (Wong & Poon, 2010, p. 153). Therefore, if the translator and the researcher is the same person, she has the advantage of gaining access to the ideas and experiences of the participants directly. As we know a translation is not a neutral and objective process. It is affected by the social context, background and worldview of the translator (Wong & Poon, 2010). My native language is Spanish. My background and worldview were aligned with the purpose of the study, and I am a bilingual person who regularly uses Spanish and English for academic/scholarly purposes. Thus, I was able to be the translator in order to reduce the possibility of losing information in the process from the translation to the communication of information with the researcher. Since the role as researcher and also as translator raises possible issues of credibility, it is of great relevance to the outcomes of this study. To maximize credibility, I did the translation process as transparently as possible. I had a group of bilingual persons who were also knowledgeable in the mathematics education field check and verify the accuracy of my translations.

Another critical issue related to language was that participants were Spanish dominant. Therefore, I was not able to check the translations with them and agree on the translation and their words. Moreover, the findings could not be also verified by the participants in the form of member checking. According to Varjas and colleagues (2005), member checking involves presenting findings and interpretations to representatives of the population to establish agreement between their words and researcher’s interpretations. Since translation is not a neutral process in which a bilingual person only changes one word of one language with another word from a language, the assignment of meanings to words in both languages was sought (Bühler, 2002). This assignment of meanings is an interpretation by the translator. Like interpretation, the member checking procedure would help to ensure authenticity of the data. However, the
participants of the study were Spanish dominant; they were not able to perform member checking of English language findings, thus, this could be one limitation of this study.

3.11 TRUSTWORTHINESS

According to Varjas and colleagues (2005), trustworthiness of qualitative data “is defined by the concepts of credibility, transferability, dependability, and confirmability, which parallel traditional (quantitative) constructs of internal validity, external validity, reliability, and objectivity, respectively.” (p. 248). Lincoln and Guba (1985) offer a set of activities that can help the qualitative researcher to follow the standards of trustworthiness in qualitative research to ensure that the study is credible and unbiased. In order to have trustworthiness in this qualitative phase, I used two of these activities. One of them was triangulation. Another was peer debriefing.

3.11.1 Triangulation

Triangulation is a procedure in which the use of multiple data collection instruments, sources, or devices is conducted in order to establish validity to the findings of the study (Lincoln and Guba, 1985). Thus, a triangulation of the data was done in each phase of the analysis. Triangulation among field notes, observations, and interviews was accomplished after all data were collected.

3.11.2 Peer debriefing

Peer debriefing is a process in which the researcher discusses emergent findings with critical friends or colleagues to ensure that analysis is grounded in the data (Lincoln and Guba, 1985). Then, I discussed the analysis of data and findings with scholars interested in this field of inquiry in other to mitigate any bias and avoid missing details. The feedback that I received from them reinforced the study.

In addition, I provided many details to describe the setting, the behavior, and everything that is under study to ensure the analysis and findings were based on data. I described the data as richly as possible, considering minimum details in order to ensure the credibility of the study.
It is important to emphasize that this qualitative phase was illustrative. This qualitative phase did not seek to make generalizations beyond the participating teachers. To identify the mathematics middle school teachers’ actions employed by four teachers during their classroom was the purpose of this qualitative phase. Based upon this identification, an understanding of the “knowing-to act” of these teachers was accomplished. The transferability of these actions to the actions of similar teachers can happen, but it was not the focus of this phase.

3.12 Ethics

Research involves several ethical considerations that go beyond the approval of an Institutional Review Board. That authorization does not allow invading the privacy of people (Stake, 2008). Although the presence of the researcher was relatively brief, as during in-depth interviews and during class observation; the researcher was entering the lives of the participants and interrupting their daily lives and practices (Marshall and Rossman, 2011). To address these ethical considerations, I asked the participants to provide their consent to be part of the study. Through informed consent the researcher has the ethical responsibility to inform the participants about: a) their participation is voluntary; b) anything that might affect the well-being of the participants; and c) the participants can decide to stop their participation at any time during the study (Glesne, 2011).

The nature of my participation as researcher was that of a non-participant observer during the classroom observation. However, during the interviews, I had social interactions with the participants to build rapport in order that they would feel comfortable sharing their experiences. To negotiate how the research was conducted, I talked with each of the participants. To do the class observations, I negotiated which date was better for them. After the class observation, I discussed with each participant any issue resulting from the observation that she or he would like to share with me. This helped me to consider all issues and aspects related to the observation at the time of the analysis. Regarding the interviews, I talked with each teacher to schedule the interviews. These interviews were semi-structured to allow the participant to share their
experience from some questions posed after watching each video recording to start the conversation.

I explained to the participants the purpose of the study and the nature of their participation. However, I did not mention specific details. I did not want to have the participants aware of the aspects that I was observing or seeking during the interviews. As Marshall and Rossman (2011) discussed, “revealing exact purposes tends to cue people to behave in unnatural ways, undermining qualitative purposes and principles” (p.113). This phase was focusing on teachers’ “knowing-to act” during mathematics instruction. If I had mentioned which situations were considered for analysis, teachers could become aware of them and acted in ways that they did not usually act. Perhaps, they were not aware of the situations that require their “knowing-to act”, and if I told them which situations I was going to be analyzing, they would become aware of them, thus undermining research validity. The results of the study were reported in English. However, considering that the participants were Spanish monolinguals, a presentation in Spanish was going to take place to present the study if they requested it. In addition, a brief written report paper-based in Spanish about the findings of the research could be done as they requested it after the finalization of the study.

The administrative or logistical concerns that I managed in order to conduct the research were formal and informal. First I requested authorization to travel to Mexico to conduct the data collection of this research. This request included specific information such as dates, locations, people who were waiting for me outside of the schools and aspects that ensured my safety. Once this request was approved, I asked permission to conduct the study in the middle schools described above. This permit was granted by the Subsecretary of Education, Sport and Culture of the city (subsecretario de educación, deporte y cultura). This process lasted about a month. Asking permission and collaboration from other Mexican education authorities such as subsystem coordinators, department chairs, and principals was the next step. Once all the permissions were granted, I applied for institutional review board approval (IRB) at the university where my doctoral studies were done (see Appendix D). The time that the IRB took to
grant authorization was about two months. Then, after securing all those approvals, I approached those teachers whose principal and educational authorities allowed me to have access to them. Then I asked them to participate in the research. Once they accepted to be part of the study, their participation was negotiated with them through the informed consent mentioned above.

3.13 Validity

There is a term in mixed methods studies to refer to validity, which is “legitimation”. However, the term validity was used in this study. Due to the absence of acceptance of the term legitimation (Creswell & Plano, 2011), validity was utilized instead of legitimation.

The validity of this mixed methods study was defined, following Creswell and Plano (2011), as “employing strategies that address potential issues in data collection, data analysis, and the interpretations that might compromise the merging or connecting of the quantitative and qualitative strands of the study and the conclusions drawn from the combination.” (p. 239)

To ensure the validity of the study, the samples of both phases were from the same population in order to make data comparable. However, the sample for the quantitative phase was larger than the qualitative phase. In order to avoid potential bias, the data collection for both phases was conducted separately. The identification of distribution of the scores was explored in order to be able to select the appropriate statistical techniques. In addition, the different procedures of validity for each type of approach were followed.

3.14 Summary

This chapter discussed a crucial component of the proposed study: methodology. This component was composed of four phases: quantitative, interim, qualitative phase, and interpretation phase. The mixed methods sequential explanatory research design that was followed in this study provided for the sequential implementation of the phases. The quantitative phase was the first phase of the design, and quantitative data collection and analysis was be done through the administration of two surveys TCKS and KtAS with 70 mathematics middle school teachers in Mexico. Based on the results of the quantitative phase, the interim phase included the
case selection (purposive sampling) as well as the development of the protocol for the classroom observations. Then, the qualitative phase was described in which 4 teachers participated. The description of the methods, data sources, and analysis used in each phase was provided in this chapter. The integration phase was described. This phase integrates the result from previous phases and is discussed below. The discussion and presentation of the findings of each phase is found in the following chapter.
Chapter 4: Results and Findings

4.1 Overview

The purpose of the study was to measure middle school mathematical teachers’ content knowledge and its relationship with teachers’ “knowing-to act” ability among a sample of Mexican mathematics teachers in a border city in the north of Mexico. In order to achieve this purpose, a mixed method study was conducted. An explanatory sequential mixed method design was implemented. It consisted of four phases: quantitative phase, interim phase, qualitative phase, and interpretation phase. Figure 3.1 in Chapter 3, on page 70 illustrates the different phases of the study and how they are related. In addition, Table 4.1 presents the research questions addressed in the study, the method and the data sources implemented to answer these questions to recall how the research design was implemented.

Table 4.1: Review of Research Questions, Methods and Data Sources.

<table>
<thead>
<tr>
<th>#</th>
<th>Research Question</th>
<th>Method</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>To what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ knowing-to act?</td>
<td>Quantitative</td>
<td>TCKS KtAS</td>
</tr>
<tr>
<td>2</td>
<td>How do teachers act in the KtA situations occurring in mathematics classrooms?</td>
<td>Qualitative</td>
<td>Classroom Observations</td>
</tr>
<tr>
<td>3</td>
<td>How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?</td>
<td>Qualitative</td>
<td>Interviews</td>
</tr>
<tr>
<td>4</td>
<td>To what extent do the qualitative findings explain the quantitative results of the study?</td>
<td>Mixed methods</td>
<td>All previous sources</td>
</tr>
</tbody>
</table>

It is important to keep in mind that a mixed method research design was conducted for several reasons; most importantly the implementation of quantitative and qualitative methods to offset the weaknesses of using each method singularly and to draw on the strengths of using both (Bryman, 2006). Also, the mixed method research design permitted a comprehensive account of the relationship between mathematical teacher content knowledge and teachers’ “knowing-to act”.”
This chapter will discuss the results of this mixed method study. Several components of the methodology will be briefly revisited to discuss the results of the phases of the study such as the sample and instruments. The chapter is organized by the research questions that guided this study. The quantitative phase will present evidence to answer research question number 1. The qualitative phase was built on the interim phase, thus, these two phases addressed research questions number 2 and 3. And the interpretation phase addressed research question 4. In addition, this chapter includes a broader description of the participants who are part of the case studies.

4.2 Research Question 1

In order to accomplish the purpose of the study, the following question was posed:

1. To what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ knowing-to act?

In this phase, the teacher content knowledge of mathematics middle school teachers was assessed. This study focused on the specific cognitive types of teacher mathematical content knowledge identified by Tchoshanov (2011). He distinguished among three different cognitive types of mathematical teacher content knowledge. The cognitive type 1 is the knowledge of facts and procedures. The cognitive type 2 is the knowledge of concepts and connections. And the cognitive type 3 is the knowledge of models and generalizations. The measurement of middle school mathematical teachers’ content knowledge was completed by examining these three cognitive types as a framework for collecting and categorizing the data through the teacher content knowledge survey (TCKS).

The quantitative phase also focused on the “knowing-to act” ability of these middle school mathematics teachers. Based on the conceptual framework developed by Mason and Spence (1999), “knowing-to act” refers to “the kind of knowledge which enables people to act freshly and creatively.” (Mason, 1998, p.245) A correlational analysis was developed to seek
any relationship among the teacher content knowledge cognitive types and teacher “knowing-to act”.

The sample used for the quantitative phase consisted of a total of 70 teachers at the middle school level. These teachers were teaching at least one mathematics course in Mexican middle schools. Teachers from Type C middle schools did not participate due to their lack of interest to be part of this research. Therefore, 70 teachers who belonged to only two out of four types are the participants of the research (A and B). Two surveys were administered to the 70 middle teachers in Mexico as mentioned in Chapter 3. These surveys are the Teacher Content Knowledge Survey (TCKS) and the Knowing-to Act Survey (KtAS).

The Knowing-to act survey (KtAS) consisted of 11 items as described in Chapter 3. However, only 10 items were considered for analysis in this survey. Item 8 was rejected for analysis, as illustrated in Illustration 4.1. This item was removed due to the lack of agreement among the members of the team who accomplished the expert analysis of the KtAS. There was not any agreement among the rankings provided. For example the choice number one of this item was ranked as 1 by one member of the team, 2 by one member, and 3 by two members. Therefore, this item is not considered for the quantitative analysis.

Illustration 4.1: KtAS Item Removed From Analysis.

The analysis of this survey considered an item to be correctly answered if the choice ranked as number 1 by the experts matched the number 1 selection of the teacher for each given item. The correctly answered item has a score of 1, and the items considered as wrong answers
have a score of 0. Then, the sum of the scores of all the items considered in the analysis is the overall score of the KtAS. The maximum score that could be obtained was 10.

### 4.2.1 Data Cleaning

The data cleaning process was done to prepare data for analysis. During the data cleaning, the scores from 6 participants were eliminated. The elimination of the scores of these participants was done because they were considered outliers or because they indicated a lack of awareness of certain items of the Knowing-to act survey (KtAS). The criterion for outliers that is widely used in exploratory data analysis is 2.32 or more standard deviations from zero (Kirk, 1995). Considering this criterion, the rejection was performed because the criterion was met in one or both scores on the surveys administered to the 6 teachers (overall score of TCKS and KtAS). Table 4.2 presents the raw scores of the participants removed from the study.

<table>
<thead>
<tr>
<th>Participant Identifier</th>
<th>TCKS Overall Score</th>
<th>KtAS Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>G042</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>G105</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>G068</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>G117</td>
<td>22</td>
<td>8</td>
</tr>
<tr>
<td>T202</td>
<td>22</td>
<td>3</td>
</tr>
</tbody>
</table>

The mean of the overall score of the TCKS is 13.531. The standard deviation for these overall scores is SD= 3.2463. Calculating the z-test for the participants G042 and G105, using the raw overall scores of the TCKS, the z-test is \(z=-2.329\) which means that the overall scores of these participants (X=6) are below the mean (\(\bar{X}=13.531\)) by more than 2.32 standard deviations. Then, the overall scores of the TCKS of the participants G042 and G105 are considered outliers which are rejected for analysis of this research.

The teacher participants G117 and T202 also were rejected for analysis. These participants obtained an overall score in the TCKS of X=22. Considering the mean score 13.531,
the raw scores of the participants are above the mean score ($\bar{X}=13.531$) by more than 2.32 standard deviations. The z-test for these participants is $z=2.609$, which means that the TCKS of the participants G117 and T202 are outliers and, therefore, are rejected for analysis.

The mean score for the KtAS is 7.2188. Participant G068 gained a raw score of $X=3$. Calculating the z-test for this participant, the z-test is $z=-2.926$. This means that the teacher’s KtAS score is below the mean score by more than 2.32 standard deviations. Therefore, this participant was also rejected for analysis.

The last participant rejected for analysis was the teacher G033. This participant was removed from the analysis because the last three items of the survey were not answered. This was because the participant was not aware of these items because these unanswered items were located at the back of the last page of the survey. Therefore, the evaluation of this teacher’s KtAS score could not be done.

4.2.2 Description of Participants

After the data cleaning process, the data analysis was done with a sample of $N=64$ middle school teachers. From the 64 teachers, there were 62.5% of teachers who were teaching mathematics at one particular grade level. Thus, 37.5% of the participating teachers were teaching mathematics in at least two grade levels of middle school. With regards to the years of teaching experience of the participants, there were 20.9% of teachers who had from 0 to 6 years of experience teaching mathematics. Teacher participants who had mathematics teaching experience from 7 to 13 years were 22.5% of the sample. There were 14.5% of teachers in the sample who had from 14 to 20 years of experience teaching mathematics. The rest of the teachers in the sample, 41.9%, had 21 or more years of teaching experience in mathematics.

Teachers who were part of this research belonged to two out of the four types of middle schools that exist in Mexico. The percentage of teachers who belong to Type A is 66%. The rest of the teachers of the sample (34% of the total) work in Type B schools. Regarding the gender of the participants, the majority of the teachers are males (56%), and 44% of the teachers are
females. The 47% of the middle school teachers in the sample are also teaching other courses different from mathematics. Some of the courses taught by these teachers are technology, civics, English, Art, visual Art, Physics, Graphic Design, Drawing, Spanish, State course, workshops, among others.

The results of the surveys applied to these 64 middle school teachers are shown below in Figure 4.1. According to the data, the mean score percentage of the performance of Mexican teachers in the TCKS and KtAS is 41% and 72% respectively.

4.2.3 Results of Research Question 1

The following result is obtained from the correlation analysis among TCKS (total score) and KtAS. The analysis showed that there is no significant correlation between teacher content knowledge measured as the total score on the TCKS and “knowing-to act” measured by KtAS score (Pearson’s \( r(64)=.17, p>.05 \)). In other words, the teachers’ performance on the TCKS is not related to their “knowing-to act” ability.

According to the data, the performance in the TCKS varies by cognitive type of content knowledge. Teachers have a higher performance level on the items that measured the cognitive type 1, knowledge of facts and procedures. The mean score percentage of the cognitive type 1 is 60%. The mean score percentage of the cognitive type 2, knowledge of concepts and connections, is 36%. And the mean score percentage of the cognitive type 3, knowledge of models and generalizations, is 28%. Figure 4.2 illustrates the performance of Mexican teachers on the TCKS by cognitive types, using the mean score percentages. This result shows that teachers possess more knowledge of fact and procedures than the other two cognitive types. It means that teachers have more procedural knowledge.
The following data are representative of the results obtained in this study, which examined the relationship between cognitive types of teacher content knowledge (T1, T2, T3) and KtAS. The correlational analysis between cognitive type T1 and KtAS is not significant (Pearson’s $r(64)=.13$, $p>.05$). In other words, the absence of relationship between T1 and KtAS adds to the discussion that the actions enacted by teachers during mathematics instruction are not related to their knowledge of facts and procedures.

With regard to the correlation analysis between cognitive type 2 and the KtAS score, the results obtained show that there is no significant correlation between T2 and KtAS (Pearson’s $r(64)=.0001$, $p>.05$). The last correlational analysis examined was between T3 and KtAS. After analyzing the results of the last two correlations, the results for this last comparison unexpectedly reported a significant correlation between T3 and KtAS ($r(64)=.27$, $p<.05$). For evaluating the effect size of this correlation, $r^2$ is computed. This correlation showed a medium effect size with $r^2 = .0729$ according to the Cohen’s (1988) standards for interpreting $r^2$. This result showed in the correlation indicates that there is a relationship between the knowledge of models and
generalizations, and the actions enacted by teachers during mathematics instructions. The medium effect size indicated by $r^2$ is informing us that this is a medium correlation.

This result adds to the discussion that teachers who possess the mathematical knowledge that allows generating and testing conjectures, making generalizations, and proving theorems, among other aspects has more possibility of knowing-to act in the moment. It is important to highlight that in order to be able to generate conjectures and testing them, certain procedural and conceptual knowledge must be possessed. Analyzing this result, the “knowing-to act” ability appears when knowledge is called in the moment required. If there is no knowledge to be called, “knowing-to act” might not be enacted. According to Mason and Spence (1999) “knowing-to act” at the moment requires more than content knowledge, but this means that at least some content knowledge is essential. Therefore, there are more possibilities of “knowing-to act” when teachers hold knowledge of models and generalizations.

4.3 **Research Question 2**

The second research question moves the study to the following phases: interim phase and qualitative phase. The second research question that guided this study is:

2. How do the teachers act in the KtA situations occurring in the mathematics classroom?

In order to collect qualitative data, a follow up was required after the quantitative phase. Because this study implemented a mixed methods sequential explanatory design, the qualitative phase is built on the quantitative phase. This follow up is conducted in this interim phase. In this interim phase, two main procedures included: the purposive sampling for the qualitative phase and the development of the observation protocol.

The purposive sampling strategy was used to make the selection of 4 participants whose performances on the surveys met certain criteria. The criteria used to select the teachers to participate in the qualitative phase are presented in Table 3.2 in Chapter 3. The four participants met one of the 4 different criteria: (a) one teacher who had a high score on the TCKS and KtAS;
(b) another teacher who got a low score on the TCKS, and a high score on the KtAS; (c) one teacher whose performance on TCKS was high, but on the KtAS is low; and (d) another teacher whose performances on both surveys TCKS and KtAS were low. There were more teachers who met these criteria. However, the follow up was done with these four teachers because of accessibility to the schools and scheduling issues with the teachers.

The term I mentioned in the above criteria is used relatively in regard to the TCKS due to the low performance obtained by participating teachers on this survey. To define what a high or low score is, the calculation of quartiles was done. The proportion of the total scores on each survey was conducted to compute the quartiles. The values of the quartiles for the proportion of the scores on the TCKS are \( x_L = .3333 \), \( x_M = .3939 \), and \( x_U = .4848 \). The values of quartiles for the proportion of the scores on the KtAS are \( x_L = .6 \), \( x_M = .7 \), and \( x_U = .8 \). Table 4.2 presents the quartiles by survey.

<table>
<thead>
<tr>
<th>Quartile</th>
<th>TCKS</th>
<th>KtAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower quartile ((x_L))</td>
<td>.3333</td>
<td>.6</td>
</tr>
<tr>
<td>Median ((x_M))</td>
<td>.3939</td>
<td>.7</td>
</tr>
<tr>
<td>Upper quartile ((x_U))</td>
<td>.4848</td>
<td>.8</td>
</tr>
</tbody>
</table>

Overall scores on the TCKS that are greater than or equal to the upper quartile \( x_U = .4848 \) are considered high scores. The term low on the performance of the TCKS includes all the overall scores that are less than or equal to the lower quartile \( x_L = .3333 \). Concerning the KtAS, the scores that are greater than or equal to the upper quartile \( x_U = .8 \) are considered high scores. And the scores that are less than or equal to the lower quartile \( x_L = .6 \) are considered low scores.

**Point of Interface for Mixing**

In this point of interface for mixing both quantitative and qualitative strands, the consideration of the previous criteria and the quartiles calculated allowed the selection of the
four teachers. Then, the participants for the qualitative phase are G036, T019, T200, and T203. These participants met the criteria mentioned above. A fictitious name is assigned to each participant to keep the confidentiality and anonymity of the participants. The name Omar is assigned to participant G036, Rosa to participant T019, Rogelio to participant T200, and Maria to participant T203. Rosa’s overall scores on the surveys were low, thus, she met the criteria of low performance on both surveys TCKS and KtAS for inclusion in the qualitative phase. Omar is another participant that met a criterion of earning high scores on both surveys. Rogelio’s scores on both surveys met the criterion of a high score in the TCKS and a low score on the KtAS. And Maria met the criterion of a low score on the TCKS and a high score on the KtAS. These four participants are the subjects for the case studies for this research. The description of each case study will be presented in the next section.

4.3.1 Case Studies

Rosa

Rosa is a middle school teacher. At the time of the study, she was teaching mathematics at the seventh grade. She taught Mathematics to three classes at the seventh grade. She has 11 years of experience in teaching mathematics. Rosa also works in another middle school, but she teaches science in that middle school. She reported that more than 90% of her students passed her mathematics courses in the last four school years. She has a Normal school teacher preparation with a specialization in mathematics. The school where Rosa teaches mathematics is a small school. The morning session is only offered in this middle school. It has a student population of 335 students. The dropout rate of this school is 0%. The fail grade rate is 16.4% (SNIEE, 2010).

Rosa’s classes are interactive. Students usually work in groups. She provides incentives such as pops, chocolates, to students in order for them to work harder and be disciplined. She uses the whiteboard or poster board to present the topic or assign a class activity. She speaks to the students using positive adjectives. In addition, Rosa poses a phrase to motivate students to
work and pay attention to the class. For instance, this motivational phrase: “the human being
does not know what he/she is capable to do until he/she tries it” (in Spanish “el ser humano
nunca sabe de lo que es capaz hasta que lo intenta”). She takes care of her students as if she
were their mother. There is an average of 21 students in each of her mathematics classes.
Calculators are not allowed in her classes.

Omar

Omar is a teacher who teaches mathematics at the eighth and ninth grade. He has 27
years of teaching experience in mathematics. Mathematics II, Mathematics III, and technology
are the courses that he was teaching at the time of the study. He went through a normal school
preparation. The most advanced mathematics course that he took was integral calculus. Omar is
around 55 years old. The school where he works is not in good physical condition. It is an old
school. This middle school is very small. The student population of Omar’s middle school is 401
students for the current academic year. This school has a dropout rate of 11.6%. However, more
alarming is the fail grade rate which is 38.9%. It means the 38.9% of the students in this school
do not pass at least one course (SNIEE, 2010). Because this middle school only offers classes in
the morning session, Omar works during the morning session. He is teaching mathematics to two
ninth grade classrooms and one eighth grade classroom. There are approximately 25 stud-

Omar’s classes are usually divided in two phases. During the first phase, he introduces
the topic. He explains the topic and provides several examples. Whiteboard or big cardboards are
used to present the topic. Calculators are allowed in his classes. After that, the second phase
starts. Omar assigns class activities to be done in groups for the rest of the class. He supervises
students’ work and is very attentive to students’ questions.

Rogelio

At the time of the study, Rogelio was teaching mathematics at the eighth and ninth grade
levels. He has 15 years of teaching experience. Rogelio is approximately 45 years old. He has a
bachelor’s degree in mathematics. He had not had any teacher preparation until he started to teach mathematics; he went through the Normal school teacher preparation during the weekends. He is enrolled in the bonus program (Carrera magisterial) that is mentioned in chapter two. This program provides incentives to teachers who are in continuous preparation and meet other criteria.

Rogelio works in a high school in the morning and in a middle school in the afternoons. This middle school offers two sessions per day: a morning and an afternoon session. Even though this middle school offers two sessions per day, the school functions in each session as if it were a different school. The student population is 561 only in the afternoon session. There are approximately 38 students per classroom in this middle school. In this middle school, in the afternoon session, there is a dropout rate of 6.52%. The fail grade rate for the afternoon session of this middle school is 14.12% (SNIEE, 2010).

Rogelio’s classes are small considering the average provided above; there is an average of 28 students in his classes. He allows students to use calculators. His way to work is using worksheets. Most of the time, he provided the worksheet to the students. He would explain one problem on the worksheet on the whiteboard for the students. Then, he allowed students to work for a certain time. After that, he solved more problems on the worksheet on the whiteboard and at the same time he was asking questions to the students about those problems.

Maria

Maria was a mathematics middle school teacher in Mexico at the time of the study. She has 7 years of teaching experience. Maria reported that above 80% of her students pass the class. She holds a bachelor’s degree in engineering. She is approximately 40 years old. She works in a middle school during the morning session. The student population of this middle school is 845. There is a dropout rate of 9.95%. The fail grades rate is 13.31% (SNIEE, 2010). She teaches mathematics and science. However, she only has one mathematics class. This class is at the 8 grade level. It is a large class with 40 students.
Maria’s classes are very disciplined. Students are engaged in the class. She asks students to work in groups. She spends at least one class explaining or introducing content. At the same time, class activities in the textbook are assigned. During this time, she supports students with explanations. After that, she assigns students to work on worksheets individually. Then, she uses a class session to have the common presentation (puesta en común). The common presentation is where students explain and share their activities: students solve problem on the whiteboard and explain their solution without help.

4.3.2 Classroom Observations

After this selection of participants for the case studies, the study moved to the qualitative phase in order to answer the second and third research questions. The second research question required a different type of data than the data presented before. Classroom observations are the data source that addressed this research question. The classroom observations were non-participant observations. It means that I was positioned as an observer. I did not have any other role in the classroom. My intention was only to observe and make sense of what was happening and how the teachers acted in those situations. There were three classroom observations performed for each of the participants, as described in Chapter 3.

Point of Interface for Mixing

In this point of interface for mixing, decisions were made to develop an instrument for collecting qualitative data. This point of interface for mixing occurred during the interim phase of the research. Based on the quantitative results the protocol observation was developed. The complete protocol is in Appendix J.

Observation Protocol

The protocol observation consisted of the same classroom situations that were presented in the KtAS. There are 11 situations in this survey. In Chapter 3, there is a categorization of these situations: (a) student misconceptions; (b) student difficulties; (c) challenge for the teacher; and (d) emerging situations. The classroom situations were clustered according to their nature in the
protocol observation. For instance, in the category of *student misconceptions*, there are three classroom situations. The first one is when a student solves a particular mathematical problem and asks if the answer or steps are correct. Another situation is when a student provides a nonsensical solution to a given problem. And the third situation under this category is when a student makes a mistake. The situations were placed in order according to their nature in the protocol observation in order to allow immediate identification.

The observation protocol included 11 classroom situations. Some white space was left below each situation in order to be able to write down notes about the phenomena observed. The difference between the observation protocol and the KtAS is the number of choices (actions) provided in the KtAS. Teachers in the KtAS ranked those choices from 1 to 5 as their most likely to do first. In the protocol observation, there are no choices to be ranked. There is a blank space to capture the teacher’s actions in those situations. Moreover, there is also more space on the back of the protocol to take notes about emerging situations where the knowing-to act should be enacted.

In Mexico, the public middle schools classes are organized by modules. Each module lasts 45 minutes. It means a class lasts 45 minutes. I observed all four teachers and their classes for more than two hours as Table 4.3 presents. There were three classroom observations done for each teacher participant. I video recorded all the classroom observations to capture teacher actions and situations as much as possible. Furthermore, the video recordings allowed me to review and observe the phenomena that were taking place in the classroom for any number of times that I needed. Thus, the videos allowed a more detailed account about what was happening in the classroom. Transcriptions of these videos were done to facilitate the data analysis.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Total Time observed</th>
<th>Number of classes observed</th>
<th>Grade level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosa</td>
<td>135 minutes</td>
<td>3</td>
<td>7th grade</td>
</tr>
<tr>
<td>Omar</td>
<td>135 minutes</td>
<td>3</td>
<td>8th and 9th grade</td>
</tr>
<tr>
<td>Rogelio</td>
<td>135 minutes</td>
<td>3</td>
<td>9th grade</td>
</tr>
<tr>
<td>Maria</td>
<td>135 minutes</td>
<td>3</td>
<td>8th grade</td>
</tr>
</tbody>
</table>
During the classroom observations, I was taking field notes based on the observation protocol. The observations allowed me to validate the information that they provided on the knowing-to act survey (KtAS) about how they would act in KtA situations. In addition, the observations served as a means to understand of the nature of those teachers’ actions. Therefore, the main focus of the classroom observations was the teachers’ actions. Furthermore, the classroom observations also helped me to understand the situations where the “knowing-to act” should be or is enacted. More specifically, I mainly tried to observe how teacher’s knowing-to act ability arises during mathematics instruction.

The classroom observation’s findings are presented as case studies. However, only the full description of Rosa’s case study is presented in this chapter. After Rosa’s case, descriptive summaries are presented for the other three cases which include the following information: the “knowing-to act” situations considered in the protocol observation that were identified; the recognition and analysis of other situations not considered in the protocol but that challenged that “knowing-to act” of teachers; and the frequency of the occurrence of the situations identified for the four case studies. Next, the focus is placed on the teacher’s actions enacted in these identified situations (see Figure 4.3). For a description of the actions enacted by Omar, see Appendix A, for Rogelio see Appendix B, and for Maria see Appendix C. Excerpts of the transcriptions done of the classroom observations exemplifies the situations identified and the teacher’s actions enacted. A coding process is conducted to analyze the teacher’s actions. The codes assigned to teacher’s actions and the frequency of these actions is also discussed in Rosa’s case. Figure 4.3 illustrates the pathways for analysis of the classroom observations. For detailed descriptions of Omar, Rogelio and Maria’s case studies see appendices A, B, and C, respectively.
Figure 4.2: Pathways for Analysis of the Classroom Observations.
Observing Rosa

In the interest of minimizing repetition and an unwieldy chapter length, Rosa will be the only case study that is fully elaborated by detailed descriptions and transcriptions of those interactions that are directly relevant to research question two. Rosa’s case is the most insightful and better illustrates the basic ideas of this research. For instance, Rosa acted in more ways than the other participants. She acted 21 different ways. Thus, Rosa’s case was an excellent instrumental case study to illustrate the teacher “knowing-to act”. However, the complete descriptions for the other three case studies may be found in appendices A, B, and C. Additionally, the discussions for the case studies for Omar, Rogelio, and Maria will include only summaries of the findings and will follow the same format used in the discussion of the case study for Rosa.

I observed and videotaped Rosa’s classroom during the spring semester of 2013-2014 academic year for a period of three classes. Those classes were 7th grade. In the first two classes observed “Proportionality” was the topic covered. Rosa assigned an activity called “The magical triangle” in the first class that I observed. Students spent the whole class finishing the activity. In the second class observed, “the magical triangle” was also covered. However, students completed the activity in 28 minutes, thus, there was time for another activity: a mathematical problem about proportionality. The day of the last observation, Rosa was still teaching proportionality. She started the class with a mathematical problem of proportionality. Then, the topic “Calculate Percentages” was covered and several exercises were assigned. In each classroom observation I was allowed to walk around the classroom with the video camera hung on my neck to try to capture as many details as possible.

The analysis of the classroom observation was based on the protocol for the observation. During Rosa’s classes that were observed and videotaped, several of the classroom situations provided in the knowing-to act survey (KtAS) were identified (see Table 4.4). One excerpt of the transcriptions of video-recordings is presented in order to exemplify each of the situations identified in Rosa’ classes. As previously mentioned in Chapter 3, the observation protocol
reflected the same situations that are included in the KtAS. The classroom situations provided in the survey are categorized in four categories: student misconceptions, student difficulties, challenge for the teacher and emerging situations and are called knowing-to act situations or KtA situations.

Table 4.4: Knowing-to act Situation Observed in Rosa’s Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct: S6: I finished teacher. R: wait wait wait, here the sum is greater that it should be. Add it. No here it is your mistake, keep looking at.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 3 KtAS. When a student provides a nonsensical solution to a given problem: I need to leave alone and remove what I do not need in that side. What I do not need with the x over there? S15: the equal symbol. R: we do not need the equal symbol =? S16: no, the 20% R: the 20%? Let’s see, you are coming from the break very sleepy.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake: R: hey guys I see that you are placing the number 15, no, you only can place the numbers from 1-9, the 1, 2, 3, 4, 5, 6,7,8,9, you should place them in a certain way that the sum of the corners must be 15 and the sides 20.</td>
<td></td>
</tr>
<tr>
<td>Student difficulties</td>
<td>Item 2 KtAS. When a student does not recognize the same pattern in a different situation: The 225, how would it be represented? I want to see more hands up. Saul, how would it be represented? S9: it could not be the same because the number is greater than mmm… (Other students were raising their hands, but the teacher said). R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 5 KtAS. When a student is unable to see an obvious pattern in the problem: R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions. How would it be represented in common fraction, Brenda? S10: as a quarter. R: no no no, my love, remember that we remove the point, it would be also 225/100 it would be the same, this was seen previously. You do not remember it?</td>
<td></td>
</tr>
</tbody>
</table>
In the category of student misconceptions, the following classroom situations were classified: (a) when a student solves a particular mathematical problem and asks if the answer or steps are correct; (b) when a student provides a nonsensical solution to a given problem; and (c) When a student makes a mistake. The table of frequencies shows how many times these KtA situations occurred in the Rosa’s classes (Table 4.5). The category student misconceptions was identified 34 times.

Table 4.5: Frequency Table of the Situations in the Category Student Misconceptions.

<table>
<thead>
<tr>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1. When a student solves a particular mathematical problem and asks if the answer or steps are correct</td>
<td>24</td>
</tr>
<tr>
<td>Item 3. When a student provides a nonsensical solution to a given problem</td>
<td>2</td>
</tr>
<tr>
<td>Item 9. When a student makes a mistake</td>
<td>8</td>
</tr>
</tbody>
</table>

With regards to the category of student difficulties, the KtA situations included in this category according to their nature are: (a) when a student does not recognize the same pattern in a different situation; (b) when a student is unable to see an obvious pattern in the problem; (c)
When a student continuously responds with wrong answers to questions posed by you; and (d) when students are having a hard time completing an assigned activity/task. There were 18 times where these situations occurred as Table 4.6 presents.

Table 4.6: Frequency Table of the Situations in the Category Student Difficulties.

<table>
<thead>
<tr>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 2. When a student does not recognize the same pattern in a different situation</td>
<td>3</td>
</tr>
<tr>
<td>Item 5. When a student is unable to see an obvious pattern in the problem</td>
<td>1</td>
</tr>
<tr>
<td>Item 8. When a student continuously responds with wrong answers to questions posed by you</td>
<td>1</td>
</tr>
<tr>
<td>Item 10. When students are having a hard time completing an assigned activity/task</td>
<td>13</td>
</tr>
</tbody>
</table>

The following situations of the KtAS that belong to the category of situations that challenge for the teacher are: (a) when you assign an activity about a previously learned topic, and some students express that they do not know how to do it; (b) when you make a mistake and a student notices the mistake; (c) When a student asks you a question from a different perspective that was not previously considered by you; and (d) when you are teaching a topic that you are not knowledgeable about and a student asks you to elaborate more on that topic. However, these types of situations were not identified during Rosa’s classes.

Table 4.7: Emerging Situations During Rosa’s Classes.

<table>
<thead>
<tr>
<th>Category</th>
<th>KtA situations</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emerging</td>
<td>when a student is explaining his/her reasoning</td>
<td>Let’s see how did you do it.</td>
</tr>
<tr>
<td>Situation</td>
<td></td>
<td>S25: I did this… (She is interrupted by the teacher)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R: Remember this is proportion. How would you solve for x? let me know how you did it?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S25: that is because I know that this result ammm emm (She is interrupted by the teacher, again).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R: remember, let me speak (she said to the student), we do not have to do it mentally.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S25: no no I did not do it mentally. I do this…</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R: ok but solve for x here. This is the 5% of 460.</td>
</tr>
</tbody>
</table>
when a student is willing to participate…  Rosa dictated the topic, after that, she wrote an example on the whiteboard. She started to explain the example when a student wanted to participate. However, she did not allow the student to participate; she asked for a moment first to provide her explanation.

when a student provides an answer to a given problem  One student answered correctly and the teacher said “very good” and she explained the response. She did not allow the student to justify her response.

Table 4.7 shows several situations that emerged during Rosa’s classes. These situations are in the category of *emerging situations*: (a) when a student is explaining his/her reasoning; (b) when a student is willing to participate; and (c) when a student provides an answer to a given problem. One example of the classroom situation for each of the emerging KtA situations is provided in Table 4.7. The frequency of occurrence of these situations is presented in Table 4.8. These situations were observed 13 times during Rosa’s classes.

Table 4.8: Frequency Table of the KtA Situations in the Category of *Emerging Situations*.

<table>
<thead>
<tr>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>when a student is explaining his/her reasoning</td>
<td>6</td>
</tr>
<tr>
<td>when a student is willing to participate</td>
<td>2</td>
</tr>
<tr>
<td>when a student provides an answer to a given problem</td>
<td>5</td>
</tr>
</tbody>
</table>

According to the data, three categories of classroom situations were identified during Rosa’s classes: *student difficulties, student misconceptions, and emerging situations*. The category that most frequently took place was the *student misconceptions*. The category of *challenge for the teacher* was not observed in Rosa’s classes. This study also considered for analysis emerging situations where the “knowing-to act” is enacted. These situations were included in the category: *emerging situations*.

**Rosa’s Action in Student Misconceptions**

After the situations were identified, the analysis of the teacher’s “knowing-to act” was done. The focus was placed on the teachers’ actions enacted during those situations. In the first
KtA situation (item 1 of the KtAS), which is when a student solves a particular mathematical problem and asks if the answer or steps are correct, most of the time, Rosa told the students where their mistakes were. Because students are not encouraged to find their mistakes by themselves, the critical thinking of the students was limited by Rosa’s action. In the following excerpt from the transcription, the student (S10) is asking if what he was doing was right:

S10: is it this?
R: no then here we got 21. Let's see add it. (S10 is adding)
R: how many?
S10: this number.
R: ok then let's see this other side. Your sum is greater, keep working on it.

In the previous segment of the transcription, Rosa is checking the student’s work. Rosa identified two possible student’s mistakes and asked the student about them, however, there was only one mistake. Rosa revealed where the mistake was “Your sum is greater, keep working on it”. She did not allow the student to find his mistake or explain from where he got those numbers.

In these types of classroom situations, Rosa decided to tell the students several times whether their work was correct or incorrect. According to Zemelman and colleagues (2005), answering students’ questions with only “yes” or “no”, or “correct” or “incorrect” might decrease the ability to communicate mathematics ideas and limit the reasoning and ability to make proofs. The following excerpt of the transcription describes a student who revised her work:

S7: teacher I got the answer!
R: did you solve it?
S7: yes, I did.
R: Let’ see. (She took the notebook and started to add the number to see if the sides of the triangle are 20 and continued checking the rest of the problem without saying anything to
the student. Then she finished checking it). Yes, it is correct, do not tell the solution to anybody (group), tell them how to do it.

Rosa preferred to check the student’s work by herself instead of asking for the student’s justification. Rosa analyzed the work and told the student the correctness of her solution “Yes, it is correct, do not tell the solution to anybody (group), tell them how to do it.” Another action enacted by Rosa that describes the lack of her “knowing-to act” was providing an explanation when she already identified a mistake or told students who went to ask if their work was correct or incorrect that their work was wrong. For instance, a student called her to check her solution.

R: 4x1, 36 we have 3, then 4x8 is equal 32 we have 3, 4x1 is equal 4 plus 3, then we have 189, and here it is not 189, so here the division is wrong, ok this part is correct but this one is wrong. It is mistaken. How much is 189 divided by 7, here you have this number, yes, from this point you are wrong. Very good.

In the previous segment of the transcription, Rosa provided an explanation to the student when she identified that there was a mistake in the division. She guided the student by telling her where she needed to make a correction: “from this point on you are wrong”. On one occasion, Rosa asked for student’s justification of her/his work. And on other occasion, Rosa did not want to provide any response to the student just “continue working”. To perform the analysis of all these actions observed, a coding process was implemented. Codes were assigned to the teachers’ actions enacted in these classroom situations. Table 4.9 presents an example of classroom situations and the code assigned to the teacher’s action.

<table>
<thead>
<tr>
<th>Teacher’s Actions Codes</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revealing mistakes</td>
<td>S10: is this? R: no then here we got 21. Let's see add it. (s10 is adding) R: how many? S10: this number. R: ok then let's see this other side. Your sum is greater, keep working on it.</td>
</tr>
<tr>
<td>Asking for justification</td>
<td>Rosa responded to the student asking her what she did it, and both together discovered the student's mistake.</td>
</tr>
</tbody>
</table>
The analysis of the classroom situations observed was done using these codes. The coding process allowed clustering and organizing data in the way that it relates to this research question. This second research question focuses on the teachers’ actions performed during mathematics instruction in which the knowing-to is enacted. Therefore, the use of these codes allows the identification of these actions. However, this is just the analysis of the first KtA situation, the analysis of the other KtA situations will be next. Table 4.10 shows the frequency of the actions enacted by Rosa when a student solves a particular mathematical problem and asks if the answer or steps are correct. There are situations where two actions were enacted simultaneously such as providing an explanation and revealing where the mistake is, or saying it is not correct and this is your mistake. These actions were coded two times. Thus, the number of occasions that the first KtA situation occurred does not coincide with the frequency of the teacher’s actions.

Table 4.10: The frequency Table of Rosa’s Actions in Item 1 on KtAS.

<table>
<thead>
<tr>
<th>Teacher’s Actions Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revealing mistakes</td>
<td>12</td>
</tr>
<tr>
<td>Asking for justification</td>
<td>1</td>
</tr>
<tr>
<td>Teacher provides explanation</td>
<td>6</td>
</tr>
<tr>
<td>Indicating correctness or incorrectness</td>
<td>11</td>
</tr>
<tr>
<td>Ignoring student's work</td>
<td>1</td>
</tr>
</tbody>
</table>
The triangulation between anticipated actions provided on the KtAS survey and the observed actions during the classroom observation was conducted. According to Marshall and Rossman (2011), “Triangulation is the act of bringing more than one source of data to bear on a single point… Data from different sources can be used to corroborate, elaborate, or illuminate the research in question. (p. 252-253)”. The triangulation process is illustrated in Figure 4.3.

Figure 4.3: Triangulation Process of the KtAS Survey and the Classroom Observations.

The conducted triangulation involved two data sources the KtAS survey and the classroom observations. In addition, the choices provided in the KtAS served as a basis for this coding. Furthermore, several of these codes are part of a subset of the actions given in the choices of the KtAS as Table 4.11 presents. In the first KtA situation (item 1 of KtAS), which is when a student solves a particular mathematical problem and asks if the answer or steps are correct, the choices provided to be ranked on the survey and the action’s codes observed as well as the frequency of those actions are presented in Table 4.11 However, the code “Ignoring student's work” is not included in Table 4.11 because this code does not fit in any of the choices provided on the KtAS survey.
Table 4.11: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations in Item 1.

<table>
<thead>
<tr>
<th>KtAS choices</th>
<th>Observed Teacher’s Actions Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tell him/her whether his/her response is correct.</td>
<td>• Indicating correctness or incorrectness;</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>• revealing mistakes;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• teacher providing an explanation</td>
<td></td>
</tr>
<tr>
<td>2. Ask another student to confirm if it is correct or not.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Ask the student to read the corresponding section of the textbook.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Ask the whole group to share what they think.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Ask the student to explain his/her solution</td>
<td>• Asking for justification</td>
<td>1</td>
</tr>
</tbody>
</table>

Based on Table 4.10 and Table 4.11, we add to the discussion that Rosa’s knowing to act is not desirable in this first KtA situation. Rosa on 12 occasions revealed the students’ mistakes. Moreover, she responded to the students indicating the correctness or incorrectness of their work on 11 occasions. This means that. Rosa’s actions enacted are not promoting critical thinking for her students.

During the second KtA situation of the student misconceptions category (item 3 of the KtAS), which is when a student provides a nonsensical solution to a given problem, Rosa provided the correct answer. According to Mason and Spence (1999) providing the solution is an action that might not be effective for increasing student’s reasoning. Rosa acted in this way on one occasion.

R: ok, there, what am I going to solve for?
S13: the 20 %
R: let’s see. Let’s see.
S14: the number 250.
R: the x because that is what we do not know.
In this situation, Rosa was solving a problem of how to calculate percentages. She wrote the equation of the problem and asked what was needed for the solution. Two students (S13 and S14) provided nonsensical answers. They did not identify what the variable was even if in the equation the variable x was there. Thus, Rosa told them the correct answer “the x because that is what we do not know.” This type of situation occurred on a second occasion. However, Rosa acted in a different way.

R: I need to leave alone and remove what I do not need on that side. What do I not need with the x over there?
S15: the equal symbol.
R: we do not need the equal symbol =?
S16: no, the 20 %
R: the 20%? Let’s see, you are coming from the break very sleepy.

In this segment of the transcription, Rosa asked students about a step in the procedure for solving a problem of percentages. She asked “What do I not need with the x over there?” in order to solve for x. Then, a student (S15) answered “the equal symbol”. Definitely this student provided a nonsensical response. Rosa asked another student and S16 gave a wrong answer. Then, Rosa ignored the students’ responses. Awareness of ignorance may relate to both correct and incorrect student’s response. Rosa asked the whole class again. However, nobody knew the answer. The KtA situation was identified in Rosa’s classes only these two times. Rosa’s actions were different in both situations. The codes that were assigned to these actions are presented in Table 4.12.

Table 4.12: Teacher’s Action Codes Item 3 KtAS.

<table>
<thead>
<tr>
<th>Teacher’s Actions Codes</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher provides the solution</td>
<td>R: ok, there, what am I going solve for?</td>
</tr>
<tr>
<td></td>
<td>S13: the 20 %</td>
</tr>
<tr>
<td></td>
<td>R: let’s see. Let’s see.</td>
</tr>
<tr>
<td></td>
<td>S14: the number 250.</td>
</tr>
<tr>
<td></td>
<td>R: the x because that is what we do not know.</td>
</tr>
</tbody>
</table>
Ignoring the students' responses

R: I need to leave alone and remove what I do not need on that side. What do I not need with the x over there?
S15: the equal symbol.
R: we do not need the equal symbol =?
S16: no, the 20 %
R: the 20%? Let’s see, you are returning from the break very sleepy.

These codes were generated based on the nature of the teacher’s action. The triangulation between the KtAS choices provided to be ranked and the observed teacher’s actions in this KtA situation: when a student provides a nonsensical solution to a given problem, is presented in Table 4.13.

Table 4.13: Correspondence of the Triangulation Between the KtAS Surve and Classroom Observations for Item 3.

<table>
<thead>
<tr>
<th>KtAS choices</th>
<th>Observed Teacher’s Actions Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ask the student why he or she thinks that way.</td>
<td>• Ignoring the students' responses</td>
<td>1</td>
</tr>
<tr>
<td>2. Ask the student to re-do the problem.</td>
<td>• Teacher provides the solution</td>
<td>1</td>
</tr>
<tr>
<td>3. Ignore the student’s response.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Tell the student how the problem should be solved step by step.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Give an extra credit for her/his effort.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the third KtA situation of the student misconceptions category (item 9 of the KtAS), which is when a student makes a mistake, Rosa mostly provided an explanation. This explanation implies the repetition of the indications of how to solve the problem or the indication of a mistake. Rosa observed a group in her class; the students did not how to solve a problem. One student was adding 15 by 15.

R: No instead of adding 15 by 15 you need to write these numbers [she borrowed a student’s pencil and wrote in the student’s notebook 1,2,3,4,5,6,7,8,9.] Yes, you can solve it. The sum of the corners must be 15. And each side must be 20.

Before the above transcription, Rosa assigned an activity called “The magical triangle”. Students were to solve it placing certain numbers in a position where the sum of the sides is equal to 20, and the sum of the corners is 15. Students in one group of Rosa’s class did not know
how to find the solution. Then, Rosa identified the students’ mistake and started to explain and repeat the directions to solve the problem as observed in the segment of the transcription above. Rosa’ action was enacted on three occasions as Table 4.15 describes.

Rosa revealed a mistake to one student who was wrong. As was previously mentioned, revealing students’ mistakes does not provide the opportunity for students to be aware of their own mistakes and look for another way to solve a problem in order to construct their knowledge. The classroom situation observed was the following:

Rosa revised the solution to a problem from one of the students in a group in her class, and she told her where the mistake was.

In this situation, Rosa was walking around the groups supervising students’ work. Then she was looking at the solution from one of the groups in her mathematics class. Then, she detected a mistake and revealed where the mistake was to the student of the group. This KtA situation was presented again during Rosa’s classes. However, Rosa decided to ask the student for justification of her/his work.

R: if 4 pencils cost 750 pesos, why are you doing this honey?
S8: this…
R: the problem is if 4 pencils cost 750 pesos, how much will 14 pencils be?
S8: aah.
R: you wrote it wrong

In this previous segment of the transcription, a problem about proportionality was assigned. Rosa was walking around the groups to see how they were working. She stopped in one group and checked how the students were doing the problem. Then, she identified a student’s mistake, and Rosa asked the student “if 4 pencils cost 750 pesos, why are you doing this honey?” Because the student needed to think to justify her/his answer, Rosa’s action is promoting the critical thinking of this student.

This KtA situation where a student makes a mistake happened on two more occasions. However, Rosa’s actions differed in both situations. In one classroom situation, Rosa decided to
ignore the student’s response. This type of action might discourage students from being involved in the class.

Rosa selected one student and the student answered incorrectly, she ignored the wrong answer because she did not do anything about it.

In the previous situation, Rosa was explaining how to solve for x in an equation to calculate percentages. She was asking questions to the whole class at the same time that she was explaining the problem. Then she asked a question to one student. Then, the student answered incorrectly, and she decided to select another student to answer her question. In another situation in Rosa’s classroom, Rosa asked one student to look at the whiteboard.

The student did not know it, and the teacher asked him to see the problem on the whiteboard and look at the procedure. He did not know, then, she allowed other students to participate.

In this situation, Rosa asked a question about a problem on the calculation of percentages. The student did not know the answer. Then Rosa asked him to observe the whiteboard (the problem and a part of the procedure were written there). After that, Rosa asked for another students’ participation. As observed in the segment above, there is no support from Rosa to address the student’s weaknesses. Thus, Rosa’s actions were coded as shown in Table 4.14.

Table 4.14: Teacher’s Actions Codes Item 9 KtAS.

<table>
<thead>
<tr>
<th>Teacher’s Actions Codes</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher provides explanation</td>
<td>R: hey guys I see that you are placing the number 15, no, you only can place the numbers from 1-9, the 1, 2, 3, 4, 5, 6, 7, 8, 9, you should place them in a certain way that the sum of the corners must be 15 and the sides 20.</td>
</tr>
<tr>
<td>Asking for justification</td>
<td>R: if 4 pencils cost 750 pesos, why are you doing this honey? S8: this… R: the problem is if 4 pencils cost 750 pesos, how much will be 14 pencils? S8: aah. R: you wrote it wrong</td>
</tr>
<tr>
<td>Revealing student’s mistake</td>
<td>Rosa checked another student’s solution, took his notebook and she said to the student “you are wrong here in this multiplication process”</td>
</tr>
<tr>
<td>Ignoring the students' responses</td>
<td>She selected one student and the student answered incorrectly, she ignored the wrong answer because she did not do anything about it</td>
</tr>
</tbody>
</table>
Referring to the whiteboard or notes

The student did not know it and the teacher asked him to see the problem on the whiteboard and look at the procedure. He did not know, then she allowed other students to participate.

These codes allowed the analysis of this type of classroom situations observed. This KtA situation when a student makes a mistake was identified during Rosa’s classes on 8 occasions. Rosa’s actions were different in the classroom situations most of the time. The frequencies of these actions are reported in Table 4.15. Rosa’s explanation was frequently provided when a student made a mistake.

Table 4.15: The Frequency of Rosa’s Actions in the Item 9 KtAS.

<table>
<thead>
<tr>
<th>Teacher’s Action Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revealing mistakes</td>
<td>2</td>
</tr>
<tr>
<td>Asking for justification</td>
<td>1</td>
</tr>
<tr>
<td>Teacher provides explanation</td>
<td>3</td>
</tr>
<tr>
<td>Referring to the whiteboard or notes</td>
<td>1</td>
</tr>
<tr>
<td>Ignoring student's response</td>
<td>1</td>
</tr>
</tbody>
</table>

In the KtAS, item 9 corresponds to the third KtA situation, which is when a student makes a mistake. Table 4.16 presents the choices provided in this item to be ranked and the teacher’s action codes that are under these choices as well as their frequencies. However, this triangulation does not include the code of “Referring to the whiteboard or notes” in Table 4.16 because it does not fit in any KtAS choice.

Table 4.16: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations for Item 9.

<table>
<thead>
<tr>
<th>KtAS choices</th>
<th>Observed Teacher’s Action Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tell the student that using a calculator is not allowed.</td>
<td>• Ignoring student’s response</td>
<td>1</td>
</tr>
<tr>
<td>2. Ignore it.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Ask the student to re-do the task.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. Explain to him/her why it is wrong.
   - Revealing mistakes;
   - Teacher provides explanation

5. Let the student explain his/her answer.
   - Asking for justification

Up to this point, the analysis within the case study of Rosa’s on the category of students’ misconceptions was already done. Next, Rosa’s actions enacted in the category of student difficulties are analyzed in each of the four types of KtA situations.

**Rosa’s Actions in Student Difficulties**

The first KtA situation is the item 2 on the KtAS: when a student does not recognize the same pattern in a different situation. In this classroom situation, Rosa allowed the assistance of one student to support the student’s learning.

R: ok, we are fine, then how is 2% represented? Let other students participate not only you (2 students). You, Jonathan, how would it be?

S7: mmmm

S8: me.

R: well, you can help him.

Rosa already provided an explanation about how to calculate percentages. However, most of the students had not understood the topic yet. She asked a question to one student and he was not able to respond. Then Rosa allowed that other student to help him. This action was enacted only on this occasion. The code “allowing or asking for help from other student” was assigned to this teacher’s action. In another classroom situation that belongs to this type of KtA situation, Rosa decided to explain the problem.

R: The 225, how would it be represented? I want to see more hands up. Saul, how would it be represented?

S9: it could not be the same because the number is greater than mmm… (Other students were raising their hands, but the teacher explained).
R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions.

In this situation, Rosa asked a student how to represent a percentage in a fraction (S9). Rosa already solved two exercises like this one. However, the student (S9) did not recognize the same pattern of the two previous exercises in this last exercise. Then, Rosa provided a brief explanation about the fractions. Rosa acted in this way only once. This teacher’s action was coded as “Providing an explanation”. Rosa’s actions are limiting the opportunity for the student to be able to find the pattern of the problem and apply the solution to another problem. But in the following excerpt from the transcription, Rosa’s action promoted the student’s thinking.

S22: if it would be solved doing this and this. (The teacher listened to her, after that)
R: ok ok I admitted that this problem is harder and I apologized for it. But you have here the procedure, it is represented, the only thing that you need to do is to put in an equation and get the proportion. What percentage is 20 out of 100? I do not know if you understand me, if you do not, let me know. She realized that her students had difficulties understanding. Hence, she tried to explain talking about money.
R: I have 20 pesos, what percentage of 100 pesos is 20 pesos?
S23: ah I got it, because my classmate told me another thing that we should do and I became confused.

Rosa assigned a harder problem. She apologized to the student because of it. However, she observed that students did not grasp how to solve it even though several problems of the same type were already solved. Thus, Rosa looked for another way to represent the problem. This action was performed only once in the classroom situation already mentioned. The code used for analysis of this action is “looking for another way to represent the problem”.

The KtA situation when a student does not recognize the same pattern in a different situation is item 2 of the knowing-to act survey (KtAS). The choices given in the survey for this situation are triangulated with the observed actions. Table 4.17 illustrates this triangulation and the frequency of these actions.
Table 4.17: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations in Item 2.

<table>
<thead>
<tr>
<th>KtAS choices</th>
<th>Observed Teacher’s Action Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Provide the solution.</td>
<td>• Providing an explanation</td>
<td>1</td>
</tr>
<tr>
<td>2. Ask another student to help him/her.</td>
<td>• Allowing or asking for help from other student</td>
<td>1</td>
</tr>
<tr>
<td>3. Look for another way to represent the problem for the student.</td>
<td>• Looking for another way to represent problem</td>
<td>1</td>
</tr>
<tr>
<td>4. Ignore it and move to another topic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Pose another similar problem to the student.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The second KtA situation of the *student difficulties* is when a student is unable to see an obvious pattern in the problem. This situation was identified on only one occasion during Rosa’s classes. Rosa was covering the topic of percentages:

R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions. How would it be represented in common fraction, Brenda?

S10: as a quarter.

R: no no no, my love, remember that we remove the point, it would be also 225/100 it would be the same, this was seen previously. You do not remember it?

Rosa made a reminder about the topic previously seen. She asked one student (S10) about the representation of 2.25 in fractions. Exercises on this topic had already been completed by the students and Rosa. However, the student was not able to identify the pattern. Thus, she could not provide the right answer. Then, Rosa provided the right representation in fractions. Rosa’s action was to tell the student what the solution is or what the pattern is.

This KtA situation is described in item 5 of the KtAS. The code assigned to the action performed by Rosa was “providing the solution” which is a subset of choice 1. Providing the solution to the problem when the student does not find the pattern is basically to tell the student what the obvious pattern is.
The third KtA situation of the category of student difficulties is when a student continuously responds with wrong answers to questions posed by the teacher. It is item 8 in the KtAS. This situation was identified in Rosa’s classes on one occasion. Rosa selected a student to go to the whiteboard to write her solution. Then the student wrote her solution. Rosa indicated that the solution was wrong. After that, she asked another student “why is it incorrect?”, and the student was not aware of the incorrectness of the solution written on the whiteboard. Thus she did not know why that solution was wrong. Rosa asked the whole class and selected another student. Then, this student went to the whiteboard to solve it. One student is selected and went to the whiteboard and wrote her solution.

R: it is a little bit incorrect, Brenda (another student) why is it incorrect?
S24: is it incorrect?
R: why Brenda?
S24: well I got this other result?
R: tell me why? (She asked the whole classroom and selected another student), ok you go to the front to see if you know as you said. Remember that you are going to solve for x, eh.

In this classroom situation, Rosa did not address any of the students’ lack of knowledge. She preferred to select another student and another until one could do it right. Rosa’s action is coded as “ignoring the students' responses” because she ignored what the students said. She preferred to select other students instead of doing something to help the students construct their knowledge. The assigned code is really choice 2 of item 8 on the KtAS.

The last KtA situation of the KtAS is when students are having a hard time completing an assigned activity/task. This situation is item 10. Table 4.18 shows an example of each of the classroom situations observed in Rosa’s classes where the KtA situation is recognized. Rosa’s actions differ in this KtA situation. Example 1 shown in Table 4.18 described the situation where a student asked Rosa something about the work, and Rosa referred her to the whiteboard to see what Rosa had already written on it. In this example, Rosa’s action was to refer students to the
whiteboard and repeat the directions to do the task. The codes assigned to these teacher’s actions are “referring to the whiteboard or notes” and “repeating directions”. Triangulating the anticipated action provided on the KtAS choices under item 10 and the observed teacher’s actions, these codes do not fit under any choice on the KtAS. However, these codes are considered for the analysis of the teacher’s actions. Qualitative research allows this flexibility to include important aspects of the phenomena studied that were not previously considered. For example, these actions were not included as choices in the KtAS.

Table 4.18: Teacher’s Action Codes under Item 10 of the KtAS.

<table>
<thead>
<tr>
<th>Teacher’s Action Codes</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Referring to the whiteboard or notes; repeating directions</td>
<td>1. One student asked her something and she referred the student to what she wrote on the whiteboard and repeated the same instructions.</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>2. R: hey guys I see that you are placing the number 15, no, you only can place the numbers from 1-9, the 1, 2, 3, 4, 5, 6,7,8,9, you should place them in a certain way so that the sum of the corners must be 15 and the sides 20.</td>
</tr>
<tr>
<td>Providing hints or more time</td>
<td>3. Rosa provided a hint in order to help her students to solve it quickly. She told the students to find out what were the numbers of the corners first to avoid difficulties. Then she was observing one group and she decided to provide another hint.</td>
</tr>
<tr>
<td>Provides the solution</td>
<td>4. When she saw her students did not know how to answer to her questions she provided the answer and explanation.</td>
</tr>
<tr>
<td>Solves key parts of the solution.</td>
<td>5. After that she asked for the solution to the first problem. She wrote on the whiteboard the proportion of the problem in order for students to solve it.</td>
</tr>
</tbody>
</table>

Example 2 shown in table 4.18 is a situation where students were having a hard time solving the problem. They had not understood how to solve it. When students asked a question to Rosa, she usually responded explaining her answer to the whole class.

R: hey guys I see that you are placing the number 15, no, you only can place the numbers from 1-9, the 1, 2, 3, 4, 5, 6,7,8,9, you should place them in a certain way so that the sum of the corners must be 15 and the sides 20.
In the previous segment of the transcription Rosa provided an explanation so students could understand what they needed to do to solve the activity. Rosa already had explained the activity, however, students had not understood. Then, Rosa provided a different explanation to see if the students grasped how the activity should be solved. Rosa’s action was coded as “providing an explanation” as Table 4.18 shows. The triangulation between this code and the KtAS choices is presented in Table 4.20.

In another class observed, Rosa assigned the activity of “the magical triangle” to her second mathematics class. In her first class, students took the entire class to solve this activity. Rosa observed that her students were having a hard time completing the activity. Then, she realized that a hint was needed to allow the students to move forward in the activity. After that, Rosa observed how one group of her class was working and decided to provide another hint as Table 4.18 presents. The action acted by Rosa is coded as “provide hints or time”. Rosa decided to provide students with hints in order to be able to solve the assigned activity.

Rosa was teaching how to calculate percentages and represent them in different ways. Then she was solving one problem. At the same time, she was explaining how to do it. Then, she asked how to solve for x in the equation that she wrote to find the percentage of a certain amount. She observed that the students did not know how to solve for x. Then, Rosa decided to provide the solution as mentioned in Table 4.18. Rosa’s action was coded as “provides the solution”. This action is related to choice 1 on the KtAS as Table 4.20 shows.

On another occasion Rosa was solving and explaining a problem on the whiteboard so students could take notes. She asked for the solution to the problem, however, she noticed that students were having difficulties doing it. Thus, she obtained the equation from the problem and told the students to solve the problem using that equation. Here, Rosa gave the students one part done of the problem in order to help the student to solve the whole problem as presented in Table 4.18. This action was coded as “solves key parts of the solution.”

The different actions mentioned above were enacted during mathematics instruction by Rosa when students were having a hard time completing an assigned activity or task. In Table
4.19, we can observe that Rosa more frequently refers students to the whiteboard or repeats the directions to help students to accomplish the activity. Moreover, she also provides hints to the students on 4 occasions.

Table 4.19: Frequency Table of Teacher’s Actions in Item 10 of the KtAS.

<table>
<thead>
<tr>
<th>Teacher’s Action Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Referring to the whiteboard or notes; repeating directions</td>
<td>4</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>2</td>
</tr>
<tr>
<td>Providing hints or more time</td>
<td>4</td>
</tr>
<tr>
<td>Provides the solution</td>
<td>1</td>
</tr>
<tr>
<td>Solves key parts of the solution</td>
<td>2</td>
</tr>
</tbody>
</table>

The triangulation between anticipated actions provided on the KtAS survey and the observed actions was conducted. This triangulation involved two data sources the KtAS survey and the classroom observations. Table 4.20 presents the triangulation between the choices of the item 10 on the KtAS and actions observed during the classroom observations as well as the frequency of these actions.

Table 4.20: Correspondence of the Triangulation Between the KtAS Survey and Classroom Observations in Item 10.

<table>
<thead>
<tr>
<th>KtAS choices</th>
<th>Observed Teacher’s Actions Codes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Complete the activity for the students and let them copy it.</td>
<td>• Provides the solution</td>
<td>1</td>
</tr>
<tr>
<td>2. Analyze students’ difficulties and provide hints</td>
<td>• Providing hints or more time</td>
<td>4</td>
</tr>
<tr>
<td>3. Do half of the assignment and leave the other half to the students.</td>
<td>• Solves key parts of the solution</td>
<td>2</td>
</tr>
<tr>
<td>4. Let the students go to a recess.</td>
<td>• Providing an explanation</td>
<td>2</td>
</tr>
<tr>
<td>5. Explain the assignment using examples or another way to explain it.</td>
<td>• Providing an explanation</td>
<td>2</td>
</tr>
</tbody>
</table>
Rosa’s Actions in Emerging Situations

The last category is emerging situations. The situation when a student is explaining his or her reasoning emerged during the observations. This situation is included in this category because the action enacted by a teacher in this situation might affect the students’ ability to communicate their ideas and think critically. Therefore, “knowing-to act” in this situation plays an important role which limits or enhances the learning environment.

During one of Rosa’s classes, one student took his work to be checked by the teacher. The student was explaining to Rosa what he did to solve the problem. However, Rosa checked her work and did not allow the student to continue explaining his work. She finished checking it and said “yes it is correct” and gave him a chocolate as an incentive to work harder as is described in Table 4.21. This action was exhibited on only one occasion. In this situation, Rosa’s action is coded as “not allowing student’s explanation”.

R: the 35% how is it represented in a fraction? Let’s see… remember it.
S5: then if, for example, an integer will be a 100%, no?
R: yes.
S5: all complete, then it would be 3, 11, mmm 1/7?
R: no no no, let’s see (she interrupted the student and did not listen to the student’s reasoning). Pay attention

In the situation presented in the segment of the transcription above, Rosa is asking one student how to represent a percentage in fractions. The student was reasoning how to get the solution. However, when the student is in that critical moment of connecting ideas, reasoning, and thinking what to answer Rosa did not allow him to explain what he was thinking. She interrupted the student and said “no no no, let’s see, pay attention”. Rosa’s action is coded as “interrupting the student’s explanation”. As in the previous action mentioned, since students are not allowed to explain and share their thinking, it decreases students’ critical thinking. This action was enacted by Rosa on three occasions.

R: what percentage is 30 out of 120?
S20: how how 30*120 or 4, what portion would it be, teacher?
R: pay attention, here is where we need to be careful (she ignored the student S20)
S20: teacher, it means that is the 30% of 120 or the 20% is 30 out of 120?
R: wait a moment, you can ask me questions next. (She ignored the student S20) she started to explain the problem.

In the situation transcribed above, Rosa was asking one student the percentage of 30 out of 120. The student was thinking and asking Rosa questions in order to know if she was right. However, Rosa did not allow her to ask questions. Rosa wanted to first provide her explanation. Rosa’s action enacted was to ignore the student’s responses and questions. The code for this action is “ignoring the student”. This action was performed by Rosa on two occasions.

When a student is willing to participate is the second type of emerging situations. After Rosa discussed percentages, she wrote an example on the whiteboard. When she was explaining it to the class, one student wanted to participate. However, Rosa did not allow the student to participate; she wanted to continue explaining and asked for a moment to finish her explanation. “Limiting student participation to provide his/her explanation” is the code assigned to Rosa’s action. She acted in this way on two occasions during the classes observed.

The last emerging situation is when a student provides an answer to a given problem. This classroom situation occurred several times during Rosa’s classes that were observed. Rosa decided to provide the reassurance to the students about the correctness or incorrectness of their work on four occasions. For instance, saying “very good”. Revealing a student’s mistake is also another action enacted by her in this situation.

R: here we need to pay attention; here it is asking what percentage is 340 out of 200?
Let’s see, what is here the 100%?
S24: 200
R: yes 200. Then it goes over the 100%, ok what is the 50% of 200?

In the previous situation, the student responded correctly to Rosa question. She did not ask the student to explain why she was right. The action was coded as “not asking for
justification”. In another classroom situation, on only one occasion, Rosa asked one student for justification of his work.

R: ok the result is correct, but I want you tell me what you did because you solved for x, but this part of the procedure is wrong

Even though Rosa asked the students for justification, she did not allow the student to discover where her mistake was: “tell me what you did because you solved for x, but this part of the procedure is wrong”. This action does not provide the time needed for the student to explain what she did because Rosa told her the mistake she made. Rosa’s action was limiting the student to think about her procedure; however, she was told that her procedure was wrong. This action reduced the complexity to solve the problem to keep the student engaged and trying to solve it to construct knowledge. Rosa’s action is coded as “revealing mistakes”. Table 4.21 presents the codes for Rosa’s actions enacted in the category of emerging situations.

Table 4.21: Teacher’s action Codes in Emerging Situations.

<table>
<thead>
<tr>
<th>Teacher’s Actions Codes</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not allowing student’s explanation</td>
<td>Another student finished the problem and went to the teacher, the student was explaining to the teacher what she did to solve it but the teacher did not allow the student to explain it because Rosa preferred to check it by herself. Rosa said yes it is correct and gave the chocolate to the student.</td>
</tr>
<tr>
<td>Interrupting the student's explanation</td>
<td>R: the 35% how is it represented in a fraction? Let’s see remember it. S5: then if, for example, an integer will be a 100%, no? R: yes. S5: all complete, then it would be 3, 11, mmm 1/7? R: no no no, let’s see (she interrupted the student and did not listen to the student’s reasoning). pay attention</td>
</tr>
<tr>
<td>Ignoring the student</td>
<td>R: what percentage is 30 out of 120? S20: how how 30*120 or 4, what portion it would be, teacher? R: pay attention, here is where we need to be careful (she ignored student S20) S20: teacher, it means that is the 30% of 120 or the 20% is 30 out of 120? R: wait a moment, you can ask me questions next. (She ignored student S20) she started to explain the problem.</td>
</tr>
<tr>
<td>Limiting student participation to provide his/her explanation</td>
<td>Rosa started to explain it. She provided the equation from the sentence. She did not allow students to obtain it by themselves. She used the previous problem to highlight the difference between them.</td>
</tr>
</tbody>
</table>
Not asking for justification
Rosa asked for the first problem, one student responded correctly and Rosa said “very good” and gave her a pop.

Revealing mistakes
R: ok the result is correct, but I want you tell me what you did because you solved for x, but this part of the procedure is wrong.

Observing Omar

During the spring semester of 2013-2014 academic year, I observed and videotaped Omar’s classrooms for a period of three classes. In the first classroom observation that I did, the topic “solution of simultaneous equations” was covered. This class was an eighth grade level. In the second and third class that I observed and videotaped, Omar was providing a review of the two previous units “linear equations” and “quadratic equations”. These two classes were ninth grade level. During each classroom observation I was allowed to walk around the classroom to try to capture as many details as possible.

Three of Omar’s classes were observed and videotaped. The different KtA situations (situations considered on the knowing-to act survey KtAS) that were identified during Omar’s classes are presented in Table 4.22. In addition, an example of each of the situation is provided in the chart. These observations were based on the protocol observation developed. The classroom situations identified were part of the three categories already described earlier in this chapter. During Omar’s classes, there were not identified emerged situations, thus, this category is not included in Table 4.22.

Table 4.22: Knowing-to Act Situations Observed in Omar’s Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td>Another student asked him to check what he was doing if his procedure was right, and the teacher said yes (OC: KtA).</td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake:</td>
<td>Omar paid attention to what that group was doing and he told them:</td>
</tr>
</tbody>
</table>
**Student Difficulties**

<table>
<thead>
<tr>
<th>Item 2 KtAS. When a student does not recognize the same pattern in a different situation</th>
<th>S11: I do not understand here teacher. O: which one? S11: the sixth equation (then the teacher explained it to her and she said:) S11: ah! Then it would be 4x O: yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 5 KtAS. When a student is unable to see an obvious pattern in the problem</td>
<td>One group asked the teacher “from where did you get this in the example?” and the teacher did not tell them in order that they could deduce it and think about it. After a few minutes, the teacher explained to them just one part and allowed the students to deduce the next part.</td>
</tr>
<tr>
<td>Item 10 KtAS. When students are having hard time to complete an assigned activity/task</td>
<td>S2: teacher, this can be moved to this side? O: yes (he checked it).</td>
</tr>
<tr>
<td>Challenge for the teacher Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>Other students asking what he needed to do to solve the equation, and the teacher answered her “look at the example” in order for the student to think about it.</td>
</tr>
<tr>
<td>Item 7 KtAS. When a student asks you a question from a different perspective that was not previously considered by you</td>
<td>When Omar is explaining the example he said that 3x is a multiplication. Then a student asked “why 3 was multiplied”. Then Omar explained to him when a number is together with a letter as 3x it is a multiplication, and he provided some examples.</td>
</tr>
</tbody>
</table>

The frequency of these KtA situations is presented in Table 4.23. Omar’s classes exhibited mostly the situations of the first category of *student misconceptions*. The situation most frequently identified in Omar’s classes was when a student solved a particular mathematical problem and asked if the answer or steps were correct. On 21 occasions Omar was exposed to this first KtA situation. The situation of when a student makes a mistake was identified on 4 occasions during Omar’s classes observed. Also, on 4 occasions the situation of when you assign an activity about a previously learned topic, and some students express that they do not know how to do it arose in Omar’s classes. There were other KtA situations that were identified several times as Table 4.23 shows.
### Table 4.23: Frequency Table of the KtA Situations in Omar’s Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>When a student makes a mistake</td>
<td>4</td>
</tr>
<tr>
<td>Student Difficulties</td>
<td>When a student does not recognize the same pattern in a different situation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When students are having hard time to complete an assigned activity/task</td>
<td>2</td>
</tr>
<tr>
<td>Challenge to the Teacher</td>
<td>When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>When a student asks you a question from a different perspective that was not previously considered by you</td>
<td>2</td>
</tr>
</tbody>
</table>

Additional descriptive statistics with accompanying descriptions of classroom interactions for Omar may be found in Appendix A. Rogelio’s case is presented in the next section.

**Observing Rogelio**

I observed and videotaped Rogelio’s classrooms during the spring semester of the 2013-2014 academic year for a period of three classes. Those classes were 9th grade. Rogelio worked on worksheets in his classes. The topics that were covered on the classes observed were “linear equations” and “real word problems about linear and quadratic equations”. The methods to solve quadratic equation that were taught were “factoring and the general formula”. I was allowed to walk around in the classroom with the video camera hung on my neck to better capture the emerged situations where knowing-to act should be or was enacted.

Several KtA situations were identified on multiple occasions during Rogelio’s classes. Table 4.24 presents the category of the knowing-to act situations, the KtA situation, and an
example of each classroom situation where the KtA situation was identified in Rogelio’s class. There were 10 different KtA situations observed in Rogelio’s classes. The three types of KtA situations of the category of *student misconceptions* were recognized.

Table 4.24: Knowing-to Situations Observed in Rogelio’s Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Rogelio’s Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct</td>
<td>S15: teacher am I doing well? R: yes, you are right, but you need to use the parenthesis. X and X in each parenthesis, if you want it to do it this way, if not…</td>
</tr>
<tr>
<td></td>
<td>Item 3 KtAS. When a student provides a nonsensical solution to a given problem</td>
<td>S2: its multiplication should be 0 R: no, it’s multiplication should be this number: 396. and he continued explaining the problem.</td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake</td>
<td>R: multiplying, so to what side is it going to be? S2: toward the x. R: no no you cannot move to the x, S3: move toward the other side. R: to what? S4: move toward the number 8.</td>
</tr>
<tr>
<td>Student Difficulties</td>
<td>Item 5 KtAS. When a student is unable to see an obvious pattern in the problem</td>
<td>S10: teacher it is divided by what? R: it is divided by the number of grades. How many grades are there? S10: 3 R: then by 3.</td>
</tr>
<tr>
<td></td>
<td>Item 8 KtAS. When a student continuously responds with wrong answers to questions posed by you</td>
<td>Then he put x on one side and he asked what are we going to write on the other side? S12: x4 R: nooo, it would be 4 times. Students provided wrong answers: “4x, 8x, 4*10” R: nooo, we need to add, heeey if I have 4 units greater S13: x+4</td>
</tr>
<tr>
<td>Challenge to the Teacher</td>
<td>Item 10 KtAS. When students are having a hard time completing an assigned activity/task</td>
<td>When Rogelio saw that students had not solved the problem, he wrote another step to the solution on the whiteboard</td>
</tr>
<tr>
<td>Unexpected Emerging Situation</td>
<td>Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>One student called the teacher and he went to the student’s seat. Rogelio is asked about how to use factoring. Then he explained to the student how to do it.</td>
</tr>
<tr>
<td></td>
<td>Item 2 KtAS. When a student is explaining his/her reasoning</td>
<td>Then Rogelio said that the other way is using the general formula. He asked students if they remember how it is, he did not provide time to answer, but he wrote it on the whiteboard.</td>
</tr>
</tbody>
</table>
When a student is willing to participate…

R: why? Because x by x is square x (he did not wait students’ response). Then the signs: + and -, + by -, then we need to seek two numbers that…

When a student provides an answer to a given problem

R: nooo, we need to add, heeey if I have 4 units greater
S13: x+4
R: yes it is. Very good.

The frequency of these KtA situations is presented in Table 4.25. Rogelio’s classes exhibit more unexpected emerging situations than the other KtA situations considered in the protocol observation. The first category of student misconceptions includes three types of KtA situations. The first type of KtA occurred more frequently in this category.

Table 4.25: Frequency Table of the KtA Situations in Rogelio Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>When a student provides a nonsensical solution to a given problem</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When a student makes a mistake</td>
<td>7</td>
</tr>
<tr>
<td>Student Difficulties</td>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When a student continuously responds with wrong answers to questions posed by you</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>When students are having a hard time to complete an assigned activity/task</td>
<td>3</td>
</tr>
<tr>
<td>Challenge to the Teacher</td>
<td>When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>5</td>
</tr>
<tr>
<td>Emerging Situation</td>
<td>When a student is explaining his/her reasoning</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>When a student is willing to participate…</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>When a student provides an answer to a given problem</td>
<td>7</td>
</tr>
</tbody>
</table>
Additional descriptive statistics with accompanying descriptions of classroom interactions for Rogelio may be found in Appendix B. Maria’s case is presented in the next section.

**Observing Maria**

Maria is the last case study of this research. During the spring semester of the 2013-2014 academic year, I observed and videotaped Maria’s classrooms for a period of three classes. The topic that she was covering was “linear equations and solution methods” during the three classes. In the first class observed, Maria worked on exercises of the textbook. In the second class, worksheets were provided to the students. And in the last class observed, students presented the problems that were on the worksheet the day before. I was also allowed to walk around the groups to be able to get a better and detailed account about the teacher’s actions.

Considering the protocol observation developed for conducting these classroom observations, there were seven KtA situations recognized in Maria’s classes observed and videotaped. These KtA situations belong to the three categories of situations on which the KtAS was based. In addition, data was also collected about unexpected emerging situations where the teacher’s knowing-to act should be or is enacted. Additionally, the category *emerging situations* was created. Maria’s classes observed exhibited two types of situations not considered in the protocol observation as Table 4.26 presents.
Table 4.26: Knowing-to Act Situations Observed in Maria’s Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Maria’s Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td>One student asked her if his work is correct and he explained to the teacher what he did. Maria asked him “why did you do this?” and the students answered her “this” and she explained hi why he is wrong.</td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake</td>
<td>She walked around the groups and stopped with one student. She was observing the student’s work and asked why he did this, then the student explained to her, but she asked the student to do first certain steps and she explained what he needed to do.</td>
</tr>
<tr>
<td>Student Difficulties</td>
<td>Item 2 KtAS. When a student does not recognize the same pattern in a different situation</td>
<td>One student was asking several questions during the class. This student went to ask the teacher again and she said “no, you should know how to do it by yourself and go to sit down”.</td>
</tr>
<tr>
<td></td>
<td>Item 5 KtAS. When a student is unable to see an obvious pattern in the problem</td>
<td>Another student asked her about one problem and Mayra said “do it by yourself, read it again and compare it to the solution on the whiteboard”</td>
</tr>
</tbody>
</table>
|                             | Item 8 KtAS. When a student continuously responds with wrong answers to questions posed by the teacher | M: let’s see Escamilla, yes, it is right, but where did you get the positive sign in this term?  
S6: ah when I did this, this number is multiplied by -36, is not it?  
Students: no. (Maria asked if somebody wanted to help Escamilla. A student went to the front, and they were discussing the solution. Then Maria asked them)  
M: Where are you struggling?  
S6: we do not remember from where we got this term, and how it became positive, if we already move the term, then, the sign changes, is not it? (Then Maria asked the whole class)  
M: what was done by your classmates is it right? 3x +5x?  
Students: no.  
M: until here we were right. Look at the problem.  
What did you do with 5x? was it moved? to where?, and then, Escamilla?  
S6: the addition of 3x+5x=8x, is equal… |
| Challenge to the Teacher    | Item 10 KtAS. When students are having a hard time completing an assigned activity/task | Students asked her about the step where they were and she explained to them what else needed to be done.                                                                                                                                                  |
|                             | Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to do it | S4: how is it?  
M: try to remember it. (Then the student explained what he thought). Do it but keep the order to avoid confusions.                                                                                                                                  |
Emerging Situation when a student is explaining his/her reasoning

One student is explaining to her what he did, and the teacher listened to him and checked his work. Then she said: M: ahaa, but remember this is already known. What does it mean? What is going to happen to this 4? (She really did not provide him time to answer her questions, she was explaining step by step and asking the student why that step, and she told him what else he needed to do. She left that group)

When a student provides an answer to a given problem

The student read the problem and solved the problem on the whiteboard.
M: Esteban, can you read again the problem please and the student read it.
M: now we pay attention to hear the explanation of the group #7. The student who wrote it on the whiteboard explained her procedure and solution.

Table 4.26 presents the KtA situations that were identified in Maria’s classes. In addition, in Table 4.26 excerpts of transcriptions or field notes taken in Maria’s observations are provided for each situation. Maria’s classes included numerous situations where the teacher’s “knowing-to-act” should be or is enacted.

The frequency of each of the classrooms situations where the “knowing-to-act” should be or is enacted by the teacher are shown in Table 4.27. The situation most recurrent was the first KtA situation which is when a student solves a particular mathematical problem and asks if the answer or steps are correct. This situation was identified in Maria’s classes observed on 40 occasions. Moreover, the situation of when a student makes a mistake was detected on 16 occasions.

Table 4.27: Frequency Table of the KtA Situations Identified in Maria’s Classes.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Misconceptions</td>
<td>When a student solves a particular mathematical problem and asks if the answer or steps are correct</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>When a student makes a mistake</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>When a student does not recognize the same pattern in a different situation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>When a student continuously responds with wrong answers to questions posed by you</td>
<td>1</td>
</tr>
</tbody>
</table>
When students are having hard time to complete an assigned activity/task 4

When you assign an activity about a previously learned topic, and some students express that they do not know how to do it 5

When a student is explaining his/her reasoning 2

When a student provides an answer to a given problem 3

### 4.4 Research Question 3

The third research question of this study is built on the previous phase. Qualitative data is collected in order to answer this specific research question:

3. How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?

This research question was answered with information collected through interviews with the same four participants whose classes were observed. Four semi structured interviews were conducted. One interview was conducted per participant. Each interview lasted approximately 60 minutes. The purpose of these interviews was to allow teachers to describe their “knowing-to-act”. The description of teachers’ “knowing-to act” is done through the actions enacted by teachers in the situations already mentioned in the analysis of the classroom observations. In addition, several differences were detected between what was observed in the classroom and what was answered in the Knowing-to Act Survey (KtAS). These differences and their reasons are described and analyzed in this section of the study. This description and analysis offer a more comprehensive insight of the “knowing-to act” processes that characterize the classroom instruction of mathematics middle school teachers in Mexico.

**Point of Interface for Mixing**

In this part of the study, a point of interface for mixing was implemented. The analysis of the teachers’ responses on the KtAS survey administered during the quantitative phase and the
findings of the observations conducted in the qualitative phase, was performed to connect both types of data: quantitative and qualitative. This mixing allowed the design of the interview questions for the qualitative phase.

### 4.4.1 Interviews

In the analysis of the classroom observations of the participants of this qualitative phase, there were four types of KtA situations that occurred in the four participants’ classes observed. These KtA situations are: (a) when a student solves a particular mathematical problem and asks if the answer or steps are correct; (b) when a student is unable to see an obvious pattern in the problem; (c) when a student makes a mistake; and (d) when students are having a hard time completing an assigned activity/task.

The interview questions were focused on these four types of KtA situations. The interview consisted of showing to the teacher a short video clip of her or his classes that showed the KtA situation and the action enacted by him or her. This is the format of the interview: watch the video clip, and ask the following questions:

1) Describe what you saw in the video clip

2) What was your thinking to act this way in the situation captured in the video clip?

3) Your response in the KtAS was the following (show the participant her/his response in the survey). It is different from the action performed in the video clip. Explain why.

These three questions were asked after each video recording showed as illustrated in Figure 4.4. The interview process shown in Figure 4.4 was conducted for each participant. The number of video recordings showed depended on the participant. In some participant’s classes, certain KtA situations were observed only once. However, other participants’ classes exhibited the same situation several times. Moreover, the question about the difference between the teacher’s action observed and teacher’s answer in the KtAS was posed to the teacher when this difference occurred. There were situations where the teacher’s action observed and the answer in the KtAS were alike, thus, there was no need to ask this question. In addition, there was the case
that most of the time the teacher acted in one way during a certain situation observed, and the teacher acted differently in the same situation at least one time. Also, there was the case that the half of the times that a situation occurred, the teacher acted in one way and the other half in other way.
Video Clip

Describe what you saw in the video clip

What was your thinking to act this way in the situation captured in the video clip?

Your response in the KtAS was the following (show the participant her response in the survey). it is different from the action performed in the video clip. Explain why?

Is there another video clip?

Yes

No

End

Figure 4.4: Individual Interview
During Rosa’s classes observed, there were 52 KtA situations identified. However, only 16% of the KtA situations reflect Rosa’s responses of the KtAS. It means that the 84% of the KtA situations observed disagree with Rosa’s responses in the survey. In the interview, there were 7 situations with disagreements between her action observed and survey response showed to Rosa. These situations were classified in the four KtA situations previously mentioned. The first KtA situation analyzed is when a student solves a particular mathematical problem and asks if the answer or steps are correct. There were two video recordings presented to Rosa for analysis in this type of KtA situation. (The complete transcription of the interview is presented in Appendix H). In one of the video recordings showed, one student called Rosa to check her solution. The teacher took the notebook and checked the problem.

R: yes, it is perfect, do not share your solution with your teammates but help them to solve the problem

Rosa told the student the correctness of her work “yes, it is perfect”. After Rosa watched the video, she was asked to describe the situation:

R: it is what I told you, I am checking the work for the most advanced student of that group in order for that student to be able to help the others without giving the answer.

This description is what I observed, thus, I classified this classroom situation as the first type of the KtA situations. Rosa told the student that her work was correct. Then I proceeded with the second question.

T: What was your thinking to act this way in the situation captured in the video clip?
R: when a student finishes and her classmates do not, then it’s there when I use my monitors who help their classmates but without giving the answer, they try to give guides or tips (clues) to them, the rest of the students will be looking by themselves for the solution, but monitors should not give it.

I could observe that there was a difference in the action performed during the classes observed with what Rosa answered in the KtAS survey and I asked her about it. Table 4.28
shows the differences in her actions, the reasons provided, the themes emerged, and the codes for these reasons. Table 4.28 is the justification of this difference. Rosa said that the items of the KtAS are not related to the situation. However, she mentioned that she answered the survey according to her teaching nature as she really is. Rosa explained that she did what came up at that moment. Analyzing her response, the emerged theme from the explanation of why the difference existed between the survey’s response and the action observed is *switching the situation*. Rosa did not focus on the situation that I was asking. She focused on the situation that the student could share her results, thus, Rosa anticipated saying “…do not share your solution with your teammates…” The code assigned to the reason of acting this way is “anticipation”. The theme *survey* encompasses all the codes assigned to the reasons given about something related to the KtAS survey as Rosa mentioned in Table 4.28. The code is “survey not applicable” because Rosa mentioned that items on the KtAS survey did not correspond to the situation watched in the video clip.

*When a student is unable to see an obvious pattern in the problem* is the second KtA situation analyzed. One video recording was presented to Rosa in this type of situation. The following excerpt of the transcription is what the video included.

R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions. How would it be represented in a simple fraction, Brenda?

S10: as a quarter.

R: no no no, my love, remember that we remove the point, it would be also 225/100 it would be the same, this was seen previously. You do not remember it?

In this situation, I observed that the student was unable to understand how to represent decimals in fractions. The pattern to be followed to solve this problem was already taught. The student (S10) could not identify how to do it when according to Rosa it would be something obvious in the problem. Rosa was asked to describe what she did in the situation observed.
R: for example here, the students do not know it and those are topics already taught, topics that I explained it and thought that they learned, and they do not learn it…

T: then you go back to the topic.

R: well to re-teach it, I do the procedure to allow them to solve it…

T: … What were you doing on the recording?

R: well I guess that I am explaining to the students but they do not get it.

Rosa described the same KtA situation that was happening in her class as I already briefly described above. Rosa provided an explanation with the solution to the problem. Then, Rosa explained her justification:

R: I was nervous.

T: what was the purpose to act in this way?

R: well, those topics were already taught, it is supposed that the students learned them. Then I started to do a review, a feedback because the topic was already covered. For example fractions and decimals, I guessed that this topic was already learned, then when students are working on the worksheets, they took the textbook and started to look at the topic again, I told them to read it, therefore the textbook is a support for learning, but it is supposed that we are doing a review because we saw it and then nothing that…

T: that’s why you told them how to do it?

R: yes.

T: if you would not do this, what?

R: I could not advance.

There was a difference between Rosa’s action observed and what she answered on the KtAS. In Table 4.28, the reasons provided for this difference are presented. Rosa justified this difference saying that she had to act in that way because “students are lazy” she had to take care of them all time. Also, she mentioned that it is faster to provide the solution to finish the class and go home earlier. The emerged themes from this justification are: student and time. The theme student includes the aspects related to the students such as attitude, mistake, response,
question, etc. In this case, the code assigned to one of the reasons given for this difference was “student’s attitude”. The theme time factor is about the aspects related to the time. The code assigned to Rosa’s reason is “time factor” as Table 4.2 presents. Rosa saved time in her classes to leave earlier and go home.

The third KtA situation analyzed is when a student makes a mistake. Two video recordings were shown to Rosa. However, only one is presented in this section (the whole transcription of the interview is presented in Appendix H). In the video recording shown, Rosa checked one student’s solution. She took his notebook and said to the student “you are wrong here in this multiplication process”. In this situation, Rosa provided the incorrectness of the student’s work saying “you are wrong here in this multiplication process”. After watching the video, I asked Rosa to describe the situation and her action:

R: when I ask students to do a problem, even when those problems are items on any test… Then students do operations to get the solution, and I think that in the operations was her mistake. That’s why I told her to check the operations.

Rosa indicated to the student where the mistake was after saying that “you are wrong”. The justification provided for her action is:

R: I am wondering why I do this if my way of thinking is this other way. Because to me students should solve the problem as they can and believe that it is needed for the solution, the problem, there is no way to check something in their notes, just do it! …

T: …why did you do that, teacher? What was the purpose for acting in that way?

R: the student can see where her mistake was, if he or she has the operation wrong, he or she needs to do it again. I think it was because she was adding something, she did it wrong in the operation, I said check the operation to find your mistake.

Analyzing the KtAS, Rosa’s answer and her action in this situation differs. Rosa said that she acted in a different way because the mistake was in a simple arithmetic operation as Table 4.28 shows. She considered that this situation did not require a different action from her because of the content. The reason for this difference allowed for this new theme emerged: content. The
theme *content* refers to any reason provided by teachers about any aspect of the content or topic that they were covering in that situation such as arithmetic operation, complexity, etc. The code assigned to Rosa’s reason for this difference was “arithmetic operation” as Table 4.28 presents.

Differences between the teacher’s action and her survey response as described previously occurred 42 times out 52 times identified. It means that most of the time what Rosa responded on the KtAS survey did not correspond to her action enacted during her classes observed. Only on 8 occasions, Rosa’s actions agreed with her response on the KtAS survey.

Table 4.28: Reason Codes for the Differences in Rosa’s Responses.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Difference in responses</th>
<th>Reason</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey, switching the situation</td>
<td>KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct. Survey's answer: ask the student to read the corresponding section of the textbook. In the recording, “you told her yes it is perfect, do not share your solution to your teammates but help them to solve the problem”.</td>
<td>R: you can see that those questions (survey’s questions) do not correspond with this situation. R: well I am like this, the thing is what is happening is what is coming up, for example, in that moment, they had to read, we will be back to the same point.</td>
<td>Survey is not applicable; anticipation</td>
</tr>
<tr>
<td>Student, time factor</td>
<td>KtAS item: when a student is not able to see or identify something obvious. Survey's answer: ask the student to continue trying until he/she get it. In the recording, “we saw that you re-taught it providing the solution.”</td>
<td>R: well, because they are lazy, I prefer to explain it in order to help them to get an understanding about it. R: maybe in this situation I considered it better to explain it again. I mean there are some students who like to read and one can ask them to look for something but there are others do not do it. Then how am I going to do it? Explaining and explaining it again. .... For example this survey (KtAS), I answered it according to my thinking as I am. Well, I can see things that maybe I do wrong because one does things to finish as soon as possible and go home, but your survey I answered it as I really am and act.</td>
<td>Student's attitude; time factor</td>
</tr>
<tr>
<td>Content</td>
<td>KtAS item: when a student makes a mistake.</td>
<td>Survey's answer: let the student explains his/her answer.</td>
<td>In the recording, “you told the student check the operation”.</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>Denial</td>
<td>KtAS item: when students are having a hard time completing an assigned activity/task,</td>
<td>Survey’s answer: analyze students’ difficulties and provide hints.</td>
<td>In the recording, “you explained them again and again.”</td>
</tr>
<tr>
<td>Denial, curriculum</td>
<td>KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct.</td>
<td>Survey’s answer: ask the student to read the corresponding section of the textbook.</td>
<td>In the recording, “you checked it and told her that it was wrong and where her mistake was.”</td>
</tr>
<tr>
<td>Student, curriculum</td>
<td>KtAS item: when a student makes a mistake.</td>
<td>Survey's answer: let the student explain his/her answer.</td>
<td>In the recording, “we saw that you explained it and repeat the directions.”</td>
</tr>
</tbody>
</table>
and he knows how to correct it or I tell him how to do it, all this to me is learning.

The four type of KtA situations included in the interview is when students are having hard time to complete an assigned activity/task. Two video recordings were shown to Rosa during the interview. The complete transcription of the interview is presented in Appendix H. One of the video recordings shown to Rosa is presented below in the excerpt from the transcription.

S22: I do not understand this
R: what?
S22: what do I need to do here?
R: ah you need to get what percentage is the first quantity out of the second quantity. It means 20 out of 100 what percentage is, yes? (Then other student asked the same question, so she realized that most of the students did not understand that part of the activity and she explained the directions again)

Rosa assigned a problem about percentages. One student told her “I do not understand this”. Rosa realized that most of the students were having a hard accomplishing the task. She provided an explanation and repeated the directions several times. I asked Rosa to provide a description of what was happening in the video:

R: no, no. The students did not read correctly. In this other I missed something in the problem because they had not understood the problem.
T: what was happening in this fragment?
R: I did not write the problem correctly because students did not understand it. Then I realized by myself when I was reading that something was missing.

Rosa argued that she made a mistake in the writing of the problem. She said that students did not understand it because of that mistake. However, during the observation of this particular class and during the analysis of the videotape, I did not identify any mistake in the part that she mentioned. Rosa’s action observed in this situation was providing an explanation. This action
differed to what she answered in the survey. Her answer was to "analyze students’ difficulties and provide hints." There is an obvious difference. Rosa claimed that she made a mistake from where this entire situation was derived. Thus, the theme emerged from the analysis of this situation is "denial." The theme "denial" encompasses the reasons for denial of what really happened in the classes observed. The code assigned to the reason provided by Rosa is “denial” because it was the main reason that she gave to justify what the video showed.

Table 4.29 illustrates the themes emerged and assigned codes for the reasons provided for the differences found. The theme "curriculum" refers to the aspects where the sequence of the topics, program of study, reform (new educational reform approved in 2013), class activity, among others are involved. The code assigned to Rosa’s reason for a difference between her answer in the survey and her action observed was “class activity” as Table 4.28 presents. The code “deny teacher action” was assigned to Rosa’s reason for the difference between how she acted in the classes observed and the answer on the KtAS. Rosa denied that she revealed the mistake to the student. Rosa said that she only asked the student to continue working, however, she told the student where her mistake was as is observed in the video recording and transcription. This code is part of the theme of "denial." The excerpt of the transcription of the interview is in Appendix H. Table 4.29 exemplifies these themes and codes briefly.

Table 4.29: Themes and Codes Assigned to Rosa’s Reasons.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Reason Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching the situation</td>
<td>Anticipation</td>
</tr>
<tr>
<td>curriculum</td>
<td>Class activity</td>
</tr>
<tr>
<td>Survey</td>
<td>Survey is not applicable</td>
</tr>
<tr>
<td>Time</td>
<td>Save time</td>
</tr>
<tr>
<td>Student</td>
<td>Student's mistake; Student's attitude</td>
</tr>
<tr>
<td>Content</td>
<td>Arithmetic operation</td>
</tr>
<tr>
<td>Denial</td>
<td>Denial, Deny teacher action</td>
</tr>
</tbody>
</table>
Interviewing Omar

During Omar’s classes observed, there were 35 KtA situations identified. However, only 31% of the KtA situations reflected Omar’s responses on the KtAS. It means that the 69% of the KtA situations disagreed with Omar’s responses on the survey. In the interview, there were 7 situations with disagreements between his action observed and survey response showed to Omar. These situations were classified in the four KtA situations previously mentioned.

There were three video recordings presented to Omar. These video recording show classroom situations which are of the type of this KtA situation: when a student solves a particular mathematical problem and asks if the answer or steps are correct. One of the video clips is described below through the transcription of what was happening in the video (the entire interview transcription is in Appendix H).

One student stood up and went to ask for a review of her work to Omar. Then Omar said:

O: it is ok but here it is 4.

S12: ah ok then it is 4.

Omar in this situation is revealing the student’s mistake when the student asked for a review of her work. Omar’s description about this classroom situation is:

O: students were revising the activity that I asked for. I was walking around the groups to see if students are working and to see if they needed some help.

T: and there your action was: yes it is right but here is 4, right? then…

O: showing her where the mistake was.

Omar identified the same situation as I did. Omar recognized that he revealed the student’s mistake. Then I asked him why:

T: … why did you do that, why do you think that telling the student where the mistake was? What was your thinking to…?

O: in that moment, it just happened; I clarified it to the student because it was right but her writing is deficient.
Analyzing Omar’s action and what he answered in the KtAS, a difference is noticed. Table 4.30 presents this difference. I asked Omar about it. He responded talking about why this difference happened. He mentioned that when he was answering the KtAS, he did not consider the same level of complexity of the topic that he was teaching during the classes observed. Omar’s reasons are part of the theme of content. The code assigned to one of the reasons was “level of complexity”. Omar insinuated that he acted differently because of the level of complexity of the problem been solved. Another theme that emerged was teacher’s exception/excuse to the KtA situation which means that Omar did not feel the need to ask the student an explanation. He provided this reason as an excuse of doing the exception in this particular situation. The code assigned to this reason was “exception”.

The KtA situation analyzed in this part of the interview with Omar was when a student makes a mistake. There were two video recordings showed to Omar for the interview about this situation. One of the videos deals with the situation of the use of a new word by Omar. Before he continued explaining the topic using that word, he asked students about the meaning of that word. This word is “transponer” transposition or transfer. Students answered Omar’s question saying that it is dividing. Omar said “no, it is like transporting, moving, to move something to another place, ok”. And then, he asked if they have questions about this term. Students did not ask anything about it.

What Omar described about this classroom situation is:

The description provided by Omar coincides with what I described above. Then, I asked Omar what was his thinking to act in that way, and he replied to me:

O: it was something important that they should know, it is a basis for equations, in matter of terms.
T: and if you did not provide an explanation they wouldn’t be able to understand it or solve it without the explanation of that term.
O: the explanation.
T: or they already knew it.
O: no

There was a difference between what Omar did during the classroom observation and his answer in the KtAS. Omar mentioned that he answered the survey thinking of a mathematical problem instead of a word definition. The theme recognized in this reason is survey. The code assigned to Omar’s reason is “survey’s interpretation”. In addition, the incorrect responses of the students when he asked about the meaning of the word “transfer” are not considered mistakes by Omar. The incorrect students’ responses are just a lack of knowledge of this word. The theme emerged in this reason is content. In this situation Omar did not consider the mistake made by students as a mistake because the content involved was not a mathematical problem. The code classified in this theme is “perception of mistake”. Omar provided his perception about what a mistake is for him depending on the content. Another theme emerged from Omar’s reason for the difference in his responses is teaching practice. This theme refers to the reasons for acting in a certain way derived from the teaching practice such as doing the easiest actions, the most logical, anticipating, and making students recall the topic by doing certain action. According to Omar, in this situation, he did the easiest action. The code assigned to this action was such “easiest action” is under the theme teaching practice.

The KtA situation of when students are having a hard time completing an assigned activity or task is exhibited in a video recording showed to Omar. In this video, students were having difficulties to solve the assigned equations. One student asked Omar how to do one part for solving an equation. Omar responded to him with an explanation of the procedure.

I showed the video to Omar. After watching the video recording, I asked him to describe what he saw in the video recording.

O: students called me again to check their work.

T: that student asked you what needs to be moved to this side of the equation. Then, you were explaining what can be moved to the other side and how to do it.

O: yes the unknown letters (variables) are moved to one side.
Omar identified the same situation as I did in that segment of the video recording of one of his classes observed. Then I asked Omar what about his thinking when acting in this way. Omar responded:

O: maybe the student did not understand what I already explained, the explanation was written on the whiteboard. At that moment the student did not pay attention when I explained that. And if he did not know: what terms needed to be transferred, which one to this side, and which one to the other side, he was not going to get the result. If he had not understood the first step to develop this type of problem, he would not be able to solve it.

I identified a difference between Omar’s action observed, which was providing an explanation, and Omar’s response in the KtAS as Table 4.30 exemplifies. Omar mentioned that he preferred to explain the topic again because it was just an equation. The theme involved here is content and the code assigned to this reason is “level of complexity”. At the time of the surveys administration, Omar was late to fill out the surveys. Thus, he did not have enough time to complete the TCKS. However, he responded completely to the KtAS. Omar said that he responded to this last survey with time pressure because he was short of time to finish it. The theme of survey is involved in Omar’s reasons for this difference. The code assigned to Omar’s reason of time was “time factor”.

Differences between Omar’s actions and his survey responses as the previous ones described occurred 24 times out 35 times observed. It means that what Omar responded in the KtAS survey mostly does not correspond to his actions enacted during his classes. Omar’s actions agreed with his responses on the KtAS survey only on 11 occasions.
Table 4.30: Reason Codes for the differences in Omar’s responses.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Difference in responses</th>
<th>Reason</th>
<th>Codes</th>
</tr>
</thead>
</table>
| Content; teacher’s exception/excuse to the KtA situation | KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct.  
Survey’s answer: ask the student to explain his/her solution.  
In the recording, “we saw that you said it’s right but here it is 4”.       | O: with what I selected in the previous survey, maybe the level of the problem was not the same that I imagined when I answered the survey.  
T: in that moment, you were looking at the linear equations.  
O: yes, how to solve for x and get the result. This student already had it solved, I do not know, I did not feel the need to ask her an explanation if she had all the procedure to solve it. | level of complexity, Exception |
| Student                                    | KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct.  
Survey's answer: ask the student to explain his/her solution.  
In the recording, “you told him yes it is right but do the procedure.” | O: the procedure, how it was the student get an answer.  
T: you were asking him.  
O: yes he needs to do the procedure and then get to the correct answer. The steps that he made to get a result. I expect that he really learned, if he is able to say I did it like this and this, and shows the procedure. | learning expectation          |
| Survey; content; teaching practice         | KtAS item: when a student makes a mistake.  
Survey's answer: let the student explains his/her answer.  
In the recording, “you told them what the correct answer was.”                                                 | O:I think that the question of the survey was about a specific activity of mathematics such as a problem and then we can see where the mistake is. But here the students did not really have a mistake. What they had was a lack of awareness of this word o term.  
O: A problem that has been practiced or it was already done, and let’s revise it, let see why there is a mistake, because in the problem we can do that. In the survey that you applied me I considered this question as it refers to this type of problem. A problem that can be evaluated, in this type of problem I asked the students to explain why, how they got to that result, and then I explained it if there was something wrong. However, if students only take for review just the answer, I cannot check all what I already mentioned. I need for students write the procedure, even in certain topics I ask students to say what the easiest and hardest part of the activity was.  
This situation to me it is not a mistake it is a lack of awareness of that word. To explain that word was the easiest action to me: how to move a term from this side to the other so there is no mistake? | survey's interpretation; perception of mistake; easiest action |}

Content  
KtAS item: when a student makes a mistake.  
Survey's answer: let the student explains his/her answer.  
In the recording, “you said check the addition, this plus this could not be this.”  
O: my justification is that: the student was starting to do the procedure of the problem, so the result should not be gotten if he was doing it wrong.  
O: it was prudent for me to clarify it, that’s it. The student’s mistake was in a simple addition.  
T: and then, for example, in the survey you answered: let the student explain his/her answer, I mean you could do this.  
O: at the end.  
Arithmetic operation,
| Content; survey, KtAS item: when students are having hard time to complete an assigned activity/task, Survey's answer: analyze students' difficulties and provide hints. In the recording, “you preferred to explain them again.” | O: I tell you, I usually I provide them a review of easy activities that they can do, I think that if… O: in certain part of the procedure maybe will be good to use hints or tips. T: for example there? O: it was just an equation and then to do a movement of the variables or coefficients to the other side. T: is it not like a hint? O: it is not a hint, what is being moved to the other side, what it is being transferring. I am usually nervous when somebody is supervising me. I do not know why. T: yes I noticed that you were nervous. O: I am not considering wrong that somebody comes to observe my class if know that I do well my work. It is fine to me. It helps me to improve my teaching and change the things that I am doing wrong such as this about providing hints T: I am not telling you that you are wrong. O: let me tell you something, remember when you came to apply the survey, I was late and you have to leave at certain time… T: yes, I do remember, even that’s why you did not finish the TCKS. O: I did not complete it. T: yes I remember it. O: I answered the survey with a little of pressure because of the time. It usually happens. | Level of complexity, time factor |

Table 4.30 describes briefly other classroom situations analyzed for Omar. The code “learning expectation” is assigned to Omar’s reason to act asking for the procedure of the problem that the student was working on. Because Omar considered that a student who really learns is able to explain how he or she did the problem, the code of “learning expectation” is a part of the theme of student. This code refers to the Omar’s thinking about a student who learned what was expected and was able to do the procedure and get a correct answer. The code presented in Table 4.30 as “arithmetic operation” refers to the same reason provided by Rosa. Omar decided to act in that different way from what he answered on the survey because the student was dealing with a simple arithmetic operation such as an addition. As previously mentioned in Rosa’s interview analysis, the code of “arithmetic operation” is a part of the theme of content. Table 4.31 illustrates the codes for Omar’s reasons for acting in a different way from what he wrote on the KtAS provided in the interview.
Table 4.31: Themes and Codes for Omar’s Reasons.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Learning expectation</td>
<td></td>
</tr>
<tr>
<td>Teacher’s exception/excuse to the KtA situation</td>
<td></td>
</tr>
<tr>
<td>Teaching practice</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Arithmetic operation;</td>
</tr>
<tr>
<td></td>
<td>Level of complexity;</td>
</tr>
<tr>
<td></td>
<td>Perception of mistake</td>
</tr>
<tr>
<td>Survey</td>
<td>Time factor; survey's interpretation</td>
</tr>
</tbody>
</table>

**Interviewing Rogelio**

During Rogelio’s classes observed, there were 31 KtA situations identified. However, only 24% of the KtA situations reflected Rogelio’s responses of the KtAS. It means that the 76% of the KtA situations observed disagreed with Rogelio’s responses on the survey. In the interview, there were 10 situations with disagreements between his action and survey response showed to Rogelio. These situations were classified in the four KtA situations previously mentioned.

There were two video recordings that exhibited the KtA situation: *when a student solves a particular mathematical problem and asks if the answer or steps are correct.* Both video recordings were presented to Rogelio. One of the video clips is described below through the transcription of what was happening in the video (the entire interview transcription is in Appendix H).

S16: teacher is this right?
R: 2*1; follow the formula
S16: then here is 16, and 7 and then this …
R: aha but here is missing something, 4ac… (Rogelio told her where her mistake was).
S16: it means this should be this
R: aha ok.
S16: ah ok I got it.
R: do you know how? Did you remember it?
S16: yes, I know how to do it.

In this video recording, Rogelio was asked if the student’s work was right. Rogelio told the student that she had to follow the formula, and he revealed the student’s mistake. I asked Rogelio to describe what he identified in the video recording.

R: I approached one group and I helped them to find the answer, after that, they asked me if that was everything for today in order to put their stuff back in their bags and leave the class.

This was Rogelio’s description about what he saw in the video. I described the situation transcribed above showing him the video slowly. Then he identified the same situation as I did. After that, I asked him:

T: the way you acted there, ¿why did you act in that way? What was the purpose to act in that way? Why did you do that?
R: ah well, in order to tell her that she had not finished yet, there were some steps that she missed, the way that she used to find the answer was wrong, she needed to know that there are other ways, more mathematics I mean she was guessing.

Rogelio acted in this situation revealing the student’s mistake. He mentioned that the student was mainly guessing in her work that is why Rogelio preferred to tell her what was wrong. There was a difference identified between how Rogelio acted during the class observed and his response in the KtAS as Table 4.32 presents. Rogelio said that this difference happened because of the topic that he was teaching. He mentioned that it is algebra, and it is complex for the students. Rogelio was teaching linear and quadratic equations. The theme involved in the reason provided by Rogelio is content. The code assigned to Rogelio’s reason is “level of
complexity”. Rogelio considered that the topic was very complex, thus, he had to provide his explanations including telling the mistakes that were made by the students.

The KtA situation of when a student is unable to see an obvious pattern in the problem was exhibited in one video recording. This video recording was shown to Rogelio. In the video, Rogelio was walking around the groups supervising students’ work.

S10: teacher it is divided by what?
R: it is divided by the number of grades, how many grades are there?
S10: 3
R: then by 3.

Rogelio provided the solution to the student who asked him a question. Rogelio’s description of this situation was:

R: he asked me by what it should be divided, by the total number of grades, I confirmed how many grades there were, he told me three, I reaffirmed his answer.

Rogelio described this situation and his action performed as telling the right answer to his question. The justification provided for the action enacted by Rogelio was:

R: because I wanted to confirm it in order for the student to be sure, and he was able to do it, he had the idea but he was unsure, thus, I just confirmed it.

Rogelio wanted to provide support to the student through the communication of the right answer. In the survey KtAS, Rogelio responded that he would modify the context of the problem in this situation. However, he provided the solution to the student. The reason given by Rogelio about this difference was because it was a simple question where only the student needed to know by what number it should be divided. Rogelio opted by telling that number. Rogelio said that “I did not find another way to react”. The theme emerged from Rogelio reason was teaching practice as previously described in Interviewing Omar. He mentioned that he acted that way because it was the most logical action. The code assigned to Rogelio’s reason was “most logical action” as Table 4.32 presents.
Another video recording was shown to Rogelio. In this video recording, the KtA situation uncovered was *when a student makes a mistake*. Rogelio was explaining and solving a problem on the whiteboard in front of the class. He asked the students a question.

R: ok because I am solving for this variable, I am going to move this number subtracting. We are going to have x=24-14.2. Do not make the common mistake of saying oh it is 10, because it is not, you need to pay attention to the .2 that 14 has. So the result is?

S7:10.2

R: no no it could not be 10.2 this is the common mistake.

S8: 9.8

R: 9.8. Then what is the grade that this person needs to get an average of 8?

Rogelio advised students about this common mistake, however, the student (S7) made that mistake. Rogelio only said “no no it could not be 10.2 this is the common mistake” and waited for somebody to answer with the right question. I asked Rogelio to describe his action and the situation and he said:

R: I was advising students before they made a mistake, but I had not told them, I should have allowed them to make a mistake and then corrected them.

Rogelio described the situation as I identified during the analysis of the observations. In addition, he regretted his action. The justification that he provided to act in this way in his class observed was:

R: to avoid that they made a mistake, (he laughs) however, they were mistaken

T: the student who made a mistake, what did you do with her because she provided a wrong answer?

R: nothing I told her that she was wrong. I did not punish or scold her, I should give a little blow (he laughs and jokes)

Rogelio recognized that he did not address the student’s mistake. Analyzing the KtAS, I discovered that there is a difference between how Rogelio acted in his class and his answer on the KtAS. Rogelio mentioned that he ignored the student because he already provided the
explanation of that common mistake before the student made it. Rogelio said that he anticipated mentioning that common mistake because in their previous classes of other school years, students made the same mistake. The theme that emerged as already identified is teaching practice. Rogelio acted anticipating to the students’ mistake because his teaching practice indicated him that students usually make a mistake in that part. The code assigned to Rogelio’s reason is “anticipation” as Table 4.32 illustrates.

Differences between Rogelio’s actions and his survey responses as the previous ones described occurred 22 times out 31 times that the KtA situation happened. It means that most of the time, what Rogelio responded in the KtAS survey did not correspond to his action enacted during his classes. Only on 7 occasions, Rogelio’s actions agreed with his responses in the KtAS survey.

Table 4.32: Reason Codes for the Differences in Rogelio’s Responses.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Difference in responses</th>
<th>Reason</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content</td>
<td>KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct. Survey’s answer: ask the student to explain his/her solution. In the recording we saw that you provided an explanation to the student.</td>
<td>R: ah because of kind of the topic, because it is algebra, it is more complex for the students, thus it is difficult for them to explain with their own words or using their previous knowledge in this type of situation. Because students were starting to see more about algebra, it is more complex, I have to explain a little bit more to the nine graders, and I have to orient them more than the 8th graders. That’s why the difference in the survey’s response, and how it was an application problem, it was even a little bit more complex.</td>
<td>Level of complexity</td>
</tr>
<tr>
<td>Content, student</td>
<td>KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct. Survey's answer: ask the student to explain his/her solution. In this situation “we saw that you preferred to explain instead of allowing the student to explain her answer.”</td>
<td>R: Because as I told you, it is algebra, and you know what quadratic equations are? Now the students are distracted, they have an instantaneous short memory, which everything learned in one class, they forget it the following day. That’s why I try to orient them and explain it again. She/he already got an answer, then, I suggested another way to do it, and it is better and safer to find the answer but it takes more time.</td>
<td>Level of complexity, student's capability</td>
</tr>
</tbody>
</table>
When students are having a hard time completing an assigned activity/task is the other type of KtAS which was explored in the interview. I presented a video recording to Rogelio where this KtA situation is exhibited. The classroom situation that was happening in the video was that Rogelio was walking around the groups. He was supervising what students were doing. Also, Rogelio was helping them. Then Rogelio went toward the whiteboard and told the students “you need to try”. He mentioned that because he saw that students were having a hard time doing the activity. After that, Rogelio started to explain to them and said “I will help you a little with one step more”; that step was written on the whiteboard. Rogelio told the students that until that point they should be able to continue working. The description provided by Rogelio when he watched the video was:
R: we were solving for x to find the quadratic equation in order to solve it, I told them how to solve for a variable in the quadratic equation.

Rogelio recognized that he provided part of the solution to the students in order to students to continue working on the problem. When I asked, “why did you act in that way?” Rogelio said:

R: because the time was over and they did not find the way to solve for x, and solve the equation.

T: do you think that if you had not provided that little step, they could not do it? R: no, they could do it, but they would need another class (45 min). Sometimes, the time is over with all the activities, but this activity we needed to finish it.

Analyzing the KtAS, a difference was detected. This difference is between Rogelio’s action in the video recording and Rogelio’s answer in the KtAS. When Rogelio was questioned about it, he said that because of the time. He mentioned that he is asked to respect the time for the activities in order to be aligned with the program, exams among others. Therefore, He helped the students by providing parts of the solution. The theme involved in Rogelio reason is time factor. The code assigned to his reason is “time factor” as Table 4.32 presents.

Table 4.33 presents a summary of the themes involved in Rogelio’s reasons for the differences detected during his classes observed and his answers in the KtAS. In addition, the codes that belong to each of the themes are also illustrated in Table 4.33. In the student theme, the code “student’s answer” refers to the answer provided by the student that caused the teacher to act in certain way as Table 4.32 presents.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Reason’s Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content</td>
<td>Level of complexity</td>
</tr>
<tr>
<td>Student</td>
<td>Student's capability; Student's answer</td>
</tr>
<tr>
<td>Time</td>
<td>Time factor</td>
</tr>
<tr>
<td>factor</td>
<td></td>
</tr>
<tr>
<td>Teaching</td>
<td>Anticipation; Most</td>
</tr>
</tbody>
</table>
Interviewing Maria

During Maria’s classes observed, there were 70 KtA situations identified. However, 39% of the KtA situations reflected Maria’s responses on the KtAS. It means that the 61% of the KtA situations observed disagreed with Maria’s responses on the survey. During the interview, there were 11 situations that presented the differences between her actions observed and survey responses showed to Maria. These situations were classified in the four KtA situations previously mentioned.

Maria was interviewed on one occasion. The interview lasted about 60 minutes. There were 11 video recordings of her classes observed presented to Maria that showed the four KtA situations that are common among the four participants. One video that covers the first KtA situation when a student solves a particular mathematical problem and asks if the answer or steps are correct, was discussed during the interview. The following excerpt is the transcription of this video recording. One student asked Maria if his work was right. Maria explained to the student.

M: you need to remember that coefficients are on one side and these ones on the other side.
S7: no no these numbers no.
M: what? What variables do you have?
S7: mmm
M: what are your variables?
S7: this one.
M: but it is only this x?
S7: mmm
M: this one will be left here and then…
Maria’s action is to explain to the student about his work and what he needed to do. I asked Maria to describe her action and the situation that was happening in the video recording. Maria said:

M: well it is work in groups, and then, for example the student who wore blue color is in, well the groups are integrated by one leader and usually the leader is the one who has more mathematics abilities to guide and direct the rest of the group; the student who is beside the leader is needing support from the leader, and they are working as the new reform says (see chapter 2 for details), it means they must be able to find a solution, whatever way, here for example we already saw the topic then they should be able to develop the activity by themselves. However, it is not always like that.

M: aha, and I am still explaining to him. An explanation is still provided in those areas where students have doubts. This activity was still done for the purpose to look for doubts or questions that students could have. This activity is previous to the evaluation of this content. Aha.

Maria recognized the KtA situation that I detected in the video recording. She described her action as explaining to the student. I asked her why she decided to act in that way.

M: analyzing what he answered maybe there was a sequence of mistakes or there were little mistakes that he did in order to be able to solve it correctly.

Maria explained to the student because the students only had little mistakes. Analyzing this action and the responses of Maria in the KtAS, I identified a difference. Maria acted explaining to the student the problem, and she responded in the survey that she would ask the student to explain his/her solution. I asked Maria about this difference. The reasons provided are presented in Table 4.34. She mentioned that this difference happened because she followed the program. Maria assigns one class to do the common presentation (puesta en común) where the student is asked for justification of his or her work. Thus, the moment of that activity that was happening in the video recording was not the moment to ask for the student's justification according to Maria. The theme emerged from Maria was curriculum. This theme involves all the
teacher arguments where they mention program, class activity, lesson plan, sequence of topics, among others. One code classified in this theme is “program”. Maria’s reason was coded as “program” in the part she mentioned because the program delineates the different processes to cover a topic. Also, the code “class activity” was assigned. This second code was assigned to the part of the reason where Maria talked about the activity that was happening at that moment which was designed to support students providing an explanation if they have questions.

Differences between Maria’s actions and her survey responses as the previous ones described occurred 42 times out 70 times that a KtA situation was identified. It means that mostly what Maria responded on the KtAS survey did not correspond to her actions enacted during her classes. On 27 occasions, Maria’s actions agreed with her responses on the KtAS survey.

Table 4.34: Reason Codes for the Differences in Maria’s Responses.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Difference in responses</th>
<th>Reason</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum</td>
<td>KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct.</td>
<td>M: you know when I selected that choice as the answer for the survey, it is like the program delineates it, it means, if it is the <em>common presentation</em> (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet? T: yes M: I told you these are the consignas (worksheet) that we work. Here is where we make the <em>common presentation</em>; here is when the student needs to go to the whiteboard and explain his solution, then we do the observations.</td>
<td>Program; class activity</td>
</tr>
<tr>
<td></td>
<td>Survey’s answer: ask the student to explain his/her solution.</td>
<td>M: you know when I selected that choice as the answer for the survey, it is like the program delineates it, it means, if it is the <em>common presentation</em> (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet? T: yes M: I told you these are the consignas (worksheet) that we work. Here is where we make the <em>common presentation</em>; here is when the student needs to go to the whiteboard and explain his solution, then we do the observations.</td>
<td>Program; class activity</td>
</tr>
<tr>
<td></td>
<td>In the recording, “we watched that you preferred to explain to the student.”</td>
<td>M: you know when I selected that choice as the answer for the survey, it is like the program delineates it, it means, if it is the <em>common presentation</em> (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet? T: yes M: I told you these are the consignas (worksheet) that we work. Here is where we make the <em>common presentation</em>; here is when the student needs to go to the whiteboard and explain his solution, then we do the observations.</td>
<td>Program; class activity</td>
</tr>
<tr>
<td></td>
<td>KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct.</td>
<td>M: you know when I selected that choice as the answer for the survey, it is like the program delineates it, it means, if it is the <em>common presentation</em> (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet? T: yes M: I told you these are the consignas (worksheet) that we work. Here is where we make the <em>common presentation</em>; here is when the student needs to go to the whiteboard and explain his solution, then we do the observations.</td>
<td>Program; class activity</td>
</tr>
<tr>
<td></td>
<td>Survey’s answer: ask the student to explain his/her solution.</td>
<td>M: you know when I selected that choice as the answer for the survey, it is like the program delineates it, it means, if it is the <em>common presentation</em> (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet? T: yes M: I told you these are the consignas (worksheet) that we work. Here is where we make the <em>common presentation</em>; here is when the student needs to go to the whiteboard and explain his solution, then we do the observations.</td>
<td>Program; class activity</td>
</tr>
<tr>
<td></td>
<td>In the recording, “we watched that you preferred to explain to the student.”</td>
<td>M: you know when I selected that choice as the answer for the survey, it is like the program delineates it, it means, if it is the <em>common presentation</em> (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet? T: yes M: I told you these are the consignas (worksheet) that we work. Here is where we make the <em>common presentation</em>; here is when the student needs to go to the whiteboard and explain his solution, then we do the observations.</td>
<td>Program; class activity</td>
</tr>
</tbody>
</table>

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<p>| KtAS item: when a student solves a particular mathematical problem and asks if the answer or steps are correct. | M: because I told you it is that I do not know then if there is another video recording where we are working the worksheets and the <em>common presentation</em> (puesta en común)? T: yes, there is a recording where the <em>common presentation</em> takes place. M: it’s there where the student has first to explain it, but in this case I will provide the same answer, yes. | Program; class activity |
| Survey's answer: ask the student to explain his/her solution. In the recording, “you did not explain anything to the student...; you told him that it was correct.” | M: because here the <em>common presentation</em> was happening; the explanation already took place, and also there was more work, then until this point the student should be able to solve the problem by himself, then that’s why I did that. | Program; class activity |
| KtAS item: when a student makes a mistake. Survey's answer: let the student explains his/her answer. In the recording, “you told the student do it by yourself without looking at the whiteboard.” | M: I told you because here it was the time for when she has to know it by herself, maybe she was a student who made a minimum effort to solve the things then, maybe that’s why my response in the class. | Student's attitude; program |
| Curriculum, student | M: no, maybe this response would apply in the other situations that we have been talking such as the work in the textbook; we must focus more on the work on worksheets than in the book. Thus, the work on the textbook I use it for this: when I provide an explanation and a set of activities, and then I see that students still have questions or doubts, is here when I asked students to work on the textbook. T: here for example, the student has the questions, doubts, why did you consider that it was better to provide him with an explanation? M: well, the student was missing little details again, and I considered critical to make my intervention since the beginning than allow him to develop the procedure of the problem that would generate a conflict in the classroom. | Class activity; student's mistakes; student's conflict |
| KtAS item: when students are having a hard time completing an assigned activity/task, Survey's answer: explain the assignment using examples or another way to explain it. In the recording, “you confirmed their answer or explained them where their mistake was.” | Student |</p>
<table>
<thead>
<tr>
<th>KtAS item: when students are having a hard time completing an assigned activity/task.</th>
<th>M: well, regularly I exceeded the times, I do not respect the time that I assign for an activity because they do not finish, maybe this is my mistake I think if they feel some pressure they will finish it, maybe is why I am behind I am always one topic behind than my colleagues. I have understood that if I assign 5, or 10, or 20, the student who want it to do it, study it, he or she will do it, but there are other students who do not make any effort.</th>
<th>Students' capabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey's answer: explain the assignment using examples or another way to explain it.</td>
<td>In the recording, “you provided more time to allow them to complete the task.”</td>
<td></td>
</tr>
<tr>
<td>In the recording, “you provided more time to allow them to complete the task.”</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Teaching Practice

| KtAS item: when a student makes a mistake. | M: no here I think that at this moment I told him what happened with this 10 I want to believe, because it is the most logical, that he told me I moved it…then I said aha you moved it to the other side thus its sign is changed. If he was saying to me I moved it, that is like to move it but you forgot the change of the sign to the opposite sign, aha. M: those words that I used are to identify: the student is telling me something, but I told him ok you did this then what it follows form there, maybe the words that I used such as but, ah, then, then I think that it is like the answer is not given but makes the student remember what we have been working on, aha. | Most logical action; recall |
| Survey's answer: let the student explains his/her answer. | In the recording, "you told him where the mistake was, and you started to explain it.” | |
| In the recording, "you told him where the mistake was, and you started to explain it.” | | |

### Denial

| KtAS item: when a student makes a mistake. | M: let the student explain his response. Because I told you here to me, I consider for example this I still consider coherent what I did in the class with this response, because at the moment that student told me “I moved this to that side” I said “but you did not change the signs” I understand that I allowed him to tell me what he did and maybe I told him or my participation is a complement such as but you did not do this, or you are missing this, but I feel that I allowed him to explain his answer first. | Consideration as the same |
| Survey's answer: let the student explains his/her answer. | In the recording, “you told them that they made a mistake in the signs even you mentioned it twice in a loud voice to allow all the class to hear it” | |
| In the recording, “you told them that they made a mistake in the signs even you mentioned it twice in a loud voice to allow all the class to hear it” | | |

### KtAS item: when a student is unable to see an obvious pattern in the problem. | M: yes yes because I was not developing the problem with the student, to me an explanation is to develop all the problem, it means sit down and we are going to work together look this and this, I say to the students the points where the mistakes are, here is where I reinforce them, aha, where the mistake is and I reaffirm what we already saw in class in order for the student to correct it. Maybe for me this is a clue. M: it is as everything in life; everybody has his/her own thinking, way, and understanding. | Consideration as clue |
| Survey’s response was: give the student clues to help him/her find the pattern. | In the recording, you were explaining to the student. | |
| In the recording, you were explaining to the student. | | |

The KtA situation of when a student is unable to see an obvious pattern in the problem was presented in a video recording to Maria. During one of the Maria classes, one student asked Maria about the assigned problem. Maria said “do it by yourself, read it again, and compare to the solution on the whiteboard”. This situation described was shown in the video recording. I asked Maria to provide the description of her action and the classroom situation observed in the video recording.
M: I was working to prepare for the *common presentation*. There is a period of time where the student has to read the worksheet, solve it by himself, read again, try to solve it.

T: here students already went to the whiteboard.

M: oh here students already went to the whiteboard?

T: Yes it was that class; they already went to the whiteboard and explained the problems.

M: aha

T: here you already told them that they needed to correct the mistakes…

M: the mistakes aha

T: then a student asked you something, and what was your action?

M: that he read it again.

In the description provided by Maria, she did not remember exactly at what moment the situation happened whether after the *common presentation* (puesta en común) or before. However, she recognized the situation and her action enacted when I mentioned to her some details of the video recording. The justification for her action was:

M: because the student must be able to do it, if I provided that response to her: do it by herself, maybe she is not making a minimum effort to try to work and solve it. That’s why I provided that response.

M: because here it is an evaluation, now after one week of working on it: with the introduction, feedback, then here students must be capable to solve it by themselves or with some help from the group during the *common presentation*, then, here I have to evaluate each group.

According to Maria she acted by asking the student to do it by herself because it was the moment to be able to know how to do it. She mentioned that by that time, there were several explanations, exercises, and time for questions provided. Analyzing the KtAS, there is a difference between the way that she acted in the class observed in this situation and her response in the survey as Table 4.34 presents. She answered in the survey that she would give the student
clues to help him/her find the pattern; in this case the way of doing the problem. Maria’s action observed was to ask the student do it as she could. There are two themes involved: student and curriculum. In the theme of student, Maria’s part of the reason was mainly about the attitude of the student. Maria thinks that this student did not make any effort to learn and solve the assigned problem. To this part of the reason the assigned code is “student’s attitude”. The other theme involved is curriculum. The reason coded under this theme was when Maria mentioned that at that moment the student should know how to do it because the program delineated that. The code assigned to this reason is “program”.

Maria watched a video recording where the KtA situation of when a student makes a mistake was presented. In the video recording, Maria was walking around the groups and stopped with one group. Maria identified a student’s mistake and asked “what happen with this -7?” She answered herself checking a student’s work:

M: oh, you moved it to the other side?
S3: yes
M: ok because you moved it, what happen with it? Aaah (she asked and explained it in order to help the student to identify his mistake) then what did you do with it?
S3: oh it should be positive (he realized what he had a wrong solution)
M: aha

Maria’s action was to provide an explanation to the student about his mistake. First, Mara helped the student to be aware of the mistake. Then, she explained why the problem should be solved. This is the description provided by Maria after watching the video recording:

M: it is the same there are students’ questions that can be considered as doubts. When he is developing the procedure and then I started to reaffirm what we already saw (the topic) and then he should remember where he made the mistake and deal with it correctly.

The action performed by Maria was to help the student to identify his mistake and explain how the procedure should be done. When I asked Maria why she acted in that way, she said:
M: why tell them where the mistake was? (she was thinking) because it is a supervision of the work that they are doing, identifying the little details that they have, well, there are weaknesses in those little details and from there, students can reaffirm or correct it to get a better grade in the evaluation.

Analyzing the KtAS and this video recording, Maria‘s action differs from her response in the survey. Table 4.34 shows this difference and its reasons provided. Maria answered in the KtAS that she would let the student explain his/her answer. However, in the video recording, Maria told the student where the mistake was and started to explain it. Maria mentioned that this difference occurred because her action in that classroom situation was the most logical action to do. The theme involved in this part of Maria’s reason is teaching practice. The code assigned is “most logical action”. In addition, Maria said that she uses certain words during her explanations that allow the student to remember the topic of the procedure uncovered in that class or previous classes. For this reason, she provided an explanation to help them to recall the topic. This reason encompasses the same theme of teaching practice because Maria’s teaching practice guides her to use those words with her students. “Recall” is the code assigned to Maria’s reason for the difference identified.

The last KtA situation presented to Maria was when students are having a hard time completing an assigned activity/task. In the video recording of one of Maria classes, at the beginning of the class, Maria said that students had 10 minutes to solve a certain number of equations. However, she provided the whole class to work on the equations, and most of the students did not finish them.

Maria provided more time than she had established at the beginning of the activity. Maria description about her action was:

M: look, I consider 10 minutes is little time but if I say 15 minutes,
T: I think there were 4 equations that you assigned
M: yes, 3 or 4 equations, yes they are solved up to one module (one class 45 minutes)
Maria recognized that she mentioned a certain time was assigned for the activity. However, she observed students having a hard time working on the activity and considered that more time was needed. Then, she provided it. Maria’s explanation about why she acted in this way is:

M: aha, I was thinking if we already worked all the week on it, they would be able to do it, even the most capable students, to solve the equations quickly, but then, this allows me to evaluate myself to be aware of it, I cannot move to the next topic until reaffirm this work. That’s why I am always behind.

Analyzing Maria’s actions performed in the class observed, there is a difference identified between her responses on the KtAS and the actions exhibited. This discrepancy exists because Maria acted in the class providing more time that she expected, and in the survey, she said that she would explain the assignment using examples or find another way to explain it. Maria was asked about this discrepancy. She said that because the students did not finish the work she could not move to the next topic. This reason makes my class be behind the other classes. Maria mentioned that students should make it within the time that she expected, however, they did not finish it. Table 4.34 presents the reasons provided about this discrepancy. The theme student is involved in Maria’s reason. Maria said that because students do not finish she has to provide more time. The code assigned to this reason is “student’s capabilities”.

Table 4.35 presents all the themes involved in the differences identified for Maria. These differences were about her actions identified and her actions that were thought in order to respond to the KtAS. In addition, codes that were assigned to the reasons provided are also shown in Table 4.35. The student theme also includes the code of “student’s conflict”. This code refers to Maria’s reason for providing an explanation to the students. One student was struggling to solve a problem in the class during the common presentation (puesta en común). This student had a conflict with the whole class, thus to avoid troubles Maria decided to limit his participation providing her explanation. The conflict with this student is that he humiliates classmates and is continuously fighting with everybody in class even with the teacher. According to Maria, this
student is always fighting or humiliating classmates. In the theme of *denial*, two more codes are included: “consideration as the same” and “consideration as a clue”. The “consideration as the same” is the code assigned to Maria’s reason for acting differently in the video recording from her answer in the KtAS. However, Maria’s action and response in the survey are considered the same according to her as Table 4.34 illustrates. The code “consideration as a clue” is assigned to Maria reason for considering her explanation where she revealed the student’s mistake as a clue as Table 4.34 shows.

Table 4.35: Themes and Codes for Maria’s Reasons.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Reason’s Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum</td>
<td>Program; class activity</td>
</tr>
<tr>
<td>Student</td>
<td>Student's capabilities; student’s attitude; student’s mistake; Student's conflict</td>
</tr>
<tr>
<td>Denial</td>
<td>Consideration as the same; consideration as a clue</td>
</tr>
<tr>
<td>Teaching practice</td>
<td>Most logical action; recall</td>
</tr>
</tbody>
</table>

4.5 **Research Question 4**

The fourth research question that guided this study was:

4. To what extent do the qualitative findings explain the quantitative results of the study?

This question is mixed methods in nature. In the response to this question, the main purpose of the mixed methods of the research design implemented is achieved. According to Creswell and Plano (2011), in the explanatory sequential research design, the primary purpose is the interpretation of the integration of the results of both quantitative and qualitative strands. In order to achieve this interpretation and integration, the pragmatism philosophical assumption was considered. The integration of the quantitative results and qualitative findings was conducted in the last phase of this study in order to respond to this question.
The integration of the results from both quantitative and qualitative phases was conducted considering the main two results of the quantitative findings. One main result was the lack of correlation between the mathematical teacher content knowledge (total scores of TCKS) and the “knowing-to act” ability (KtAS). The second main result was a significant correlation between the knowledge of models and generalizations (T3) and the “knowing-to act”. These two main results were integrated to the findings of the qualitative phase. The qualitative phase helped to explain these two results. However, because this question includes the interpretation of the integration of the results of the previous research questions presented in this chapter, the answer to this fourth research question will be provided in Chapter 5 in the section of Discussion of Findings for Research Question 4.

4.5 SUMMARY

This chapter discussed the results of this mixed method study. The results were presented for each research question. The first research question is:

To what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ knowing-to act?

A quantitative analysis was conducted to answer this question. The main results reported are: there is not a significant correlation found between the mathematical teacher content knowledge (total scores of TCKS) and the “knowing-to act” ability (KtAS) of Mexican teachers at the middle school level. Nonetheless, a significant correlation was identified between the knowledge of models and generalizations and the “knowing-to act” ability of Mexican middle school teachers in mathematics.

A comprehensive description of the participants who are part of the case studies and the selection process was presented. The study of these cases allowed answering the other two research questions. The second research question is:

How do the teachers act in KtA situations occurring in the mathematics classroom?
Findings from the classroom observations revealed the KtA situations identified in the classrooms of each participant. Moreover, other situations that challenged that “knowing-to act” of teachers emerged from the observations conducted. The different actions enacted by Rosa in each identified situation and the coding process applied were described. The detailed descriptions of the teacher actions and the coding process implemented as well as descriptive statistics for the other three participants are included in the appendices A, B, and C, respectively. In addition, frequencies of these actions and the situations identified were fully illustrated in this chapter only for Rosa. However, some frequencies about the situations identified in the other three cases were reported in this chapter.

The third research question addressed in this study was:

How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?

Findings obtained from the interview with each participant revealed the justification of the “knowing-to act” ability of the teacher. Several differences were identified between the teacher’s action and response in the KtAS. Teachers provided their reasons for these differences,

This chapter presented the four research question of this study:

To what extent do the qualitative findings explain the quantitative results of the study?

However, this question was not addressed in this chapter because it includes the interpretation of the integration of the results reported in the first three research questions. Therefore, only a brief introduction of what was considered to respond to it was provided in this chapter.

I noticed in the findings reported in this chapter that from the case studies, teachers who obtained a higher score in the KtAS survey differ less in the correspondence of their actions with their responses on the KtAS than teachers who got lower scores in the KtAS. This might be related to the strong “knowing-to act” of the teachers. Teachers with a high score on KtAS demonstrated to know how to act during their teaching without acting arbitrarily. Thus, these
teachers differed less in the correspondence of their actions with their responses on the KtAS survey. Also, I noticed that the mathematical content knowledge is not a synonym of having “knowing-to act” to teach mathematics. These findings will be discussed in Chapter 5 with details.
Chapter 5: Discussion and Conclusion

5.1 Overview

This chapter presents key components of this research. The purpose of the study, the research questions, the methodology and the theoretical framework will be discussed in this final chapter. Reviewing the key ideas presented in previous chapters, the main findings of the study will be discussed and interpreted.

The final chapter encompasses two sections. The first section will discuss the results obtained in response to each research question. The second section will summarize the theoretical framework, the research design, the research questions and the discussion. In addition, this chapter provides the implications, directions for future research, the contribution to this field of inquiry, and several recommendations for policy and practice. Furthermore, the limitations of this study will be reviewed in this final chapter.

5.2 Discussion of Findings for Research Question 1

5.2.1 Teachers’ performance on the TCKS and Knowing-to Act Survey

The first research question addressed in this study was as follows:

To what extent is the cognitive type of middle school teachers’ knowledge associated with teachers’ knowing-to act?

Two main results were obtained that answered this research question. In order to answer this question a correlational analysis was conducted. The data that was used for analysis of the mathematical teacher content knowledge was the total scores on the teacher content knowledge survey (TCKS). For analyzing teachers’ knowing-to act, the scores on the knowing-to act survey (KtAS) were used. The analysis showed that there is no correlation between the mathematical teacher content knowledge and their “knowing-to act” during teaching mathematics. In other words, the content knowledge possessed by a teacher is not an indication of “knowing-to act” at the moment. As Mason and Spence (1999) discussed the content knowledge is part of the “knowing-about” which is accumulative knowledge that can be possessed, however, it does not
imply that it can be used in a classroom situation such as the “knowing-to act” situations considered in this study. “knowing-about” is the knowledge that includes these three types of knowing: “knowing-that” “knowing-how”, and “knowing-why”. These types of knowing refer to the factual knowledge, the knowledge of knowing how to do something such as techniques, and having an argument in order to structure actions and from which to reconstruct actions (Mason & Spence, 1999). Mason and Spence (1999) recognize that “knowing-about” is immersed in Shulman’s categories (1987).

Shulman (1986, 1987) mentioned that content knowledge is the knowledge that grows in the minds of teachers. He placed an emphasis on the content and distinguished seven categories (see chapter 2 literature review). One of these categories is subject matter content knowledge which involves knowledge that goes beyond just facts or concepts; it includes the understanding of structures of the subject matter (Shulman 1986). Furthermore, Tchoshanov (2011) specified that mathematical content knowledge includes knowledge of facts and procedures, knowledge of connections and concepts, and knowledge of generalizations and models. However, Mason and Spence (1999) considered that more than these categories including the mathematical teacher content knowledge are necessary to enable a teacher to act at the moment required. “Knowing-about” is considered as static knowledge meaning knowledge possessed by a person, but it does not indicate that she/he is able to act creatively in a particular situation. Also, Skemp (1979) made this distinction. He distinguished between having knowledge about something and being able to use a technique in a fresh situation or at the moment. Therefore, the absence of a correlation between the mathematical content knowledge measured by the TCKS and the knowing-to act examined by the KtAS was already inferred in the literature review and conceptual framework was supported by the results of this study.

Rogelio’s case study exemplifies this result. He possessed knowledge of mathematics as evidenced by the survey that measured mathematical content knowledge; he got one of the highest scores of the surveyed teachers. Despite this evidence of having the mathematical
knowledge, he did not convey in his teaching the sort of actions that promote students critical thinking.

In addition, correlational analyses were conducted to determine to what extent the cognitive types of mathematical content knowledge were related to the “knowing-to act” of Mexican teachers at the middle school level. The three cognitive types of mathematical content knowledge identified by Tchoshanov (2011) were measured through the TCKS. The items on this survey are categorized according to the cognitive type measured. The cognitive type 1 (T1) is the knowledge of facts and procedures. Cognitive type 2 (T2) is the knowledge of concepts and connections. And cognitive Type 3 (T3) refers to the knowledge of models and generalizations.

The examination of cognitive Type 1 and the scores on the KtAS were not correlated. In other words, the absence of a relationship between T1 and KtAS adds to the discussion that the actions enacted by teachers during mathematics instruction are not related to their knowledge of facts and procedures. Moreover, the results obtained from the correlation analysis between T2 and KtAS showed that there is no significant correlation between them. The fact that a teacher possesses conceptual knowledge that enables her/him to make a connection among mathematical concepts does not indicate that the teacher knows how to act in the moment. The correlation analysis of T3 and KtAS provided the second main result addressed in the first research question. A significant correlation was obtained between T3 and KtAS scores. This result will be discussed and interpreted next.

5.2.2 Correlation Between Cognitive Type 3 and Knowing-to Act

Mason and Spence (1999) mentioned that “knowing-to act” is less possible when there is no knowledge. Since “knowing-to act” is the process where knowledge is called at the moment required, and if there is no knowledge, then, it cannot be called. In this study, results reported that teachers who possessed more knowledge of mathematics models and generalizations (cognitive Type 3) were better able to think about how to act desirably at the moment than
teachers with a limited knowledge of models and generalizations. In the correlational analysis conducted, a significant correlation was found between the cognitive type 3 (T3) of mathematical content knowledge and the scores on the knowing-to act survey (KtAS). This result is measured as a medium effect size correlation according to the interpretation of the $r^2$ recommended by Cohen’s standards (1988). It indicates that the correlation between the scores on the KtAS and the scores that measured only T3 is significant but with a medium effect size.

The cognitive type 3 of mathematical content knowledge is more theoretical (Tchoshanov, 2011). This type of knowledge is about models and generalizations. It includes conjecturing, generalizing, proving theorems, etc. The significant correlation between T3 and KtAS obtained is a relevant result that means that there is a relationship between the knowledge of models and generalizations and the actions enacted by teachers during mathematics instruction.

This interesting result adds to the discussion that teachers who possess the mathematical knowledge that allows generating and testing conjectures, making generalizations, and proving theorems, among other abilities, has more possibility of “knowing-to act” in the moment. It is relevant to highlight that in order to be able to generate conjectures and test them; certain procedural and conceptual knowledge must be possessed. According to Tchoshanov (2011) cognitive type 3 of mathematical content knowledge (T3) involves teachers’ knowledge and thinking to be able to generalize mathematical statements, design mathematical models, make and test conjectures, and prove theorems. Therefore, teachers’ performance on the knowing-to act survey (KtAS) was generally higher when teachers possessed higher knowledge of mathematical models and generalization (T3).

According to Mason and Spence (1999) “knowing-to act” at the moment requires more than content knowledge, but this means that at least some content knowledge is essential. The cognitive type 3 of mathematical content knowledge possessed by the Mexican teachers allowed the process in which “knowledge that enables people to act creatively rather than merely react to
stimuli with trained or habituated behavior” (Mason and Spence, 1999, p.136) evolved. Therefore, teachers have more possibility of “knowing-to act” when they hold knowledge of models and generalizations. In Tchoshanov’s study (2011), two middle school teachers were interviewed to analyze their cognitive type 3 (T3) of mathematical content knowledge. One of the participants who had a higher T3 knowledge showed more confidence to develop class activities on fraction division. This teacher would have more opportunity to know how to act at the moment of implementing these activities. T3 offers possibilities for “knowing-to act” at the moment.

Furthermore, we can add to the discussion that the significant correlation between the cognitive type 3 (T3) of mathematical teacher content knowledge and their “Knowing to act” measured by KtAS might also be related to the finding that teachers with high T3 knowledge feel more comfortable and confident during their teaching as reported in Tchoshanov’s study (2011). According to Fennema and Loef (1992), they pointed out that “when a teacher has a conceptual understanding of mathematics, it influences classroom instruction in a positive way” (p. 151). Even though T3 knowledge does not refer only to the conceptual understanding of mathematics, T3 involves this conceptual understanding of mathematics in order to be able to test conjectures, theorems or make mathematical models and generalizations. This claim supports the significant correlation reported in this study between T3 and KtAS. Teachers with knowledge of models and generalizations have more possibility to provide learning opportunities during mathematics instruction which increase the possibilities that their “knowing-to act” could be enacted.

5.3 DISCUSSION OF FINDINGS FOR RESEARCH QUESTION 2

With regards to the second research question that guided this study:

How do the teachers act in KtA situations occurring in the mathematics classroom?

I found out the KtA situations that more commonly take place in the mathematics classrooms in Mexico. Analyzing the four case studies, the KtA situation that more frequently
occurred during the classes observed is when a student solves a particular mathematical problem and asks if the answer or steps are correct which is item 1 of the Knowing-to act Survey (KtAS). This situation was identified 97 times. The following next most frequent KtA situation that took place in the four participants’ classrooms is when a student makes a mistake which is item 9. It was identified 35 times. The third most frequent classroom situation where the KtA is enacted is when students are having a hard time completing an assigned activity/task which is item 10 of the KtAS. This situation was identified 22 times.

The actions of the teachers enacted depended on the classroom situation that was happening at the moment. The findings of the teachers’ actions in all the KtA situations considered on the observation protocol and the emerging situations were presented in Chapter 4. However, I focused on four KtA situations that were common among the four participants in order to do a cross-case analysis to discuss the findings. Table 5.1 shows the actions more frequently enacted in a certain KtA situation as well as the name of the teacher who performed those actions.

Table 5.1: Actions Most Frequently Enacted.

<table>
<thead>
<tr>
<th>KtA situation</th>
<th>Teacher’s Action</th>
<th>Teacher(frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct</td>
<td>• Revealing mistakes;</td>
<td>Rosa (12), Rogelio (4)</td>
</tr>
<tr>
<td></td>
<td>• Asking for student’s justification</td>
<td>Omar (7), Maria(15)</td>
</tr>
<tr>
<td>Item 9 KtAS. When a student makes a mistake</td>
<td>• Providing an explanation</td>
<td>Rosa (3), Rogelio(4)</td>
</tr>
<tr>
<td></td>
<td>• Revealing mistakes</td>
<td>Omar (2)</td>
</tr>
<tr>
<td></td>
<td>• Asking for student’s justification</td>
<td>Maria (7)</td>
</tr>
</tbody>
</table>
Item 5 KtAS. When a student is unable to see an obvious pattern in the problem

- Providing the solution: Rosa (1), Rogelio (1)
- Providing time and hints: Omar (1)
- Revealing mistakes; asking student to keep trying; providing an explanation: Maria (1)

Item 10 KtAS. When students are having a hard time completing an assigned activity/task

- Referring to see the whiteboard or notes; providing hints or more time: Rosa (4)
- Providing an explanation: Omar (2), Maria (3)
- Asking for verification: Rogelio (1)
- Providing part of the solution

Analyzing the findings obtained from the classroom observations, we added to the discussion that the Mexican teachers acted a few times showing a desirable “knowing-to act”. This analysis considered the teacher actions enacted in the KtA situations included in the observation protocol and the emerging situations for the four participants. Table 5.2 presents the number of actions enacted by each teacher, and how many actions are considered as most desirable or non-desirable “knowing-to act”. In Rosa’s case, she acted most of the time showing a non-desirable “knowing-to act” as her responses on the KtAS survey already indicated. Only 14% of Rosa’s actions were considered as desirable “knowing-to act”.

With regards to the actions enacted by Omar, we observed that the 37% of his actions showed a desirable “knowing-to act”. It means that the 63% of his actions are considered as non-desirable “knowing-to act” (see Table 5.2). However, Omar was the teacher who acted showing the most desirable “knowing-to act”. This finding is reaffirmed by his score on the KtAS survey which was the highest score of the four case studies. Most of Rogelio’s actions were classified as non-desirable “knowing-to act” because the 97% of his actions were under this consideration. Thus, the rest of his actions enacted during the classes observed were considered as desirable teacher “knowing-to act”.

According to the analysis of the classroom observations conducted in Maria’s classes, 36% of Maria’s actions were showing a desirable “knowing-to act”. This means that the other
64% of the actions performed by Maria are considered as non-desirable “knowing-to act”. However, she had the second highest number of actions classified as desirable “knowing-to act” from the four case studies. Also this finding was reaffirmed with the score on the KtAS survey which was the second higher score of the four participants as Table 5.2 illustrates.

Table 5.2 presents the number of KtA situations observed during the classes of the four participants. Analyzing cross-cases, we can see that Maria’s classes presented more KtA situations than the others (70 situations) and Rogelio’s classes observed only 31 KtA situations occurred. Moreover, the analysis of the actions enacted and the KtAS survey was done to see if there was an agreement among their responses and actions. Most of the actions enacted by the four teachers did not reflect their responses on the KtAS survey as illustrated in Table 5.2. As could be observed in Table 5.2 that less than 40% of the actions enacted by the four participant reflected what they responded on the KtAS survey. There is the case of Rosa where only 16% of her actions reflected her answers on the survey. For instance, 39% of Maria’s actions reflected what she responded on the survey. These differences between actions enacted and responses on the KtAS were addressed in the interviews.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Total KtA Situations Observed</th>
<th>#Teacher actions reflected on the survey</th>
<th>#Teacher actions that differ from the survey</th>
<th>#Teacher actions Desirable KtA</th>
<th>#Teacher actions Non Desirable KtA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosa</td>
<td>52</td>
<td>8 (16%)</td>
<td>42 (84%)</td>
<td>7 (14%)</td>
<td>44 (86%)</td>
</tr>
<tr>
<td>Omar</td>
<td>35</td>
<td>11 (31%)</td>
<td>24 (69%)</td>
<td>13 (37%)</td>
<td>22 (63%)</td>
</tr>
<tr>
<td>Rogelio</td>
<td>31</td>
<td>7 (24%)</td>
<td>22 (76%)</td>
<td>1 (3%)</td>
<td>28 (97%)</td>
</tr>
<tr>
<td>Maria</td>
<td>70</td>
<td>27 (39%)</td>
<td>42 (61%)</td>
<td>25 (36%)</td>
<td>44 (64%)</td>
</tr>
</tbody>
</table>
5.4 DISCUSSION OF FINDINGS FOR RESEARCH QUESTION 3

The third research question that guided this study was:

How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?

During the individual interviews conducted to each participant, teachers provided a description of the classroom situation that occurred as well as the action enacted by them. Analyzing these actions in the situations, they are in agreement with what was observed during the classroom observations conducted. However, most of the actions enacted did not reflect the responses of the participants on the KtAS survey.

During the individual interview, participants were questioned about the difference between their actions and responses on the survey. There were several themes that emerged from the reasons provided of the differences such as curriculum, survey, time factor, and teaching practice, among others. Table 5.3 presents the themes emerged, the codes included in that theme, and the name of the participant who provided the reason for that theme emerged. For instance, the theme Student emerged from the reasons provided by the four participants. Rosa said that she could not act the way she responded on the KtAS survey “because they are lazy”. She referred to the students’ attitude. Omar’s reason to act differently was because he considered primordial to do what he did than acted as he said that would act in the KtAS survey in order to see that the student learned. Omar mentioned that he asked for the procedure instead of only the result from the student because of his learning expectation of the student. He considered that a student who is able to do the procedure correctly also gets the correct result which indicated for him that the student really learned. Thus, he did not act as he responded on the survey because he preferred to ask the student for the procedure to be sure that the student learned.

In Rogelio’s case, the theme student emerged in his reason for preferring to explain to the students the problem solved by them instead of asking the students how and why they solved the
problem. He acted that way because “now the students are distracted, they have an instantaneous short memory; everything learned in one class is forgotten for the following day.” This reason was coded as “student’s capabilities” and it is under the theme student as Table 5.3 shows.

Maria’s reason where the theme student emerged was when she asked the student “do it by herself”. In the KtAS, she responded that she would help students by providing clues to solve the problem or identify the mistake. However, she acted by asking the student do it without any assistance. Maria mentioned that the student “was a student who did not make a minimum effort to solve the things [problem]”. According to Maria, she acted this way because of the student’s attitude. Thus, from this reason the theme student also emerged. Table 5.3 illustrates the themes obtained from the teachers’ reasons to act differently as they responded on the KtAS survey. In addition, we can observe in Table 5.3 that there were themes that emerged from only one reason provided just by one teacher such as switching the situation and teacher’s exception/excuse to the KtA situation.

Table 5.3: All Themes and Codes of Teachers’ Reasons for Differences in their Responses and Actions.

<table>
<thead>
<tr>
<th>Theme</th>
<th>Codes</th>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum</td>
<td>Class activity; program</td>
<td>Rosa, Maria</td>
</tr>
<tr>
<td>Denial</td>
<td>Deny teacher action; denial; consideration as the same; consideration as a clue</td>
<td>Rosa, Maria</td>
</tr>
<tr>
<td>Survey</td>
<td>Survey is not applicable; survey’s interpretation; time factor;</td>
<td>Rosa, Omar</td>
</tr>
<tr>
<td>Switching the situation</td>
<td>Anticipation</td>
<td>Rosa</td>
</tr>
<tr>
<td>Student</td>
<td>Student’s attitude; student’s mistake; learning expectation; student’s conflict; students’ capabilities; student’s answer</td>
<td>Rosa, Omar, Maria, Rogelio</td>
</tr>
<tr>
<td>Time Factor</td>
<td>Time Factor</td>
<td>Rosa, Rogelio</td>
</tr>
</tbody>
</table>
Content | Arithmetic operation; level of complexity; perception of mistake; | Rosa, Omar, Rogelio
Teacher’s exception/excuse to the KtA situation | Exception | Omar
Teaching practice | Easiest action; recall; most logical action; anticipation | Omar, Maria, Rogelio

5.5 DISCUSSION OF FINDINGS FOR RESEARCH QUESTION 4

5.5.1 Meta-inferences

After discussing the results and findings that answered each research question that guided this research, the focus is placed on the meta-inferences. This process was conducted in the integration phase, which is the last phase of the study. This phase was supported by the philosophical assumption of pragmatism. Meta-inferences means the interpretation of the integration of the results of the quantitative phase (research question 1) and the qualitative phase (research questions 2 and 3). This interpretation addressed the fourth research question: To what extent do the qualitative findings explain the quantitative results of the study?

This section discusses how the qualitative findings help to explain the qualitative results. It is one of the main features of the research design implemented: The explanatory sequential design.

After collecting data from 4 teachers, we found out that these data from the case studies reflect the major quantitative results. One of the major quantitative results was the lack of correlation between the mathematical teacher content knowledge and their “knowing-to act” during teaching mathematics. This finding is reflected by the data collected from the case studies. For instance, Rogelio obtained the highest performance on the teacher content knowledge survey TCKS as Table 5.4 shows. However, he got a low score on the KtAS survey. These quantitative results are reflected in the classroom observations and interview conducted. 97% of Rogelio’s actions enacted during the observations were considered as non-desirable
“knowing-to act”. Despite the evidence that Rogelio possessed one of the highest scores on the TCKS survey, he acted most of the time by limiting the critical thinking of his students.

Maria’s case also reflected the absence of a correlation between the total scores on the TCKS and the KtAS surveys found in the quantitative phase. According to the quartiles explained in Chapter 4, Maria obtained a performance on the TCKS survey considered as low. However, she got a high performance on the KtAS survey as Table 5.4 illustrates. These quantitative results were exhibited during the classroom observations and interviews. Maria was one of the four participants who acted with a most desirable “knowing-to act”. The 36% of her actions were considered as desirable “knowing-to act” in comparison of the 3% of Rogelio’s actions showing a desirable “knowing-to act”. I add to the discussion that even though Maria did not possess a strong mathematical content knowledge, she acted in her classroom with more actions that promote the critical thinking of her students. Therefore, the lack of correlation between the mathematical teacher content knowledge and their “knowing-to act” during teaching mathematics indicates that the mathematical content knowledge is not a synonym of having a desirable “knowing-to act” to teach mathematics as discussed at the beginning of this chapter.

Doing a post hoc analysis, I noticed in the findings reported in Chapter 4 from the case studies that teachers who obtained a higher score on the KtAS survey differed less in the correspondence of their actions with their responses on the KtAS survey than teachers who got lower scores on the KtAS. For instance, Omar and Maria, in Table 5.2, we can observe that Omar and Maria had a small number of actions enacted that differed from what they responded on the KtAS. If we look at Table 5.4, we will observe that Omar and Maria were also those whose performances in the scores on the KtAS survey were higher. Thus, I add to the discussion that these two teachers differed less in their responses on the KtAS with their actions observed because their strong “knowing-to act” allowed them to know how to act without arbitrariness during their mathematics teaching.
The second main result of the quantitative phase was the significant correlation between the cognitive type 3 of mathematical content knowledge (T3) and the teacher’s “knowing-to act” measured by the KtAS survey. In this study we found that teachers who possessed more knowledge of mathematics models and generalizations (cognitive type 3), were able to know how to act at the moment more times than teachers with limited knowledge of models and generalizations as discussed earlier in this chapter. Table 5.4 presents the raw scores on both surveys as well as the raw scores for each cognitive type of knowledge. Moreover, the percentages of the scores obtained out of the total of scores of the surveys are shown between parentheses. We can observe that teachers who possessed higher percentages on T3 obtained higher percentages on the performance on the KtAS survey. For example, Omar got a 38% on the scores that examined T3 out of the total scores of the TCKS. He acted with the most desirable “knowing-to act” from the four participants of the case studies. Thirty seven percent of his actions promoted critical thinking during mathematics instruction. Also, this quantitative result is reflected in Maria’s case. She got a high score on the KtAS survey as Table 5.4 shows. Maria’s cognitive Type 3 mathematical knowledge was 27% out of the total scores that measured her mathematical content knowledge. As discussed earlier in this chapter, teachers who possessed knowledge (T3) to generate conjectures and test them, certain procedural and conceptual knowledge must be also possessed by them. Moreover, teachers with T3 knowledge are more confident during mathematics instruction which increase the possibilities that their “knowing-to act” could be enacted as reported in this study.

Due to the nature of qualitative research, a generalization of the qualitative findings should not be done. However, we can discuss the transferability of the findings of the qualitative phase to the results of the quantitative results. Interpreting the findings and results of this study, we found out that our four case studies have this property of transferability to the quantitative analysis as observed throughout the discussion in this section and in the following paragraph.
Table 5.4 shows that Omar and Maria got a high score on the KtAS survey. These two participants were also those whose number of actions considered as desirable “knowing-to act” was higher. Since the scores on the KtAS survey of these teachers were reflected on the actions enacted during the classroom observations conducted, this added to the discussion that the KtAS survey examined the “knowing-to act” of these teachers accurately. For instance, teachers who have higher scores on the KtAS survey acted similarly. And teachers who scored low on the KtAS survey acted similarly as Table 5.4 illustrates.

Table 5.4: Raw Scores and Percentages of the TCKS, T1, T2, T3, and KtAS.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>TCKS total</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>KtAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rose</td>
<td>10 (30%)</td>
<td>5 (50%)</td>
<td>3 (30%)</td>
<td>2 (20%)</td>
<td>5 (50%)</td>
</tr>
<tr>
<td>Omar</td>
<td>18 (54%)</td>
<td>8 (44%)</td>
<td>3 (17%)</td>
<td>7 (38%)</td>
<td>9 (90%)</td>
</tr>
<tr>
<td>Roger</td>
<td>19 (57%)</td>
<td>8 (42%)</td>
<td>7 (37%)</td>
<td>4 (21%)</td>
<td>6 (60%)</td>
</tr>
<tr>
<td>Mary</td>
<td>11 (33%)</td>
<td>6 (54%)</td>
<td>2 (18%)</td>
<td>3 (27%)</td>
<td>8 (80%)</td>
</tr>
</tbody>
</table>

5.6 Conclusion

The topic of teacher knowledge is critical for teaching, learning and culture. The process of teaching and learning is a decisive aspect in shaping culture. Therefore, to be focused on components of this process such as teacher knowledge is relevant for the educational field. Since the knowledge possessed by teachers has an impact on students’ learning, it is an educational issue worthy of study.

Due to the complex nature of the mathematical knowledge for teaching, investigators are challenged to research and define with precision each kind of teacher knowledge as presented in Chapter 2. Interactions among these kinds of knowledge are essential as a part of the knowledge base for teaching mathematics and, as such, some scholars have recognized and studied it (e.g. An et al., 2004; Koehler and Mishra, 2009). Thus, further research is needed about the nature of the interaction among specific kinds of teacher knowledge such as mathematical teacher content knowledge and “knowing-to act”. Knowing what kinds of knowledge have a direct influence on
teaching practices is critical in order to enhance teacher education programs and teaching practices. Policy makers and teacher education programs can make decisions about how teachers should be prepared in order to help students to learn mathematics by focusing on research in the area of teacher knowledge.

This study was conducted in a Mexican city located on the border with the United States. An existing lacuna of research at the elementary and middle school levels that analyzes specific mathematical knowledge and instructional skills required for teaching effectively were identified (National Mathematics Advisory Panel, Report of the Task Group on Teachers and Teacher Education, 2008b). Of great relevance is to point out that in Mexico, research on the middle school level is critical because this level has had for many years the greater dropout rate from the basic education system which includes middle school.

Considering the relevance of studying the specific kinds of teacher knowledge and the research context, this study attempted to measure middle school mathematical teachers’ content knowledge and its relationship with teachers’ “knowing-to act” in Mexico. The broader educational purpose of this study was the understanding of how mathematical teacher content knowledge influences teachers’ instructional decisions to act in particular ways. For achieving this purpose, the following research questions were addressed in this study:

1) To what extent is the cognitive type of teachers’ knowledge associated with teachers’ knowing-to act?

2) How do the teachers act in KtA situations occurring in the mathematics classroom?

3) How do middle school mathematics teachers describe and justify the knowing-to-act processes that characterize their classroom instruction when their actions are not aligned with their responses on the Knowing-to-Act survey?

4) To what extent do the qualitative findings explain the quantitative results of the study?

This study analyzed two kinds of teacher knowledge: mathematical teacher content knowledge and teachers’ “knowing-to act” (Mason and Spence, 1999) possessed by Mexican in-
service teachers in mathematics classrooms at the middle school level in Mexico. Shulman’s model (1986) provided the theoretical foundation for this study. The study is immersed in one of the categories of Shulman’s model: content knowledge. Analyzing previous research with regard to the content knowledge category, the researcher identified the different cognitive types of mathematical content knowledge in Tchoshanov’s studies (2008, 2011): cognitive type 1 (teacher content knowledge of facts and procedure); cognitive type 2 (knowledge of concepts and connections); and cognitive type 3 (knowledge of models and generalizations). These cognitive types allowed a deeper examination of the mathematical teacher content knowledge. Therefore, an instrument developed by Tchoshanov (2011): the teacher content knowledge survey (TCKS) was used to measure the mathematical content knowledge through several items that measured the different cognitive types.

For the analysis of the “knowing-to act”, Mason and Spence (1999) proposed a framework where they identified a different kind of knowledge “knowing-to act” that refers to “active knowledge which is present at the moment when it is required.”(p.135). Therefore, this kind of knowledge depends on the situations or context during mathematics instruction, and the level of awareness possessed by teachers. Following this framework, the exploration of teachers’ “knowing-to act” was conducted after the identification of several situations. According to Mason and Spence (1999), the situations identified and analyzed are relevant to teaching mathematics and challenge the “knowing-to act” of the teachers.

Considering Mason and Spence’s study (1999), 11 situations were identified and included in the development of a survey that examined the teacher “knowing-to act”. This survey was named knowing-to act survey (KtAS). The outcome of this survey provides a snapshot of the teacher actions most likely to occur in the situations provided where a desirable or non-desirable “knowing-to act” is shown. Mason and Spence’s framework of “knowing-to act” (1999) guided the design and development of the observation protocol. Based on this protocol,
classroom observations were conducted. Moreover, this conceptual framework allowed the understanding of the actions enacted by the teachers observed.

Based on the theoretical framework and diverse nature of the research questions of the study, a mixed methods study was the most appropriate approach to conduct the study. Moreover, conducting a mixed method study provides credibility due to the integrity of the findings (Bryman, 2006). This mixed methods study was composed of three phases: quantitative, interim, and qualitative phase. A mixed methods sequential explanatory research design was implemented in this study. It included the sequential implementation of the phases. In addition, the mixed methods research approach allowed the qualitative phase to extend the breadth of the relationship between the mathematical teacher content knowledge and “knowing-to act” analyzed in the quantitative phase.

The quantitative phase is the first phase of the design; quantitative data was collected through the administration of two surveys TCKS and KtAS to 70 mathematics middle school teachers in Mexico. This phase looked critically at the correlation of the mathematical teacher’s content knowledge and the teachers’ “knowing-to act” at the moment. Then, the interim phase encompassed the case selection (purposive sampling) of four teachers as well as the development of the classroom observation protocol based on the results of the quantitative phase. After that, during the qualitative phase, classroom observations and individual interviews of 4 teachers were conducted in order to explore the “knowing-to act” enacted during mathematics instruction.

In the quantitative phase of the study, the first research question was addressed. The quantitative analysis allowed the examination of the mathematical content knowledge of middle school teachers through the cognitive types and the association of this kind of knowledge with teachers’ knowing-to act. This analysis reported two major results: there is no correlation between the mathematical teacher content knowledge (total scores of TCKS) and the “knowing-to act” ability (KtAS) of Mexican teachers at the middle school level. Nonetheless, a significant
correlation was identified between the knowledge of models and generalizations T3 and the “knowing-to act” ability of middle school teachers in Mexico.

During the discussion of result of the absence of a correlation between mathematical teacher content knowledge (total scores of TCKS) and the “knowing-to act” ability (KtAS) in this chapter, we could see that the content knowledge possessed by a teacher is not an indication of knowing-to act at the moment. According to Mason and Spence (1999), more than Shulman’s categories including the mathematical teacher content knowledge are necessary to enable a teacher to act at the moment required. This quantitative result is also well supported by the analysis of the case studies of the qualitative phase. We could see in Rogelio’s case that despite the evidence of a higher score on the survey that measured the mathematical content knowledge, he was not able to know how to act at the moment in a way that promoted the critical thinking of his students. Only one action enacted by Rogelio during the three classes observed was considered as desirable “knowing-to act”. The contrasting case occurred with Maria. Even though Maria possessed a weak mathematical content knowledge, she acted more desirably during her classes than Rogelio.

The other major result reported a significant correlation between the knowledge of models and generalizations T3 and the “knowing-to act”. This correlation means that a relationship exists between the knowledge of models and generalizations and the actions enacted by teachers during mathematics instruction. We added to the discussion of this result that teachers have more possibility of “knowing-to act” when they possess knowledge of models and generalizations because this last type of knowledge involves certain factual and conceptual knowledge. Tchoshanov’s study (2011) reported that teachers with high T3 knowledge feel more comfortable and confident during their teaching, which might also be related to this result.

The second research question analyzed the teachers’ actions enacted during situations that challenge the “knowing-to act” of the teachers. These situations are named knowing-to act situations (KtA situations). The findings allowed a comprehensive description of the classroom
observations where KtA situations were identified and the analysis of the actions performed in these situations was conducted for each case study. In addition, the study allowed the inclusion in the analysis of classroom situations not considered in the observation protocol (emergent themes). In the section discussion of findings for research question 2 in this chapter, Table 5.1 provided a precise report of the actions more frequently enacted in the four KtA situations that occurred in the four participants’ classes. We could observe that Rosa and Rogelio acted by revealing to the students their mistakes when students solved a problem and asked if their answer or steps were correct. However, in this situation, Maria and Omar preferred to ask their students for a justification of their solution. For more details of these findings review Table 5.1 in this chapter or appendices A, B, and C, or see the section titled observing Rosa in chapter four for a full description of Rosa’s case study.

Findings presented to answer the third research question showed how the four mathematics middle school teachers described their “knowing-to act” during their teaching. In addition, differences between the teacher’s actions observed and responses on the KtAS were identified and addressed in the individual interviews conducted. During the interviews, teachers provided their accounts about the KtA situations, the actions enacted by them, and the reasons for acting that way and answered differently on the KtAS survey. Table 5.3 showed the themes and codes of teachers’ reasons for the differences identified in their responses and actions.

Furthermore, several meta-inferences were presented. Those involved the interpretation of the integration of both quantitative results and qualitative findings. After collecting data from four teachers, the major quantitative results were viewed through the qualitative analysis done of the data. As previously discussed in the section above, the post hoc analysis of the four case studies reflected the lack of correlation between mathematical teacher content knowledge and “knowing-to act” as Rogelio and Maria’s cases exemplified. With regard to the significant correlation between T3 and “knowing-to act”, Maria and Omar’s data reflected this result after analyzing the observations and interviews. More meta-inferences were obtained in the study.
For more detailed descriptions of these meta-inferences see the previous section. This study focused on the analysis of middle school mathematical teachers’ content knowledge and its relationship with teachers’ “knowing-to act” in the Mexican borderland with the United States. In the following section, several implications are discussed for further research as well as contributions to the field of mathematics education and recommendations for policy and practice.

5.7 IMPLICATIONS

The study implies that professional development in Mexico can be strengthened by focusing on cognitive processes with relationship to teachers’ actions in the classroom that most impact student learning. For example, when pre-service teachers are asked during their teacher preparation to observe in-service teachers, the observation can be focused on the “knowing-to act” of in-service teachers during their classroom instruction. For in-service teachers, the analysis of classroom situations that are familiar to them can be done in order to identify the teacher’s actions that most impact their students. This analysis could be done during meetings, professional developments or during the academies.

In terms of the very challenging logistics for the implementation of mixed method studies in Mexico, the implication is that in order to facilitate a higher level of research in Mexico with respect to teaching and learning in the school systems, a system or protocol needs to be developed and put in place. This will greatly facilitate additional research and thereby the improvement of education in Mexico. Even though the author of this study belongs to this Mexican society and knows the educational system in Mexico, several challenges were faced in conducting the research. For instance, the time spent in looking for a protocol to ask access to the middle schools; the unavailability of the Mexican authorities to talk with the researcher about the study; the permission granted for one authority was not enough to have access to the school even though that authority provides the permission; principals and administrative staff at the schools made complicated the process because of the lack of organization or communication among educational authorities and school administrators, among others.
Another implication is with reference to the language used in international research. The further refinement and development of the KtAS taking into consideration diverse languages and cultures would facilitate research and analysis. The translation of an instrument is not always the most adequate; rather what is needed is a conversion of an instrument that is aligned with the language usage and culture of the society where it is to be used.

This implication could also be applied to any protocols that are used in research. For example, during the interview protocols that took place in this research. One of the interview questions was: what was your thinking to act this way in the situation captured in the video clip? Basically this question is asking the teacher why he or she acted that way. This kind of question is not frequently used in the Mexican culture. Teachers may feel uncomfortable with this kind of questions because they feel questioned about their actions and may think that one is criticizing their teaching practices. This feeling was perceived during the interviews conducted in this study. To minimize this tension, the researcher tried to rephrase the question and constantly reminded the teachers that she was not indicating that their actions were wrong or correct.

This research has several contributions to the field of mathematics education research. Some of these contributions apply only for Mexico. However, this study also contributed to this field of inquiry on an international level. In addition, this study has several implications for policy and/or practice and for conducting further research.

5.8 **CONTRIBUTION TO THE FIELD OF MATHEMATICS EDUCATION RESEARCH**

There are several aspects of teacher knowledge that undoubtedly have an impact on student learning. Therefore, this study contributes to the field of mathematics education with the analysis provided of certain kinds of teacher knowledge: mathematical content knowledge and “knowing-to act”. This knowledge added by this study may help to restructure mathematics teacher education programs, engaging pre-service teachers to focus on the “knowing-to act” during the observations and activities that pre-service teachers are asked to do in their
preparation as well as during their practice teaching. And hopefully it impacts the teaching and learning process in Mexican middle schools.

As we observed in Chapter 2, few studies in Mexico focus on teacher content knowledge and its relationship with teaching practices at the middle school level. Thus, this study contributes to the literature review on types of teacher knowledge by studying mathematics teachers from perspectives not considered in Mexico (Pinto & Gonzalez, 2008), such as “knowing-to act”. This research has a greater contribution to the educational field of research in Mexico at the middle school level in regards to mathematics education. Moreover, this study provided evidence of the relevance of studying teachers’ attributes in an effort to enhance the preparation of Mexican teachers. It also provided a framework for discussing several educational issues in Mexico such as the design for evaluating teachers, weaknesses of teacher preparation, student’s achievement, and high course failure rates in middle schools.

The implementation of this mixed methods study provided an understanding of the relationship between teacher content knowledge and “knowing-to act”. As chapter two showed, there is a dearth of studies that use a mixed methods approach to analyze teacher content knowledge and “knowing-to act”. Therefore, this study contributed to the mixed methods community promoting the use of this approach.

5.9  **RECOMMENDATIONS FOR POLICY AND/OR PRACTICE**

This research provided in-service teachers and other participants in the education field with awareness about the active knowledge that is needed to enact the teachers’ knowing-to act in teacher preparation programs in Mexico that can be used to support teachers and students in the United States.

Research about teacher knowledge, particularly mathematical content knowledge and “knowing-to act”, provides tools that could be incorporated in teacher education programs in order to prepare mathematics teachers with a stronger knowledge that directly has an influence on students’ acquisition of knowledge.
There are also several recommendations for policy/practice that arose from this study. One recommendation is that educators should be aware of the attention that needs to be placed on the “knowing-to act” of teachers and students. Another recommendation is that we need to educate in-service or pre-service teachers about the “knowing-to act” processes that characterize their teaching in mathematics classrooms. This could be achieved through the implementation of professional development that includes activities that allow teachers to identify their “knowing-to act” and reflect about their actions enacted in “knowing-to act” situations. Teachers need to pay attention in particular situations that arise in their classroom that either limit or promote critical thinking.

Another implication for policy and/or practice that arose from the implementation of this study concerns professional development and teacher preparation programs. The evaluation of teacher professional development and teacher preparation can be done by focusing on the “knowing-to act” and/or mathematical content knowledge. Using the instrument developed for this study KtAS survey to measure the “knowing-to act” or the TCKS survey to examine the mathematical content knowledge may be a viable way of enhancing professional development for mathematics teachers in Mexico.

Of great relevance is to promote the influence that “knowing-to act” has for teaching and learning mathematics. This may be accomplished through the implementation of research focused on “knowing-to act”. Publications and/or presentations at conferences about this type of research are needed to achieve this purpose. An important point is how this study can also impact further research.

5.10 Future Research Directions

Further studies are needed in which the association and exploration of other kinds of knowledge for teaching mathematics and students’ learning can be analyzed. For instance, research on “knowing-to act” in the United States or other countries can also be worthy of a study; how would teachers act in KtA situations during their mathematics instruction in the USA,
Canada, and Russia? In addition, this study allows comparisons among Mexico and countries where data is already collected in regards to teacher knowledge in the area of Mathematics, such as Russia, U.S., Latin American countries, and other countries that participated in the TEDS-M Study 2012. These comparisons will let policy makers, teachers, and scholars learn about other teacher education programs, and situate the Mexican teacher preparation internationally, in order to be able to make improvements in the preparation of Mexican mathematics middle school teachers.

Furthermore, research on the extent to which pedagogical content knowledge is associated with “Knowing-to act” will be of great relevance for this field of inquiry. It is critical to understand what influences teachers’ instructional decisions to act in particular ways during specific moments while teaching mathematics in their classrooms. Therefore, more studies that focus on how the “knowing-to act” of mathematics teachers during their instruction matters in producing student learning are essential.

This study could be replicated in other parts of Mexico. This study was conducted on the Mexican borderland. It would be interesting to see how the mathematical content knowledge of teachers throughout the country is such as in a central zone, or in the capital, or in the other borderland in southern Mexico, or in another geographic area. Moreover, to conduct this study including Mexican teachers who belong to rural areas, or private schools, or indigenous communities would be another great opportunity to contribute to the field of mathematics education. However, the logistics for access to schools in Mexico is challenging.

The logistics for gaining access to public middle schools in Mexico is very complex. First of all, permission to conduct the research must be granted by the subsecretary of education, sport and culture of the city (person in charge of the public education in the city). This approval lasted at least 3 weeks. After that, several visits were done to find the coordinator of one out of the three possible types for participating in the study. Once the approval from the coordinator of one kind was granted, the permission and coordination with the teaching chair of the kind was done.
A calendar and time was provided to visit each school that belongs to Type A of middle schools. However, all these approvals did not indicate that teachers would be willing to participate in the study. The access to several schools was difficult to the particular location of the schools; some of them were located in the poorest areas of the city and/or the farthest zones such by the city limits.

Once the researcher was in the school, the principal granted permission to survey teachers. Several times the principal did not allow the administration of the surveys on the day and time assigned by the teaching chair for different reasons such as the administrations of exams, the absence of other teachers or staff, the principal needed to first ask teachers if they would like to participate, suspension of activities, etc. Finally, the access to the teachers was accomplished, and teachers started to read the consent form and fill out the surveys. However, many teachers did not want to participate in the study.

A different logistic was implemented for Type C middle schools. After the permission granted by the subsecretary of education, sport and culture of the city, the coordinator of this type of schools asked me to implement the survey during a meeting called the academy. Teachers are requested to attend an academy every two months. An academy is for all teachers of that particular kind of middle schools and a specific content area. Then, the researcher was asked to administer the survey during an academy of mathematics. However, mathematics teachers from Type C approximately 80 teachers did not want to participate in this study. Therefore, participants of this study did not belong to this type of middle schools. All this process is presented here to provide an awareness of what conducting research in Mexico involves.

5.11 LIMITATIONS

This study provided a snapshot of the mathematical content knowledge and “knowing-to act” of Mexican teachers as well as the attempt to explore the “knowing-to act” and content knowledge of four participants to provide a better understanding of the entire population.
However, the results of this study do not indicate that all teachers in Mexico possessed the same level of mathematics content knowledge and act in their classroom as the participants of this study acted. If this study were to be replicated in other part of Mexico, results might be different. There were several limitations identified to the study such as language, the instrument, and sample.

5.11.1 Language

Language was a key issue involved throughout the study. The native language of the participants of the study was Spanish. Thus, the first language issue found was writing the study in English. Then, several translations were done. The translator plays an important role in the research: “The researcher gains access to the ideas and experiences of the participants through the translator, and it is through the translator that the research participants’ voices are heard” (Wong & Poon, 2010, p. 153). Therefore if the translator and the researcher are the same person it has the advantage of gaining access to the ideas and experiences of the participants directly. As we know a translation is not a neutral and objective process. It is affected by the social context, background and worldview of the translator (Wong & Poon, 2010). Since there is always the possibility to lose information in the translation process, the translation process of this study is a limitation. Moreover, the participants of the study were Spanish dominant; they were not able to do member checking, thus, this could be one limitation of the study. These limitations could be overcome if the study would analyze and report data in Spanish, and the findings could be verified by the participants in order to do a member checking. However, the complex accessibility to these schools because new permissions need to be granted to have access to talk to these teachers to do a member checking and the new schedules of the teachers because of the new academic year made not possible to do a member checking.

Another limitation related to the language is the TCKS instrument for surveying teacher content knowledge. This instrument was developed in English for American teachers at the middle school level. Therefore this instrument was translated to Spanish and a group of persons
(mathematics teachers) verified the translation done. However, there is the possibility that certain meaning could be lost during the translation and might affect teachers’ interpretation of the items.

5.11.2 Knowing-to Act Survey KtAS

The KtAS survey is an instrument developed for the implementation of this study. It was designed by the author of this study and two of her committee members to evaluate the ways in which teachers recall their active knowledge to act in the moment when it is required. In other words, it was developed to examine the “knowing-to act” of teachers. However, this instrument included only 11 situations where the “knowing-to act” of teachers is challenged. This is one limitation because there are more classroom situations that challenge the teachers’ “knowing-to act”. For instance, this instrument and the observation protocol do not consider situations or factors out of the classroom context such as meetings, or exam administrations mentioned by Rogelio. However, due to the nature of a classroom, it is not possible to include all the situations that could occur. In addition, this survey included the situations that according to Mason and Spence (1999) are relevant for teaching and learning mathematics.

5.11.3 Sample

Another limitation of this study is the sample size. According to the information provided by the coordinator of each middle school, at the time of the approval process, there were approximately 235 mathematics teachers among the three types of middle schools (federal, state, and technical). The sample size analyzed in this study was 70 teachers at the beginning, but data cleaning led to the removal of several participants. The final sample size of this study was 64 mathematics teachers at the middle school level. This sample is 27% of the entire population of Mexican teachers teaching mathematics in this city of the Mexican borderland at the middle school grades. Having a greater sample size would provide results that allow making generalizations within the participants.
Another limitation to the study is that teachers who participated in this study belonged to two out of three possible types of middle schools in Mexico. Teachers from the three types would provide a better understanding and examination of the teacher knowledge of Mexican teachers in the border. However, this could not be achieved in this study because teachers from one type C middle schools were not willing to participate in the research. I tried to include teachers from the three types of middle schools. However, the lack of interest to participate in the study did not allow me to have a bigger sample. Therefore, the sample was only composed of 70 teachers who belonged to the participating schools mentioned above.

Another limitation of the study is that the sample was a convenience sample. As it was mentioned previously, the intention to include a bigger sample was not possible because teachers did not want to be part of the study. The sample of the study included 70 teachers who were willing to participate and were teaching at least one mathematics course at the middle school level at the time of the study. Therefore, this sample might be not representative and it is considered a limitation of the study.
References


testing. Paper presented at the research pre-session of the 79th annual meeting of the National Council of Teachers of Mathematics. Orlando, FL.


Appendix A: Omar’s case study

Observing Omar

During the spring semester of 2013-2014 academic year, I observed and videotaped Omar’s classrooms for a period of three classes. In the first classroom observation that I did, the topic “solution of simultaneous equations” was covered. This class was an eighth grade level. In the second and third class that I observed and videotaped, Omar was providing a review of the two previous units “linear equations” and “quadratic equations”. These two classes were ninth grade level. During each classroom observation I was allowed to walk around the classroom to try to capture as many details as possible.

Three of Omar’s classes were observed and videotaped. The different KtA situations (situations considered on the knowing-to act survey KtAS) that were identified during Omar’s classes are presented in Table A1. In addition, an example of each of the situation is provided in the chart. These observations were based on the protocol observation developed. The classroom situations identified were part of the three categories already described earlier in this chapter. During Omar’s classes, there were not identified any emerged situation, thus, this category is not included in Table A1.

Table A1: Knowing-to act Situations observed in Omar’s classes

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td>Another student asked him to check what he was doing if his procedure was right, and the teacher said yes (OC: KtA).</td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake:</td>
<td>Omar paid attention to what that group was doing and he told them:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O: it is not complete here.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S1: what do I need the operation?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O: the operation.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S1: that’s it?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>O: yes.</td>
</tr>
</tbody>
</table>
Item 2 KtAS. When a student does not recognize the same pattern in a different situation

S11: I do not understand here teacher.
O: which one?
S11: the sixth equation (then the teacher explained it to her and she said:)
S11: ah! Then it would be 4x
O: yes

Item 5 KtAS. When a student is unable to see an obvious pattern in the problem

One group asked the teacher “from where did you get this in the example?” and the teacher did not tell them in order that they could deduce it and think about it. After a few minutes, the teacher explained to them just one part and allowed the students to deduce the next part.

Item 10 KtAS. When students are having hard time to complete an assigned activity/task

S2: teacher, this can be moved to this side?
O: yes (he checked it).

Challenge to the Teacher

Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to do it

Other students asking what he needed to do to solve the equation, and the teacher answered her “look at the example” in order for the student to think about it.

Item 7 KtAS. When a student asks you a question from a different perspective that was not previously considered by you

When Omar is explaining the example he said that 3x is a multiplication. Then a student asked “why 3 was multiplied”. Then Omar explained to him when a number is together with a letter as 3x it is a multiplication, and he provided some examples.

The frequency of these KtA situations is presented in Table A2. Omar’s classes exhibited mostly the situations of the first category of student misconceptions. The situation most frequently identified in Omar’s classes was when a student solved a particular mathematical problem and asked if the answer or steps were correct. On 21 occasions Omar was exposed to this first KtA situation. The situation of when a student makes a mistake was identified on 4 occasions during Omar’s classes observed. Also, on 4 occasions the situation of when you assign an activity about a previously learned topic, and some students express that they do not know how to do it arose in Omar’s classes. There were other KtA situations that were identified several times as Table A2 shows.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Difficulties</td>
<td>Item 2 KtAS. When a student does not recognize the same pattern in a different situation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S11: I do not understand here teacher.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>O: which one?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S11: the sixth equation (then the teacher explained it to her and she said:)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S11: ah! Then it would be 4x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>O: yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 5 KtAS. When a student is unable to see an obvious pattern in the problem</td>
<td></td>
</tr>
<tr>
<td></td>
<td>One group asked the teacher “from where did you get this in the example?”</td>
<td></td>
</tr>
<tr>
<td></td>
<td>and the teacher did not tell them in order that they could deduce it and think about it.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>After a few minutes, the teacher explained to them just one part and allowed the students to deduce the next part.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 10 KtAS. When students are having hard time to complete an assigned activity/task</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S2: teacher, this can be moved to this side?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>O: yes (he checked it).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Challenge to the Teacher</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Other students asking what he needed to do to solve the equation, and the teacher answered her “look at the example” in order for the student to think about it.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 7 KtAS. When a student asks you a question from a different perspective that was not previously considered by you</td>
<td></td>
</tr>
<tr>
<td></td>
<td>When Omar is explaining the example he said that 3x is a multiplication. Then a student asked “why 3 was multiplied”. Then Omar explained to him when a number is together with a letter as 3x it is a multiplication, and he provided some examples.</td>
<td></td>
</tr>
<tr>
<td>Student misconceptions</td>
<td>When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>When a student makes a mistake</td>
<td></td>
</tr>
<tr>
<td>Student Difficulties</td>
<td>When a student does not recognize the same pattern in a different situation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td></td>
</tr>
<tr>
<td></td>
<td>When students are having hard time to complete an assigned activity/task</td>
<td></td>
</tr>
<tr>
<td>Challenge to the Teacher</td>
<td>When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td></td>
</tr>
<tr>
<td></td>
<td>When a student asks you a question from a different perspective that was not previously considered by you</td>
<td></td>
</tr>
</tbody>
</table>

**Omar’s actions in student misconceptions**

After identifying the situations identified in Omar’s classes, the emphasis is placed on the actions enacted by Omar in these situations. In the KtA situation of when a student solves a particular mathematical problem and asks if the answer or steps are correct, Omar acted in five different ways. This KtA situation is item 1 in the KtAS. Omar assigned several equations to be solved in groups.

S4: do I have to solve for x?
O: yes (then the student did it and)
S4: that is it?
O: why?

One student asked Omar if he needed to solve for x for that equation. Then Omar told her “yes”. The student was solving for x the equation. When she finished it, she asked Omar if “that is it”. Omar responded to her asking “why”. Then, the student was thinking until she understood why he questioned her. Omar’s action was to ask the student why she did that or think that. This
action allowed the student to think critically in order to know what she was doing and why she needed to do it.

During one of the Omar’s classes, a student asked the teacher to revise her work. Then, Omar checked it and said “yes it is right”. This classroom situation describes how Omar provided the correctness of one student solution. According to Mason and Spence (1999), this teacher’s action limits the student to think more about what she did and be able to know if what was done is correct or not.

On another occasion during Omar’s classes, one student asked Omar about his work. The student wanted to know if his work was right or wrong. Then, Omar started to revise what the student did. The teacher identified a mistake. Omar decided to tell him that there is a mistake in the x. Omar’s action was to tell the student about a mistake that he made. The Omar’s “knowing-to Act” is not desirable in this classroom situation according to the expert team that evaluated the situations and the possible actions presented in the knowing-to act survey (KtAS) and according to the best teaching practices recommended by Zemelman and colleagues (2005).

The following chart (Table A3) presents the codes assigned to the different actions performed by Omar when a student solves a particular mathematical problem and asks if the answer or steps are correct. Also an excerpt of the transcriptions for the classroom observations is provided as an example of the teacher’s actions that were coded.

<table>
<thead>
<tr>
<th>Teacher’s Action Code</th>
<th>Example of Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicating correctness or incorrectness</td>
<td>Another student asked him to check what he was doing if his procedure was right, and the teacher said yes</td>
</tr>
<tr>
<td>Asking for student's justification</td>
<td>Another student asked him whether his solution was correct, and the teacher asked him to explain his solution.</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>Another student asked him if he was doing it right, and the teacher said that he needs to write the complete procedure.</td>
</tr>
</tbody>
</table>
Ignoring student's work

Another student asked the teacher why his solution was wrong, but the teacher looked at the solution and he did not say anything and accepted the activity.

Revealing mistakes

One student stood up to ask for a review of her work. Then the teacher said:
O: it is ok but here it is 4.
S12: ah ok then it is 4.

These teacher’s actions that coded are based on the choices given in the KtAS. However, teacher’s actions that are not considered in the survey were considered in the analysis and coded as it is shown in Table A3 under the code of “Ignoring student’s work”. This data source which is qualitative in nature, allows the inclusion of different teacher’s actions that are not considered in the survey (KtAS) but that allow a response to the second research question, which ask how do the teachers act in KtA situations occurring in mathematics classroom? The relationship between the choices under item 1 and the teacher’s actions codes is that the code “indicating the correctness or incorrectness” is a subset of the choice 1. Choice 1 is to tell him/her whether his/her response is correct. The code “providing an explanation” is also a subset of choice 1 because it was guiding the student to the correctness or incorrectness of his or her solution. Additionally, “revealing mistakes” is a teacher’s action that is a subset of choice 1 of the KtAS. To reveal a mistake is implying that the student’s work is not correct.

The frequency of these coded actions is presented in Table A4. Omar acted most frequently asking for student’s justification when a student solved a particular mathematical problem and asked if the answer or steps are correct. Omar’s action was enacted on seven occasions during the classes observed and videotaped. However, Omar also decided on five occasions to indicate the correctness or incorrectness of student’s work. The other different actions performed are in Table A4 along with their frequencies.

<table>
<thead>
<tr>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table A4: Frequency Table of Omar’s actions under item 1 of the KtAS</td>
<td></td>
</tr>
</tbody>
</table>
One of the situations provided in the KtAS as item 9 is when a student makes a mistake. Omar’s classes exhibited this situation on four occasions. There were different actions observed that fell under this situation. In one class, Omar was looking at what the group was doing and then he said:

O: it is not complete here.

S1: what operation do I need?

O: the operation.

S1: that’s it?

O: yes.

In this segment of the transcription, Omar indicated the mistake to the student. Then the student was unsure about what was needed. She asked if the operation was missing and Omar said “yes”. This action revealed to the student where the mistake was. The knowing-to act of Omar was not called to promote critical thinking in this student. Omar’s action is coded as “revealing mistakes”. This action occurred two times during Omar’s classes. Omar’s action is a subset of the choice 4 which is to explain to him/her why it is wrong. To indicate the mistake to the student might be a way to help the student understand why his or her work is wrong.

When Omar used a new word in his explanation about the topic of linear equations, he usually continued explaining the topic and asked students about the meaning of that word. The word was “transponer” meaning transposition or transfer. Students answered saying that it is dividing, and Omar said “no, it is like transporting, moving, to move something to another place,
ok”. After that, he asked students if they had questions about this term. Students did not ask anything about it.

The described situation above shows how students did not know the meaning of the new word used by Omar in his explanation. He realized that student did not know because they made several mistakes when guessing the meaning. However, when they made that mistake, Omar decided to provide the definition of the word. The code assigned to this action is “providing an explanation”. This code is basically under choice 4 of explain to him/her why it is wrong. When students did not know the meaning of the word he explained the meaning. Thus, this teacher’s action is related to the choice. This action only was enacted on one occasion by Omar. During one of the classes, Omar was observing a work group and asked:

O: what operation is that? x is equal to?

S1: 10 ....

O: 10 what?

S1: 10/2

O: and what is the result of 10/2?

In this situation, Omar was asking the student about the work that he did. The student was justifying his solution. Omar made the student think about what he did. This action performed by Omar in this KtA situation was identified on only one occasion during Omar’s classes observed. The code assigned to this teacher’s action is “asking student for explanation”. According to the expert team and Zemelman and colleagues (2005), this action might increase the reasoning of students. This action is related to choice 5. It is to let the student explains his/her answer. In the classroom situation above, we saw how Omar asked the student about his work. Thus, this
teacher’s action is referenced under choice 5. The teacher’s actions described and analyzed above were enacted in the first category of KtA situations.

**Omar’s Actions on students’ difficulties**

The second category of the KtA situations is *students’ difficulties*. In this category only three different types of KtA situations occurred. The first type of situation identified is when a student does not recognize the same pattern in a different situation. This situation was identified in Omar’s classes only on one occasion.

S11: I do not understand here teacher.

O: which one?

S11: the sixth equation (then the teacher explained it to her and she said:)

S11: ah! Then it would be 4x

O: yes

In one class, Omar explained how to solve linear equations and assigned several exercises to be solved by students in groups. After a certain time of students working on it, one student called Omar. This student (S11) expressed her lack of understanding of the topic. Then Omar explained the problem to her in a way that she would grasp it. Finally, the student understood it. Omar’s action was to provide an explanation to the student in a different way than what he had already explained to the class. This action was coded as “providing an explanation”. This was coded as choice 3 under item 2 of the KtAS. Choice 3 is to look for another way to represent the problem for the student. Omar explained to the student using another way to represent the problem in order for the student to make sense of it.

The situation when a student is unable to see an obvious pattern in the problem was identified on only one occasion during Omar’s classes. He explained the topic covered, and he
usually provided one or several examples about it before any class activity or worksheet was distributed to the students. Then, one group of his class asked the teacher “from where did you get this in the example?” Omar listened to them. However, he did not tell them anything and he still was there with the group. This action was performed in order for the students to be able to deduce and think about it because after a few minutes, Omar explained just one part of the example to the students and allow the students to deduce the next part.

The Omar’s action in this situation is coded as “providing time or hints”. This teacher’s action is a subset of choice 2 under item 5 of the KtAS. Choice 2 says: *give the student clues to help him/her find the pattern*. Omar was providing an explanation that helped the student to deduce the way to get the solution. In other words, Omar’s explanation could be seen as a kind of clue to assist the student to find the pattern. In this case the clue was to understand how the example was solved.

The other situation identified during Omar’s classes that belongs to this category is when students are having a hard time completing an assigned activity/task. This KtA situation was detected on only two occasions. In both occasions he acted in the same way. After the explanation about linear equations, Omar assigned several equations to be solved on a worksheet. This work could be done by groups. Some students were having a hard time solving the equations. One student was trying to solve an equation and after several minutes of working on it, the student called the teacher.

*S10: listen teacher. This can be moved to this side?*

Omar saw that this student was having difficulties to do the work. When this student (S10) asked him about it, Omar explained the procedure to him. Omar’s action was coded as “providing an explanation”. This code is related to the KtAS. Item 10 has five choices. Choice 5
is where Omar’s action is classified. To explain the assignment using examples or another way of explaining it is choice 5 in the KtAS. Table A5 shows the teacher’s actions codes and the frequencies of the actions performed by Omar in the category of student difficulties. Omar’s action enacted more frequently was “providing and explanation”. It appears on two times on Table A5 because this action was performed in both types of KtA situations. However, if this action enacts Omar’s “knowing-to act” ability or not depends on the situation.

Table A5: Frequency Table of Omar’s Action Codes in The Second Category

<table>
<thead>
<tr>
<th>KtA situation</th>
<th>Teacher’s Actions Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>When a student does not recognize the same pattern in a different situation</td>
<td>providing an explanation</td>
<td>1</td>
</tr>
<tr>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>providing time and hints</td>
<td>1</td>
</tr>
<tr>
<td>When students are having a hard time completing an assigned activity/task</td>
<td>providing an explanation</td>
<td>2</td>
</tr>
</tbody>
</table>

Omar’s Actions in Challenge to The Teacher

The third category of KtA situations identified during Omar’s classes is challenge to the teacher. The first KtA situation identified in this category is when you assign an activity about a previously learned topic, and some students express that they do not know how to do it. During the classes observed, this situation happened and was detected four times. Omar acted in two ways. There is one unit that is a review of two previous units in the curriculum.

S3: how?
O: look at the example.

This short segment of the transcription shows how a student did not know how to solve an equation. The topic of linear equations was already taught in one previous unit. In addition, Omar provided a review of this topic. He explained it in a new way. Examples were provided. The student (S3) asked Omar how to do it. Omar referred the student to the whiteboard to see
the example provided. To refer to the example is Omar’s action in this situation. This KtA situation is described in Item 4 of the KtAS. The code assigned for Omar’s action is “referring to the whiteboard or notes”. This code is related to the KtAS in choice 5, item 4. Choice 5 is to ask students to review the topic by themselves and do the activity. Analyzing Omar’s action of referring the students to see what is written on the whiteboard is forcing them to review how and why it was done. Omar’s Action was enacted twice during classes.

During Omar’s classes, students worked in groups. Students were asked to work solving equations. Omar was walking around the groups supervising students’ work. A student (S1) asked “teacher, how is this? “ Then Omar only saw her work. He did not respond anything. However, he was still with that group. Other teammates of the same group explained to the student (S1) how to do it while Omar was paying attention to them. Then Omar said “yes, it is done like that.”

In the situation described above, Omar acted in a way that allows students to help their teammate. The group of students had to think about the problem by discussing it among themselves to recall how to solve that equation. When they used their recalls and were doing fine the task, Omar confirmed the students’ work was correct. This teacher’s action is coded as “allowing the teammates to help the student”. This action is a subset of choice 2 of item 4 of the KtAS. Choice 2 is to ask students do it using their recalls. Omar’s action is related to choice 2 because he allowed the students to help their classmate and discuss what they knew and had in their minds at that moment. This action was enacted two times in Omar’s classes.

KtA situation, item 7 is when a student asks the teacher a question from a different perspective that was not previously considered by him or her. This situation was identified twice
in Omar’s classroom during the observations. The same action was enacted by Omar on both occasions.

S1: why are they called linear, because they are like a line or what? (Then many of his classmates laughed and said “ashhh,” “ooooo”)

O: the linear equations or first grade equations are called like that because the letter that they have (variable) are to the power 1. That’s why it is a first grade equation, if we have a letter to the power 2, it would be a second grade equation or quadratic equation or if it is to the power 3 it would be a third grade equation, and so on. The greater the exponent in the equation indicates the type of equation that it is. (Omar provided an example about this). Linear equations are called like that because at the moment that we represent it in a graph we have a line. (Then Omar drew a line on a graph on the whiteboard. Also, he drew a parabola for quadratic equations). Then you know why it is called linear equation.

Do you have another question?

Omar was teaching linear equations. He was explaining the topic to the whole class. Suddenly, one student called Omar. He allowed the student to ask his question. This question looked at something obvious. However, this student did not know why those equations were called like that. This simple question was done from the student’s perspective which might be not have been considered by Omar before. The action done by Omar was to provide an explanation about the name of linear equations according to his knowledge. Thus, the assigned code for this teacher’s action is “providing an explanation” which is a subset of choice 3 of item 7 of the KtAS. This choice includes trying to respond to a student’s question to the best of your knowledge. Basically, the action described under choice 3 was enacted by Omar when he provided the explanation.
Table A6 presents the frequency of the actions performed by Omar during the classes observed and videotaped. The types of KtA situations and teacher’s actions enacted in those situations are also presented in Table A6.

<table>
<thead>
<tr>
<th>KtA situation</th>
<th>Teacher’s Actions Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>Referring to the whiteboard or notes</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Allowing the teammates to help the student</td>
<td>2</td>
</tr>
<tr>
<td>When a student asks you a question from a different perspective that was not previously considered by you</td>
<td>Providing an explanation</td>
<td>2</td>
</tr>
</tbody>
</table>
Appendix B: Rogelio’s case study

Observing Rogelio

I observed and videotaped Rogelio’s classrooms during the spring semester of the 2013-2014 academic year for a period of three classes. Those classes were 9th grade. Rogelio worked on worksheets in his classes. The topics that were covered on the classes observed were “linear equations” and “real word problems about linear and quadratic equations”. The methods to solve quadratic equation that were taught were “factoring and the general formula”. I was allowed to walk around in the classroom with the video camera hanged on my neck to better capture the emerged situations where knowing-to act should be or was enacted.

Several KtA situations were identified on multiple occasions during Rogelio’s classes. Table B1 presents the category of the knowing-to act situations, the KtA situation, and an example of each classroom situation where the KtA situation was identified in Rogelio’s class. There were 10 different KtA situations observed in Rogelio’s classes. The three types of KtA situations of the category of student misconceptions were recognized.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Rogelio’s Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct</td>
<td>S15: teacher, am I doing well? R: yes, you are right, but you need to use the parenthesis. X and X in each parenthesis, if you want it to do it this way, if not…</td>
</tr>
<tr>
<td></td>
<td>Item 3 KtAS. When a student provides a nonsensical solution to a given problem</td>
<td>S2: its multiplication should be 0 R: no, its multiplication should be this number: 396. And he continued explaining the problem.</td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake</td>
<td>R: multiplying, so to what side is it going to be? S2: toward the x. R: no no you cannot move to the x, S3: move toward the other side. R: to what? S4: move toward the number 8.</td>
</tr>
<tr>
<td>Student Difficulties</td>
<td>Item 5 KtAS. When a student is unable to see an obvious pattern in the problem</td>
<td>S10: teacher it is divided by what? R: it is divided by the number of grades. How many grades are there? S10: 3</td>
</tr>
</tbody>
</table>
Item 8 KtAS. When a student continuously responds with wrong answers to questions posed by you

Then he put x on one side and he asked what are we going to write on the other side?

S12: x4

R: nooo, it would be 4 times. Students provided wrong answers: “4x, 8x, 4*10”

R: nooo, we need to add, heeey if I have 4 units greater

S13: x+4

Item 10 KtAS. When students are having a hard time completing an assigned activity/task

When Rogelio saw that students had not solved the problem, he wrote another step to the solution on the whiteboard

Challenge to the Teacher

Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to do it

One student called the teacher and he went to the student’s seat. Rogelio is asked about how to use factoring. Then he explained to the student how to do it.

Unexpected Emerging Situation

When a student is explaining his/her reasoning

Then Rogelio said that the other way is using the general formula. He asked students if they remember how it is, he did not provide time to answer, but he wrote it on the whiteboard.

When a student is willing to participate…

R: why? Because x by x is square x (he did not wait students’ response). Then the signs: + and -, + by -, then we need to seek two numbers that…

When a student provides an answer to a given problem

R: nooo, we need to add, heeey if I have 4 units greater

S13: x+4

R: yes it is. Very good.

The frequency of these KtA situations is presented in Table B2. Rogelio’s classes exhibit more unexpected emerging situations than the other KtA situations considered in the protocol observation. The first category of students’ misconceptions includes three types of KtA situations. The first type of KtA occurred more frequently in this category.

Table B2: Frequency Table of the KtA situations in Rogelio Classes

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>When a student provides a nonsensical solution to a given problem</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When a student makes a mistake</td>
<td>7</td>
</tr>
<tr>
<td>Student Difficulties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------------------</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>When a student continuously responds with wrong answers to questions posed by you</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>When students are having a hard time to complete an assigned activity/task</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Challenge to the Teacher</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Emerging Situation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>When a student is explaining his/her reasoning</td>
<td>3</td>
</tr>
<tr>
<td>When a student is willing to participate...</td>
<td>14</td>
</tr>
<tr>
<td>When a student provides an answer to a given problem</td>
<td>7</td>
</tr>
</tbody>
</table>

**Rogelio’s Actions in student misconceptions**

The first “knowing-to act” situation, item 1 in the KtAS is when a student solves a particular mathematical problem and asks if the answer or steps are correct. Rogelio provided worksheets to the students to work on. After a certain time, a student said that she finished it and asked Rogelio for revision. Then, Rogelio looked at the student’s notebook and checked the solution. He said “ok, but nobody else has finished. Thus, let the rest of the class finish it”. After that, Rogelio asked the student about the verification of the solution of the problem, and she showed her verification. Finally, Rogelio said that “it is correct”. In this classroom situation, Rogelio acted asking for verification of the solution obtained by the student. This teacher’s action is coded as “asking for verification”. This action was performed in only one time during Rogelio’s classes in this KtA situation as Table B3 shows.

Another action enacted by Rogelio in the same KtA situation is described in the following excerpt of the transcription of the observations conducted.

S15: teacher, am I doing well?
R: yes, you are right, but you need to use the parenthesis. X and X in each parenthesis, if you want it to do it in this way, if not…(Rogelio is looking in the student’s notebook notes about the general formula to solve quadratic equations, but the students did not have it, hence the teacher wrote it on her notebook).

One student (S15) asked Rogelio about his work. The student wanted to know if his task was right. Rogelio told the student that his work until that point was correct. The student obtained correctly the equation from the real world problem assigned. Thus, Rogelio told him that “you are right”, however, the next step of the problem was to solve the equation. Therefore, Rogelio suggested to him two ways of solving the equation. The action performed by Rogelio was to indicate the correctness of the steps done by the student. Rogelio’s action enacted in this situation was coded as “indicating the correctness or incorrectness” of the student’s task. This action was done 3 times as table B3 presents. In Rogelio’s classes, students work in groups. One student called Rogelio to ask him something.

S16: teacher is this right?
R: 2*1 follows the formula (ktA)
S16: then here are 16, and 7 and then this …
R: aha but here is missing something, 4ac… (Rogelio told her where her mistake was).
S16: it means this should be this
R: aha ok.
S16: ah ok I got it.
R: do you know how? Did you remember it?
S16: yes, I know how to do it.
One student (S16) asked Rogelio if her problem was right. Rogelio approached the student and asked her to follow the formula. Then, the student concluded what should be in the problem’s solution. However, she had a mistake, and Rogelio told her what the mistake was. Then, the student continued asking questions to reaffirm what she understood. Rogelio’s action was to reveal the student’s mistake. The code assigned to this action is “revealing mistakes”. This action was performed on four occasions during Rogelio’s classes observed as Table B3 illustrates. Most of the time, Rogelio was supervising students’ work. He walked around the groups and helped students with questions.

S30: teacher, please come here (the student showed his work to Rogelio)
R: this information (a, b y c) needs to be substituted, now we need to make operations, this is at the power 4x4=16 it would be 16… then the result would be…

The student (S30) called Rogelio when the student had already obtained the equation from the word math problem. The student did not know what the next step was to solve the whole problem. Then Rogelio started to explain what the student needed to do in order to solve the problem. The action enacted in this situation by Rogelio was to explain to the student what he or she needed to do. This action was coded as “providing an explanation”. Rogelio provided an explanation to the students in this type of KtA situation on three occasions as Table B3 described.

<table>
<thead>
<tr>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asking for verification</td>
<td>1</td>
</tr>
<tr>
<td>Indicating the correctness or incorrectness</td>
<td>3</td>
</tr>
<tr>
<td>Revealing mistakes</td>
<td>4</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>3</td>
</tr>
</tbody>
</table>

Rogelio acted by indicating students’ mistakes when they asked about the correctness or incorrectness of their work. Also, in several times during the observations, he decided to provide
an explanation or indicate the correctness or incorrectness of the student’s task. These codes were created based mainly on the choices of the KtAS. The teacher’s action code “asking for verification” is a subset of choice 5, item 1 of the KtAS. Choice 5 is to ask the student to explain his/her solution. To ask for verification is leading to the student to know how and why he or she gets a certain result. The code “indicating the correctness or incorrectness” is mentioned under choice 1 of this item. Choice 1 is to tell him/her whether his/her response is correct which is basically what Rogelio did on 3 occasions. Revealing a mistake allows the student to discover the incorrectness of his or her task, “revealing mistakes” is a teacher’s action code which is a subset of choice 1. Rogelio’s action of “providing an explanation” is related to the choice 1 because Rogelio provided an explanation to the students with enough information in order to allow them to grasp the correctness or incorrectness of the task.

The KtA situation when a student provides a nonsensical solution to a given problem is item 3 of the KtAS. This situation was only identified once during Rogelio’s classes. Rogelio assigned a mathematics real world problem about area where a quadratic equation was involved. The equation had to be equal to the area of a rectangular land which was 396. However, one student said:

S2: its multiplication should be 0

R: no, its multiplication should be this number: 396. (Rogelio continued explaining the problem.)

Students were trying to get the quadratic equation that represents the area of the land. The students said that dimensions multiplied must be equal to zero “its multiplication should be 0”, thus, the student’s answer (S2) did not make any sense. Then Rogelio told the student the correct answer should be 396. Rogelio’s action was coded as “providing the solution”. This action is
related to the KtAS item 3 with choice 4 which is to tell the student how the problem should be solved step by step. Rogelio’s action in this situation is considered a subset of choice 4 because Rogelio told the student what should be done and explained the problem to be solved.

When a student makes a mistake is a KtA situation identified in Rogelio’s classes seven times as Table B2 described. In those situations, Rogelio acted in three different ways. Rogelio provided an explanation, or ignored students, or revealed student’s mistakes. Rogelio was solving and explaining a mathematics real world problem of the worksheet among the class. This problem involved areas of land.

R: Ok, then the dimensions of the land are? I cannot write negative numbers for the land’s dimensions, right?

Students: no.

R: then its height is 18 and what about the width, how many will it be?

S8: 22.

R: why?

S9: because it is negative.

R: no. according to the problem, what the measure is of the greater side is x+4. 22.

S10: but it is negative.

R: ok here we got a negative number -22 +4, because there are not negative measures of land or when have you seen a negative area? Of course not; anytime that you are dealing with lands, shapes, we will never have negative numbers.

Rogelio told the student since the beginning of his explanation a key concept that would promote critical thinking. However, he provided it before students could discover it. Even though Rogelio told them lands are not negative numbers, three students were mistaken about it. Then,
Rogelio provided an explanation about why areas of lands could not be negative. The action enacted by Rogelio was to give an explanation to clarify this key concept to the students. This action was coded as “providing an explanation”. This teacher’s action was observed four times during Rogelio’s classes when a student makes a mistake as Table B4 presents. This code is related to choice 4 which is to explain to him/her why it is wrong. Rogelio acted in the same way that choice 4 described. Choice 4 is given under item 9 of the KtAS.

R: ok because I am solving for this variable, I am going to move this number by subtracting. We are going to have \( x=24-14.2 \). Do not do the common mistake of saying oh it is 10, because it is not, you need to pay attention to the .2 of the 14. So the result is?

S7:10.2

R: no no it could not be 10.2 this is the common mistake.

S8:9.8

R: 9.8. then what is the grade that this person needs to get an average of 8?

Rogelio was solving and explaining a mathematics problem on the whiteboard. He tried to involve all the class; thus, he was asking students procedural or operational questions. Rogelio was explaining the problem about a linear equation to get a grade average without knowing one grade out of three. Rogelio asked the class what the result of a subtraction with decimals is. He advised students about a common mistake made in previous classes. However, the students mistaken were in the same area that Rogelio had discussed. Then, Rogelio said this is the common mistake and waited for a correct response from any student in the class. Finally, one student provided the right answer and Rogelio continued with his explanation of the problem.

Undoubtedly, in the situation above described, Rogelio decided to ignore the students because he already had discussed the concept with them. Thus, Rogelio waited for a right answer.
The code assigned to this action is “ignoring the student's response”. The situation described is identified as the ktA situation of item 9. Rogelio’s action code is related to choice 2 of this item. Choice 2 indicated “ignore it.” which is clearly the action enacted twice by Rogelio when a student made a mistake. The frequency of this action is shown in Table B4.

Rogelio was formulating the problem assigned about the area of a rectangular land. He was analyzing the information provided in the problem to obtain the dimensions of the land. He continued explaining the problem, he talked about the dimensions that one side is x and the larger side is 4 units greater than the other side. Then, Rogelio wrote x on one side and asked:

R: what we are going to write in the other side?
S12: x4
R: nooo, it would be 4 times

One student (S12) responded incorrectly to Rogelio’s question. Rogelio without any doubt replied to the student with a “nooo” and then he explained why the student was wrong. Rogelio revealed to the student the mistake made and explained why it was wrong. This action limited the opportunity for the student to think critically because Rogelio did not allow the student discovered his mistake and deduced why was wrong. This action is coded as “revealing mistakes”. Rogelio’s action is related to choice 4 of item 9 of the KtAS. Choice 4 is to explain to him/her why it is wrong. The action described in this choice is exactly what Rogelio did once as Table B4 presents. Rogelio’s “knowing-to act” is considered as not desirable in this KtA situation because Rogelio’s actions limited the student to think critically. Here, we can see what is mentioned in Zemelman and colleagues (2005) as teaching by telling was mostly enacted by Rogelio.

Table B4: Frequency Table of Rogelio’s Actions in item 9 of the KtAS
<table>
<thead>
<tr>
<th>Teacher's Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ignoring student’s response</td>
<td>2</td>
</tr>
<tr>
<td>Revealing mistakes</td>
<td>1</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>4</td>
</tr>
</tbody>
</table>

**Rogelio’s Action in student difficulties**

In the category of student difficulties, there are three KtA situations identified during Rogelio’s classes as Table B2 presents. The first situation recognized in this category is when a student is unable to see an obvious pattern in the problem. It is item 5 of the KtAS. This KtA situation was seen only once during Rogelio’s classes as Table B5 illustrates. Rogelio assigned a worksheet. One of the problems included in the worksheet was about a linear equation to get the average of three grades without knowing one grade.

S10: teacher it is divided by what?

R: it is divided by the number of grades, how many grades are there?

S10: 3

R: then by 3.

In the excerpt of the transcription above, the topic of average was dealt with. To know how to calculate an average is a topic previously learned at the ninth grade level. The segment of the transcription above mentioned how one student (S10) asked Rogelio by what; the sum of the grades should be divided. He did not make students think. He did not allow students to make conjectures. Rogelio told him the number of grades. He provided the right answer to the student’s question. Rogelio’s action is coded as “providing the solution”. This action is related to choice 1 of item 5 of the KtAS. *Tell the student what the obvious pattern is*, refers to choice 1. In this case, the obvious aspect in the KtA situation identified was not a pattern, but it was an obvious part of the procedure to calculate an average. Rogelio told the student the obvious part of the procedure worked by him. Rogelio’s action was performed once as Table B5 shows.
When a student continuously responds with wrong answers to questions posed by you is one KtA situation recognized under the category of *student difficulties*. This situation was identified twice during Rogelio’s classes. Rogelio was obtaining the value of the dimension of a rectangular area of land. Then, he put x on one side of the rectangle, and he asked what are we going to write on the other side?

S12: x4

R: nooo, it would be 4 times. (Students provided wrong answers: “4x, 8x, 4*10”)

R: nooo, we need to add, heeey if I have 4 units greater

S13: x+4

In the excerpt of the transcription above, several students replied to Rogelio’s question. However, students’ answers were wrong again and again. Thus, Rogelio decided to help the students and said “we need to add, heeey if I have 4 units greater”. Then, one student immediately grasped it and provided the correct answer to Rogelio’s question. Rogelio’s action was to help the students by giving them a hint. It was the word “add”. The teacher’s action code assigned was “providing hints or time”. Rogelio’s action is not related to any choice under item 8 of the KtAS. This teacher’s action was enacted twice as Table B5 shows.

Another KtA situation identified during Rogelio’s classes observed was when students are having a hard time completing an assigned activity/task. This KtA situation is item 10 in the KtAS. Rogelio acted in two ways: one was asking for verification and providing one part of the procedure for solving the problem. The following excerpt shows one classroom situation classified as this KtA situation.

S11: teacher so we have to add and divide what?
R: we already know the value of x, I add the 3 grades, this, this and this. And then divide it by 3 to see if we get 8. That is the verification of the problem is it right?

Rogelio was walking around the groups to supervise students’ task. Then, one student (S11) called him. Rogelio observed what the student had and responded to him asking for verification. Rogelio explained to him briefly how to verify it. And then, Rogelio asked if it was right. But this question would be answered after the verification. Rogelio’s action was coded as “asking for verification”. This teacher’s action is not related to any choice under item 10 of the KtAS. In the following segment of the transcription, Rogelio noticed that students were having a hard time accomplishing the task.

R: ok let me help you a little.
S8: it is what is inside.
R: inside of what?
S8: of the shape?
R: ah ok, it is all the space of the shape, very good and what shape is it?
S9: a rectangle.
R: a rectangle, then all the area is.
S10: 396 square meters.
R: what are the dimensions?
S11: it is the perimeter.
R: the sides and if I do not know them, I look the x, but it says that the longer side is greater 4 units than the other.
S12: x+4.
R: very good, then if the smaller side is x and the greater side is x+4.
Rogelio realized that the students were having difficulties solving the problem, thus, he decided to help them. Rogelio was explaining and asking students questions to get to the formulation of the problem in order to get the quadratic equation. In other words, Rogelio helped students doing one part of the problem. It was the interpretation of the information in the problem to obtain the equation. The code assigned to this action was “providing part of the solution”. Rogelio did one part of the problem in order to help students solve the problem. Rogelio’s action is related to choice 3 of the item 10 of the KtAS. Choice 3 mentioned that do half of the assignment and leave the other half to the students. In this case, Rogelio did not do the half of the assignment. However, he did one part of the problem. Therefore, Rogelio’s action is a subset of choice 3 which is wider. Table B5 presents the frequency of this action. Also, Table B5 shows the frequencies of Rogelio’s actions enacted during the KtA situations of the category of student difficulties.

<table>
<thead>
<tr>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Providing the solution</td>
<td>1</td>
</tr>
<tr>
<td>Asking for verification</td>
<td>1</td>
</tr>
<tr>
<td>Providing parts of the solution</td>
<td>2</td>
</tr>
<tr>
<td>Providing hints or time</td>
<td>2</td>
</tr>
</tbody>
</table>

**Rogelio’s Actions in challenge to the teacher**

The category of challenge to the teacher was also presented in Rogelio’s classes. This situation is when you assign an activity about a previously learned topic, and some students express that they do not know how to do. In one of the classes, students were asked to work on a problem that involved the solution of a quadratic equation. Rogelio started to explain the problem and how to use the general formula to solve the quadratic equation. The general formula was a topic taught previously. He asked students to make the substitution of the numbers in the
formula. He provided 10 minutes to do just the substitution of the coefficients in the formula.

One student said:

S28: I do not know how to do it.

R: you did not pay attention. You only need to exchange the numbers by letters.

S28: then I will do this?

R: yes, it is.

Although a previous explanation was given in that class and the topic was already taught before, one student (S28) did not know how to do it. Rogelio decided to explain to the student what he needed to do to make the substitution of the coefficients on the formula. Rogelio’s action was to explain how to do it. This action was coded as “providing an explanation”. Rogelio acted in this way five times when students expressed that they did not know how to do it even though the topic was previously covered. The code “providing an explanation” is a subset of choice 1, item 4 of the KtAS. Choice 1 is to re-teach the previous topic. The action described under choice 1 is clearly what Rogelio did in this situation.

Rogelio’s Actions in emerging situations

The last category of situations identified in Rogelio’s classes observed was emerging situations. This category includes the classroom situations not considered in the protocol observation. There are three types of situations that emerged in this category: when a student is explaining his/her reasoning, when a student is willing to participate, and when a student provides an answer to a given problem.

When Rogelio found himself in the situation of when a student is explaining his/her reasoning, he acted in two manners. One action performed by Rogelio was interrupting the student's explanation. Not providing time for answering was the other manner of acting of
Rogelio. In one of Rogelio’s classes, Rogelio selected a student to solve a problem on the whiteboard in front of the class. Then, the student was solving the problem in front of the class. However, Rogelio interrupted the student several times because she was skipping steps according to him.

In this situation described above, Rogelio was interrupting the student to explain the steps or what the student missed. The student was solving and explaining the problem assigned, however, she was not allowed to provide her complete explanation. Rogelio’s action is coded as “interrupting student’s explanation”. This teacher’s action was enacted once. Due to this situation emerging during the observation and analysis, it is not included in the KtAS or in the protocol observation.

R: x by 4 is? 4x. Then we have an equation, it is taking shape, here I can solve for x, this can be moved to the other side, if in this side was adding the other side will be?

Students: subtracting.

R: and then, miss, what type of equation is it? It is a quadratic equation and it is solved in two ways: which are they? (he did not provide time for answer the question)

In the previous excerpt of the transcription of the observation conducted, Rogelio is asking the student a simple multiplication between a variable and an integer. He continued explaining how to solve for x in the equation. He asked students how one term that is moved to the other side is, and the students answered correctly. Then Rogelio asked one student (he called her by miss) about the type of equation that they got, however, Rogelio did not provide enough time for the student to answer his questions. Rogelio’s action is described as not allowing the student to answer him because he continued talking, and he answered his own questions. The
code assigned to Rogelio’s action was “not providing time for answering”. This action was performed by Rogelio twice.

When a student is willing to participate is one of the emerging situations in Rogelio’s classes. Most of the time, Rogelio assisted the students by providing an explanation of one key part of the problem in order for students to be able to solve the problem completely without any difficulty just following what he said. The following classroom situation exemplifies this action.

Rogelio started one of the classes assigning one problem on the worksheet. Rogelio already had written the problem on the whiteboard. Rogelio began to explain it. Essentially, the hardest part is to get a linear equation in order to discover one unknown grade. The question on the problem is “what is the other grade in order to have an average of 8.0?” Rogelio started to solve it and explained it step by step. Then, he obtained the equation from the problem without allowing students to make any attempt to get the equation.

This situation described exhibit the lack of knowing-to act of Rogelio. Limiting the student’s participation just to provide his own explanation or the procedure or solution of the problem is a weakness of the teaching practice of Rogelio. This action was coded as “providing a key part or concept of the solution”. Rogelio’s action was enacted 11 times as Table 32 presents.

In the similar type of situation identified, Rogelio also acted in a different way. Rogelio assigned a problem to be solved by the students. Then, one student asked Rogelio something about the equation. Rogelio replied to the student saying that “they need to solve from that point” Rogelio was solving the problem on the whiteboard but he left it incomplete, thus, he was asking for the solution from the point where he left off. Then, Rogelio said “you need to solve it; if you cannot in few moments we are going to solve it all together”.

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The last comment made by Rogelio in this situation indicated the lack of promoting critical thinking. Rogelio did not encourage students to try intensely to learn mathematics. He provided the hardest part of the problem to the students, thus, students did not have to make their best effort to solve the problem and construct their mathematical knowledge. Rogelio’s action in this situation is coded as “not promoting critical thinking”.

Only once in this type of emerging situation, Rogelio asked students to participate as can be seen in the following excerpt of the transcription of the class observation conducted.

R: the x is 9.8. be quiet Carrillo, is there any way to verify if it is correct? Of course if I did it (he was joking). Who want to verify it?

Students said “you teacher”. (Then because nobody wanted to verify Rogelio did it.)

In this example of the classroom situation, Rogelio asked students to participate in the class by going to the front and doing the verification of the problem on the whiteboard. However, students did not want to participate and Rogelio opted for doing the verification. Rogelio’s action was asking students to verify the problem. This action was coded as “asking for student’s participation”. This action was enacted only once as Table B6 illustrates.

The last type of unexpected emerging situation identified in Rogelio’s classes observed is when a student provides an answer to a given problem. In this type of emerging situation, Rogelio acted in two opposite ways: one is asking for student’s justification and the other is “not asking for student’s justification”. The following example of this classroom situation showed the “knowing-to act” of Rogelio.

R: ok it will multiply the 8. We could do it earlier, but in order to keep the sequence we are going to do it now. 8*3, then it is 24. Then we write it. And we almost finished the problem, no? What more?
S5: subtract.

R: but why am I going to subtract?

S6: because this number is adding so it should be moved to the other side subtracting

In this brief excerpt from the transcription, we can see how Rogelio is explaining and solving a problem. Rogelio asked students “what more?” what else was needed to be done. One student (S5) answered him correctly. However, Rogelio asked the student “why”. Then, the other student (S6) provided the justification of Rogelio’s question. Rogelio’s action was to ask students for justification of their responses. This teacher’s action was coded as “asking for justification”. Rogelio acted in this way twice in this type of emerging situation as Table B6 presents.

In other side, the opposite action was done five times by Rogelio in the same type of situation. Rogelio was explaining and solving one problem assigned on the whiteboard. One student was asking questions about the problem. Rogelio asked this student to go to the whiteboard to continue solving the problem. Then, the student asked:

S15: will I just write it?

R: yes, and I will explain it next

S15: it is easy. (Rogelio was asking students to be quiet.)

S15: a+c= 399 (he wrote it)

Rogelio asked the student (S15) to pass to the whiteboard to solve the problem. He was not interested in listening to any student’s justification of his solution. Rogelio asked the student just to write the problem on the whiteboard and the explanation of the solution was going to be provided by Rogelio. This example of a classroom situation clearly shows how Rogelio did not promote the students’ critical thinking. This teacher’s action was coded as “not asking for
justification”. Rogelio usually teaches by telling; Zemelman and colleagues (2005) mentioned that this action decreases the opportunity for students to engage in reasoning about their work.

The frequencies of the teacher’s actions enacted in the category of unexpected emerging situations are presented in Table B6. In the emerging situations, Rogelio usually provided a key part or concept of the problem to be solved. This action was enacted 11 times. The second more frequently action performed by Rogelio in this type of situation was “not asking for justification”.

Table B6: Frequency Table of Teacher’s Action Codes for The Category of Emerging Situations.

<table>
<thead>
<tr>
<th>Teacher's Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Providing a key part or concept of the solution</td>
<td>11</td>
</tr>
<tr>
<td>Asking for participation</td>
<td>1</td>
</tr>
<tr>
<td>Not promoting critical thinking</td>
<td>2</td>
</tr>
<tr>
<td>Interrupting student's explanation</td>
<td>1</td>
</tr>
<tr>
<td>Not providing time for answering</td>
<td>2</td>
</tr>
<tr>
<td>Asking for justification</td>
<td>2</td>
</tr>
<tr>
<td>Not asking for justification</td>
<td>5</td>
</tr>
</tbody>
</table>
Appendix C: Maria’s Case Study

Observing Maria

Maria is the last case study of this research. During the spring semester of the 2013-2014 academic year, I observed and videotaped Maria’s classrooms for a period of three classes. The topic that she was covering was “linear equations and solution methods” during the three classes. In the first class observed, Maria worked on exercises of the textbook. In the second class, worksheets were provided to the students. And in the last class observed, students presented the problems that were on the worksheet the day before. I was also allowed to walk around the groups to be able to get a better and detailed account about the teacher’s actions.

Considering the protocol observation developed for conducting these classroom observations, there were seven KtA situations recognized in Maria’s classes observed and videotaped. These KtA situations belong to the three categories of situations on which the KtAS was based. In addition, data was also collected about unexpected emerging situations where the teacher’s knowing-to act should be or is enacted. Under this distinction of situations, the category of unexpected emerging situations was created. Maria’s classes observed exhibited two types of situations not considered in the protocol observation as Table C1 presents.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Example of Maria’s Classroom Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student misconceptions</td>
<td>Item 1 KtAS. When a student solves a particular mathematical problem and asks if the answer or steps are correct:</td>
<td>One student asked her if his work is correct and he explained to the teacher what he did. Maria asked him “why did you do this?” and the students answered her “this” and she explained hi why he is wrong.</td>
</tr>
<tr>
<td></td>
<td>Item 9 KtAS. When a student makes a mistake</td>
<td>She walked around the groups and stopped with one student. She was observing the student’s work and asked why he did this, then the student explained to her, but she asked the student to do first certain steps and she explained what he needed to do.</td>
</tr>
</tbody>
</table>

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| Student Difficulties | Item 2 KtAS. When a student does not recognize the same pattern in a different situation | One student was asking several questions during the class. This student went to ask the teacher again and she said “no, you should know how to do it by yourself and go to sit down”.

Item 5 KtAS. When a student is unable to see an obvious pattern in the problem | Another student asked her about one problem and Mayra said “do it by yourself, read it again and compare it to the solution on the whiteboard”

Item 8 KtAS. When a student continuously responds with wrong answers to questions posed by the teacher | M: let’s see Escamilla, yes, it is right, but where did you get the positive sign in this term?
S6: ah when I did this, this number is multiplied by -36, is not it?
Students: no. (Maria asked if somebody wanted to help Escamilla. A student went to the front, and they were discussing the solution. Then Maria asked them)
M: Where are you struggling?
S6: we do not remember from where we got this term, and how it became positive, if we already move the term, then, the sign changes, is not it?
(Then Maria asked the whole class)
M: what was done by your classmates is it right? 3x +5x?
Students: no.
M: until here we were right. Look at the problem. What did you do with 5x? was it moved? to where?, and then, Escamilla?
S6: the addition of 3x+5x=8x, is equal…

Challenge to the Teacher | Item 4 KtAS. When you assign an activity about a previously learned topic, and some students express that they do not know how to it | Students asked her about the step where they were and she explained to them what else needed to be done.

Emerging Situation | When a student is explaining his/her reasoning | One student is explaining to her what he did, and the teacher listened to him and checked his work. Then she said: M: ahaa, but remember this is already known. What does it mean? What is going to happen to this 4? (She really did not provide him time to answer her questions, she was explaining step by step and asking the student why that step, and she told him what else he needed to do. She left that group)

When a student provides an answer to a given problem | The student read the problem and solved the problem on the whiteboard.
M: Esteban, can you read again the problem please and the student read it.
M: now we pay attention to hear the explanation of the group #7. The student who wrote it on the whiteboard explained her procedure and solution.
Table C1 presents the KtA situations that were identified in Maria’s classes. In addition, in Table C1 excerpts of transcriptions or field notes taken in Maria’s observations are provided for each situation. Maria’s classes included numerous situations where the teacher’s “knowing-to-act” should be or is enacted.

The frequency of each of the classrooms situations where the “knowing-to-act” should be or is enacted by the teacher are shown in Table C2. The situation most recurrent was the first KtA situation which is when a student solves a particular mathematical problem and asks if the answer or steps are correct. This situation was identified in Maria’s classes observed in 40 occasions. Moreover, the situation of when a student makes a mistake was detected on 16 occasions.

<table>
<thead>
<tr>
<th>Categories</th>
<th>KtA situations</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student misconceptions</strong></td>
<td>When a student solves a particular mathematical problem and asks if the answer or steps are correct</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>When a student makes a mistake</td>
<td>16</td>
</tr>
<tr>
<td><strong>Student difficulties</strong></td>
<td>When a student does not recognize the same pattern in a different situation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>When a student continuously responds with wrong answers to questions posed by you</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>When students are having hard time to complete an assigned activity/task</td>
<td>4</td>
</tr>
<tr>
<td><strong>Challenge to the Teacher</strong></td>
<td>When you assign an activity about a previously learned topic, and some students express that they do not know how to do it</td>
<td>5</td>
</tr>
<tr>
<td><strong>Unexpected Emerging Situation</strong></td>
<td>When a student is explaining his/her reasoning</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>When a student provides an answer to a given problem</td>
<td>3</td>
</tr>
</tbody>
</table>
Maria’s Actions in Student’s Misconception

The first category of student’s misconception was exhibited more frequently than the other categories. The situation of when a student solves a particular mathematical problem and asks if the answer or steps are correct was observed on multiple occasions as Table C2 illustrates. Maria acted in five ways in this first type of situations: asking for justification, explaining, indicating the correctness or incorrectness of student’s work, telling the mistakes, and asking for verification. Maria in one her classes, one student took his paper to be revised by the teacher. Then Maria started to check it and said:

M: why -2?
S5: it is -12.
M: here I see this -12, from where does this number come up?
S5: from this side
M: and where did you leave the other term?... 3x-x? (then the student is justifying his response and Maria continued asking questions) and where on each side should these coefficients be?

Maria was revising student’s work. She was asking the student questions such as “why -2? From where does this number come up? Where did you leave the other term?” Among others. The student was answering all her questions. Maria’s action was asking the student why and how he or she got certain numbers. The action performed by Maria is promoting critical thinking for her student. This teacher’s action is coded as “asking for justification”. This KtA situation is item 1 of the KtAS. The relationship between the survey’s choices and Maria’s action is through choice 5. Choice 5 is said to ask the student to explain his/her solution. Maria was actually doing this when a student solved a particular mathematical problem and asked if the answer or steps are correct.
Another action was performed by Maria in the same type of situation mentioned above. One student asked Maria if her work was right. Maria explained to the student that “you need to remember that coefficients are on one side and these ones on the other side”

S7: no no these numbers no.
M: what? What variables do you have?
S7: mmm
M: what are your variables?
S7: this one.
M: but it is only this x?
S7:mmm
M: this one will be left here and then…

Maria was explaining to the student what the student had at that point and what was still needed. Maria provided that explanation to the student. Therefore, the code assigned to Maria’s action was “providing an explanation”. To provide an explanation in this case leads to the student knowing what is correct or mistaken in her work. Thus, Maria’s action is a subset of choice 1 of item 1 of the KtAS. This choice indicates that the teacher tells him/her whether his/her response is correct.

The following short excerpt of the transcription of the observations conducted in Maria’s classes exemplifies the situation of when a student solves a particular mathematical problem and asks if the answer or steps are correct. One student (S4) approached to Maria and asked”

S4: teacher is it correct? (Maria checked it)
M: yes, you just need to circle the result.
The student asked Maria if his work was right. Maria checked it mentally and answered the student’s question “yes, you just need to circle the result”. Maria confirmed the correctness of the student’s work. This action is coded as “indicating the correctness or incorrectness”. Maria acted exactly as choice 1 of item 1 of the KtAS indicates. This choice is said to tell him/her whether his/her response is correct which was the action enacted by Maria.

In one of the Maria’s classes observed, one student revised his work. Maria checked it and explained to him that he had a mistake and why he had a mistake:

M: this coefficient -10 you moved it to the second term then it should be positive, remember that it moves to the other side with the opposing sign.

In this brief example of Maria’s classroom, Maria decided to reveal to the student a mistake that the student made. This action enacted by Maria limited the opportunity for the student to be able to identify by himself his own mistakes which involves critical thinking. The code assigned to Maria’s action was “revealing mistakes”. This action is a subset of choice 1, item 1 of the KtAS because to reveal a mistake indicates that incorrectness of the student’s response. Choice 1 said that to tell him/her whether his/her response is correct.

Only once did Maria ask for verification. Several students asked Maria to review their work. She was checking it one by one. When Maria saw a mistake or something that was wrong, she said to the students “check it again” without saying where the mistake was. Maria’s action was coded as “asking for verification”. This teacher’s action is a subset of choice 5, item 1 of the KtAS. Choice 5 indicates to ask the student to explain his/her solution. Maria’s action was leading the students to be able to see how they got a certain result.
Table C3 present the frequency of each of Maria’s actions coded when a student solves a particular mathematical problem and asks if the answer or steps are correct. Maria’s more frequent action was asking for justification when they asked for a review of their work.

Table C3: Frequency Table of Maria’s action codes in Item 1 KtAS situation

<table>
<thead>
<tr>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asking for justification</td>
<td>15</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>11</td>
</tr>
<tr>
<td>Saying correctness or incorrectness</td>
<td>4</td>
</tr>
<tr>
<td>Revealing Mistakes</td>
<td>9</td>
</tr>
<tr>
<td>Asking for verification</td>
<td>1</td>
</tr>
</tbody>
</table>

The second situation identified during Maria’s classes observed is: when a student makes a mistake. Maria also acted in five different ways: providing an explanation, referring to the whiteboard or notes, asking for justification of student’s work, involving the whole class, and pointing out the mistakes. One example of a classroom situation where Maria provided an explanation when a student made a mistake is described below.

M: what happened with this -7? oh, you moved it to the other side?

S3: yes

M: ok because you moved it, what happened with it? Aaah (she asked and explained that to help the student to identify his mistake) then what did you do with it?

S3: oh it should be positive (he realized what he had a wrong solution)

M: aha

Maria was walking around the students’ groups. Then, she identified a mistake in one group. Maria decided to stop at that group. She asked the student “what happened with this -7?” and she also answered herself by checking the student’s work. However, she continued explaining to the student the problem until the student saw his mistake. Maria’s action was explaining to the student what he did to solve the problem and how it should be done. The
teacher’s action was coded as “providing an explanation”. This action is evidently choice 4 which is to explain to him/her why it is wrong. This choice is given in item 9 of the KtAS.

Maria was referring to one student to see what was written on the whiteboard when she identified his mistake. One example of a classroom situation where Maria’s action can be identified in the same type of KtA situation is provided.

M: observe what you have and what it is on the whiteboard, identify your mistake, I know that there were students who have mistaken signs. Then circle your mistake. One student said: ah yes, I know where my mistake was.

M: ok then make the correction by yourself

In this classroom situation, a common presentation (puesta en común) had already occurred. Students went to the whiteboard to write and explain the solution to the problems assigned. After that, students are asked to correct their mistakes considering their classmates’ explanation and what was written on the whiteboard. Therefore, when Maria identified a mistake, she referred the student to see the whiteboard. Maria’s action was coded as “referring to the whiteboard or notes”. This action is related to choice 3, item 9 of the KtAS. Choice 3 pointed out ask to re-do the task. Maria’s action led deciding whether to redo the task after observing the whiteboard and determining what correction was needed.

The following excerpt of the transcription of the observation conducted is an example of one of Maria actions when a student makes a mistake. Maria was walking around the groups. She was supervising students’ work. She identified a student’s mistake and stopped with that group.

M: why -11? 8-6?

S11:-2
M: -2, no (Maria explained it and said) but also here, then 160-60 how much is it?

S11: 100

M: Ah ha and what did you write there?

Maria detected a student’s mistake and she decided to ask the student “why -11? 8-6?” the student (S11) responded wrong. Then, Maria continued asking about what the student did. The action enacted in this situation by Maria was to ask for student’s justification. The code assigned to Maria’s action was “asking for justification”. This action is clearly choice 5, item 9 of the KtAS. This choice indicates to let the student explain his/her answer which was what Maria did.

Maria involved the whole class twice when she identified a student’s mistake. The common presentation (puesta en común) was taking place. Students were going to the whiteboard to explain and write the solution to the problem assigned. One student already did it. Then, Maria asked another student to explain the same problem.

M: Clara can you repeat the explanation of your classmate?

S2: 2x is obtained multiplying the side by the perimeter.

M: right students? Do you agree?

Students: no.

Maria had already detected the student’s mistake. She asked one student to explain the problem again, then, Maria asked the whole class if they agreed. Maria’s action was involving the whole class. The code assigned to this action was “involving the whole class”. This action does not have any relation to any choice under item 9 of the KtAS. “Involving the whole class” when a student makes a mistake was an action not considered in the development of the survey.
Maria was reviewing students’ work because students asked for that review. She revised a student’s work where she identified a student’s mistake. This mistake was about the law of signs. Maria decided to say in a loud voice to the whole class “recall how the signs are managed”. Maria’s action was to inform the class about the common mistake that was been made by students. This action was coded as “revealing mistakes”. It is a subset of choice 4 item 9 of the KtAS. Choice 4 is said to explain him/her why it is wrong. Maria revealed to the students why their work was wrong by indicating their mistake.

The frequency of the actions performed by Maria when a student makes a mistake is presented in Table C4. We can observe that the action enacted by Maria more times in this situation was asking for justification.

<table>
<thead>
<tr>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asking for justification</td>
<td>7</td>
</tr>
<tr>
<td>Providing an explanation</td>
<td>5</td>
</tr>
<tr>
<td>Referring to the whiteboard or notes</td>
<td>1</td>
</tr>
<tr>
<td>Revealing Mistakes</td>
<td>1</td>
</tr>
<tr>
<td>Involving the whole class</td>
<td>2</td>
</tr>
</tbody>
</table>

**Maria’s Actions in Student difficulties**

The next category analyzed in Maria’s classes observed is student difficulties. The first type of situation of this category is when a student does not recognize the same pattern in a different situation. Maria’s classes exhibited only one time this type of classroom situation. Maria was teaching the topic of linear equations. She assigned several exercises. Students were working individually even they sat down in groups. One student was asking Maria several questions during the class. After a certain time, this student went to ask the teacher again, and Maria said “no, you should know how to do it by yourself and go sit down”
In the classroom situation described above Maria ignored the last question of the student because she considered that the student should know how to do it after several times that he asked Maria. However, this student was not able to identify the same pattern of the previous exercises that Maria assigned in the problem that he was working on. Maria’s action was coded as “ignoring the student”. This action is related to choice 4, item 2 of the KtAS. Choice 2 said to ignore it and move to another topic. Maria is obviously ignoring the student; however, Maria did not move to the next topic. Thus, Maria’s action is a subset of choice 2.

When a student is unable to see an obvious pattern in the problem is a classroom situation identified in Maria’s classes observed. In this situation, Maria’s actions differ. She acted revealing mistakes, or asking student to keep trying, or providing an explanation. The following describes a classroom situation that occurred in Maria’s classes. One student was repeatedly asking Maria about her solution. Maria was checking part by part and asking the student for calculations, she spent some time revising this student’s problem until she identified a mistake and told the student “pay attention here this number is negative and how did you move to the other side?”

S6: negative

M: and it should be positive (then the student saw her mistake and went to her seat)

Here, the student (S6) was not able to identify an obvious pattern in the problem assigned after several explanations that Maria provided previously. Maria revealed where the student made a mistake. Then, the student went to her seat to continue working on it. The code assigned to Maria’s action was “revealing mistakes”. This action is a subset of choice 1 item 5 of the KtAS. This choice mentioned the action of telling the student what the obvious pattern is.
The evidence that shows Maria asking the student to keep trying when unable to see an obvious pattern in the problem is described in the following excerpt of one Maria’s classes. Several students explained and solved a task on the whiteboard. One student asked Maria about a problem that was already presented in the common presentation (puesta en común) that took place earlier. Then, Maria said “do it by yourself, read it again and compare it to the solution on the whiteboard”. Maria’s action was basically asking the student to keep trying and then compare it with what was written on the whiteboard. The code assigned to the action enacted was “asking student to keep trying”. Maria’s action is undoubtedly choice 4, item 5 of the KtAS. Choice 4 is to ask the student to keep trying it until he/she gets it.

One Maria’s class observed exhibited the common presentation (puesta en común). The common presentation was described briefly above. After this presentation finished, students were supposed to be able to make the corrections needed on their work, however, one student asked Maria how to make corrections in a certain problem if she was not able to find her mistake. Then Maria explained what she needed to correct, but she did not provide the right solution. Noticeably, the student was unable to see an obvious pattern in the problem, in this case a mistake. Maria opted by providing an explanation. This action was coded as “providing an explanation”. Maria’s action is related to choice 2 which is to provide the student with hints to help him/her find the pattern. Maria is a subset of this choice because she helped the student to know what she needed to do.

Another type of KtA situation identified in Maria’s classes observed is when a student continuously responds with wrong answers to questions posed by the teacher. This situation only happened one time in Maria’s classes.
M: let’s see Escamilla, yes it is right but where did you get the positive sign?
S6: ah when I did this, it is multiplied by -36, yes or no?
Students: no. (Maria asked if somebody wanted to help Escamilla. One student went to the front and they were discussing the solution. Then Maria asked them:
M: Where are you struggling?
S6: we do not remember from where we got this and how it became positive, if we change it the sign changes right or no? (Then Maria asked the whole class
M: what did your classmates do was right? 3x +5x?
Students: no.
M: until here we were right. Look at, what did you do with 5x, it was moved to where, and then Escamilla?
S6: the addition of 3x+5x=8x, is equal…

In this classroom situation transcribed above, one student was solving the problem incorrectly. He has certain parts right. Then, Maria started to ask him “where did you get the positive sign?” and the student responded wrong. Then Maria asked the class if anyone wanted to help the student. One student went to the front to help his classmate. Both were discussing the problem. Then Maria asked them “Where are you struggling?” and then, the student indicated where. Maria continued asking for justification of what students were doing and also involving the class to find the mistake. Maria’s action was asking for justification to the students. The code assigned to this action was “asking for justification”. This code is related to choice 3, item 8 of the KtAS. This choice provided the action of asking the student to justify his/her answer. The action describe in choice 3 is exactly what Maria did.
The last type of KtA situations of the category of student difficulties is when students are having hard time to complete an assigned activity/task. Maria acted in two ways in this type of situation: one was providing an explanation, and the other action performed by her was providing more time to accomplish the task. In one of Maria’s classes observed, one student was having hard time to solve the problem assigned. Then he went to Maria’s desk to ask: “do I have to move this number to the other side like this?”

M: yes, remember you are going to multiply only what you have in the parenthesis.

Maria’s responded to the student with a brief explanation about what the student needed to do. Maria’s action was coded as “providing an explanation”. This teacher’s action is related to choice 5, item 10 of the KtAS. This choice said to explain the assignment using examples or another way to explain it. This action listed under choice 5 was what Maria did in her class.

In this KtA situation Maria also acted in a different way. In one of the classes, she assigned four linear equations to be solved. Maria assigned 10 minutes of time to solve those equations. However, Maria noticed that students were having a hard time accomplishing the task. Then, she decided to provide more time. She provided the entire time of that class (45 minutes) to work on the equations. However, most of the students did not finish the equations. Maria’s action was to extend the time for students to accomplish the task. This action was coded as “providing time or hints”. Maria’s action is a subset of choice 2, item 10 of the KtAS. This choice is to analyze students’ difficulties and provide hints. Although Maria did not provide hints, she analyzed students’ difficulties before deciding to extend the time of the activity.

The KtA situations of the second category of student difficulties and the actions performed in each of the situations are illustrated in Table C5. In addition, the frequency of each action enacted by situation is provided in Table C5. The KtA situation of when students are
having hard time to completing an assigned activity/task happened Maria’s classes was the most frequently observed situation.

Table C5: Frequency Table for Maria’s Action Codes In The category of Student difficulties

<table>
<thead>
<tr>
<th>KtA Situation</th>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>When a student does not recognize the same pattern in a different situation</td>
<td>Ignoring the student</td>
<td>1</td>
</tr>
<tr>
<td>When a student is unable to see an obvious pattern in the problem</td>
<td>Revealing mistakes</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Asking student to keep trying</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Providing an explanation</td>
<td>1</td>
</tr>
<tr>
<td>When a student continuously responds with wrong answers to questions posed by you</td>
<td>Asking for justification</td>
<td>1</td>
</tr>
<tr>
<td>When students are having a hard time completing an assigned activity/task</td>
<td>Providing an explanation</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Providing time or hints</td>
<td>1</td>
</tr>
</tbody>
</table>

Maria’s Actions in Challenge to the Teacher

The third category of classroom situations where the “knowing-to act” should be or is enacted considered in the protocol observation is challenge to the teacher. In this category only one type of KtA situation was identified in Maria’s classes. This type of situation is when you assign an activity about a previously learned topic, and some students express that they do not know how to do it. Maria acted in two ways in this situation.

Maria provided an explanation to the student (S5) about how to solve a problem which required the calculation of a perimeter. This type of problem was already seen in the two previous classes. However, one student asked:

S5: how to do this?
M: how do you get the perimeter of a shape? (Then the student pointed to something in the worksheet.)

M: ah ok, you only need this, it is the same. (Indicating in the student’s notebook)

Maria explained to the student briefly what he needed to do. This action is coded as “providing an explanation”. This teacher’s action was performed on three occasions. Maria’s code is a subset of choice 1 which is to re-teach the previous topic. Maria did not re-teach the topic. However, she provided a brief explanation that allowed the students to know what they had to do. The choice that described this action is choice 1, item 4 of the KtAS.

The other action enacted by Maria in this KtA situation is making students to recall the topic already taught. After covering a topic, she assigned a worksheet to be answered individually and without any help. The situation that occurred was that Maria taught how to solve linear equations for three days. When students are asked to answer the worksheet, one student asked Maria:

S1: teacher can we use our notebook to recall what we did?

M: no, first try it like this, alone, remember what we did last Friday, Tuesday and yesterday.

The student did not remember how to solve those equations on the worksheet. Thus, he asked for permission to use the notebook to recall the topic. However, Maria did not allow him to use the notebook. She told him “no, first try it like this, alone, remember what we did last Friday, Tuesday and yesterday”. Maria’s action is coded as “asking to use their recalls”. This action was enacted on two occasions. Choice 2, item 4 of the KtAS is to ask students to do it using their recalls. This is what Maria did.
Maria’s Actions in Emerging Situations

The last category created to classify the situations where the teacher’s “knowing-to act” should be enacted is emerging situations. Two different types of situations were identified during Maria’s classes. One type of situation is when a student is explaining his/her reasoning. And the other type identified is when a student provides an answer to a given problem.

When a student is explaining his/her reasoning is a classroom situation that occurred two times in Maria’s classes. One student was explaining to Maria what he did in the assignment, and the teacher listened to him and checked his work. Then, Maria said:

M: ahaa, but remember this is already known. What does it mean? what is going to happen to this 4? (she really did not provide him time to answer her questions, she was explaining step by step and asking the student why that step, and she told him what else he needed to do. She left that group)

Maria’s action was allowing the student to share his explanation with her even though that student was wrong. After giving the chance to the student to explain what he did, Maria explained to him what was wrong. Also, she was explaining to the student step by step and asking him why that step was done. Maria told him what else he needed to do. Maria’s action is coded as “allowing the student to share his/her point”. This action does not have any relationship with the survey because the situation where this action was enacted was not considered for the development of the KtAS.

Interrupting the student when he or she is participating in the class is the other action performed by Maria one time. The following excerpt from the transcription of the observations exemplifies Maria’s action in this emerged situation. One student was explaining his work to Maria to see if his work was right.
M: let’s see the perimeter of the rectangle how did you represent it? (He was explaining to Maria’s, but she interrupted and said)

M: no, what is here 12 or 2

S10: 12

M: but where did you get it?

S10: from here

M: ah ok the coefficients 6+6, that it is right but you have this, (the student explained her) equal to the perimeter, perimeter is what we are working; when you have this part you will get the solution.

Maria interrupted the justification of the work of the student (S10). She interrupted him when Maria identified a student’s mistake. After that, she continued asking the student about his procedure. Maria’s action was to interrupt the student’s explanation, and was coded as such.

Classified in this category of unexpected emerging situations, there is this classroom situation found: when a student provides an answer to a given problem. In this type of situation Maria acted in one way on three occasions that this situation was detected.

Maria asked one student to go to the whiteboard to solve a certain equation. When the student finished it, Maria said:

M: is it right? Are you sure? (Then Maria asked the student to explain it).

S10: well here 5 is multiplied by what it is in the parenthesis, also in this part. Then, below, we are developing each side, then we add them, then this provides the result.

M: let’s see guys, do you have any comment?

The student (S10) was asked to solve the problem on the whiteboard. When he finished it, Maria asked him if he was sure of the solution provided and explained what he did to solve the
problem. Maria’s action was asking for the student’s justification of his solution. The action performed by Maria was coded as “asking for justification”.

Table C6 presents the type of situation, the code assigned to the teacher’s action, and the number of times that the action was enacted during Maria’s classes observed. We can observe that the presentation of these situations was not frequent in Maria’s classes. This category was exhibited only five times as Table C6 presents.

Table C6: Frequency Table for Maria’s Action Codes In The category of Emerging Situations

<table>
<thead>
<tr>
<th>KtA Situation</th>
<th>Teacher’s Action Code</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>When a student is explaining his/her reasoning</td>
<td>Interrupting the student’s explanation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Allowing the student to share his/her point</td>
<td>1</td>
</tr>
<tr>
<td>When a student provides an answer to a given problem</td>
<td>Asking for justification</td>
<td>3</td>
</tr>
</tbody>
</table>
Appendix D: IRB Proposal

Use this template to provide a description of your research proposal. All applications for review should contain the following information, presented in paragraphs prefaced by the number of the item and the underlined descriptive phrase. When not applicable, DO list the heading and then indicate N/A.

Please note that if this study is part of an NIH funded grant proposal, you will need to attach ONE copy of the complete grant proposal, in addition to the information requested below.

I. Title:
Relationship between cognitive types of teacher content knowledge and knowing-to-act: A Case Study of Mexican borderland middle school teachers.

II. Investigators:
Principal investigator. Maria Cruz Quinones
Co-Investigator. Mourat Tchoshanov

III. Hypothesis, Research Questions, or Goals of the Project

The purpose of the study is to measure the cognitive type of middle school teachers’ content knowledge and its relationship with teachers’ “knowing-to-act”. Knowing-to act is the process in which “knowledge that enables people to act creatively rather than merely react to stimuli with trained or habituated behavior” (Mason and Spence, 1999, p.136) is involved. Also, the broader educational purpose focuses on understanding what influences teachers’ instructional decisions to act in particular ways during specific moments throughout the processes involved in teaching mathematics in their classrooms. Therefore, it is crucial to delve deeper into the teachers’ actions performed in specific situations emerged during the classroom instruction that challenge teachers to act in a certain way. The qualitative component of this exploration is going to be conducted in order to provide an understanding and descriptions of these actions. The hypothesis is that teachers with a poor content knowledge (cognitive type 1 T1) have less opportunity to act in a mathematical moment.

The following are the research questions for the study:
What kind of teacher content knowledge is critical for teaching effectively?
To what extent is the cognitive type of teachers’ knowledge associated with teachers’ knowing-to-act?
What are the cognitive types of teacher knowledge that have a strong impact on teachers’ knowing-to-act?
What are the teachers’ actions in specific moments during instruction of middle school courses?
How do (five) purposefully selected middle school teachers describe the knowing-to-act that characterizes their classroom instruction?

IV. Background and Significance:

There are several studies published that focus on teacher knowledge over the last twenty five years (e.g., An, Kulm, & Wu, 2004; Davis, & Simmt, 2006; Tchoshanov, 2011). The pioneering work that broke new ground was the well-known Shulman model (1986). Shulman (1986, 1987) listed several categories of teacher knowledge in which researchers
have studied individually or consider two categories or modify aspects of them, and even add more categories. These categories are known as types of teacher knowledge or knowledge for teaching mathematics: teacher content knowledge, pedagogical content knowledge, knowledge of curriculum, knowing-to act, among others. Teacher content knowledge includes the knowledge “that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems” (Hill, Ball & Schilling, 2008, p.377-378). Then, a researcher identified three cognitive types of teacher content knowledge: knowledge of facts and procedures, knowledge of connections and concepts, and knowledge of generalizations and models (Tchoshanov, 2011). Teacher knowledge has been measured through teacher certification, mathematics coursework, and a content knowledge test for teachers (National Advisory Panel, 2008b). Regarding knowing-to act, there are researchers that have identified this fundamental component of teacher knowledge, but these studies theorized this component using several approaches (Mason and Spence, 1999). Also, there is a study that shows how mathematics and science pre-service teachers struggled to act in the mathematical moment (Wilhelm, Sherrod and Walters, 2008). But this research did not take place in classrooms in where pre-service teachers were teaching. Therefore, studies that research how the knowing-to act of teachers during their instruction in the mathematical classroom matters in producing student learning are critical.

**Significance**

The insights of this investigation will provide institutions of higher education in Mexico and around the world with valuable information to contribute to the expansion and development of teacher education pre-service programs, especially in the area of secondary school mathematics. Additionally, I am confident that the understanding and description presented in this study will provide in-service teachers and other participants in the education field, with awareness about the active knowledge that is needed to enact the teachers’ knowing-to act in teacher preparation programs in Mexico that can be used to support teachers and students in the United States.

In the United States, there is a large and growing number of Mexican students who have left Mexico for several reasons (Pew Research Center, 2009). These students are incorporated in U.S. public schools at different grade levels in the entire country. Teachers frequently do not have background knowledge on the educational context in Mexico where these students studied (cite). This study aims to provide a perspective in regards to the lived realities of Mexican public schools in mathematics courses at middle school levels. Additionally, the student may help teachers understand Mexican students’ practices in order to offer learning environments in which students feel comfortable to construct their knowledge.

**References:**


V. Research Method, Design, and Proposed Statistical Analysis:

In connection with the literature review and the research questions, mixed methods sequential nested design is going to be implemented in this study. The mixed methods sequential nested studies are “products of the pragmatist paradigm and that combine the qualitative and quantitative approaches within different phases of the research process.” (Tashakkori & Teddlie, 2008, p.22). The research sample will include Mexican mathematics middle school teachers (N=100) who are teaching at least one of the mathematics courses at the middle school level: Mathematics I, or Mathematics II, or mathematics III.

Data sources will include the following: (1) interviews with mathematics teachers; (2) classroom observations; (3) field notes which will be made on teacher behaviors and interactions that occurred during instruction observed (4) video recording of mathematics classes (5) teacher content knowledge survey (TCKS), and (6) knowing-to act survey (KtAS).

In Mexico, an appointment will be set up with the Mexican Subsecretary of Education, Culture and Sport who is in charge of the public education in Juarez city. The purpose of this meeting is to explain the purposes of this study, the procedures to be used, and to obtain authorization for conducting the study. Then, several middle schools are going to be selected in order to accomplish the research sample size. After that, appointments with the principals of the middle schools are going to be set up. The principal investigator will be responsible to
contact the middle schools in Mexico and apply the two surveys to the Mexican middle school teachers. Later, after the data analysis of these surveys will have been finished, class observations and individual interviews are going to take place. There will be five middle school teachers selected to be interviewed and observed. The selection of these teachers is going to be based on the results of the data analysis done to the surveys previously applied. The strategy sampling is going to be presented in the next sections. The types of data analysis will be used on the data sources and research questions. The qualitative data will be fully transcribed and then analyzed in order to codify pattern and themes related to the research questions.

VI. Human Subject Interactions

The subject population involved in this study will be composed of a group of Mexican middle school teachers in Juarez city. These in-service teachers are from different public middle schools. Participants will be recruited and selected according to the purpose of the study. The participants will be invited to participate in this project and they will be entirely voluntary. We will meet with the participants to describe the purposes of the study, and to explain the roles of the informants. We will ask each participant to complete and sign a consent form (enclosed). We will do all we can to protect participants’ rights and welfare. We expect the human subject involvement in this project to begin in the summer of 2013 and to end in the spring 2014.

All of the teacher participants will be non-English speakers, their native language is Spanish. Therefore, the consent forms will be translated into that language. This translation may be completed after IRB approval of the study and consent forms (The IRB committee may want to review the translation of the documents before, rather than after they approve).

B. Describe the procedures for the recruitment of the participants are the following: Step One: One hundred mathematics middle school teachers will be selected to fill out two surveys. The criterion for selection of these teachers will focus on what they are currently teaching; all participants must be currently teaching at least one mathematics course at middle school level.

Step Two: A survey (TCKS) will be administered to these (100) teachers to measure their content knowledge. A second survey is about the teachers’ actions enacted during teaching mathematics at particular moments. Based on results of these two surveys, the researcher will select 5 mathematics’ teachers who performed in a specified way in the surveys in order to be interviewed and their teaching observed and video recorded (only two classes). For instance, a teacher who has a high score in the cognitive type 1, and a low score in the cognitive type 2 and 3 will be one of the five teachers. These scores are going to be provided by the TCKS data analysis.

C. Describe the procedure for obtaining informed consent.

The researcher will conduct a visit to the schools to ask for permission to approach teachers and be able to talk to them about how they act in specific moments during instruction. Also, another two visits to the schools are going to be made to ask for permission to conduct class observation and video record those class. The potential participants are not considered members of a group that may be construed as stigmatized, so my contact with participants will not be done through flyers or advertisements.
During the first meeting the researcher will introduce him or herself to each mathematics teacher, the consent form will be given to them and the researcher will explain each part of the consent form and will clarify any doubt or comment that the mathematics teachers may have.

The consent form will include the following information: purpose of the research; time involved in completing the research; risks and benefits to participation; safeguards for protecting data; protection of confidentiality of participants’. Also the consent form will inform that the participation is voluntary.

**D. Research Protocol.** Three strategies will be used to collect data for this study: survey, interview, and observation. Data collected from the surveys will be used to select participants for the observation and interview. Participants will be given two surveys. The first survey is the TCKS which consists of two sections: the first one requires demographic information, the second section is composed of 33 items which measure the teacher content knowledge through their three different cognitive types. This survey is going to take 90 minutes to be answered. The second survey is composed of 15 items about the knowing-to-act of the teachers in certain situations. This survey is going to take 30 minutes to be answered.

Another strategy that will be used to collect data is the Interview. Individual teacher interviews will take approximately 60-90 minutes and will be semi-structured. The interviews will take place at each middle School, at the convenience of the participant. It is expected that of the entire sample of participants, approximately 5 will be selected to be interviewed based on the criteria described above.

Classroom Observations are the other strategy to be used for data collection in this study. The investigator will conduct observations. The observations will be guided by the research questions of this project. Field-notes are going to be taken based on specific teachers’ actions enacted during teaching mathematics. These class observations are going to be video recorded. However only, data from the participants who consent being videotaped will be utilized.

**E. How will you protect the privacy and confidentiality of participants?** The participation of the participants in this study is confidential. None of the information is identified by name. All records will be coded to maintain anonymity. The coding system will be stored in a password-protected file, and the researcher will be the only one with access to the codes. Each participant will be assigned a generic number code and the researcher will use only that code on his/her work. No names will be revealed in order to protect the privacy of the participants.

**F. Discuss the procedures that will be used to maintain the confidentiality of the research data.**

All the recordings will be stored in a locker, under lock and key in the EDU 100. The researcher will have the only key to open the locker. Electronic versions will be stored on the researcher’s laptop and will be password protected. The researcher will be the only with access to it. The researcher will used this information to do data analysis and will retain it after the study concludes because this information can served to further publications. However, the confidentiality will not be affected since only the researcher will have access to it. The recordings will not be related by the name of the participants, they will be associated with codes.
G. Please describe your research resources. The researcher has a laptop, an office space in EDU 100, with access to printing.

VII. Describe any potential risks (physical, psychological, social, legal, or other).

The possible risk associated with this research is the loss of confidentiality of the participants' responses. The information obtained will be shared with a faculty advisor; however, the researcher will do everything possible to keep the information confidential. Personal information obtained in this study may be disclosed if required by law. Organizations that may inspect and/or copy of the investigation records for quality assurance and data analysis include, but are not necessarily limited to:

- The sponsor or agent for the sponsor
- Department of Health and Human Services
- UTEP Institutional Review Board

The results of this research may be presented at meetings or in publications, however, his identity is not revealed in the presentations.

VIII. Describe and assess the potential benefits.

There will be no direct benefits for taking part in this study. However, this study will benefit to the field of Mathematics Education to understand the relationship between different kind of teacher content knowledge and teachers’ knowing-to act in order to know what really matters to teach mathematics effectively in a manner in which students can build their knowledge.

IX. Indicate the specific sites or agencies involved in the research

The Mexican participants are employed at middle schools in Juarez city. A letter from Subsecretary of Education in cd. Juarez that granted the researcher permission to conduct the research is attached.

The researcher and co-researcher have access to formal and informal authorities who approve this study such as the subsecretary of Education in Cd. Juarez, Mexico, and the principals of the middles schools. The letters in which granted their permission to conduct the research will be attached.

X. The project has not had or will not receive review by another IRB. In Mexico, there is not an institutional review board. The person who provides authorization to develop projects is the subsecretary of education in each city. Once that permission is granted the schools allow the entry of the researcher to the setting. This study is already granted to conduct the research in cd. Juarez by the Subsecretary of education.
Appendix E: Knowing-to Act Survey (KtAS)

Thank you for taking the time to complete this survey. This survey should take only about 20 minutes of your time. The primary purpose of this survey is to obtain personal information regarding the actions/behaviors/decisions that you frequently do in a specific moment during instruction. Please rank the options listed below in each of the situations presented according to your most likely action to do first. Jot the number 1 next to the action that you would do first in that situation, number 2 next to your second choice, and so forth. This survey or any of its components is NOT intended to evaluate your teaching. The results of this survey will be kept confidential.

1. When a student solves a particular mathematical problem and asks if the answer or steps are correct, which of the following would you most likely do? Rank in order from 1 to 5 (1 - you most likely do first, 5 - you least likely do)
   - Tell him/her whether his/her response is correct.
   - Ask another student to confirm if it is correct or not.
   - Ask the student to read the corresponding section of the textbook.
   - Ask the whole group to share what they think.
   - Ask the student to explain his/her solution.

Explain the choice you most likely do first below:

2. When a student does not recognize the same pattern in a different situation, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)
   - Provide the solution.
   - Ask another student to help him/her.
   - Look for another way to represent the problem for the student.
   - Ignore it and move to another topic.
   - Pose another similar problem to the student.

Explain the choice you most likely do first below:
3. When a student provides a nonsensical solution to a given problem, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

   ____ Ask the student why he or she thinks that way.
   ____ Ask the student to re-do the problem.
   ____ Ignore the student’s response.
   ____ Tell the student how the problem should be solved step by step.
   ____ Give an extra credit for her/his effort.

Explain the choice you most likely do first below:

4. When you assign an activity on a previously learned topic, and some students express that they do not know how to do it, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

   ____ Re-teach the previous topic.
   ____ Ask students to do it using their few recalls.
   ____ Review the first topic of the textbook.
   ____ Skip to another activity.
   ____ Ask students to review by themselves the topic and do the activity.

Explain the choice you most likely do first below:

5. When a student is unable to see an obvious pattern in the problem, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

   ____ Tell the student what the obvious pattern is.
   ____ Provide the student with hints to help him/her find the pattern.
   ____ Modify the context of the problem.
   ____ Ask the student to keep trying it until he/she gets it.
   ____ Provide the student with necessary instructional materials.

Explain the choice you most likely do first below:

6. When you make a mistake and a student notices the mistake in front of the whole class, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

   ____ Respond by saying there is no mistake.
   ____ Ignore the student’s notification.
Re-do the task in front of the whole class.
Thank the student; explain where your mistake is and why it happened.
Ask the student to read the corresponding chapter of the textbook.

Explain the choice you most likely do first below:

7. When a student asks you a question from a different perspective that was not previously considered by you, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

Tell the student to follow your perspective only.
Allow the student to make a presentation on his/her perspective.
Try to respond to student’s question to the best of your knowledge.
Look for an excuse to not respond.
Ask the student for more details in order to be able to make a point.

Explain the choice you most likely do first below:

8. When a student continually responds with wrong answers to questions you pose, which of the following would you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

Rephrase the question or problem again.
Ignore the student’s wrong answers.
Ask the student justify his/her answer.
Send the student to the principal’s office for detention.
Analyze why the student is doing that and look for another way to explain the material.

Explain the choice you most likely do first below:
9. When a student makes a mistake, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

- Tell the student that using a calculator is not allowed.
- Ignore it.
- Ask to re-do the task.
- Explain him/her why it is wrong.
- Let the student explain his/her answer.

Explain the choice you most likely do first below:

10. When students are having hard time to complete an assigned activity/task, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

- Complete the activity for the students and let them copy it.
- Analyze students’ difficulties and provide hints.
- Do half of the assignment and leave the other half to the students.
- Let the students go to a recess.
- Explain the assignment using examples or another way to explain it.

Explain the choice you most likely do first below:

11. When you are teaching a topic that you are not knowledgeable at and a student asks you to elaborate more on that topic in front of the class, what of the following you most likely do? Rank in order from 1 to 5 (1 – you most likely do first, 5 – you least likely do)

- Tell that this topic is not very important.
- Ask the student to research the topic independently.
- Elaborate on the topic to the best of your knowledge.
- Tell the student that you will elaborate more on the topic the following day in order to better respond to his/her request.
- Change the topic.

Explain the choice you most likely do first below:
Appendix F: Teacher Knowledge Survey

STUDENT NAME _______________________________ DATE__________________

1. **What grade level you are teaching** (check appropriate box below)?
   - a. Grade 5
   - b. Grade 6
   - c. Grade 7
   - d. Grade 8
   - e. Other/ Not Applicable

2. **Gender:**
   - ____ Male
   - ____ Female

3. **Race/ Ethnicity** (check appropriate box below):
   - a. White, Non-Hispanic
   - b. Black, Non-Hispanic
   - c. Hispanic
   - d. Asian/Pacific Islander
   - e. American Indian/Alaskan Native
   - f. Other

4. **Teaching experience** (number of years in the teaching profession) ______________

5. **What subjects are you teaching?**
   __________________________________________________________
   __________________________________________________________

6. **What are your students’ average TAKS passing rates?**

<table>
<thead>
<tr>
<th>Year</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAKS passing rate (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. **What was your academic Major?** _________________________________
8. **Are you certified?**  
   ___ Yes  ___ No  
   
   If “Yes”, what is the field you were certified in?  
   a. EC-4  
   b. 4-8 (_____ Generalist; _______ Mathematics)  
   c. 8-12 (_______ Mathematics; ______ Other)  
   
   **Which program you were certified through?**  
   - Traditional teacher preparation  
   - Alternative teacher certification program (ATCP)  

9. **Your Grade Point Average:**  
   Undergraduate School ___________________________  
   Graduate School _______________________________  

10. **How many mathematics content courses have you taken?**  
    None  
    1 to 4 courses (no more than 12 credit hours)  
    5 to 8 courses (more than 12, but not more than 24 credit hours)  
    9 to 12 courses (more than 24, but not more than 36 credit hours)  
    More than 12 courses (more than 36 credit hours)  

11. **What is the highest level mathematics class you have taken so far?**  
    ____________________________________________  
    ____________________________________________  
    ____________________________________________
Appendix G: Teacher Content Knowledge Survey

Name/ Number __________________________
School __________________________
Date __________________________

(1) From the following set, how many numbers are irrational numbers?

S = \{\sqrt{4}, \sqrt{6}, \sqrt{8}\}

A. 0
B. 1
C. 2
D. 3

Use the following diagram to answer questions (2) and (3).

(2) Which of the following measurements associated with the triangles in the diagram above can be expressed as a non-terminating, non-repeating decimal?

A. AD
B. AC
C. BC
D. DC

(3) If point C moves continuously along the ray \overrightarrow{BD}, the length of segment AC takes on values that are:

A. Mostly rational
B. Mostly irrational
C. Equal amount of rational and irrational
D. Insufficient information provided

(4) A number set is dense if and only if for any elements \(a\) and \(b\) there exists an element \(x\) such that \(a < x < b\). According to this definition, which of the following sets is dense?

A. Integers
B. Natural numbers
C. Rational numbers
D. Complex numbers

(5) What topic does the following geometric illustration address?

A. Comparing Fractions
B. Pythagorean Theorem
C. Similarity Property
D. Pick’s Theorem
(6) Which of the following fractions is/are located between $\frac{a}{b}$ and $\frac{c}{d}$

I. $\frac{ad + bc}{2bd}$

II. $\frac{a + c}{b + d}$

III. $\frac{ad + bc}{bd}$

IV. $\frac{a + c}{bd}$

A. I, II and III
B. I and II
C. II and IV
D. III only

(7) What is the rule for fraction division?

A. $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$

B. $\frac{a}{b} \div \frac{c}{d} = \frac{ab}{cd}$

C. $\frac{a}{b} \div \frac{c}{d} = \frac{cd}{ab}$

D. $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$
(8) Which of the following problems represents the operation below?

\[ 1 \frac{3}{4} \div \frac{1}{2} = ? \]

A. Juan has a piece of rope \(1 \frac{3}{4}\) feet long and cuts it in half. At what length should he cut the rope?

B. Maria has \(1 \frac{3}{4}\) liters of juice. How many \(\frac{1}{2}\) liter containers can she fill?

C. A boat in a river moves \(1 \frac{3}{4}\) miles in 2 hours. What is the boat’s speed?

D. Daniel divides \(1 \frac{3}{4}\) pounds of coffee evenly between 2 customers. How many pounds of coffee will each customer get?

(9) Is \(\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}\) ever true?

A. Always true

B. Sometimes true

C. Never true

D. Not enough information to tell
(10) Which of the following equations best describes the function $y_3$?

A. $y = ax^2 + bx + c$
B. $y = ax^2 + bx + 1$
C. $y = ax^2 + 1$
D. $y = x^2 + 1$

(11) The function $y_3$ is translated 4 units left and 7 units down. Which of the following equations best describes the new function?

A. $y = ax^2 + 11x + 28$
B. $y = ax^2 + 4x + 7$
C. $y = ax^2 + 8ax + c$
D. $y = x^2 + 28x + 11$

(12) The diagram shows a family of functions in the form $y = ax^2 + bx + c$. Which of the following statements best describes the changes in the values of the coefficients as the graphs vary from $y_1$, to $y_2$, to $y_3$?

A. $a$ is increasing, $b = 0$, and $c$ is increasing
B. $a$ is increasing, $b = 0$, and $c$ is decreasing
C. $a$ is decreasing, $b$ is increasing, and $c = 0$
D. a is decreasing, b is decreasing, and c = 0

Use the information below to answer questions (13), (14), and (15).

In a small town in East Texas, Ms. Steger always keeps her students engaged by appropriately incorporating technology in interesting activities.

During one activity, students collected data (foot length and height, in centimeters). After collecting the data, students worked cooperatively to analyze their data. They organized their data in the table below and used the available technology of their choice.

<table>
<thead>
<tr>
<th>Point</th>
<th>Foot Length (cm)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Independent</td>
<td>Dependent</td>
</tr>
<tr>
<td>E</td>
<td>22</td>
<td>121</td>
</tr>
<tr>
<td>F</td>
<td>29</td>
<td>125</td>
</tr>
<tr>
<td>G</td>
<td>30</td>
<td>125</td>
</tr>
<tr>
<td>H</td>
<td>31</td>
<td>140</td>
</tr>
<tr>
<td>I</td>
<td>38</td>
<td>150</td>
</tr>
<tr>
<td>J</td>
<td>39</td>
<td>150</td>
</tr>
<tr>
<td>K</td>
<td>40</td>
<td>160</td>
</tr>
</tbody>
</table>

(13) Which scatterplot below best describes the data in the table above?

A. ![Scatterplot A]

B. ![Scatterplot B]

C. ![Scatterplot C]

D. ![Scatterplot D]
(14) Which linear function best describes the line of a best fit for data in the table?
A.  \( y = -2x + 100 \)
B.  \( y = \frac{9}{4}x + 70 \)
C.  \( y = 2x + 75 \)
D.  \( y = \frac{9}{4}x + 50 \)

(15) What pair of lines is parallel?
A.  (EH) II (FJ)
B.  (FI) II (GJ)
C.  (GK) II (HI)
D.  (EJ) II (FJ)

Use the diagram below to answer the question that follows.

(16) Quadrilateral ABCD is a parallelogram. What are the coordinates of the point A?
A.  (4, 7)
B.  (3, 7)
C.  (4, 6)
D.  (3, 6)
Use the diagram below to answer the question that follows.

(17) Quadrilateral ABCD is a parallelogram. What is an equation of the line AB?

A. $3y - 4x = -23$
B. $3y - 4x = -14$
C. $3y + 4x = 23$
D. $3y + 4x = 26$

(18) The vertices of a polygon drawn on an x-y coordinate system are given by the ordered pairs below.

$(1, 2), (2, 4), (5, 2), (4, 4)$

What type of polygon is described by these coordinates?

A. Kite
B. Rectangle
C. Rhombus
D. Trapezoid
(19) Find the ratio of the areas of Figure A to Figure B

A. 1:1.5
B. 1:2.25
C. 1.5:1
D. 1:3

(20) Figures A and B are similar. The ratio of their areas is 1:4. Figure A has an area of 60 units^2. What are the dimensions of Figure B?

A. 10 x 24
B. 30 x 8
C. 12 x 10
D. 20 x 12
Use the diagram below to answer the question that follows.

Figure A

(21) Figure A was transformed by scaling 1:2 in a horizontal direction and 1:5 in a vertical direction. The ratio of the area of B to A is

A. 4:1  
B. 1:4  
C. 1.5:1  
D. 1:1
(22) Find the mean of {-3, 0, 15}

A. 0  
B. 4  
C. 5  
D. 6

(23) If 3 is added to each of the values of a dataset, which of the following will NOT increase?

A. Mean  
B. Mode  
C. Range  
D. Median

(24) Judah collects a 5-number dataset and announces that the mean is 5, the median is 6, and the mode is 7. Larry says “the range could be 6”. Laurie says “5 could be in the dataset”. Who is correct?

A. Larry only  
B. Laurie only  
C. Larry and Laurie  
D. Neither Larry nor Laurie

(25) Suppose your test score was at the 90th percentile. What does that mean?

A. You got 90% of the questions right.  
B. 90% of the other students scored lower than you did.  
C. 90% of the other students scored higher than you did.  
D. You have a 90% chance of getting the same score if you took the test again
(26) From the following picture, which distribution would have a larger 90\textsuperscript{th} percentile?

A. A  
B. B  
C. The 90\textsuperscript{th} percentiles of A and B would be equal  
D. More information needed to determine

(27) Joe and Maria take the same test. Joe’s test score (x) represents an improvement of 10 percentile points (e.g., from the 20\textsuperscript{th} to the 30\textsuperscript{th} percentile, or from the 70\textsuperscript{th} percentile to the 80\textsuperscript{th} percentile, etc.). Maria’s test score (y) represents an improvement of 5 percentile points. What can we conclude?

A. x > y  
B. x < y  
C. x = y  
D. Any of the above are possible
The orbit of a satellite is approximately 250 miles above the surface of the earth (the diameter of the earth is approximately 8,000 miles). If the satellite travels a total about 4,000,000 miles in space during a 240-hour period, it will orbit the earth approximately once every:

A. 90 minutes
B. 96 minutes
C. 180 minutes
D. 192 minutes
(29) Imagine that you have a long enough rope to circumscribe the earth (the diameter of the earth is about 8,000 miles). Approximately how much longer rope do you need if you want to have a 1 yard gap between the earth and the rope?

A. 1 yard  
B. 6 yards  
C. 1 mile  
D. 6 miles
Use the diagram below to answer the question that follows.

\[ OA = O_1A_1 = r, \quad OB = O_1B_1 = R \]
\[ AA_1 = 2\pi r, \quad BB_1 = 2\pi R \]
Since \( ABA_1B_1 \) is a rectangle, then:
\[ 2\pi r = 2\pi R \]
\[ r = R \]

(30) The argument above is given by a student in order to claim that all circles have the same radius. Which of the following responses would best address the student's claim?

A. \( AB \ A_1B_1 \) is not a rectangle
B. The claim is correct.
C. Visualize a trace of points A and B.
D. The lengths are not \( 2\pi r \) and \( 2\pi R \), but \( \pi r^2 \) and \( \pi R^2 \), respectively.
(31) On the geoboard above, each measurement unit is a rectangle equal to 2 square units. What is the area of the triangle in measurement units?

A. 21.000  
B. 10.500  
C. 5.250  
D. 2.625

(32) On the geoboard above, each unit square has an area equal to 1. Which of the following quantities associated with triangle ABC is an irrational number?

A. Area  
B. Perimeter  
C. Length of $\overrightarrow{BC}$  
D. Midpoint of $\overrightarrow{AC}$
On the geoboard above, each measurement unit is a rectangle equal to 2 square units. What is the area of the given shape in measurement units?

A. 4  
B. 5  
C. 6  
D. 7
Appendix H: Interviews’ Transcription

*Rosa’s transcription*

June 26, 2014

R: I do not know if I told you before, no i did not comment to you, we have a notebook with worksheets that we work.

T: oh yes you told me.

R: we have for example...well we have a notebook where the lesson plans are included also some considerations are mentioned to be followed by us the teachers.

T: are those the worksheets (consignas)?

R: the worksheets, those are for the student, well i do not want that this would be recorded.

T: I am recording but this material is just for transcribing it for me to analyze it.

R: do not say anything. .

T: do not worry!

R: well, according to my understanding, one must make the student to be autodidact, it means that the student by himself should construct his knowledge. But if I teach like that, to me the new reform does not work, it works only for 3 or 4 students. Then, what I do, the topic that I am teaching, I write it on the whiteboard, if I do not finish covering the topic in one class I use the next class. I teach good the topic until most of the students do not have doubts or question, then I use the notebook. I deem the previous consideration (of the notebook) or I implement an activity and then they work on it.

T: listen to this first recording. Well, until, it is the first segment.

T: What did you listen to? Tell me what is happening there? What happened there, can you describe it?

R: That some students are working and others don’t.

T: What else, can you tell me? About the interaction between you and the student? One student called you and you approached to her, for what?

R: she asked me for revision.

T: To check it, and then you said to her what?

R: She said that she finished, I approached to her and it wasn’t true, I mean it was wrong. She said this is ready and nothing.

T: Did you check it?

R: yes.

T: Then you checked it and analyzed where her mistake was. Well, I don’t know, if you remember, well I am also talking about the things that I observed and everything, then…

R: tell me everything that is related to learn it. (para agarrar colmillo).

T: No no teacher, do not believe that I will judge you, I won’t do it, neither anything like that, i identified fragments like this one and I want to see if what I see is the same that you see, as we see the same here: the student called you, you checked it and told her is it was …

R: Correct o incorrect.

T: yes, what do you think that was happening in that moment when you checked it, what do you think that you told the student, here is your mistake, or here is wrong?
R: aah because she has to find the result, it means I tell her where the mistake is and she continued trying, changing the numbers.
T: well, like this is as I am going to show you the recordings, you tell me what you see and why did you do it? I do not know if you remember teacher that I applied a survey long time ago.
R: yes.
T: there was a question like this situation, for example, the question is: when a student solves a particular mathematical problem and asks if the answer or steps are correct, that was the first question of the survey.
R: yes
T: then analyzing this fragment, the student asked you if the problem was correct because she already finished it. Then I checked the survey, and you selected the choice: ask the student to read the corresponding section of the textbook. Then my question is here in the recording you checked it and told her that it was wrong and where her mistake was.
R: well, I did not tell her where the mistake was, i just checked it and told her continued looking at the mistake alone.
T: ah ok well, why did you do that and you responded in the survey with the choice of asking the student to read the corresponding section of the textbook.
R: because it was not something that was on the textbook, it was on the worksheet, it was something that she had to seek and seek.
T: then that’s why, here she did not have to seek anything?
R: no, it was an activity.
T: ok, that was the first recording, the next one is.

(we listen to the recording) from there teacher, what happen in the recording?
R: when a student finishes and her classmates do not, then it’s there when i use my monitors who help their classmates but without giving the answer, they try to give guides or tips (clues) to them, the rest of the students will be looking by themselves the solution, but monitors should not say it.
T: and how they are in the same group.
R: exactly, because I told them what, when I assign homework, you do not have to cheat it, it’s better that they ask how can I do in order to do it by myself. There is support. Even I have some students that are the monitors and when their classmates are behind and they help their classmate to get a good grade, I give them an extra credit.
T: and the whole class knows who the monitors are?
R: yes, there is one monitor per group. There is one advanced student and two students that are behind.
T: the groups are done by you?
R: yes I do it, they do not integrate the groups as they want it. I do the groups according the first diagnostic and evaluation, I am getting close the students and I am distributing them because most of the time they want to do the group just among the smartest student, or the undisciplined students, then not, I do the groups.
T: Listening to that fragment teacher, what exactly you were doing?
R: it is what I told you, I am checking the work to the most advanced student of that group in order she can help their classmates without giving the answer.
T: well, we saw that you talked with a student who finished, and then you checked her work and it was right, there was not any mistake. Then this situation fits on the same previous item 1: when a student solves a particular mathematical problem and asks if the answer or steps are correct. Somebody called you to ask you if the problem was right or not, and then you have the same answer in the survey than in the previous situation: Ask the student to read the corresponding section of the textbook. But here, the student has the answer correct.
M: no, there was not a mistake.
T: you did not have the need to ask her to read something, that’s why you told her that.
R: you can see that those questions (survey’s questions) do not correspond with this situation.
T: I was going to tell you that.
M: because for instance, somebody tells me your boss is coming, and then at the beginning I said this does not work with the new reform, well the reform do nor match with me, and she told me it is because you are resistant to the change, and no, that’s no the case, the thing is, for instance, I have to give the student the procedures to allow him to continue doing that in order he can do it, because only 3 students are going to success but the rest of the class is going to cheat, no no you continue working, but to me that’s not right because I prefer to do just the half of the notebook (notebook used by teacher to teach their classes) but the topics to be understood by the students instead of completing the notebook.
T: but if you see these questions are very general, these questions do not specify any topic. We listened to the recording and in the survey you answer asking the student to read the corresponding section of the textbook. Why what we saw in the recording is different with your response in the survey? You selected in the survey ask the student to read the corresponding section of the textbook and in the recording we saw that you checked the student’s work and you told her that it was right and asked her to help her classmates. Why do you think that this happened and in the survey you answer this other thing? Why this difference happens?
R: well I am like this, the thing is what is happening is what is coming up, for example, in that moment, they have to read, we will be back to the same point.
T: very good teacher.
R: because even in the test nobody wants to do anything, we were working on the guide for the ENLACE exam since the last 6 months, they have done all the problems, one by one went to the whiteboard all the time, and now their performance was low in the exam. This guide consisted of 30 items and many students perform low, they do not care about it, they do it without enthusiasm, they want to go on vacations, they do not want to read anymore. One of the things, I tell them read the problem, they say teacher I do not understand it, go and read it twice, teacher I do not understand, let see what it says, ay I have not read it, they want my explanation, no go and read it and if after that third time they do not understand, draw it, then they draw it and say I got it!. Because drawing the problem and read it several times is different.
T: yes, speaking of problems teacher, in this class you assigned a problem of the magical triangle, well can you describe that problem, that activity consist of?
R: i do not remember so well, but it is a triangle were the sum of the corners need to one number and the sum of the lines other.
T: yes, the sum of the corners must be 15 and the sum of the sides 20, and the students have to place the numbers from 1 to 9 in the circles of the triangle, this activity you assigned it in one of the classes, and you said that you were teaching …
R: proportionality, what happened there is, even persons who supervise me made me several observations and they told me that I have to assign problems about the topic that I am teaching in that moment, but I think different: if the students are learning mathematics and learning whatever topic they should be able to solve whatever problem, I think different.

T: and that problem you assigned it for the topic of proportionality
R: no no no I assign, I open my book, for example, the topic that I want, the problem that I see in the book is assigned in my class.
T: were you seeing proportionality?
R: yes we were seeing proportionality, at the end is when i apply the problems of the topic that i am teaching, but sometimes I select any problem of the book, if I like it, I assign it in my class to get the attention of the students or wake them up.
T: and this magical triangle s related to proportionality?
R: to me not, but I, it's what i tell you, i get problems from the guide, for example now i have four different guides for the ENLACE exams, 2013, 2012, el 2011, I select any problem.
T: that problem is in …
R: no, but it is in the math books that I have, I also have activities and challenged activities that are just problem, I mean I assign any problem from there.
T: and is that to involve the students?
R: to make them to be active, I mean because they always come sleepy, then with this problem I wake them up and make them pay attention
T: let’s listen the recording.
R: yes, they made me the observation: ey what you start with certain topic but…
T: who did the observation teacher?
R: now when supervised me.
T: they supervised to..
R: let me look for it, the paper with the observations should be saved in the principal office.
T: it means somebody came to observe your class?
R: Yes because they supervise us regularly and they tell us what they see. One of the comments that they did is that one, I told you colleague that I have to assign problems that correspond to the topic that I am teaching.
T: but it is a good problem, it makes one to think.
R: yes I also think that.
T: it was hard.
R: sometimes students say that it is hard but for others is not; you see how it changes from one class to other.
T: let listen to this fragment of the recording teacher.

R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions. How would it be represented in common fraction, Brenda?
$\frac{9}{4}$
R: no no no, my love, remember that we remove the point; it would be also 225/100 it would be the same, this was seen previously. You do not remember it?

R: for example here, the students do not know it and those are topics already taught, topics that I explained it and I think that they learned it, and they do not learn it. Then one has to use the
notebook, that’s why I told you to look into the notebook, for example the topic of fractions is there in one unit and then in other units ahead is presented again with problem using fractions.
T: and they do not remember it.
R: they do not remember it.
T: then you go back in the topic.
R: well to re-teach it, I do the procedure to allow them to solve it. Looking the mistakes here, I wanted to teach 3 topics in one class, it was too much. In other class I divided the three topics by three to get the conclusion faster, then for the other class I divided the topics in sections and I teach them by hour, sometimes I planned the class and I say certain time for this because those are topics already taught and then nothing.
T: it takes more time that you have it planned. Well I think up to there.
R: according to the new reform this should not be done.
T: no?
R: no, no I should start to explain this, to involve them into this, to read the worksheets that we saw the previous day and let’s do it. Recall it, recall it and nothing.
T: what is it happening in the recording?
R: I see that the students do not are engaged, I do not get their attention.
T: and what about you? What were you doing on the recording?
R: well I guess that I am explaining to the students but they do not get it.
T: you were explaining and then you asked a question.
R: nobody answered me.
T: yes one student.
R: that topic we saw it in the second unit, they have to remove just the point, after the point, it means those are hundreds and should be put below (denominator) and that is what I told them.
T: we saw the same, right, you were explaining something then you asked something and one student answered something wrong and you continue explaining (ignored the student), why did you do this teacher? What was you thinking to act in this way?
R: I was nervous.
T: what was the purpose?
R: well, those topics were already taught, it is supposed that student learned them. Then I started to do a review, a feedback because the topic was already covered. For example fractions and decimals, I guessed that this topic is already learned, then when students are working on the worksheets, they took the textbook and started to look the topic again, I told them read it, therefore the textbook is a support for learning, but it is supposed that we are doing a review because we saw it and then nothing that…
T: that’s why you told them how to do it?
R: yes.
T: why if you do not do this? what?
R: I could not advance.
T: they will not do the activity, and then I was analyzing this fragment…
R: also when we see a new topic, I say read it, but they do not want to read it neither. For example you have the habit of Reading?
T: well at this time yes.
R: because I previously took a book and fell sleep, then how I am going to inculcate this habit to my student if I do not have it. In my science class that I am teaching in the technical middle school I…
T: ah really,
M: I asked students to do reports about interesting topics, topics that called their attention to relate them to the class. They do not want to read neither. Then I start the class asking them to read 15 minutes, then we discuss it, but here it is different. But we have to make them to read and given credits for it.
T: well teacher, I related this fragment of the recording to the item that says when a student is not able to see or identify something obvious. In this case, you already taught the topic. Then, here in the item, I asked what do you do? And you answered me: ask the student to continue trying until he/she get it. But here in the recording we saw that you re-taught it, why do you think that it happened, why you did not ask them to read or why you did not tell them look it until you get it, why did you decide that it was better to re-teach it or explain it again?
R: you mean why I did not ask them to read?
T: or that they continue trying until they get it, why did you consider better to provide an explanation again?
R: well, because they are lazy, I prefer to explain it in order to help them to get an understanding about it.
T: I am telling you what I see in the recordings, that’s not meant that it wrong or right, those are fragments of recordings that I am focusing on them.
R: maybe in this situation I considered better to explain it again. I mean there are some students who like to read and one can ask them to look for something but there are others do not do it. Then how I am going to do it: explaining and explaining it again. There are some students who look for me during the recess and tell me: “teacher I do not understand”. Then I tell him/her, “read it and tomorrow we discuss it”. Then at the following day, the student tell me “I read it, teacher”, even sometimes I give them videos of you tube to the 8 graders, because I am also teaching this new class at the end of the academic year. Then I ask them to watch videos, I think that I last two weeks to achieve that all the students watch the video and do the task that I asked them. They do the homework but they do not want to watch videos. They do not have to read just to watch a video and click it on the link and that’s it, but they do not want to work, it is not the same thing. For example this survey (KtAS), I answered it according to my thinking as I am. Well, I can see things that maybe I do wrong because one does things to finish as soon as possible and go home, but your survey I answered it as I really am and act.
T: let ’se listen to this fragment of the recording teacher.

(We listen to it). Did you listen to it fine?
R: no
T: there, the groups are revising their work, right?
R: yes, but there are some students who do not know what they are doing.
T: what did you listen to it? What do you think that happened in that fragment?
R: they did not understand it.
T: yet?.

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R: because I had to repeat the same and the same: they should add, the corners must be…
T: then what were you doing there?
R: Repeat the indications.
T: in the survey teacher, you was aware of your students had not understood the problem, you were walking around the groups, and you saw a mistake that’s why you stopped with one group. Then the item of the survey is: when a student made a mistake what do you do? And you answered it: let the student explain his/her response. But in the recording we saw that you explained it and repeat the indications.
R: because he was wrong.
T: why do you think that it was better to repeat the indications when you saw the student’s mistake? And here in the survey you answered: let the student explain his/her response.
R: because he gave me his answer, I do not know, I mean the student misunderstood, should I leave him?
T: no no.
R: yes, it is why you did that, I was providing the indications again because I saw that the student was wrong. In this survey, I do not know, you looked at them end of the class (plenary), three students went to the whiteboard to solve and explain the problem, I mean to that, it is an activity that I have to finish it in five minutes according to me, but it last more time. Then I plan my class with the student on the class C…
T: where Rebeca is, yes I remember it.
R: Rebeca talks so much, then in the plenary of the class is when students explain why they did that, students say “no because this and this” each one does it different and here is when students explain it, each one separately, sometimes they do it wrong and everybody looks it, and then they have to make the corrections.
T: each teacher knows her/his students and the way that they work, I am not judging you.
R: no no, but as you said, why you did this and answered this in the survey. I tell you because in the plenary, each group, I provide the indications, everyone does it as he/she believes, even the monitor is who does the task first. Then students go to the whiteboard. I do not know if you see that sometimes students are wrong, but I say well explain how you did it, how you get it, I meant to this. For example there, the student, mathematics is a try and error. To me in daily activities, if the student does not get the right answer but he/she gets a result and he knows how to correct it or I tell him how to do it, all this to me is learning.
T: well that’s the way to learn, we make mistakes and we learn.
R: the student needs to see where his mistake is.
T: Let see this other fragment of the recording.

R: I am wondering why I do this if my way of thinking is this other. Because to me that problem students should solve it as they can and believe, there is no way to check something in their notes, just do it!.
T: in the recording, you were supervising students’ work by groups, then one student showed you her work and you told her.
R: What I said?
T: you are wrong here. Check this part of the problem, you told her that, in what you could listen to that fragment, what can you describe of it?
R: I did not listen it.
T: it says check this part of the problem.
R: when I ask students to do a problem, even when those problems are items of any test, I say delete the item if it is repeated because sometimes the same problem is twice. Then students do operations to get the solution, and I think that in the operations was her mistake. That’s why I told her check the operations.
T: well it was a student’s mistake. Your mistake is here. Why did you do that, teacher? What was the purpose to act in that way?
R: the student can see where her mistake was, if he/she has the operation wrong, he/she needs to do it again. I think it was because she was adding something, she did it wrong in the operation, I said check the operation to find your mistake.
T: this situation fits on the same item of the survey: when a student makes a mistake, and here you told the student “check the operation” and in the survey you answered: let the student to explain his/her answer. This answer is the same of the previous one.

R: i mean, if a student shows me an operation and it is wrong. In this type of problem, I check it quickly and I say: you are wrong my king (mi rey), mijo check it again to find the mistake. I think in this case I do not have to explain it again. For example in certain problem, during the plenary (puesta en comun), I realize where the student’s mistake is, it could be at the beginning, then I identify if the student make a mistake because of the problem or because he do not understand it, then the student explain me.
T: what about this case?
R: in this case, it was an addition, the student was adding wrong. Do it! Do it again to find the mistake.
T: teacher, let see this other fragment of the recording.
Did you hear?
R: no, no. The students did not read correct. In this other I missed something in the problem because they had not understood the problem. Then, in that part, I did not allow the student to tell the answer because the rest of the class does not do anything if they know the answer. Students need to do it first in groups.
T: what was happening in this fragment?
R: I did not write the problem correctly because students did not understand it. Then I realized by myself when I was reading that something was missing.
T: what did you do when you realize about it? .
R: I clarified it to the students, I read it right as it should be, because I realized it when I did not understand it when I read it. I wrote the problem but I do not know what happened it, then I identified the mistake.

T: for example here, i see in this situation that students were having hard time to complete the task. Then I related it to the survey with the item: when students are having hard time to complete an assigned activity/task, what do you do? And you answered: analyze students’ difficulties and provide hints.

R: yes, but here I identified my mistake.

T: then that’s why you did that?

R: yes, yes, yes, yes because the problem was written incorrectly so how they would solve it.

T: let’s move to the last recording.

R: there were the lazy students, some students asked me for a pencil, ah question for you, you have been visiting other schools, there is any good or bad difference among us?

T: both differences.

M: tell me the negative, because I would like to know to eliminate the negative, i asked you because as i start my classes, I wonder maybe nobody do it like me. In this aspect many teachers are tired about all this.

T: about what to follow the new reform?

M: yes as it is.

T: let’s finish this interview and then we talk about it. In this recording you assigned an activity, students were working on it, and you are supervising, then you provide a hint (tip). This activity is the magical triangle.

R: yes because they could not do it, and some they did not want to work on it. And then again I have to engage them.

T: in this fragment what can you describe?

R: I did not have the students engaged in the class.

T: let’s start by parts, you assigned a problem and then students were working on it.

R: some were talking, playing, well i think they were talking, they did not pay attention to the class, i was on the class but they were not.

T: and what were you doing?

R: well, I was walking around the groups trying to make them work, but I did not make students to work. There I was very flexible with them because I provided a big hint: the number 9. But this did not happen with the other class. You observed both groups, right.

T: yes, you assigned this activity to both classes.

R: I assigned it to the three groups.

T: and why do you think? What was your thinking to act in that way? Why did you consider better to provide that hint?

R: to this group because they did not want to work and I had to involve them a little bit more. First, they have the attitude of not work, then as I can and gad gives me to understand I have to make them to work.

T: and look at them like that.
R: yes I achieved that they work a little, because they had that attitude. Sometimes, there are situation in which they do not want to work, no, no, no, but one has to make them work and engage the students.
T: in this situation, teacher, the students do not want to work. Then you provided tips or hints and yes, this was you response in the survey: analyze students’ difficulties and provide hints, but in this case, they do not want to work, in the survey it was because they were having hard time. These are the differences of being in the class and answering the survey.
R: aha.
T: that’s it. Thank you, I do not know if you want to add a comment.
R: no no, use whatever you need of this and think that it will help you if not remove it.
Omar Transcription

June 26, 2014

T: more than anything, I want to thank you for your time. This is an interview, a talk. I will show you recording and then we will talk about them. I am not going to judge you, neither evaluate you because I do not know your students as you know them. This is confidential, Ruben is not going to have access to this information. Try to recall as you can to answer my questions please. I told you I have never taught to this grade level, I cannot tell you what it is right or wrong because I do not know this context. Then let’s listen to the recording.

T: these are fragments of 25 seconds. Some of them are hard to understand. However, you remember that I was observing and taking notes, thus, I can understand better the recording. What do you thinking that it is happening in this fragment?

O: students were revising the activity that I asked. I was walking around the groups to see if students are working and see if they need some help.

T: there, the students went to your place.

O: yes she is Ileana.

T: the students went to your place to ask for your revision, and you told her that it was right but there was 4, and the student said ah ok it is 4. Then here, for example, what can you describe that it is happening, in general terms, in this fragment, what did you do in this fragment of the recording?

O: well I do not know, clarify it, I do not remember it.

T: yes I know, and then the sound is not good. Are You supervising the students were working?

O: in order to the students do it right.

T: their work?

O: their work

T: and there your action was: yes it is right but here is 4, right? then…

O: showing her where the mistake was.

T: yes, you exactly did that, it is what I understood in this fragment. Maybe you do not remember, but why did you do that, why do you think that telling the student where the mistake was? What was your thinking to…

O: in that moment, it just happened, clarify it to the student because it was right just her writing is deficient.

T: but she has a 4 or other number?

O: yes she has a 4 but it did not look as a 4.

T: ok teacher, I do not know if you remember.

O: since the beginning, I thought that she was a student with a special problem, she is very distracted, she cannot pay much attention in the topics, but it was the only thing that happened with her, and her writing is horrible as she were a little kid.

T: I do not know if you remember that in September or October of the previous year, i came to apply a survey, in the last section of the survey, I provided a classroom situation and you write a number 1.

O: yes if what I do first and then.
T: then, I relate this fragment of the recording to the item 1: when a student solves a particular mathematical problem and asks if the answer or steps are correct. The student approached to you to ask for revision of her work. Here in the survey, your response was: ask the student to explain his/her solution. Here in the recording we saw that you said it’s right but here is 4, what do you think that this happened? Well, here in the survey you said to ask the student explain her answer and in the recording you said it is right but here is 4, what do you think about this difference?

O: with what I selected in the previous survey, maybe the level of the problem was not the same that I imagined when I answered the survey.

T: in that moment, you were looking linear equations.

O: yes, how to solve for x and get the result. This student already had it solved, I do not know, I do not feel the need to ask her an explanation if she had all the procedure to solve it.

(AWARENESS)

T: also if the mistake was because of her writing.

O: yes it was of writing, to recall data.

T: if the recording would have a better sound I would show you a bigger fragment to help you to recall it.

O: or you should come before such as the following day of the observations.

T: no, I could not do it because look I had to transcribe it everything. Then to translate it in English, doing this I last too much time because I had problem to hear the recording, Let’s listen to this other recording.

T: look this activity, you already explained them simultaneous equations.

O: no you were when I brought material and asked the students to go to the whiteboard to solve and explain the problem.

T: no, i did not see that you asked any student to go to the whiteboard. We have problems to hear the recording. But one student approach to you to ask you for revision of his work, and you told him that he needed to do the procedure. I do not remember, but it was the student said. What can you describe about it?

O: yes, students need to solve the problem and write the procedure, how they solve it for x and the result.

T: you always ask them the procedure in your tasks?

O: yes, I ask for the procedure in order to tell them where they have mistaken, look at this part,… to know if the way that they use to get to the result is wrong. From here, I can be based on the procedure to say look you are right until here, in this part is your mistake. And without procedure, mentally we cannot do it.

T: here for example, this situation I relate it to the same item, the student asked you if his work was right and you told him yes it is right but do the procedure. In the survey you selected the answer: ask the student to explain his answer. This is not the same.

O: the procedure, how it was the student get an answer.

T: you were asking him.

O: yes he needs to do the procedure and then get to the correct answer. the steps that he made to get a result. I expect that he really learned, if he is able to say I did it like this and this, and shows the procedure.
T: well in that fragment, teacher, what happen?
O: he gave me the activity that he completed.
T: and in that moment, you revised it alone or you already revised it with the student?
O: we already clarified what was missing.
T: here for example, you admitted the work, but I do not know if always that they finish the work, they have to turn it in or you revise it first or... I mean you admit the activities knowing what it is wrong or right or whatever they do and you turn in the revised activity the next day?
O: if there is not more time to do the common presentation or explain everything, I revise it and turn it in the following day to continue working on it with the same topic and with their mistakes. I imagine that in that moment the class was almost over and I think that I revised some students’ works and commented about it.
T: because for example this fragment it is the same than the others, the student took his work for your revision and you did not tell him yes it is right or no it is wrong.
O: yes it was because of the time, because I would revise the work in that moment. I would say: explain me again, how did you get to this answer. Yes if you have it right, you did it is very good, but yes, I think what happened was the time, the class was over, in minutes I told them turn in their work.
T: yes, the class was over.

O: I did not recognize my voice
T: what can you describe of that fragment?
O: well, we were starting the class with a topic, and they did not remember the topic at all. We already saw it previously, it was linear equations.
T: you mentioned a new term that is transposition or transfer (transponer o transposición) and then you asked the students what it is the meaning of the term, right? and students answered: it is something to divide, among other answers. Because they said two or three wrong answers. Then you told them what it is. I do not if you remember it but why did you act in that way in that moment? Why do you think that it was the most convenient or what was the purpose to say that to the students?
O: it was something important that they should know it, it is a basis for equations, in matter of terms.
T: and if you did not provide an explanation they won’t be able to understand it or solve it without the explanation of that term.
O: the explanation.
T: or they already knew it.
O: no
T: and always that you are teaching if there is a new term.
O: when it a new term, up to an hour, if it is a topic already seen previously, using their own experience to get answers, I explain them and clarify much of the things in the activities. But if it a new topic or term I need to explain it using the simplest way in order to students understand little by little the topic, and they will be gaining knowledge whether they have not seen that topic.
T: here for example in this part of the recording, I related it to the item 9: when a student makes a mistake, for example here, the student made a mistake to define transposition, right? And then I asked you here in the survey what you do when a student made a mistake, and your answer was: let the student to explain his/her answer. But here in the recording we saw that you told them what the correct answer was. Why do you think that happened in the class and your response in the survey? Why do you think about this difference? Here the students made a mistake and here your action was saying what was correct, and here in the survey you answered let the student to explain her/his answer, why is it different what we saw in the recording and your answer in the survey?
O: I think that the question of the survey was about an specific activity of mathematics such as a problem and then we can see where the mistake is. But here the students did not have a really mistake. what they had was a lack of awareness of this word or term.
T: Is it not the same to know the meaning of a word and solving a problem?
O: a problem that has been practiced, or it was already done, and let’s revise it, let see why there is a mistake, because in the problem we can do that. in the survey that you applied me I considered this question as it refers to this type of problem. A problem that can be evaluated, in this type of problem I ask the students to explain why, how they get to that result, and then I explain it if there is something wrong. However, if students only revise the answer, I cannot check all what I already mentioned. I need that students make the procedure, even in certain topics I ask students to say what the easiest and hardest part of the activity was.
T: ah yes, I saw that you asked them for comments about the activity.
O: yes, comments, therefore, that Word (transposition or transfer) is not a mistake, it is a lack of awareness of this word. They were not going to know the meaning of that word in that moment if I had not told them and explained it. For example, while solving any mathematics problem, we can see the mistakes and why we did a mistake to solve the problem. This situation to me it is not a mistake it is a lack of awareness of that word. To explain that word was the easiest action to me: how to move a term from this side to the other so there is no a mistake?
(We listen to other recording)

T: in this recording, well, we cannot hear very well. Students were working on groups and you were walking around the groups. Then you stopped with one group an you told them the addition is incorrect. For example, I told you what I saw, that’s the description. What can you tell me...
about your action? I mean you are walking around the groups and when you see a mistake you tell the students where the mistake is or you say what happened here?
O: yes, students usually ask me or if I identify a mistake, I focus on the mistake and we clarify it. There, the student was work on certain thing, it was an easy thing. So what he had wrong was an addition.
T: yes, you were seeing some equations, you stopped and told the students of one group, I do not remember the student’s name but 15 + twenty something is this? I heard that and I took notes.
O: yes they should analyze it, check fine what we did.
T: for example this situation also fits on the item 9, here the student made a mistake. In the recording we saw that you said check the addition, this plus this could not be this, that is what you commented. Here in the survey, it is the same item than in the previous situation so it is the same answer: let the student to explain his/her answer. Why did you think that this situation does not fit on the same item? Why you preferred to tell him this addition is wrong instead of asking the student how did you get this?
O: my justification is that: the student was starting to do the procedure of the problem, so the result should not be gotten if he was doing wrong.
T: it means, you were walking and you saw a student’s work and you told him you are mistaken in the addition. Why there? I am not saying that it is better to do that but...
O: it was prudent to me to clarify it, that’s it. The student’s mistake was in a simple addition.
T: and then, for example, in the survey you answered: let the student to explain his/her answer, I mean you could do this.
O: at the end.
(we listen to other recording)

One group asked the teacher “from where did you get this in the example?” and the teacher did not tell them in order they can deduce and think about it. After certain minutes, the teacher explained them just one part and allow the students to deduce the next part.

T: what do you think that it was happening here?
O: it was an interchange of ideas about how they were doing their work: asking them to check it fine.
T: in this situation what it happened is the student called you and asked you from where did you get certain part of the example provided, she tried to do the problems that you assigned. Then she looked at the example to try to solve the problem, but she did not know from where you got certain number in your example. Thus she asked you, but you did not want to tell her in order to… you did not tell her. after several seconds, the three students of that group were discussing about it. Then, they explained what they thought it to you, and then you started to explain them from where that number was obtained. That is what I see, why do you think that it was your action? Why did you act in that way? What was the purpose to act in that way? For example there, the student asked you about the example and you did not want it to tell her but after some time you told her, what happened here? Why did you act in this way? Why?
O: I wanted to make them to analyze it more time,
T: the problem. And them why you decided to tell them after certain time, why after certain time you told them from where that number was?
O: because they started to make conjectures, they tried to do it, they already had thought about it, that’s why I provided the explanation of it in that moment. In this specific case, they started to see what they needed it, after that, they analyzed it. I let them to check it by themselves. Then I asked them to be agreed into their group to see what they can do and analyze to see if it is right or not, they interchange ideas, and they try to give their opinion and participate. But there are also students who join with a smart student or who has more knowledge than they to ask only this student to do the work without working on groups.
T: they are in groups, but everyone
O: to me that was my action, you try to know it, tell me why do you think that the procedure is, then when I saw that they make several attempts, I explained them.
T: this situation I related to the item 5: when a student is unable to see an obvious pattern in the problem, here it was not a pattern. But in this situation you already explained the example, and when they were working on the problems, they could not continue because they had doubts. They asked you about it, but you let them to think few time. Then after you listened to them, to their conjectures you started to explain it. For example, in the survey, your response wa that, yes, yes it coincides with what you did and you answered. Then let move to the next recording.

T: what happened in this recording?
O: students called me again to check their work.
T: that student asked you what needs to be moved to this side of the equation. Then, you were explaining what can be moved to the other side and how to do it.
O: yes the unknown letters (variables) are moved to one side.
T: why this was your action? What was your thinking to act in this way? The student asked you, teacher, this can be moved to this side and you explained how to do it, why did you act in that way in this situation?
O: maybe the student did not understand what I already explained, the explanation was written on the whiteboard. In that moment the student did not pay attention when I explained that. And if he did not know that: what terms need to be transferred, which one to this side, and which one to the other side, he was not going to get the result. If he had not understood the first to develop this type of problem, he was not able to solve it.
T: then, in class, if a student does not understand, and the class is solving problems, you usually explained him “teacher how to do this” even you already explained it.
O: yes, i explain them many times. Generally, I think maybe because you were there, I told them let’s pay attention who has doubts.
T: you provide an explanation to the whole class?
O: yes, i explained it again to help the students who did not understand. I see other example, and I explain it again. With a second explanation, students learn how to do the procedure to get the result. Then we analyze it to see if it’s right.
T: and for example, the second explanation provided is the same than the first one, you explain it as the first one or do you look for another way to do it?
O: yes, generally we do that, maybe because you were there, nobody wanted to ask. Because I know that there are many students who did not understand, we do it with a different way, a
simplest way. And I change a little bit the explanation that I already provided to make it more comprehensible, do the same steps, sometimes I try to do it shorter. and that is what I do because I know that there are many students who did not pay attention and asked…

T: you are right because one thing is explaining the problem to the student who did not pay attention and other thing to explain it to the student who is trying and trying but he/she is having hard to understand it. I guess that you have identified your students?

O: yes, I know them, and yes, I do the explanation.

T: to both kind of students even if he/she does not pay attention?

O: even if the student did not pay attention I explained it. One student asked me to explain it again and I do it even the student who asked me that did not pay attention, I explain it in front of the class to allow everybody can hear. I solve another problem on the whiteboard and I explain it. This problem is simpler. Then, what bothers me is when the same student who asked me for this explanation is not paying attention. I tell him what happen? I am explaining and you are not paying attention.

T: and is it happened?

O: yes, yes, the students are restless, I would like to have them with the attitude of a person who want to learn.

T: that they pay attention and quiet.

O: they come to the school to look for a girlfriend, they are critical; they come with the negative attitude of not learning. But I usually explain them; I am very patience, if they have doubts.

T: yes i noticed that, for example this fragment of the recording I related it to the item 10, when students are having hard time to complete an assigned activity/task.

O: I already started with this problem and they have not understood yet, the first about the term that we need to move to the other side.

T: for example here in the survey, you selected: analyze students’ difficulties and provide hints. And in the recording we saw that you explain it again, why do you think that you answered that in the survey and in the classroom you preferred to explain them again instead of providing hints or tips to help them to know how to do it?

O: I tell you, I usually I provide them a review of easy activities that they can do, I think that if…

T: no I mean it would be more convenient to provide hints in that moment? I told you this because you answered that in the survey not because I think that. Would it be better to provide hints, I do not know, or it would be more convenient explain the topic or what the student did not understand again?

O: in certain part of the procedure maybe will be good to use hints or tips.

T: for example there?

O: it was just an equation and then to do a movement of the variables or coefficients to the other side.

T: is it not like a hint?

O: it is not a hint, what is being moved to the other side, what it is being transferring. I am usually nervous when somebody is supervising me. I do not know why.

T: yes I noticed that you were nervous.

O: I am not considering wrong that somebody comes to observe my class if know that I do good my work. It is fine to me. It helps me to improve my teaching and change the things that I am doing wrong such as this about providing hints

T: i am not telling you that you are wrong.
O: let me tell you something, remember when you came to apply the survey, I was late and you have to leave at certain time…
T: yes, i do remember, even that’s why you did not finish the TCKS
O: i did not complete it.
T: yes i remember it.
O: i answered the survey with a little of pressure because of the time. It usually happens.

(he received a call)
T: just to finish teacher, can you describe me how your classes are, how you start tou class, what the students do, how they work?
O: yes of course, I start calling the attendance list; it is recommended to start with this.
T: how are your classes?
O: the attendance list, we start with the topic, if the topic is new i provide an explanation, i highlight the important points of the topic, the explanation, the practice…
T: you always ask for activities to the students
O: i explain the topic and then i see what it is next, we use the worksheets (consignas), the students need to do that worksheet by their own, if they cannot do it, the common presentation takes place, it is with the whole class.
T: all this happens in a class?
O: yes, well sometimes we do not finish with a topic, then the following day we continue with the common presentation and review the topic. Sometimes we finish in one class, I provide them 15 minutes to solve the activity. They need to solve a problem using their experiences, their knowledge, and an interchange of ideas. They have different books form different editorials. In one group all the students have a different book. They need to bring always their books. They look the information on the book then they interchange the books and compare the information among them. Well sometimes, the worksheets have exercises that are hard to solve because of their level of difficulty. These worksheets are designed and provided from Mexico City the SEP. the worksheets are the started activity. We start with the topic and then we use the worksheets. They should solve it without help just using their books, and then the common presentation is the next part of the class.
T: that common presentation is at?
O: at the end.
**Rogelio’s Interview**

R: Let see my messy students
( I showed him the first recording )

T: I do not know whether you heard?
R: it is easy.
T: did you listen to it?
R: yes
T: can you describe what did you see in the recording?
R: I approached to one group and I helped them to find the answer, and then they asked me that’s it in order to put their staff back to go out?
T: when everybody was leaving one student approached to you to ask you something?
R: aja
T: and you told her yes, you only miss a 4 or one part, right?
R: yes
T: well it was what I …
R: perceived? (he interrupted me)
T: the way as you acted there, ¿why did you act in that way? Which was the purpose to act in that way? Why did you do that?
R:ah well, in order to tell her that she have not finished yet, there were some steps that she missed, the way that she used to find the answer was wrong, she needed to know that there are other ways, more mathematics I mean she was guessing.
T: the student asked you: is it right or is it right certain part of the problem? You told her, you started to explain her how she should do it and then everybody was leaving and she asked you again: then is it right like this? And you told her that she was missing a 4.
R: aja
T: did you remember that I applied some surveys?
R: yes
T: this fragment of the recording I related it to the first item of the KtA survey: when a student solves a particular mathematical problem and asks if the answer or steps are correct. In this survey you answered was: ask the student to explain his/her solution. But here in the recording we saw that you provided an explanation to the student. Why do you think that this difference between what you answered in the survey and what you did in the recording happens?
R: ah because of kind of the topic, because it is algebra, it is more complex for the students, thus it is difficult for them to explain with their own words or using their previous knowledge in this type of situation. Because students were starting to see more about algebra, it is more complex, I have to explain a little bit more to the nine graders, and I have to orientate them more than the 8
graders. That’s why the difference in the survey’s response, and how it was an application problem, it was even a little bit more complex.

T: do not think that I am judging you or evaluating to you.

R: no no

T: I have never taught at this level, but how there are fragments that coincide with the items of the survey, that’s why.

R: yes, the thing is to adjust it, that’s the detail.

T:yes that’s why I am doing this interview to clarify these things.

(I showed him the recording 2)

R: yes he/she is factoring, what was inside the parenthesis, because using the general formula is more complex well its process is longer, to use the method of the parenthesis (factoring) is faster but sometimes it is not because of the numbers.

T: in this fragment, the students asked you if what they were doing was right and you told them yes that they can do it like that or look for another method, and this also fits on here (in the same first item of the survey) and your answer was the same: ask the student to explain his/her solution, in this situation we saw that you preferred to explain them instead of allowing the student explains her answer, why do you think that this situation happened?

R: because of the same that I told you, it is algebra, and you know what the quadratic equations are. Now the students are distracted, they have an instantaneous short memory, that everything learned in one class, they forget it the following day. That’s why I try to orientate and explain it again. She/he already got an answer, then, I suggested another way to do it: it is better and safer to find the answer but it takes more time.

T: she solved using one method and you suggested other method, why did you do that if she solved it correctly?

R: in order to make her recall that there are two methods, and that, there is more than one way to get the answer. It was what I commented, that is the advantage of mathematics, there are different methods to get an answer, and it is repetitive you only need to change the numbers and signs.

R: ah you have the recording by times.

T: yes, If you know all the time that it took me to do this, listen to the recordings several times. This is the next recording it is short

We listen to the recording 3
T: what did you observe in this fragment?
R: he asked me by what it should be divided, by the total number of grades, I confirmed how many grades were, he told me three, I reaffirming his answer.
T: why did you do that?
R: because I wanted to confirm it in order to the student was sure, and he was able to do it, he had the idea but he was unsure, thus, I just confirmed it.
T: I related this fragment to the item: when a student is unable to see an obvious pattern in the problem. In this case, it was not a pattern; the student could not see a mistake or know by what it should be divided, right?
R: aja
T: your survey’s response was: modify the context of the problem. What we saw in the fragment of the recording is that you told him what he had to do.
R: well he had to calculate the average of those grades, there were two grades given and the average should be 8, there should be three grades, and he asked me by what he had to divide and I told him three because there should be three grades, I confirmed it that’s it. I did not find another way to react and how it was not a pattern it was not the need to modify the example.
Let listen to the recording 4

T: what can you describe about it?
R: we were solving an equation, I was helping them to solve it in order to make them recall it, again in regards of the quadratic equations, we already saw how to solve for certain variable, we saw several ways to solve it and we are, I was telling them [making them recall] as I already said they forget everything, then we were recall the topic again, I was explaining it again.
T: what you were doing in that fragment of the recording?
R: I was asking them about the answer and how to solve the equation.
T: it coincides with what you answered in the survey.
T: before we continue with the next recording, why did you do that, why explain, why to ask that to the students?
R: in order to involve them in the solution of the problem and make them to recall how to solve it, and at the same time, they were solving it in their notebooks. It was also with the purpose that making them to work because I do not have to do everything myself, they also have to do their procedure, calculation even mentally.
Let listen to the recording 5
T: what can you describe of this fragment of the recording?
R: students were looking for multiples that been multiplied to have the number 300 exactly… because it was a second grade equation, 360 I think so, additions and subtraction were done in order to calculate those number, students were working on it mechanically and subtracting correctly, one of them told me if it should be a 346, and I said no you need to continue looking.
T: why you told him? Why did you provide that explanation to the student?
R: because the student had not gotten the result yet, it was 360, so he needed to continue looking for other numbers.
R: I am congruent with what I did and said?
T: yes, sometimes there are other factors, one thing is to describe and other is when you are in class, right?
R: yes, yes
Listen to the recording 6

T: what happen in this recording?
R: I was asking how much is x by x, but students said 2x, they were adding x+x, in order to make them recall how to multiply it, I did not tell them anything until they get it. Then somebody recalled it and said the correct answer.
T: somebody told you the correct answer?
R: yes somebody answered correctly.
T: why did you act in that way?
R: because I did not want to give them the answer, I did not want to facilitate them the problem, I want to make them recall it and they were recalling everything, all the procedure to solve the multiplication of variables and find the values of that quadratic equation that could be solved using parenthesis I mean factoring.
Let’s listen to the next recording 7

R: ok because I am solving for this variable, I am going to move this number subtracting. We are going to have x=24-14.2. do not do the common mistake of saying oh it is 10, because it is not, you need to pay attention to the .2 that 14 has. So the result is?
S7:10.2
R: no no it could not be 10.2 this is the common mistake.
S8:9.8
R: 9.8. then what is the grade that this person need to get an average of 8?
T: what can you describe about this fragment of the recording?
R: I was advising students before they make a mistake, but I have not told them, I should allowed that they make a mistake and then correct them.
T: however, one student mistaken and said
R: 10 point something
T: 10.2 something like that.
R: ah yes, it was not 10.2 it was 10.8, generally they want to subtract en multiples of ten and it looks easier but they forget the decimals also need to be subtracted.
T: which was the purpose to act in this way?
R: to avoid that they make a mistake, jejeje however, they mistaken.
T: the student who made a mistake, what did you do with her because she provided a wrong answer?
R: nothing just I told her that she was wrong. I did not punish or scold her, I should give a little blow jejeje
T: ok, this situation fits on the item 9: when a student makes a mistake, as she made it, you answered in the survey: explain him/her why it is wrong and here in the recording we saw that… what did we see?
R: that I told them before what was the common mistake, I explained them before.
T: aja you told them but anyway the student made the mistake, and what did you do with her, I mean you told her where the mistake was, right?
R: yes
T: what she said was not right, you explained her why she made a mistake and you wait to somebody said the correct answer, right?
R: yes
T: why did you answer this in the survey and you did this?
R: well because I was ahead, I told them why they could make a mistake instead of waiting that they make a mistake and then to tell them where the mistake is. In this point, I was ahead, and I anticipated it because of the same practice of teaching, students have made that mistake before, thus, I told them about it and at the end, I did not explain why they were wrong because I already anticipated it.
R: it was a common mistake in all my classes.
T: ¿yes?
Let listen to the recording 8

S11: teacher so we have to add and divide what?
R: we already know the value of x, I add the 3 grades, this, this and this. And then divide it by 3 to see if we get 8. That is the verification of the problem (OC: kTA he repeated the same directions to the students, he did not allow students to look for solution or procedures.) is it right?

T: what can you describe about the fragment?
R: we were working on the problem of the grade average and they asked me if they were doing fine in the procedure and I told them yes, they have a number + number + unknown number, which are the three grades, it is equal to 8, it is the more accurate way to describe the procedure of the problem. They only needed to verify those three numbers, adding them and dividing the
result by 3, and it should be 8, that’s the correct answer. They already got the answer that’s why I said that.

T: I was going to tell that, in that time, students were having hard time to find the answer, that’s why students asked you something and you explained it, yes?
R: aja
T: why did you act in that way?
R: as I told you, they already have the answer they only wanted my confirmation of the correctness or incorrectness. They have it correct, I only asked them to verify their answer because they only needed to add them and divided by 3 and it must be 8.

T: this situation fits on the item: when students are having hard time to complete an assigned activity/task, here what I observed was that students were having hard and they were asking you again and again, that’s why I deduced that they were having hard time and they asked you something right?
R: yes
T: then you saw that students were having hard time and you confirmed their answer or explained them where their mistake was. In the survey you answered: analyze students’ difficulties and provide hints in order to they will be able to solve the problem, and in the recording we heard that you said that it was right, why did you answered that in the survey?

R: because their answer was right, and the corroboration was only missing in order to they can see what they found was right, other way was to re-write and re-do the contextualization of the problem.
T: then…
Let see the recording 9

T: aquí que me puedes decir de este pedacito?
R: estábamos despejando para encontrar la ecuación cuadrática y poder solucionarla y les recordé como despejar para la forma de ecuación cuadrática.
T: ahí en ese pedacito estabas tú caminando entre los grupos y estabas viendo lo que estaban haciendo y ayudándolos y luego cuando ibas caminando rumbo al pizarrón, les dijiste hay que hacerle el intento, porque viste que unos estaban batallando y unos no podían verdad? Y luego empezaste a explicarles algo y les dijiste les voy a ayudar con un poquito más, que puede ser el paso que escribiste en el pizarrón y hasta ahí se los dejaste para que ellos continuara verdad, porque actuaste de esa forma?
R: porque se nos terminaba el tiempo y no encontraban la manera de despejar, solucionar la ecuación.
T: tu crees que si no les hubieras dado ese pasito, no lo hubieran hecho?
R: no si lo hubieran hecho, solo que hubiéramos llevado otro modulo y en veces el tiempo nos come las actividades y ese si teníamos que terminarlo.
T: este… ahí por ejemplo hay una pregunta cuando los estudiantes están terminando un trabajo o una actividad igual contestaste que analizarías las dificultades que los estudiantes presentaran y les daría pistas y porque pusiste eso en la encuesta y aquí les diste un pasito más?
R: por lo que comento que nos exigen cumplir con el tiempo porque hay suspensiones por exámenes, por reuniones etc. Y también si nos llevamos más tiempo dándoles pistas nos vamos ir atrasando a terminar los temas a cómo deben de ser.
T: tu consideres que ese pasito mas es como una pista?
R: si lo considero como una pista.
T: porque?
R: porque no le estoy dando la solución completa solo les estoy recordando otra vez como hacer el despeje y no les dije completamente bien por eso les pregunte y sobre lo que ellos me dijeron fui haciendo los procesos.

Video 10

T: me puedes describir que es lo que está pasando?
R: les estoy ayudando a visualizar el problema, es un problema del área de un rectángulo y nos dan el área de rectángulo y nos piden encontrar las dimensiones de los lados que un lado es 4cm mas grande que el otro y estamos contextualizando les estoy diciendo que se imaginen el rectángulo, que es el área hasta encontrar la representación algebraica.
T: y cual fue tu propósito para actuar de esa manera?, ¿Por qué lo manejas así?
R: es muy difícil para casi todos los alumnos y mas los de secundaria ubicar un problema contextualizarlo en lenguaje matemático, se batalla demasiado.
T: entonces prefieres irlos ayudando?
R: si yo prefiero hacerlo así, porque he tratado que ellos lo hagan solos pero se llevan bastante tiempo de 2 a 3 días en los que les estoy recordando lo que es el cuadrado, lo que es el rectángulo lo que es el algebra realmente, uno, dos tres lo logran desarrollar pero los demás no de hecho se ve que algunos logran contextualizar el problema escrito en el cuaderno.
T: este ya para terminar, ya se acabaron los videos, describeme, yo solo fui a 3 clases tuyas, describeme como son tus clases regularmente?
R: pues generalmente vemos un tema, les explico el tema, si hay cosas que recordar, les recuerdo con uno o dos ejemplos a si como los videos los explico yo y con ayuda de ellos, T: esos son de las consignas?
R: si son de las consignas y luego yo los dejo a ellos solos, terminamos las consignas porque son eficientes y les dicto algún ejemplo o vemos ejemplos o hacemos ejercicios, cuando se terminan
los ejercicios del libro o cuando no son suficientes, yo busco ejercicios de otros libros del tema, al final del tema esperamos juntarlo con otro tema o lo que paso regularmente son similares, después ellos se dedican a resolver los problemas por ellos solos, si tienen alguna duda me hablan o el equipo o ellos se acercan donde estoy yo.

T: siempre están en equipos?
R: sí es un requisito que nos han pedido con la reforma que hubo en el 2006, que se empezaron a poner las consignas y que tienen que estar en equipos de 4.
T: tú los haces?
R: depende cuando iniciamos les doy oportunidad de que escojan uno o dos y después yo les agrego otro y luego les aplico un test para que queden variaditos.
T: no pues muchas gracias Rogelio ya se acabo la entrevista nose si quieres agregar un comentario o algo.
R: no nada.
T: pues muchas gracias Rogelio
R: no por nada, por nada
Maria’s Transcription
June 30, 2014 at 8:15 am.

T: for example, here in this fragment of this audio recording, (We listen to it),

One student asked the teacher if her work is right and explained to the student that “you need to remember that coefficients are in one side and these in the other side.”

S7: no no these numbers no.
M: what? What variables do you have?
S7: mmm
M: what are your variables?
S7: this one.
M: but it is only this x?
S7: mmm
M: this is one will be left here and then...

what can you describe about what you listened?
M: well it is work in groups, and then, for example the student who wore blue color is in, well the groups are integrated by one leader and usually the leader is who has more mathematics abilities to guide and direct the rest of the group, the student who is beside the leader is needing support from the leader, and they are working as the new reform says, it means they must be able to find a solution, whatever way, here for example we already saw the topic then they should be able to develop the activity by themselves. However, it is not always like that.

T: here for example teacher, we cannot see but there is a student that went to your place to ask you if there work was correct. (we watched the video again)

M: aha
T: he is you are talking with him.

M: aha and I am still explaining him, an explanation is still provided in the parts where students can have doubts, this activity was still done for the purpose to look for doubts or questions’ students, this activity is previous to the evaluation of this content. Aha.

T: I do not if you remember teacher, when I came to apply surveys?
M: I have an ugly voice (we laugh)
T: other teacher told me “that’s me? I speak very nice” and he asked me for a copy of the audiorecording.
M: one time I was audio recording and when I listened to it; I said that whose voice so horrible is this? And I rewinded several times until I realized that it was my voice.
T: no teacher, of course not. I do not if you remember when I came to apply surveys.
M: aha
T: this is the KtAS survey, in this survey I provided classroom situations, and you ranked from 1 to 5, the number 1 the most likely to do first, and 5 the least likely you do. For example here this student who was talking with you
M: yes
T: he went with you to revise his work and then you started to explain him, right? for example why did you do that, teacher? Why you decided to explain him.
M: analyzing what he answered maybe there was a series or there were little mistakes that he did to solve it correctly.

T: here for example in the first question of the survey is: when a student solves a particular mathematical problem and asks if the answer or steps are correct, this is what the student did.

M: aha
T: here in the survey you answered: Ask the student to explain his/her solution
M: aha
T: but here in the video, we watched that you preferred explaining to the student,

M: aha yes,
T: why do you think that this discrepancy happened?
M: you know when I selected that choice as answer for the survey, it is like the program delineates it, it means, if it is the common presentation (puesta en comun) as we called it. It is the explanation provided by the student. Maybe here because it was an activity of the textbook, then this activity is to reinforce the topic and clarify the questions or doubts that the students could have. Did you remember that I gave you a worksheet?
T: yes
M: I told you these are the consignas (worksheet) that we work. Here is where we make the common presentation is here where the student before that I can tell him, he needs to go to the whiteboard and explain his solution, then we do the observations.
T: oh. Then we move to the next fragment of the audiorecording. (we listened to it). 5:05

Here in this fragment, teacher, what can you describe?
M: again I am making observations to the students about the little mistakes that they are doing, I would like to identify who was the student. (I rewinded the recording), this activity is for feedback, when we work in the textbook is to reinforce the topic that we are covering.
T: then in this fragment what specifically are you doing, teacher?
M: I was correcting the mistakes that the student has, or clarifying the student’s doubts.
T: And what was your thinking to act in this way?
M: I tried to reaffirm the topic before the evaluation that is where we are using the worksheets (consignas) that is as we have established in the program and then the student would be able to provide the explanation of the solution of the problem during the common presentation.
T: also this fragment fits on the same item of the survey, teacher, so your answer is the same: ask the student to explain his/her solution. Here we can see that the student asked you if his work is correct or not.
M: aha
T: and then you explained him where his mistake was or what he has to do and in the survey your response was to ask the student to explain his answer, why do you consider in the class in that specific moment that it was better to do that
M: because, I do not know if… for example, in the students’ doubts, they asked me “ I am doing fine until here, yes?” and there is when I show them where the mistake was, maybe here my answer is based on the program when we work with the worksheets, but no when we work with activities for feedback and it is here where there is not concordance with what I answered in the survey and what I did in the class.
T: do not worry teacher. Let me see… this is the next recording (we listened to it) 8:12

Here we have some problems to listen to this fragment, but here a student asked you for revision and then you checked it and you told him that it was correct that he needs just to circle the result. There for example what can you tell me about your action? Why did you tell the student that or why did you do that?
M: well because if the student got the right answer, it means that the student has everything cleared, and he is ready to the next step that is the common presentation of the solution of the worksheets.
T: again this fragment fits in the same item of the survey.
M: aha
T: but here in this case the student asked if his response was correct not the procedure.
M: aha
T: here you answered the same choice but here you did not explain anything to the student, it would be the first choice provided in this item, you told him that it was correct.
M: aha
T: and why did you consider that here (in the survey)was different that in the recording (class) with this student? Because to this student you only told him “yes, yes it’s right just circle the answer”
M: because I told you it is that I do not know then if there is other video recording where we are working the worksheets and the common presentation?
T: yes, there is a recording where the common presentation takes place.
M: it’s there where the student has first to explain it, but in this case I will provide the same answer, yes.
T: do not worry teacher. Do not think that I am judging you.
M: I am a woman of few words (she laughs). No, of course not.
T: then we moved to the next recording (we watch it) 11:12

In this fragment what can you describe that is happening, teacher? With you and the student. 
M: with this student or with whoever, well, it is the same thing, I could hear that it is an explanation of the doubts that students have, it is to reinforce, in this activity, for example, there are activities when we work with the textbook, they started to work with the textbook in order they can be observed by me to see what they can understand by themselves. You know that there are things that mathematically are logical, and there are others that need, the new reform says students must be able to solve any problems by themselves, then, students need a guide or a previous explanation in certain things (topics), then when I see what my students are able to solve by themselves, I provided the explanation of the topic or an introduction of the topic, then, students start to work with the textbook again to continue solving what they could not do it before. Then, this is an explanation to the students’ doubts for example, the management of the signs when you move one term to the other side of the equation, those are details that students are correcting in order to continue with the common presentation.
T: do you believe, for example, that without that explanation students will not be able to…
M: there is content that yes students can deal with it by themselves, but there is other content where students need the support, the explanation before they see the topic, not when they are dealing with it, an introduction before students deal with the topic.

T: here for example, in this situation, teacher, what I identified fits on the item 5 because I do not know if you heard there were several times that the student was asking you why not? It looks like she did not understand it, thus, this situation fits on this item: when a student is unable to see an obvious pattern in the problem, because you provided an explanation and again you were explaining her, here for example in the survey your response was: give the student clues to help him/her find the pattern. It means to find what she does not understand or identify.

M: aha
T: this was your response and what we already saw is that you were explaining to the student, why your response in the survey differs with what we already saw. What do you think?
M: this is my response: give the student clues to help him/her find the pattern.
T: and the recording we saw that you preferred to give the explanation.
M: it was an explanation.
T: or do you consider that you explanation could be a clue?
M: yes yes because I was not developing the problem with the student, to me explanation is develop all the problem, it means seat down and we are going to work together look this and this, I say to the students the points where the mistakes are, here is where I reinforce them, aha, where the mistake is and I reaffirm what we already saw in class in order to the student can correct it. Maybe for me this is a clue.
T: and this clue does it include the solution or yes?
M: no no
T: this is the purpose of the interview to have you explaining me.
M: to clarify everything.
T: yes yes (we were laughing)
M: it is as everything in life; everybody has his/her own thinking, way, and understanding.
T: yes, exactly. And you know your students better than any else. (I show her the next recording 16:21)

Here teacher...

M: I was working to prepare for the common presentation. There is a period of time where the student has to read the worksheet, solve it by himself, read again, try to solve it.
T: here students already went to the whiteboard.
M: oh here students already went to the whiteboard?
T: Yes it was that class, they already went to the whiteboard and explained the problems.
M: aha
T: here you already told them that they need to correct the mistakes...
M: the mistakes aha
T: then a student asked you something, and what was your action?
M: that he read it again.
T: for example that was your response: the student should read it again, and do it by herself, why teacher?
M: because the student must be able, if it was the response that I provided her, to do it by herself because maybe she is not making a minimum effort to try to work and solve it. Because of that I provided that response.

T: and for example, here, you considered it: the student should do it by herself, ok?
M: aha
T: for example in the other recordings why you decided to provide an explanation and here not.
M: because here it is an evaluation, now after one week working on it: with the introduction, feedback, then here students must be capable to solve it by themselves or with some help of the group during the common presentation, then, here I have to evaluate each group.

T: oh, this fragment also is identified in the item 5, her because as you said, the topic was already covered, an explanation already was given, the common presentation took place, and the student does not identify her mistake or problem yet. Then in the survey you response was: give the student clues to help him/her find the pattern. Here the difference is your action, why did you tell the student do it by herself and you selected to give clues in the survey.
M: I told you because here it was the time for her where she has to know it by herself, maybe she was a student who does make a minimum effort to solve the things then, maybe that’s why my response in the class.
T: and for example here teacher, after all this if the student cannot do it by herself, after you told her do it by herself and if she cannot do it, what happen with her
M: ah then, with her, I sat down with her not to provide clues, but for solving the problem until we solve it.

T: that affects the student in the evaluation?
M: yes
T: oh. Here in this other recording, we cannot hear it that’s why I wrote it the transcription. “a student asked you how to correct a problem if she cannot identify her mistake” this was after the common presentation. Then you started to explain her the problem without giving her the solution. Here why you did that? why did you consider it more convenient to do that with this student?
M: how to correct a problem if she did not find the mistake? (she was thinking) and then

T: here you told the students to correct their mistakes, students already went to the whiteboard, then a student told you “teacher no…” this is because I was taking notes that why I know it without understanding well the recording, “teacher how I am going to correct it if I do not know where my mistake is” she told you that then you approach to her and started explaining.
M: the solution, it means solving the problem together in order she could find the solution at the end.

T: more than finding the solution, it was to help her to find her mistake because she did not see it.

M: aha, because she already did a procedure, it means that she was able to provide a result, maybe she did not realize her mistake that’s why I was solving it together to allow her to observe and deduce whether she did this even if it was wrong she got an answer, then, but you said without giving the solution to the problem or without giving the students…?
T: explaining but without the solution of the problem.
M: ah
T: explain her in order to the student see the mistake.
M: yes because it is obvious that even if she did a mistake she was going to be able to solve it identifying her mistake.
T: here in the survey, the student did not find the mistake the problem, and your response in the survey is the same as the previous one because is the same response for the same item: give the student clues to help him/her find the pattern, and here you were explaining to the student, did you consider the same answer here than in the previous situation?
M: aha
T: here why your action was different than in the previous situation? The previous situation was when you told the student do it by herself.
M: here I tell you this maybe the previous student she did not make an effort to solve the things, you can do it wrong but you tried and together we can look for the mistake. But this student does not make any effort for making at least the procedure to realize herself what she can do or definitely she can say I did understand anything. Even we worked on it with feedback and she would not say “I have questions or doubts” aha.
T: Let me show you another recording 23:52 (we watch the other recording)

| Mayra was walking and stopped with one student and asked |
| "why -11? 8-6?" |
| $11: 2 |
| M: -2, no (Mayra explained it and said) but also here, then |
| 160-60 how much is it? |
| $11: 100 |
| M: Ah ha and what did you write there, |

what can you describe that it is happening in the class?(24:52)
M: it is the same, an explanation to clarify the little doubts that students have, they are working on groups or maybe they are integrated in groups but working individually.
T: here teacher, you were walking around the groups to supervise their work and you stooped in one group and identified a mistake that students of that group were doing, then you stopped with them and you told them why, I do not remember exactly what it was, I think they did a mistake in one addition and then students responded to you something and then you replied them saying “but this and this” here for example why did you do that? why did you consider more convenient to act in this way?
M: because after they mentioned how they got that answer then is where I told them what the mistake was in order to they can continue working on the problem.
T: this situation is identified as the item 9, when a student makes a mistake, this student made a mistake and you identified when you were walking around them. Here in the survey your answer was: let the student to explain his/her answer. And we could observe that you asked the student why this and then you started to discuss it, I do not know if you relate your action in the class with your answer in the answer or did you see it different?
M: yes I consider that they are same, considering the student’s explanation and if I know that it is not right, let him know to see that is wrong, then tell him where his mistake is. It is guessed that this is the purpose of all the work that we do. (I showed her another recording)
Then she stopped with other student and asked “what happen with this -7?” and she also answered herself checking student’s work “oh, you moved it to the other side? S3: yes M: ok because you moved it, what happen with it? Aeah [she asked and explained that to help the student to identify his mistake] then what did you do with it?
S3: oh it should be positive (he realize what he had a wrong solution)
M: aha

T: can you hear all?
M: aha
T: what can you describe that happened?
M: it is the same there are students’ questions and those can be considered as doubts when he is developing the procedure and then I started to reaffirm what we already see (the topic) and then he should remember where he made the mistake and deal with it correctly.
T: there, we listen to it, you also were walking and you stopped with one group and asked “why this -10?” and I do not remember if somebody answered you…(I repeat the recording)
M: aha
T: here for example, you said “what happened with this -10 ah you move it to the other side” and you continue explaining to the student, there for example, you, why do you act in this way, teacher? Why to say where the mistake was if they do not realize about it?
M: why to say to them where the mistake was? (she was thinking) because it is a supervision of the work that they are doing, identifying the little details that they, well, there are weaknesses in those little details and from there students can reaffirm or correct it to get a better grade in the evaluation.
T: and that situation also fits on the item that says: when a student makes a mistake, and here you told him where the mistake was, and you started to explain it and here in your response in the survey was let the student explains his/her answer, there is a difference between this and what you did in the class.
Why do you consider that it happens?
M: aha, yes, maybe when I told the student what happen with this 10 or something like that, right? and yes if the student told me something that’s why I told him aha you moved it to the other side, maybe there the student told me “I moved it to the other side”, then I said how you moved it to the other side the sign is changed, that’s why I told that.
T: why? Because the student did not know how to explain it or what?
M: no here I think that in this moment I told him what happen with this 10 I want to believe, because it is the most logical, that he told me I moved it…then I said aha you moved it to the other side thus its sign is changed. If he was saying to me I moved it, that is like to move it but you forgot it the change of the sign to the opposite sign, aha.
T: let me show you other recording. (we watch it)
(when we were listening to the recording she said)
M: those words that I used are to identify: the student is telling me something, but I told him ok you did this then what it follows form there, maybe the words that I used such as but, ah, then, then I think that it is like the answer is not given but make the student to remember what we have been working on it, aha.
T: for example in that situation of the fragment of the recording what can you describe about what you and your students are doing?
M: it is the same to find the solution to the problems, the mistakes that students make, I told you I interpreted that as it is, students know what they are doing, making little mistakes, I asked them and they told me I moved those coefficient to the second term (other side of the equation) but they did not change the sign, then I only told them: they need to change the sign, and I reaffirm just those little details that they skip.
T: and why did you do that reaffirmation? Why did you consider that it is needed?
M: because those are the little weaknesses that are left of the content, and with the purpose that in their evaluation and in the moment of the common presentation they do not…. I know that they know how to do it, those are just little details that they missed, then it should not affect their continued evaluation.
T: there in that fragment it is the same, you saw that the student was doing a mistake in regards of the signs, and your response in the survey was the same: let the student explain his response and what we can listened was that you told him that they make a mistake in the signs even you mentioned it twice in loud voice to allow all the class could hear it: be careful with the signs, remember the management of the signs. Here for example why was it different, what you did in the class with the survey’s response?
M: let the student explain his response. Because I told you here to me, I consider for example this I still consider coherent what I did in the class with this response, because in the moment that student told me “I moved this to that side” I said “but you did not change the signs” I understand that I allow him to tell me what he did and maybe I told him or my participation is a complement such as but you did not do this, or you are missing this, but I feel that I allow him explain me his answer first.
T: let’s do the next one. (we listen to it).

What can you describe that I is happening in the class, teacher?
M: there is work on the content with worksheets, also the common presentation, all this to evaluate the group and individual who is working the worksheet …
T: who is writing on the whiteboard?
M: yes, that student,
T: there for example, the student already solved the problem on the whiteboard, you asked one student, I think …(we listened again the recording).
M: here the explanation of the student who was sat down even she knows how to do it, but verbally she did not explain it correctly. And she is able to solve it. And what was on the whiteboard was right. But at the end, that’s why I reaffirm asking them if they are sure, because sometimes oh the student who is solving the problem on the whiteboard is one of the most
intelligent, then the students said what that student (on the front)did is correct without paying attention, that’s why several time I asked them: are you sure about it?

T: yes I was going to ask you about it.

M: yes because there are students that say because Juanito is in front of the class solving the problem on the white and he is the most intelligent of the class, then, they say yes without observing, analyzing anything, because the students who are in the class sat down say “he is the most intelligent his solution should be correct”, without analyzing it. That’s why I do it “are you sure about it”

T: here we saw your response that it was the same than in the survey: let the student explain his/her response. And here more than letting the student explains her response you made a question for the class, but you are told me why. We moved to the next recording (we listen to it)

What can you describe about what we are listening to?

M: the evaluation is taking part, it is an self-evaluation, what the student did, what it is correct considering the common presentation and look at what you did, correct it, compare it, and circle your mistakes, it is with the purpose that when students are in other evaluation, they can be aware of their weakness or mistakes that they made.

T: the student by him/herself?

M: aha

T: that’s why when a student asked you something, you told him do it by yourself without looking to the whiteboard?

M: yes

T: for example there, you do not allow them to look at the whiteboard or…?

M: no when the explanation of the common presentation was already done, and there were not more students’ questions and doubts, everything was well for the students, then at the moment to make the corrections the student understand the explanation of the other students, then the student should take note in a side of the problem when the common presentation is taking part. After that, compare it with what was on the whiteboard just to corroborate what the student did is correct listening other explanation or he/she is still making mistakes in the solution process of the problem.

T: after that, students can look at the whiteboard?

M: yes yes yes, aha.

T: that situation fits on the same item: when a student makes a mistake, that’s why the student asked you something, and here your response was the same than in the previous situation: let the student explain his/her answer. But here we saw that you told the student do it by yourself without looking at the whiteboard.

M: Aha

T: why is it different, teacher?

M: because here the common presentation, the explanation already took place and also there was more work, then until this point the student should be able to solve the problem by himself, then that’s why I did that.
T: ok let listen to the next recording (45:05)

what can you describe about it, teacher?
M: the common presentation is taking place, and the student should be finished his worksheet before I can tell him something, then the student made several mistakes, and that’s why I considered that to say something to the student was convenient. Also because certain characteristics of the student and some situations between the student and the class, I considered that the most prudent was to make my intervention. The things is that there is a conflict between this student and the class, there is a derision and it’s lead to indiscipline in the class, and therefore I preferred to tell the student about his mistakes, before all that happens.
T: oh, this situation fits on the item 10 that is: when students are having hard time to complete an assigned activity/task. As we saw in your class, the student was having hard time to do it, we saw your intervention, explanation, and what we see here in the survey is that your response was: explain the assignment using examples or another way to explain it. Did you see any relation between what you did in your class with your response in the survey? Or do you think that are different?
M: no, maybe this response would apply in the other situations that we have been talking such as the work in the textbook, we must focus more on the work on worksheets than in the book. Thus, the work on the textbook I use it for this: when I provide an explanation and a set of activities, and then I see that students still have questions or doubts, is here when I asked students work on the textbook.
T: here for example, the student has the questions, doubts, why did you consider that it was better to provide him an explanation?
M: well, the student was missing little details again, and I considered critical to make my intervention since the beginning than allow him to develop the procedure of the problem and it will generate a conflict in the classroom.
T: why students do not like this student or what? Nobody?
M: no, the majority does not like him.
T: he is a problematic student or what?
M: mmm, he is very smart but in certain point he is arrogant, yes, then he spends his time with older people than him, then he generate conflict inside the classroom,
T: he feels superior than his classmates?
M: aha, even he feels superior to me.
T: then the next question for you is, well I do not have a recording but in the notes that I took when I was observing your class, it was during the first class that I observed, at the beginning
you said that students have 10 minutes to solve the equations that you assigned, however, it took all the class for solving those equations and most of the students did not finish it. Why do you think that it happens, teacher? 48:41
M: look, I consider 10 minutes is few time but if I say 15 minutes,
T: I think there were 4 equations that you assigned
M: yes, 3 or 4 equation, yes they are solve up to one module (one class 45 minutes)
T: those are equations of the textbook,
M: aha, I was thinking if we already worked all the week on it, they would be able, even the most capable students, to solve the equations quickly, but then, this allows me to evaluate myself to be aware of it, I cannot move to the next topic until reaffirm this work. That’s why I always behind.
T: why do you think about why the students could not solve the equations?
M: because if we assign the same equations we will see the same little mistakes, the change of the sign, those are the little mistakes that students continue making, but those are more of memorization not of logic or reasoning, those are of memorization, the management of the signs, the multiplication and division with sign, these things should be memorized by the students, aha.
T: there for example, this situation fits also on the same item 10: when students are having hard time to complete an assigned activity/task, even in your class most of the students did not complete the task, right?
M: aha
T: here what you did was to provide more time, you saw than 10 minutes were not enough and it took the whole class, right, maybe sometimes it does not took the whole class, but the half time of the class or more than you expected. Then here you did that providing more time, and your response in the survey was the same: explain the assignment using examples or another way to explain it, well here there is not too much relation between your response, but you provided more time to allow them complete the task. Just to finish this interview teacher, how can you describe your math class, how is it regularly? What do you do first, what do you after that? how does it work?
M: well, regularly I exceeded the times, I do not respect the time that I assign for an activity because they do not finish, maybe this is my mistake I think if they feel some pressure they will finish it, maybe it generates that I am behind I am always one topic behind than my colleagues. I have understood that if I assign 5, or 10, or 20, the student who want it to do it, study it, he or she will do it, but there are other students who do not make any effort. The way that I work, I always try to respect what is established in the program, but sometimes I cannot do it, because I provide an explanation of the content before started with it formally, because it is supposed that we as teachers automatically we have to start with the topic, to give to the students, we do not have to provide any explanation and they should solve it by themselves. Then we as teachers just to be the guide, but I consider that there are topics that need an introduction before allowing the students to start to solve it. If doing like this we could not move more fluent, that’s not true, the 10 or 15 minutes that I can provide them, maybe they have mistaked one or two equations but for the four equations, 10 minutes were not enough.
T: what you mentioned happen every class,
M: yes
T: I mean one class you explain, other you work on the worksheets…?
M: oh yes, for example, when you arrived there was an explanation from me to solve equations with some exercises, then in the moment that you arrived, we worked on the textbook, after that, we worked the common presentation and then the evaluation of the content. T: the evaluation of the content is what we saw after the common presentation? Is where students correct their problems?
M: no, it is an auto-evaluation, at the end of the unit we have to auto-evaluate, co-evaluate, this is the continued evaluation, after that there is an evaluation to each content
T: how is it done?
M: it is through a test.
T: one content is as a unit or what?
M: it is a topic, the contents are topics, aha.
T: do you have test every…?
M: yes every month, and bimonthly too, it is part of this and it has a value of 15%, or some times it is accumulative,
T: ok, thanks you so much, thank you so much for your time.
Appendix I: Field notes Expanded Observations

Rosa’s Observation 1

On April 9, 2014, I visited the technical middle school #38 at 7:10am. I went to the principal’s office to ask for Rosa (the teacher who is participating) and the principal told me that Rosa had not arrived yet to the school, hence I waited for Rosa. The bell of the school was ringing at 7:20 am. It means the classes were started and Rosa had not arrived. After 7 minutes she arrived and told me hi. Then she took me to the classroom where the mathematics class was going to be taught. She was late for her class. She told me that she could not open her door at her house because her nephew took the keys that morning, so she could not be on time. it is a 7 grade level classroom. There were 20 students.

We arrived to the classroom and she asked her students to be in groups. She said that everyone should know in which group need to be integrated. Then, she asked where the group #1 was, she asked students who belong to #1 group to raise their hands. Then she asked where the group #2 was and harried up the students to be in their groups. There were 6 groups. In each group there were 4 students. While students were looking at their groups the teacher pasted on the wall a big cardboard. After that she called them to look at the wall. In the cardboard, there was drawn a triangle with circles like this.

![Image of a triangle with circles](image)

Rosa said that to start the class students had to solve the problem of the magic triangle that was drawn in the cardboard. She read the instructions that were also in the cardboard in order to all students could hear. She said that students need to put the numbers from 1-9 in the 9 circles that shaped the triangle. But the sum of the numbers that should be in the circles that were on the corners of the triangle need to be 15. And the sums of the numbers of each side of the triangle need to be 20.

Rosa asked students to draw in their notebooks the problem. She asked the students to solve it. Students were discussing in their groups but they were unsure how to solve it. Rosa said remember we are in the topic of proportionality. She asked a student to be integrated in the group that corresponded to her. Rosa said that students only have few minutes to solve the problem. She harried up the students most of the time. she asked if students started to solve it because she said that she did not see them to work. Then, she said you are losing time if you only need to place the numbers. She asked to the groups to raise their hands if they solved it, but nobody solved it at that time. When she saw that she explained the problem again in the same way that she did it the first time. Then an student ask Rosa

S1: teacher, does it mean that the corners have a value of 15?
R: the sum of the three corners is 15, each side of the triangle should be 20 and you have to place the number from 1-9 in the circles.
S1: ah ok.
S2: I do not understand
R: what is what you do not understand S2?
Rosa went to the seat of S2 and said everybody do not lose time drawing the circles of the triangle just write the numbers, check how many circles are. She was saying that the problem required few time and students were taking more time that it should be. She was walking around the groups. She stayed with one group to observe what they were doing. A student stood up and asked something to the teacher. She said ok stayed in your seat and raise your hand first. And Rosa went to the student’s group. She explained to the group again in the same way that she previously did with the whole group.
S3: teacher the sides of the triangle should be 20?
R: the sides of each triangle should be 20,
S3: then this side could have these numbers.
R: mijo you only can use numbers from 1-9. You must look and place the numbers in a certain way that the sum of the numbers of the corners must be 15. And the sum of the numbers of each side must be 20. Yes?
S3: yes
Rosa said this problem should be solved faster. She continued walking around the groups. And attending questions that students had and Rosa replied again with the same explanation about the sum of the corners must be 15 and the sides 20. The teacher offered incentives to motivate students to solve the problem. She asked students to raise their hands if they finished. Again she said that they should finish by that time.
After 10 min of class a group solved the problem. Rosa continued walking around the groups and she explained that the sum of the 3 corners is 15. She used the same words in the explanations. Rosa observed other group; the students did not how to solve it. The student was adding 15 by 15.
R: No instead of adding 15 by 15 you need to write these numbers (she borrowed student’s pencil and wrote in the student’s notebook 1,2,3,4,5,6,7,8,9.) Yes, you can solve it. The sum of the corners must be 15. And each side must be 20.
Students had hard time to solve the problem. They had not understood how to solve it. They had not followed the instructions given by the teacher again and again. When students asked a question to Rosa, she usually responded explaining to whole class her answer.
R: ey guys I see that you are placing the number 15, no, you only can place the numbers from 1-9, the 1, 2, 3, 4,5, 6,7,8,9, you should place them in a certain way that the sum of the corners must be 15 and the sides 20.
S1: look teacher I understood this, this side must be 20, this other side 20 and this last one 20.
R: exactly.
S1: a ok
When students had more than 15 minutes working on the problem and the teacher saw that they could not solve it, she made the decision to provide a hint. She told students to place the number 9 in one of the corners.
S6: I finished teacher.
R: wait wait wait, here the sum is greater that it should be. Add it. No here it is your mistake, keep looking at.
She continued providing time to solve the problem.
S7: am I doing ok in the problem?
R: continue working.
Rosa interrupted the activity to ask one student to read a phrase that Rosa pasted on the wall in a little cardboard. This phrase was a motivational quote. Rosa said the she provide a phrase weekly to their students. “El ser humano nunca sabe de lo que es capaz hasta que lo intenta” (the human being does not know about what he/she is capable to do until he/she try it.) She asked the same student to say what is the meaning of phrase. Everybody listen to the student. Then Rosa said “very good” and she asked to the whole class, “what are the words that are prohibited in this classroom?” a student said “I cannot”. Rosa said of course not, here all can be done, all is possible. And she encouraged their students to continue working on the problem.

She offered a chocolate for the first student who solved the problem. Then a student called the teacher.

S8: is the problem solved correctly?
R: let me see, no no no, 9, 1, here the sum is 17, it should be 20, keep working, you can do it.
Then she continued walking around and saying you can do it. Other student called her.
S9: teacher, look it.
R: the sum of the corners must be 15, here your sum of the corners is greater.
Rosa continued walking around. She said “I cannot believe that you are lasting too much time to solve this problem, keep working”. “you can do it” “do not give up”
Again
s8: asked if it is correct.
R: mija the sum of the corners must be 15. Look for the numbers of the corners.
Rosa saw that students could not solve the problem, so she provided another hint. She said “you have to look first for the 3 numbers of the corners”. She was embarrassed because her students could not solve the problem faster that this situation was the first time that occurred.
Then she said that they could not repeat numbers since they are only 9 circles. Then a student called the teacher.
R: let me see. What is this?
S10: this is,
R: no then here we got 21. Let’s see, add it.
S10 is adding.
R: how many?
S10: this number.
R: ok then let’s see this other side. Your sum is greater, keep working on it.
Rosa responded to the students asking her what the student did. And both together discovered the student’s mistake.

Another student finished the problem and went to the teacher, the student was explaining the teacher what she did to solve it but the teacher did not allow to explain it because Rosa preferred to check by herself the problem solution done by the student. Rosa said yes it is correct and gave the chocolate to the student. After 25 minutes with the problem only one student solved it. Rosa told her students that they were working too slowly. Then, Rosa asked the student who solved correctly the problem to explain it and help her group.

S11: teacher the sum of the last side is different to 20, it could not be 20.
R: of course, it could be 20. You classmate already did it. What numbers are missing.
S11: we only have 6, 12
R:9 ,10, 15 yes it is correct here. And then, here oh your sum is greater in this side. You passes the 20.
S11: no this is 3.
R: oh you are also fine here. Then a student interrupted the teacher and Rosa left that team. A student showed his work to the teacher and she checked it and said it is correct. Other student called her to check her work and Rosa checked the student’s solution and said yes it is correct. 
She asked one student to explain the problem to other student that had not understood yet. There was interaction in this class, the teacher was walking around the groups to see what students were doing or respond students’ questions. Also students were engaged in the problem, they were working on it in groups, discussing and trying to help among them.
Rosa used positive adjectives to call her students such as beautiful guero, little heart, mijo, etc. she was caring of her students.
She used a dice to select the group and the team member who is going to go to the whiteboard to solve the problem. Then the dice was rolled and the student selected did not solve the problem.
So the teacher asked another student from the same group to solve the problem in the whiteboard.
Then Rosa said that it is correct. She said that only two students did not solve it. She asked the students to work faster next time.
Rosa provided all the time of the class to solve the problem rather than only few minutes as she said at the beginning of the class that the problem’s solution should be obtained in few minutes. The class was over at 8:05am.
After this class, we went together to other classroom where her next class was going to take place. She told me that her students never last too much time solving a problem that this was the first time that they did this. She told me that she prefer to teach the students slowly in order they can learn even though this could implicate that she does cover all the curriculum, because she said that if she follows the curriculum her students do not learn since the few time that is provided and she could continue teaching without students’ knowledge of the topics. Hence we talked only 2 or 3 minutes before her next class.

Rosa’s Observation 2
On April 9, 2014, I went to the second class of this day of Rosa to observe her. This class was 7 grade. There were 22 students in the classroom. The class started at 8:05, she started the class making 6 groups. There were 3 or 4 students per group. While students were looking their groups, Rosa was putting on the wall the phrase of the week. It was written on a little cardboard. “El ser humano nunca sabe de lo que es capaz hasta que lo intenta” (the human being does not know about what he/she is capable to do until he/she try it.). She harried up the students to be in groups. When the students were in groups, Rosa asked one student to read the phrase. After the student read it, Rosa asked what students think about the meaning of the phrase, five students share their interpretation. Then Rosa said “very good”. After that, Rosa asked which words are prohibited in this mathematics classroom: one student said “I cannot”, other said “do not approve”, and other said “give up”.
The topic was proportionality.
Rosa started the activity prepared for the class. She explained what students need to do to solve the problem. After that, she asked if they have questions let her know. Then a student asked:
S1: the sum of all the numbers of the triangle must be 20 or how should it be?
R: the numbers added of each side of the triangle are equal 20.
S1: each side like this.
R: the numbers are not repeated, they are only from 1-9.
S1: it means 20, 20 and 20.
R: yes. And the sum of the numbers of the corners of the triangle is 15.
S2: does it mean that the value of the corners is 15.
R: yes, well the sum of the numbers of the corners of the triangle is 15. And each side of the triangle is 20.
Rosa said that “I know that you are super smart, you should not last too much time because it only needs few time. The group that has finished it, raise their hands. Also if you have a question, raise your hand”.
Rosa told the students to write the activity in their section of proportionality. The teacher used incentives to motivate her students. She offered pops to the first group that finished the activity and one chocolate to the first student who solve it. She said that students should solve it individually; however, they can be helped by their teammates.
Rosa was walking around the teams to observe what students were doing and to harry up the student to solve it.
She was supervising the work of the students. She only waited two or three minutes to provide a hint in order to help her students to solve it quickly. Rosa told the students to find out what were the numbers of the corners first to avoid difficulties. Then she was observing one group and she decided to provide another hint. Rosa told one corner, she said one corner should have the number 9. She provided this number to help students to solve the problem faster. One student asked if the numbers should be in order.
R: no, the numbers cannot be repeated. You can only use the numbers 1,2,3,4,5,6,7,8,9. (She wrote them on the whiteboard).
S3: teacher, should it be number 9 a corner?
R: No, if you do not want it to place it there, do not do it. Just I am giving you a tip to write it on one corner.
Rosa explained that they need to find out how to place the numbers in the circles that shaped a triangle. She said that each circle must have a number. These numbers cannot be repeated.
She continued walking around the groups and asking if students have questions. One student asked her something and she referred the student to what she wrote on the whiteboard and repeated the same instructions. Rosa encouraged the students to do it saying you can do it, remember this problem should be solved quickly.
She was supervising students’ work. Then, she stopped in one group and was checking the problem. She revised a problem solution of one of the students of that group and she told her where the mistake was. She checked another student. She was revising part by part in front of the student until she identified where the mistake was.
One student called the teacher to check her solution, the teacher took the notebook and checked the problem and she said “yes, it is perfect, do not share your solution to your teammates but help them to solve the problem”. This student was the first one to solve it and last 14 minutes. (KtA Sit.).
Rosa encouraged students to work saying words such as I know that you can, you are very smart, you have to do it, etc.
She is called by one student
S4: I finished it. (The teacher took the notebook)
R: No no no, here 9 and 8 is equal 17, here you did not get 20. 6 and 10 is equal 16 and this you got 20. So look for the solution. (Then other student from the same group asked the teacher)
S5: Add this and this is equal 15, then which corners?
R: add them, what? Which corners? This is a triangle. Hence there are 3 corners, this, this and this one. Then, which are the corners? (Other student from the same group said)
S6: these ones.
R: ok, and then keep solving it.
Rosa continued walking around the groups and checking the students’ work. Then a student called her
S7: teacher I got the answer!
R: did you solve it?
S7: yes, I did.
R: Let’ see. (She took the notebook and started to add the number to see if the sides of the triangle are 20 and continued checking the rest of the problem without saying anything to the student. Then she finished of checking it). Yes, it is correct, do not say the solution to anybody (group), say them how to do it.
Rosa did not give the solution but she said where the mistakes were or if it is correct. (OC: sit KtA the teacher did not provide opportunity to the students to explain what they did, or figure out themselves where the mistakes were).
She continued walking around the groups. She started to harry up the students to solve the problem. Then several students called her to review their solutions. She checked by herself and said “very good”, or “it is correct” or “perfect”. Then she was providing the incentives to the students that were solving the problem.
When a student solved the problem, Rosa asked her/him to help the rest of the students in the group without showing the correct answer to them.
Another student asked for teacher’s revision. Rosa was revising and adding using a loud voice. After 21 minutes of class, Rosa asked who solve it, and most students of the class solved it. Then, she rolled the dice to know who was going to write the solution on the whiteboard. Rosa asked the student just to put the numbers on the circles of the triangle not to explain anything. The student wrote the solution on the whiteboard. Then, the teacher asked who has a different solution. Hence another student went to the whiteboard and wrote her different solution. A student was asking the teacher to check the problem. Rosa was checking and she did not understand a number, she asked the student what number it was. Then you need to do the triangle correctly. Other student asked the teacher to check her solution because she thought that her solution was different from the others. The teacher checked and said “yes, it is different”.
After 27 minutes, Rosa assigned other activity and commented that it was going to be the last activity to close the topic of proportionality. She dictated a problem: “if 4 pencils cost 750 pesos, how much will be 14 pencils?”
R: Raise your hand when your group finished.
She read it again several times. She harried up the students and asked not to say the answer to wait for the rest of students to finish it. She recalled the definition of direct proportion. She said:
R: remember there are 2 variables, if one increases the other also increases, if one decreases the other also decreases.
Rosa was walking around the groups to see how they were working. She stopped in one group and checked how the students were doing the problem.
R: if 4 pencils cost 750 pesos, why are you doing this honey?
S8: this…
R: the problem is if 4 pencils cost 750 pesos, how much will be 14 pencils?
S8: aah.
R: you wrote it wrong (she left and continued walking around the groups).
Rosa checked other student’s solution, took his notebook and she said to the student “you are wrong here in this multiplication process”
She noticed that almost everybody finished it, she told students that if one student was selected by the dice, and he/she did not solve the problem, a credit was going to be deducted from her/his grade’s group.
A student asked the teacher about the answer of the problem, the student was explaining to the teacher what she was doing:
S9: The sum of these numbers is greater than it should be.
R: how much was it?
S9: 181 times 14
R: what? Do not tell me lies, multiply correctly. That is because you use the calculator. I told you to use calculator makes people idiot. First, we need to understand mathematics, then we can use calculator. You are that kind of people who use calculator right? No you do not use it here. (She left that group and continued walking around).
There was interaction in her classroom; students were engaged to solve the activities.
Rosa harried up the students to solve the problem. Then a student called her to check her solution. Rosa started to check the student’s solution, making all the mathematics operations mentally just looking the student’s notebook. Also, the student was doing some of the addition and she talked in a loud voice. Then the teacher:
R: 4x1, 36 we have 3, then 4x8 is equal 32 we have 3, 4x1 is equal 4 plus 3, then we have 189, and here it is not 189, so here the division is wrong, ok this part is correct but this one is wrong. It is mistaken. How much is 189 divided by 7, here you have this number, yes, since this point you are wrong. Very good. (OC KtA Sit.)
A student asked the teacher if her problem was right, and the teacher said “yes”. (OC KtA sit.)
Rosa asked three students to go to the front to write on the whiteboard their answers. A student rolled the dice to see who students are going to be selected. Three students wrote their procedures and answers on the whiteboard. The teacher asked the student to explain their work.
S10: well, I know that 4 pencils cost 750 pesos, and then I divided 750 by 4
R: but why? To what purpose did you divide it? And then why did you multiply?
S10: to know how much each pencil was. And then that price multiplies by 14.
R: ok very good. Then Carlos is your turn.
S11: I divided 750 by 4, I got 187.5 that is the price of each pencil. Multiplied 187.5 by the 14 pencils and the result that I got was 2617.
R: very good, anybody who solved it in a different way? (Somebody said “Renato”) honey go ahead.
S12: no I did not have it different
S13: yes you have it.
R: remember to feel fear it does not help you in anything. Go to the front, you know that you must go to the front. (the student went to the front and wrote on the whiteboard).
S12: I use a 3’s rule I multiplied 14 times by 750 and I divided it by 4.
R: perfect. Ok. (then the class was over).
Teacher and I, we were taking at the end of the class. She said that she ask students to read several times the problems in order that they can understand it (ktA sit), if the students do not understand it yet, she ask them to draw it. She is concern about the performance of the students
in the Enlace test, she thinks that her students become to feel pressure. Also she commented that she preferred not to follow the new reform as it should be because she thinks that her students will be left behind. In addition, she does not allow students to use the calculator because they need to know first the mathematics without it then at the end of the academic year she allows them to use it. She mentioned the importance of using incentives to engage students to work and learn. Also, she said the most important thing is that students learn mathematics that why she uses activities. She usually explains to students several times the instructions of the activities in order to get understanding about it. She explained very detailed the problem in order to grasp the solution. She often say correct or incorrect, or indicate where the student’s mistake is. She tried to make students think harder but when she notice that students are having hard time, she provided a hint, and then another one, or extend the time for solving the problem. When students asked her if their solutions are right, the teacher verifies the solutions. After that, she said, if the solution is correct or not. Also, she told the students where the mistake is. (OC she did this several times).

**Rosa’s Observation 3**

On April 29, 2014 I visited the technical middle school #38. I went to observe the third class of the participant Rosa. Her class started at 8:50, however, I arrived at 8:40 to ask permission to observe the Rosa’s class and locate the classroom. Hence I arrived to Rosa’s classroom at 8:45 am. Rosa allowed me to be in the classroom while she was teaching that class which finishes at 8:50. Then that class was over, students were leaving the classroom and different students were entering to the classroom. There were 21 students. This class was a 7 grade. I have already observed Rosa’s teaching with those students in the first observation that I did. Rosa asked students what topic was previously taught (direct proportionality). Students said the activities that they did, but Rosa said what topic was. Then she assigned an activity to solve a problem. She put on the wall a cardboard with the problem written. She asked students to write it on their notebooks. The title of the problem is “piano keys”.

Again Rosa commented to their students about the previous topic and she briefly explained the topic. Then, she was back to the assigned problem, she said that only 5 minutes are needed to solve the problem, and asked one student to use her chronometer. Rosa read the problem in loud voice:

Consider a piano with 84 keys: 7 out of 12 keys are white and the rest are black. How many keys are of each color?

She read the problem again and asked student to do it. After 2 minutes, a student called her:

R: did you finish?

S1: yes. (Rosa took her notebook and checked it)

R: let’ see, mmm no no no mija, if there are 84 keys, check it correctly, 7 out of 12, check it right. (left the student) (OC: kTA sit.)

Rosa harried up the students to finish the problem. She explained again the problem in the same way that she did it. Then, other student asked if the problem is solved correctly?

R: mmm no no, no sir, read it correctly, it says 7 out 12 keys are white, and 84 are the total of keys. (she left the student) (OC: kTA SIt.)

The activity was individual. Other student called the teacher:

S2: like this teacher?
R: mmm no, no those are 84, read it the problem correctly, those are 84 and 7 out of 12 are white, the rest are black. (left the student to work) (OC: ktA sit.)
S3: ready teacher. (She showed her work to Rosa)
R: perfect. (OC: ktA sit.)
Rosa offered incentives to solve the problem. She gave pops for her students when they finished the problem. Rosa encouraged students who were behind to work. Rosa said “it does not matter if you solve the problem using sticks or points but you need to give the answer”. Then 5 minutes were over. Rosa asked who finished should raise their hands. Then, a student raised her hand and scream:
S4: 35 teacher!
R: no you do not scream the answer, you know that you have to write it, do not tell me the answer, write it, and called me and I will go with you to check it. In addition you are giving only one answer and the problem required two answers. Do it in you notebook and I will go to revise it.
Rosa read in loud voice the problem again. She asked one student to go the front, solve the problem and explain it. The student did it and when she finished it, Rosa said “very good”. Then Rosa asked if somebody solved the problem in a different way. Nobody did it different, so Rosa asked for a clap for the student who explained and solved on the whiteboard.
Then she started to teach new topic. She dictated the title “calculate percentages”. She dictated what the symbol of percentage is and it represents the hundreds that are being used of a number or quantity. After that the teacher wrote an example on the whiteboard. And she asked students:
R: the 35% how is it represented in fraction? Let’s see remember it.
S5: then if, for example, an integer will be a 100%, no?
R: yes.
S5: all complete, then it would be 3, 11, mmm 1/7?
R: no no no, let’s see (she interrupted the student and did not listen to the student her reasoning). pay attention
S6: I, I, teacher.
R: yes, you, miss. Let me listen to her.
S6: it would be like a 1/3?
R: let’s see, no no no, we are wrong, pay attention, the 35%, how I would represent it. It would be 35 out of 100, 35/100.
“Ah yes” all the students said.
R: ok, we are fine, then how is 2% represented? Let participate other students not only you (2 students). You, Jonathan, how it would be?
S7: mmmm
S8: me.
R: well, you can help him.
S8: 2 out 100.
R: 2/100 very good. The 225, how would it be represented? I want to see more hands up. Saul, how would it be represented?
S9: it could not be the same because the number is greater than mmm… (Other students were raising their hands, but the teacher said).
R: pay attention, I can have 2.25 in decimals; remember that we already saw how decimals are represented in fractions. How would it be represented in common fraction, Brenda?
S10: as a quarter.
R: no no no, my love, remember that we remove the point, it would be also 225/100 it would be the same, this was seen previously. You do not remember it? Well write it (ktA): The percentage of a quantity is obtained through a proportion, an example. How much is the 20% out of 250? After she dictated that she wrote on the whiteboard the example. She started to explain the example when a student wanted to participate. However, she did not allow the student to participate; she asked for a moment first to provide her explanation (OC: ktA). Rosa usually asked students to take notes. She said that it is good to do operations mentally but she said that it is important to have an ability to write on the notebook. Then Rosa asked if someone knows Do you know how to solve for...x or y or the variable z (despejar)?

S11: to leave free a number.
R: other person
S12: remove what I do not need.
R: ok, there, what am I going solve for?
S13: the 20 %
R: let’s see. Let’s see.
S14: the number 250.
R: the x because that is what we do not know. I need to leave alone and remove what I do not need in that side. What I do not need with the x over there?
S15: the equal symbol.
R: we do not need the equal symbol =? (OC: ktA, this answer does have sense)
S16: no, the 20 %
R: the 20%? Let’s see, you are coming from the break very sleepy. Well, what is not needed with the x?
S17: 250
R: yes 250. Ok then pay attention, I need to solve for x, it means to leave it alone. And all the terms are going to be kept as they are. What I removed?
s18: the 250.
R: the 250 remember the equality, when I remove something from one term and I move it to other term, how it will be?
S19: multiplying?
R: why Brenda?
S19: because it is dividing.
Rosa continued explaining how to solve for x in that equation. When she saw her students do not know how to answer to her questions she provided the answer and explanation. She solved the equation and she only asked students for simple operation such as division, multiplication. Then she asked students to raise their hands in they do not understand. One student said that she understood so so. Then the teacher continued with the next kind of problem with percentages which is how to calculate the percentage of any quantity in regards of other quantity. she said that this could be solved using proportions. The she dictated an example:
R: what percentage is 30 out of 120?
S20: how how 30*120 or 4, what portion it would be, teacher?
R: pay attention, here is where we need to be careful (she ignored the student S20)
S20: teacher, it means that is the 30% of 120 or the 20% is 30 out of 120?
R: wait a moment, you can ask me questions next. (She ignored the student S20) (OC: KtA )
Rosa started to explain it; she said that:
R: in this kind of problem the unknown is the percentage not the quantity as it was in the previous problem. So in this example we are looking for the percentage? How would it be the proportion?
Rosa started to explain it. she provided the equation from the sentence. She did not allow students to obtain it by themselves. She used the previous problem to highlight the different between them. She asked what is it needed to solve for what term?
Various students said “the x”
Then she said “how it would be solve for x?” one student responded it and the teacher wrote it on the whiteboard and asked her what more? What is missing? She asked other students to raise their hands to answer it. She selected one student and the student answered incorrectly, and Rosa selected other student (OC: KtA, she ignored the wrong answer because she did not do anything about it) the student did not know it and the teacher asked him to see the problem on the whiteboard and look at the procedure. He did not know, then she allowed other students to participate. One student answered correctly and the teacher said “very good” and she explained the response. She did not allow the student to justify her response. (OC: KtA).
Rosa started to solve the example and she is asking to the students for simple calculations of the division or multiplication. Then she finished it and asked if somebody has doubts. Nobody had doubts. Suddenly one student said that she had solved it in a different way, and the teacher said how did you solve it? But she did not allow the student to respond when she said “look this is my explanation, nobody had questions”. (OC: KtA she ignored the different way.) Rosa assigned another activity, she asked to solve several exercises and she said who solve it different let her know at the end of the activity. She highlighted that there is not any problem if students use another way to solve the problem while they get the right result. She pasted on the whiteboard a big cardboard with the activity. There are 12 exercises:

Rosa said let’s see if you really learn, this activity is individual. She read the exercises and explained the directions to complete the activity. She offered candies for the students who solve it as incentives. She did not allow using calculators. She asked if they have a question let her know raising their hands.
One student asked the teacher how to do it, and she asked him to show his notes to her. However, the student did not take any note, then the teacher explained him the procedure to solve the problem. Also other student did not take notes and she asked him to write all that they are doing in class and be sat down in other seat.

R: what are you doing sweetie?
S21: this and this.
R: but I already told you, you have the example.
S21: I am doing this other problem.
R: not this problem is over, do this activity now. You did not take any note about the entire class, you are going to be here at the classroom during the recess. (She punished the student because he did not take notes)
S22: I do not understand this
R: what?
S22: what do I need to do here?
R: ah you need to get what percentage is the first quantity out of the second quantity. It means 20 out of 100 what percentage is, yes?

Then other student asked the same question, so she realized that most of the students did not understand that part of the activity and she explained the directions again (OC: KT A she saw that students did not understand what they need to do and she explained them again and again in the same way). After that she asked how the solution of the first problem is. She wrote on the whiteboard the proportion of the problem in order to students can solve it (OC: KT A this is such as providing hints).

She started to explain other problem, she provided another hint because she got the equation the proportion in order it can be solve by the students just following her procedures. However, in the last problem it was more challenged than the others. One student asked:
S22: if it would be solve doing this and this. (The teacher listened to her, after that)
R: ok ok I admitted that this problem is harder and I apologized for it. But you have here the procedure, it is represented, the only thing that you need to do is to put in an equation and get the proportion. What percentage is 20 out of 100? I do not know if you understand me, if you do not let me know.
She realized that her students have difficulties to understand. Hence, she tried to explain talking about money.
R: I have 20 pesos, what percentage of 100 pesos is 20 pesos?
S23: ah I got it, because my classmate told me other thing that we should do and I become confused.
R: ok, I have 100 pesos what percentage is 20 pesos out of 100? (She asked this question several times and asked students to raise their hands if they do not understand).
S23: teacher I need to do the last one. 340 out of 200.
R: here we need to pay attention, here is asking what percentage is 340 out of 200? Let’s see, what is here the 100%?
S24: 200
R: yes 200. Then it goes over the 100%, ok what is the 50% of 200? (she did not ask student to explain why she is right OC: KT A)
S25: 100 pesos
R: then we have the 150% because 200 pesos is 100% and 100 pesos is 50%, then 40 is missing in order to have 340, what percentage is 40 out of 200?
Several students said oh I got it, and then the teacher asked them to finish it. (OC: ktA she provided more and more hints until she almost solve it). She walked around the students just for a second, and she asked students to start saying the answers using loud voice. She harried up the students. And she asked for the first problem, one student responded correctly and Rosa said “very good” and gave her a pop (OC: ktA she did not ask students for justification). Then other students are raising their hands to participate. Two more students said their answers and Rosa said “very good” and gave them pops.

R: ah ha just because I am giving you candies you are doing right, ah ha.

Three more students provided their answers correctly and again she said is correct (OC: KtA) and gave them candies. Students were encourage to participate almost everybody raised their hands. Then for the next problems of the activity she asked students to go to the front and write on the whiteboard the procedure and result. One student is selected and went to the whiteboard and wrote her solution.

R: it is a little bit incorrect, Brenda (other student) why is it incorrect?

S24: is it incorrect?

R: why Brenda?

S24: well I got this other result?

R: tell me why? (she asked the whole classroom and selected other student), ok you go to the front to see if you know as you said. Remember that you are going to solve for x, eh. Rebeca harry up. Rebeca solve it correctly on the whiteboard and the teacher said:

R: ok the result is correct, but I want that you tell me what you did because you solved for x, but this part of the procedure is wrong (OC: ktA teacher did not allow the student to realize where her mistake was.) let’s see how did you do it.

S25: I did this… (She is interrupted by the teacher)

R: Remember this is proportion. How would you solve for x? let me know how did you do it?

S25: that is because I know that this result ammm emm (She is interrupted by the teacher, again).

R: remember, let me speak (she said to the student), we do not have to it mentally.

S25: no no I did not do it mentally. I do this…

R: ok but solve for x here. This is the 5% of 460.

Then the student started doing that and she asked a question to the teacher. Rosa did not answer what student asked, Rosa asked the student just to solve for x there. You must leave alone the x. Rosa look around to see if anybody else can solve it, but she realized that most of the students did not understand her. She allowed students to check their notes. While she allowed other student to solve the next problem, she thinks that students do not understand because they do not take notes. Again she asked the student s25 to explain her solution for x.

S25: I divided this between 100 (She is interrupted by the teacher again)

R: no no no. we will have to do the next class. (She started to see what the other student is solving and she took the marker and started solving the problem. she was writing the problem incorrectly. Then she was checking the student’s notebook and she realized that she was doing incorrect the part of the proportion. She started to do it again and explaining in loud voice while students are getting all their staff because the class was over. She did not allow students to leave because she asked students to pay attention and answer simple questions such as to calculate division or multiplication that she was doing while solving the problem, and she asked students to do the whole activity as a homework. Hence she did not finish solving it.

Rosa most of the time she did not allow students to justify their procedures or answers, she is constantly harry up the students. When she sees that students are not doing well she starts to
provide hints or providing part of the solution in order to guide them to solve the problem (OC: this is the topaz effect). When she is explaining a problem she involves students but only for answering simple questions such as additions, subtraction, divisions, and multiplication. At the end of the class she did not allow student to present their different way to solve the problem when she told them to do that at the end of the class. In addition, she most of the time provides the correctness or incorrectness of the problem, she did not allow the student to realize if what they do is correct or not by themselves. She provides where the mistake is or she said “very good”.

Rosa has 4 years teaching at the 7 grade level.
Omar’s Observation 1 May 8, 2014

I visited the state middle school # 8347 at 7:25 am. I went to look for the principal but he had not arrived at that time. So I asked for Omar (the teacher who is participating). I found him; he was talking with other teachers before class started. Then the bell rang at 7:30am. Omar and I went together to his classroom. I briefly asked him if he knew that I was going to observe his classes. He told me “yes the principal told me something but exactly I do not know what you are going to do”. I explained him that just I was going to observe his classes without any interruption. It is an eighth grade class (2-G) and there were 24 students.

Omar started the class at 7:36 because students were arriving late or moving chairs. Then one student asked him about me and the teacher introduced me to the class. After that he started the class naming students from the attendance list one by one to see if they are in class. (OC: I noticed that the teacher was nervous.)

Omar continued teaching the previous topic, which are simultaneous equations (linear equations). He explained what the simultaneous equations are briefly. he provides a context to understand what these equations are, he mentioned about the ages of two persons. Two equations can form a simultaneous equation. He mentioned that these equations can be solved through the use of different methods. He mentioned these methods and described them briefly: equality method, substitution method, graphic method, and the other is the determinants method (he said that this method is more used at the high school level). However, he used only the reduction method because of its practicality. He asked one student to help him to paste on the whiteboard two flip chart papers. It already has a problem solved step by step.

Omar started to explain the problem part by part. While he was explaining the problem, he also asked questions to the students such as sign laws, variables, additions, coefficients, . One student answered one of his question and Omar and the student shake their hands. Several students were answering what he was asking. Then Omar asked some students if they understand. Nobody had questions. Then he continued explaining the same problem. When he used a new word before he continued explaining he asked students about the meaning of that word. This is word is “transponer” transposition or transfer. Students answered saying that it is dividing, and Omar said “no, it is like transporting, moving, to move something to another place, ok”. He asked if they have questions about this term. Students did not ask anything about it.
Omar continued explaining the problem. He mentioned that transfer is the same in physics as solving for certain variable. He continued explaining the problem and asking questions to the students simultaneously. He finished solving it and explaining it very detailed.

Omar asked student to be in groups of 3-4 persons, and each group must assign a person per group to go with him to receive directions about the activity. This activity started after 27 minutes that the class began. The teacher gave them a sheet with 2 problems.

Groups started to work the activity.

One student asked him if his solution was correct and the teacher told him where the mistake was, however, the teacher asked the student to justify what he did. (OC: ktA)

A group said that they finished and the teacher asked them to write their names and wait for a moment.

The teacher was walking around the groups. Then one student asked him if it is correct his solution and the teacher said you must verify your result. (OC: ktA teacher asked student to verify his response).

Other student asked him to check what he was doing if his procedure was right, and the teacher said yes (OC: KtA).

Other student asked him if it right or wrong what he did and the teacher identified the student’s mistake and told him that it is in the x. (ktA)

Other student asked him whether his solution was correct, and the teacher asked him to explain his solution. (OC: KtA)

Omar continued walking around the groups. Other student asked him if he is doing right, and the teacher said that he needs to write the complete procedure. (OC: KtA)

Other student asked him if his solution is correct and the teacher asked her why she did that. And the student explained him. (OC: ktA)

Other student asked him if it is right and he turned in the activity.

The teacher was checking a student’s solution together with the student.

Other student asked the teacher why his solution is wrong, but the teacher looked at the solutions and he did not say anything and accept the activity.

He was walking around the groups and he stopped with one, he paid attention to what that group was doing and he told them:

O: it is not complete here.
S1: what I need the operation?
O: the operation.
S1: that’s it?
O: yes.

He continued looking at the work of that group and he asked:

O: what operation is that? x is equal to?
S1: 10 ....
O: 10 what?
S1: 10/2
O: and what is the result of 10/2?

Students were solving that operation and the teacher left that group. Omar continued walking and supervising students around the classroom. He stopped to tell what part they still need to do.

Then the class was over at 8:15 am, and the teacher asked students to put the chairs back in order.
He last too much talking because he does it slowly because he speaks at a slow pace. He never provided the answers to the students.

**Omar’s Observation 2**

On May 8, 2014, I visited to the Omar’s second class at 8:15. There were 25 students. It was a nine grade class. He asked me if I am studying, where, and what. Students were arriving to the classroom. He supervised how students are taking their seats. After 5 minutes he started his class. He wrote on the whiteboard the topic of today that it is a review of the different types of equations that were previously seen in the past units. He said that it is supposed that students should know to solve those types of equations (linear equations). He said that he was going to start with the easiest to the hardest equations. Today Omar was going to review linear equations.

He called student by student to see if they were present in the class. He talked with one student about one test that she failed.

Omar explained that real world problem can be represented by equations. He said that “to know how to solve an equation is not difficult. The difficulty comes up when you represent a real world problem as an equation. To do that is hard. Hence, having an equation is easier to solve it than getting an equation from a math word problem”. He read a short fragment about the solution of equations to the whole class. He mentioned three types of equations, explained briefly each one and provided one example on the whiteboard:

- **Linear equations**  
  ex: \(5x+8=2x+20\)
- **Simultaneous equations**  
  ex: \(4x+4y=-200\)  
  \(2x+y=250\)
- **Quadratic equations**  
  ex: \(x^2-2x-15=0\)

He said that in this class they only were going to work on the linear equations. He said these are the three types of equations that you can find in a problem. (ktA or CK). He mentioned that to represent an equation is the hardest part. When we are done with the representation, the rest is easier to solve the equation.

Oscar is back to explain the example of the linear equations when a student asked him a question:

S1: what they are called linear, because they are like a line or what? (then many of his classmates laugh of him and said “ashhh, ooooo”)

O: the linear equations or first grade equations are called like that because the letter that they have (variable) are to the power 1. That’s why is a first grade equations, if we have a letter to the power 2, it would be a second grade equation or quadratic equations or if it is to the power 3 it would be a third grade equation, and so on. The greater exponent in the equation is which said the type of equation that it is. (He provided an example about it). Linear equations are called like that because at the moment to represent it in a graph we have a line. (Then he drew a line on a graph on the whiteboard. Also he drew a parabola for quadratic equations). Then you know why it is called linear equation. (OC: KtA)

Do you have another question?

Then he was back to the example. He started to explain how to solve the equation. He said that students need to transfer to one side certain terms. However, he used a specific vocabulary for it “transponer” (transfer, move, transport) and he explained the vocabulary used to the students. Then he explained step by step. At the same time he asked students how to do certain steps. Omar said that this topic was already taught, however, students seen as they did not know the topic, as it would be a new topic.
He provided some time for students to take notes about the staff that is on the whiteboard. Then, he started to explain the simultaneous equations. He provided an example and mentioned the methods to solve them. He asked students if they have questions. One student asked when they are going to see this type of questions. Then Omar told him the next class. (OC: ktA) He did not solve the example that he wrote on the whiteboard. He moved to the quadratic equations and wrote an example on the whiteboard. He explained it very briefly and said that those equations are solved by the general formula, and he presented the formula.

Next several activities of linear equations will be assigned. He asked students to expose their doubts or questions. He asked students to be in groups in order. There were 4 students per group. While students were doing the groups, Omar was writing the equations on the whiteboard. He wrote 10 equations to be solved in 10 minutes. He provided a blank sheet per group to write their solutions.

Omar told students do not spend too much time talking because you only have 10 minutes to solve the equations.

One student called the teacher to ask:
S2: is this equation solved right? (the teacher checked it and said)
O: it looks like it is. (OC: KtA this was to make the student to think about his response)
Other student asked:
S3: how?
O: look at the example. (OC: KtA )
Other student:
S4: is this right?
O: yes (OC: ktA here the class was almost over).

Students were writing the equations and their notebooks the equations posed on the whiteboard. He asked students to solve some equations while their teammates write other equations. The teacher was walking around the groups to observe students’ work.

Omar continued walking around the groups to supervise students’ work. One group asked the teacher “from where did you get this in the example?” and the teacher did not tell them in order they can deduce and think about it. After certain minutes, the teacher explained them just one part and allow the students to deduce the next part. (OC: ktA)

Other students asking what he needed to do to solve the equation, and the teacher answered her “look at the example” in order to the student thinks about it. (OC: ktA)

Omar realized that the class was almost over. So he asked students when the bell rang to put back the chairs in order and left the sheet where they solved the equations on his desk.

He mentioned one teammate should solve one equation and other teammate another equation. His peace to talk is very slow.

The class was over and the teacher asked students to put the chairs back in order. Omar spent so much time to explaining the students the topic. He is a traditionalist. He prefers to explain the topic and the assigned exercises about the topic. However, his class in divides into 2 sections: section 1 is where the teacher is calling students to see if they are present, then organizing students, and then presenting the topic. This section last 28 minutes. Then the rest of the time of the class is for students to work on activities. Students spent time writing the exercises and then they only have 10 minutes to solve them before the class was over.
**Omar’s Observation 3**

On May 8, 2014 I visited the state middle school #8347 at 7:20 am. The third class of Omar started at 9:13 am. This was a nine grade class. There were 26 students. One student asked the teacher why I was there. Then the teacher introduced me to the class. After that Omar called each student to see if they were present.

One student asked him “what are we going to do in the class?” and the teacher said: “we are going to see the 5th unit, it is a review of the previous units”.

The teacher started to write on the whiteboard about the equations. He mentioned three types of equations: linear equations, simultaneous equations, and quadratic equations. Then he provided an example of each type of equation.

Omar explained the linear equations and mentioned methods for solving this type of equation. He said that they can solve it by transposition of terms. He explained what it means. Then he solved the example but at the same time he was explaining and asking questions to the whole group such as calculations of simple arithmetic operations.

One student interrupted the class to ask him for a signature to the documents. When Omar is explaining the example he said that 3x is a multiplication. Then a student asked “why 3 is been multiplied.” Then Omar explained to him when a number is together with a letter as 3x is a multiplication and he provided some examples. (OC: KtA)

Then Omar asked if there were questions. After that, he explained simultaneous equations and provided an example and mentioned which method for solving is recommended, he said that reduction method is easier for solving this type of equation. But he did not solve his example. Then he moved to the quadratic equations, he explained it and wrote an example. He mentioned that using the general formula is the more recommended method for solving this equation.

After these explanations he asked students to be in groups of 3-4 students. There were 7 groups. Omar provided instructions about what students are going to do and the information that they need to write on the blank sheet provided. He wrote 10 equations on the whiteboard. Omar said that “one student solve one equation while others write the other equations”.

One student asked him if what she was doing was right and or she was doing it backwards. Then Omar said “backwards”. (ktA)

Students were writing what Omar wrote on the whiteboard. At the same time students were talking.

Omar was walking around the groups.

One student asked Omar:

S1: teacher, how is it this? Then Omar only saw her work, he did not respond anything. Then other student from the same group explained to her classmate and the teacher was paying attention to them. Then he said:

O: yes, it is done like that. (KtA)

Then the teacher continued walking around groups and supervising students’ work. He stopped with on student and told him “the addition done here is wrong?” (The student did not ask anything the teacher when was checking the students’ work identified the student’s mistake. OC: ktA)

Omar allows students to use calculators.

The teacher approached to me to tell me about one student’s situation.
S2: teacher, this can be moved to this side?
O: yes (he checked it). (ktA)
One student asked him:
S3: it is like this?
O: yes, it is. (ktA)
One student asked:
S4: do I have to solve for x?
O: yes (then the student did it and)
S4: that is it?
O: why? (the student was thinking until she got it! OC: ktA)
Then one student asked the teacher:
S5: teacher I do not understand. (Then the teacher allowed the student’s group help him and seek the way to solve it together. ktA)
One student asked Omar:
S6: is this correct?
O: yes. (ktA)
Other student asked:
S7: is it right?
O: no (he told him where his mistake was. OC: ktA)
One student asked him:
S8: teacher is it correct? (the teacher checked it and said:)
O: no, it is wrong. (other student of the same group asked him)
S9: why?
O: tell me what did you do?
Then the student was saying what she did when the other teammate realized where the mistake was. (ktA)
One student called the teacher:
S10: listen teacher. This can be moved to this side? (the teacher explained him the procedure. OC: KtA)
The students were working on the equations and the teacher was walking around the students.
S16: I do not understand here teacher.
O: ¿to which one?
S11: to the sixth equation (then the teacher explained her and she said:)
S11: ah! Then it would be 4x
O: yes, (ktA)
One student stood up to ask for revision of her work. Then the teacher said:
O: it is ok but here is 4.
S12: ah ok then it is 4. (ktA)
One student asked the teacher to revise her work. Then, the teacher checked it and said “yes it is right” (ktA).
One student went with the teacher to check his work he turned it in and the teacher was revising it.
Then the class was over and the teacher asked to put the chairs back in their place.
Rogelio’s Observation 1

Observation 1 Rogelio May 2, 2014
I arrived to the technical middle school number 1 on this day at 3:10pm. I went to the principal assistant to ask for permission to access to Rogelio’s classroom in order to observe his class. Then I was at 3:15 pm at Rogelio’s classroom. He was teaching math to other class. I entered to the classroom and took a seat. I was preparing my staff for the next two classes that I was going to observe that day. Rogelio is the teacher who is participating in the study. This day he has two classes consecutively with the same students. He taught only nine grade. In his classroom, there were 22 students.

Rogelio allowed students to be in groups with whoever they want. He introduced to me to the class.

Students were arriving late to the class. One student went to said good bye to the teacher because she was suspended definitely.

A student said a message to her classmates about the mother’s day festival. The teacher asked to use their worksheets (consignas). The teacher spent much time trying to control the class such as be quiet, sit down, your worksheet.

He asked students what activity they did the previous class. He started the class with one problem of the worksheet that he already had written on the whiteboard. He began to explain it. The problem is they have 3 grades, but they only know 2. The question is what is the other grade in order to have an average of 8.0?

He started to solve it and explain what he was doing. He got the equation from the problem, he did not allow students to spend time to get it. He asked questions to the class such as what is the unknown, how we calculate the average, students did not have any difficulty to answer these questions. Then Rogelio continued solving the problem and asked what follows and students answer “to add”. He also asked simple operations such as additions, subtractions.

R: From this point we need to solve for x. How to solve for x. what is it going on with the 3? This number 3 is dividing how it is going to be in the other side. (He waited some seconds to see if students respond)

S1: multiplication
R: multiplying, so to what side is going to be?
S2: toward the x.
R: no no you cannot move to the x,
S3: move toward the other side.
R: to what?
S4: move toward the number 8.
R: ok it will multiply the 8. We could do it since this part, but in order to keep the sequence we are going to do it now. 8*3, then it is 24. Then we write it. And we almost finish the problem, no? what more?
S5: substract.
R: but why am I going to subtract?
S6: because this number is adding so it should be moved to the other side subtracting.
R: ok because I am solving for this variable, I am going to move this number subtracting. We are going to have x=24-14.2. do not do the common mistake of saying oh it is 10, because it is not, you need to pay attention to the .2 that 14 has. So the result is?
S7:10.2
R: no no it could not be 10.2 (OC: ktA) this is the common mistake.
S8: 9.8
R: 9.8. then what is the grade that this person need to get an average of 8?
Several studnets said “9.8”.
R: how we can prove it?
S9: adding the three grades and divide them by 3
R: ok, do it by yourself, add it and then divide it and let me know what you get (OC: topaz effect) if you get an 8. If we do not get an 8, it means that we make a mistake in same part of the problem.
The teacher provided some time to do this verification. While students are working on it, the teacher asked some students about a football team. Then a student asked:
S10: teacher it is divided by what?
R: between the number of grades (OC: ktA he did not make students think, he did not allow students to conjecture) how many grades are?
S10: 3
R: then by 3.
S11: teacher so we have to add and divide what?
R: we already know the value of x, I add the 3 grades, this, this and this. And then divide it by 3 to see if we get 8. That is the verification of the problem (OC: ktA he repeated the same directions to the students, he did not allow students to look for solution or procedures.) is it right?
Students said “yes”.
The teacher assigned another problem # 2. It is about the dimensions of a rectangular piece of land. He drew it on the whiteboard, and started to explain it. He asked students:
R: how is a rectangular piece of land?
Student said “it is like a rectangle”.
He continued explaining the problem, he talked about the dimensions that one side is x and the larger side is 4 units greater than the other side. Then he put x in one side and he asked what we are going to write in the other side?
S12: x+4
R: nooo, it would be 4 times (OC: ktA he explained why the student is wrong).
Students provided wrong answers: “4x, 8x, 4*10”
R: nooo, we need to add, eeeh if I have 4 units greater (OC: ktA he provided a hint with the word add)
S13: x+4
R: yes it is. (OC: ktA) very good. Then, we need to recall how to calculate the area? (OC: ktA practically he is solving the problem step by step, because he is saying what is it next)
Students said multiply the length by the width.
R: yes multiply the length by the width. What is the width (base)?
Students: x+4
R: what is the length (height)?
Students: x

Rogelio continued explaining and solving the problem, until he got the equation, after that, he said that students should solve by themselves the equation. (OC: he made all the procedure; he did not allow students to make conjectures to obtain the equation) he provided time for students to solve the equation.

Rogelio walked around the groups and asked about why some students did not attend the previous class. Then a student said that she finished. Rogelio looked at the student’s notebook and check the solution, he say ok, but nobody else has finished, let them finish it. He asked the students about the verification and she showed where she did it. Rogelio said that it is correct. Then Rogelio walked around the groups, and provided 15 minutes to solve the problem. He said “if you cannot I can help you”.

A student asked something to the teacher. He noticed the confusion of the student, and then he went to the whiteboard and explained to the entire group. He made the next step in the solution of the problem. Then a student asked whether after that they need to multiply x by 4. And the teacher said that “then x by 4”.

R: how much it is x by x?
S14: 2x

R: noooooo, (students are saying their answers until one students said the correct) yes, square x. Then he continued solving the problem on the whiteboard asking simple questions to the students such as x by 4, x by x, etc. then he left the equation ready just to solve for x, he told students just moving the 396 in order we have the equation equal to cero. Now, we have an equation that you can solve it using the general equation or factoring, whatever you want it. (OC: he said students which method they need to use to solve it when students are dealing with these equations before.)

Rogelio called several students to talk about their failed previous tests. He was talking during 6 minutes with them. One student called the teacher and he went to the student’s seat. Rogelio is asked about how to use factoring. Then he explained the student how to do it.

One student asked Rogelio about my studies, my background, Rogelio explained him all my educational background, then he started to talk about himself, his studies. After that, Rogelio continued walking around the students, he said that students need to do at least an attempt to solve it. Immediately after that, the teacher went to the whiteboard and asked students to recall about which terms they need to move to the other side in the equation. But he said which term needs to be moved and how to do it. As Rogelio saw that students had not solved the problem, he made another step of the solution on the whiteboard (un paso más! OC: hints).

S15: teacher I am doing well?
R: yes, you are right, but you need to use the parenthesis. X and X in each parenthesis, if you want it to do it in this way, if not...(Rogelio is looking in the student’s notebook notes about the general formula to solve quadratic equations, but the students did not have it, hence the teacher wrote it on her notebook).

One student asked something about the equation and Rogelio said that they need to solve from that point that he left in the solution. He said “you need to solve it, if you cannot in few moments we are going to solve it all together”. (OC: he did not encourage students to try several times). He continued walking around the groups and supervising students’ work.
One student asked Rogelio if her procedure is right, he checked the procedure and said that “mmm well, you need to do this first” he spent some time explaining what she needed to do (OC: KtA).
Other student called the teacher and asked him what she needs to do and Rogelio said:
R: you need to solve this equation with the general formula (Rogelio looked for the notes about general formula, but he did not find it, he found the notes about factoring). Well, you can solve it in this way. (Other students lend him his notebook and he showed her the general formula, he told them that they also did several examples.)
Rogelio said that it is almost time to go to the recess, when they were going to be back, one student already had a solution that they would see if it is correct to the problem. And then we would see another one. Then Rogelio allowed students to leave the class earlier. However, one student asked:
S16: teacher this is right?
R: 2*1 follow the formula (ktA)
S16: then here is 16, and 7 and then this …
R: aha but here is missing something, 4ac… (OC: ktA Rogelio told her where her mistake was).
S16: it means this should be this
R: aha ok.
S16: ah ok I got it.
R: do you know how? Did you remember it?
S16: yes, I know how to do it.
Then students left the classroom. Rogelio said everything is forgotten by these students to the following day. He erased the whiteboard.
He said that these students work harder than students in other schools that he worked before. He showed me a letter that said that he had to administer a new test for the students who failed previous tests.
During the recess, we were talking and he told me that there are a lot of factors that implies a low achievement of these students, he told me that students usually are alone in their home during the morning so nobody is taking care of them. Therefore they do not do homework; they wake up too late, just to be ready to go to the school. Also he said that most of the students live with grandpas or single mother, those are people who work most of the time, hence they cannot be paying attention to the school’s tasks of the students. In addition to this there is a critical low level of attendance; usually 10-12 students miss class every day. He said because of that he could advance quicker than he does. This day 10 students are missing the class.
OC: I observe a critical level of indiscipline; most of the students do not pay attention. All time students are talking among them. Rogelio does not have any control over the class. He had to be screaming in order to explain a problem. Rogelio usually make a substantial part of the problem, he allows students to make just procedural parts which critical thinking is not too much involved. He assigned a problem, he started to explain it and solve it until he almost does it or at least the hardest part he does.
He focuses on students who work on his class, if students do not pay attention. He provides little attention to them. He allows solving a problem in a different way. Also when students do not how to solve the problem, Rogelio provides a quick review in order to be able to solve it. (ktA) when students are going to start thinking to solve the problem or even before it, Rogelio tells them which tools they need to use in order to solve it. (ktA). When students asked him if their solution or procedure was right, he told them where the mistake was. He provides certain time to
complete the assignment, but he did not follow it, for example: he said that students only had 15 minutes to finish but at the end he provided more time. Rogelio asked something to one student did not know, then he asked to another student and she neither knew, hence he preferred to solve it and said the answer to what he was asking to those students.
I stayed with him in his classroom during the recess.

**Rogelio’s Observation 2**
On May 5, 2014 at 4:20 the second class of Rogelio started. He already has one mathematics class with the same students. When the first class finished, students were to recess and then were back to the classroom to have the second class with Rogelio. There were 22 students. There were working in groups. It was a nine grade class. Students continued working with the problema #2 that Rogelio assigned in the previous class. R: who solved it? Write the result on the whiteboard (he asked one student to go to the front. The student wrote the result, then Rogelio started to explain it, he said:)
R: there is a way of solving it called factoring. Once we have an equation, we can used this way, it means to separate in factors the equation, if you recall it and if not it does not matter. We said to open parenthesis and what is going to be inside?
S1: x.
R: why? Because x by x is square x (he did not wait students’ response). Then the signs: + and -, + by -, then we need to seek two numbers that...
S2: its multiplication should be 0 (this does not make sense)
R: no, its multiplication should be this number:396. In this case the signs should be different, and the subtractions should be 4. Then we can start to look for the numbers. Usually, we begin as 20x10=200 and we move close, 22x12=264, until we find those numbers: 18 y 22, (he wrote the number inside the parenthesis on the whiteboard and he also said:) but here we have a mistake, the biggest number should be positive in order to get a positive number when the subtraction is done. 22 y 18, if 18 is subtracted from 22, the result is 4, 22x18=396. Like this i show we get the result. (OC: 2 ktA without sense, and he did not allow student to detect his mistake he told them).
R: the other way to solve is the General formula, it is longer, we also are going to use it. write this:
Rogelio provide some time for students to write the problem that he was solving and explaining on the whiteboard.
R: observe the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Rogelio started to explained how to use the general formula and he asked which are the coefficients a, b, and c. Then Rogelio asked one student to go to the whiteboard to solve the problem using the general formula. He asked students to take note of everything to recall it later. The student was solving the problem in front of the class. The Rogelio interrupted her several times because she was skipping steps. (ktA)
The students asked Rogelio:

S3: teacher that is the same problema? profe, esa es la misma.

R: yes it is but now we will solve it with the general formula.

Then Rogelio allowed the student to do one part of the problem. However, he helped her several times saying indications such as: “now what is the square of 4, the square root need to be calculated.” Then he summarized the part that the student did and explained it to the class. He used a brief example of an elevator to provide students understanding about the subtraction of numbers with signs. And completed the problem. He said:

R: very good. Look both solutions. Both solutions are the same even if we used different methods. Obviously the general formula is easier than factoring. Factoring is harder and you need to have more ability with the numbers while the general formula is longer but we are going to find the result without looking for certain numbers that its multiplication among them and subtraction or addition should be 396. Also not all the time factoring will be possible, because sometimes we will have decimals, fractions thus, to use the general formula is better to find those types of numbers. Yes the procedure is longer but at the end you put in practice some arithmetic.

Ok then the dimensions of the land are?, I can write negative numbers for land’s dimensions, right? (he told them this key concept of negative numbers in areas of real world problems)

Students: no.

R: then its height is 18 and what about the width, how many will it be?

S8: 22.

R: why?

S9: because it is negative.

R: no. How many the greater side is measured according to the problema x+4. 22.

S10: but it is negative.

R: ok here we got a negative number -22 +4, because there is not negative measures of land or when have you seen a negative área? Of course not, always that you are dealing with lands, shapes, we will never have negative numbers. Then we consider it as positive, it is 18. 20:23.

R:ricardo assigned other problema (#3) that is more complex, he read it as: “a car goes over 8 km per gasoline liter in the city and 12km per liter in the highway. If a car went over a total of 399 kms and used 36 liters of gasoline. How many kilometers the car went over in the city and how many kms it went over in the highway?”
He said that the problem required an equations’ system (ktA he did not allow the students to think about it). He mentioned that they saw the equations’ system two or three classes ago. He recalled it briefly in loud voice for the students. Then he asked the students to solve the problem. And he recommended using the data provided in the problem. He was going to be walking around the groups to help students. He provided some time

Then one student called Rogelio then he arrived with the student:

R: what’s up?

S11: yes or not teacher? It should be in the highway or in the city -8, right?

R: yes highway and city to avoid confusion you can write it as…

S11: X and Y?

R: a= highway and c= city, instead of X and Y, hence, we can identify it faster.

Rogelio continued walking around the students he sat down with one group and he started to explain the problem to the students, he told them what they needed to do h explained it with a lot of details to lead to the solution. (ktA)

He harried up the students saying that they have 10 more minutes to finish it.

One student asked Rogelio when they need to turn in the activity and he said when the class ends.

R= no lo acabarían.

One student asked Rogelio to check his work and he identified a mistake and said

R: yes, but this is equal 4. (ktA he mentioned where the mistake was).

He asked one student who was asking questions to Rogelio to go to the whiteboard to solve the problem.

S15: just I will write it?

R: yes and next I will explain it

S15: it is easy.

Rogelio was asking students to be quiet.

S15: a+c= 399 (he wrote it)
Rogelio asked one student to pay attention and be looking to the whiteboard. The Rogelio explained the procedure to get the equations’ system. Rogelio provided the two equations and asked the students to solve the system using the methods for solving equations’ systems that they saw two or three days ago. He mentioned that students can use the substitution method or method of equality (simultaneous).

He provided time for solving the equations’ system. One student asked him about the method of equality and he explained her how it is and mentioned its advantages of using it.

One student asked him if he could do that and Rogelio said “yes but you need to do this firstly, and the easiest way to do it is this. (ktA)

Rogelio usually does not provide the solution of the problem to the student even if they ask questions, however, he helped them to do the procedures, find the equations, and he lead the students to get the solution with all the details provided such as what they need to do, what is it easier to do, why not to do this, among other details.

**Rogelio’s Observation 3**

On May 8, 2014 I visited the technical middle school #1. I arrived to the school during the recess at 5:00pm. The class started at 5:05 pm. The teacher was talking with several students while the rest of the students of the class. This talk was not about the class. It is a nine grade class. There were 31 students.

Rogelio asked students to use their notebooks and worked in groups of 3-4 students.

R: let’s work.

Students continued walking. Before the class started when students arrived. Rogelio introduced me to the class. He told them that I was going to observe his class and students’ work to make research.

R: ok guys, we will work on the part 5 “ consigna #5” today because the 4, we already did it in the classroom. I already told you that you are going to work alone in this part (5) but what do you believe that part 5 has 3 problems.

One student was asked to go to the whiteboard to solve one problem that was homework. While students were working and talking. Then Rogelio said to the student who was on the whiteboard:

R: read the problem, but read it correctly no as the last time.

The student read the problem: one student knows two grades, the question is: which third grade the student need to have to get an average of 8?

R: this means that the student has two grades, we add them one more grade (third grade) to get an average of 8. The two grades are 6.4+ 7.8, plus the third grade that we do not know which it is x, then we divided by 3 it must be equal 8. (Then he asked the student why she wrote 14.2 in the equation).

S1: because I added 6.4+7.8.

R: very good and why she wrote here 24. (OC:ktA) (he explained the problem asking the whole group part by part as it is seen below.)

S2: because I multiplied by 3.

R: what did you multiply by 3?
S2: the 8.
R: but why? If it is here, why you multiplied over there.
S2: because I solved it for x.
R: very good, you got 14.2 plus this quantity is equal 24 and next what did you did?
S3: I moved it, I solved for x.
R: the x is 9.8. be quiet Carrillo, is there any way to verify if it is correct? Of course if I did it
(he was joking). Who want to verify it?
Students said “you teacher”. Then because nobody wanted to verify Rogelio did it. (ktA)
R: I add the three grades; 6.4+7.8+9.8, then I divide by 3, what is the result?
S4: 24.
R: very good, 24/3.
S5: 8.
R: that is the result, very good.
R: ok now let’s do the problem #2, can you read it please (Rogelio asked one student).
S6: problem #2, eh quiet, the area of a rectangular land is 196 square meters, and the longest side
is 4 meters greater than the other side, which are the dimensions of the land?
R: this problem is more complex i do not remember that someone had solved it during the
previous class. Anybody wants to do it?
S7: me
Then the student went to the front and wrote just the result not all the procedure on the
whiteboard.
R: Yadira provided the result but we need to see why, how and when. The teacher started to
explain it. First we need to revise the information. (the teacher asked student to be quiet then) to
analyze the problem we need to know what is the area of the land.
Then Rogelio started to solve it, but at the same time he is explaining and asking the students
about the procedure, Rogelio provided to the students the key parts of the procedures in order
students could solved it, as it is seen below.
Students said “is the above part.”
R: ok let me help you a little.
S8: it is what is inside.
R: inside of what?
S8: of the shape?
R: ah ok, it is all the space of the shape, very good and what shape is it?
S9: a rectangle.
R: a rectangle, then all the area is.
S10: 396 square meters.
R: what are the dimensions?
S11: it is the perimeter.
R: the sides and if I do not know them, I look the x, but it says that longer side is greater 4 units
than the other.
S12: x+4.
R: very good, then if the smaller side is x and the greater side is x+4. Let’s do a pause. Then,
how do we know that these dimensions are the sides; if I do not know the x and add 4 units to the
greater side. How we calculate the area or surface of the rectangle?
S13: side by height.
R: yes it is the width by height, which is the formula?
S14: x+4 by x.
R: it is equal to what?
Students: 396 square meters.
R: we have the area, but we want to find the dimensions, it means the sides. What we have here, which operations there are:
S15: multiplication.
R: a multiplication, then x is multiply by 2, x by x is?
Students: square x.
R: x by 4 is? 4x. Then we have an equation, it is getting shape, here I can solve for x, this can be moved to the other side, if in this side was adding the the other side will be?
Students: subtracting.
R: and then, miss, what type of equation is, it is a quadratic equation and it is solved in two ways one is? (he did not provide time for answer the question KtA)…
S16: solving for x.
R: solving for x, and what is factoring? (KtA).
S17: those are two numbers that multiplied to provide the result.
R: so so. (He explained what is factoring and at the end he said “recall it”). (KtA the students said something that is not factoring but Rogelio said that and Rogelio explained the correct procedure OC: ktA)
S18: we have two pairs of parenthesis.
R: we have two pairs of parenthesis, very good, what is going to be inside the parenthesis)
Students: x.
Rogelio continued explaining the problem. At the same time he was asking students about it such as calculation and signs. For example: he asked what we are going to do next to add or rest? Two students responded:
S20: addition. (ktA Rogelio did not ask why this student thought that)
S21: subtraction.
R: why subtraction? Ok we said that the product of two number should be 396 and the addition or subtraction of those two numbers should be 4. Why subtraction? To add the number. what is the condition?
S22: the two positive numbers.
Rogelio continued explaining how to do factoring. At the end he said that “this method sometimes does not work because to find the numbers is harder. Or if we have decimals or fractions to get those number is even harder and takes too much time. that why there is another method, I know that you do not remember” (ktA)
S23: substitution.
R: no. (he provided time for students to write on their notebooks).
Los alumnos están platicando, y algunos están apuntando lo que les pidió el profe.
Then Rogelio said that the other way I was using the general formula. He asked students if they remember how it is, he did not provide time to answer, but he wrote it on the whiteboard.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
Rogelio started to explain it. He asked students about substitution of the numbers in the formula. He provided 10 minutes to do just the substitution of the coefficients in the formula. One student said:

S28: I do not know how to do it.

R: you did not pay attention. You only need to exchange the numbers by letters.

S28: then I will do this?

R: yes, it is.

Rogelio was walking around the groups to answer students’ question about it. There were students who did not use their notebooks when they should be working on their notebooks with the general formula.

One student went to the teacher’s place to ask for revision to his work.

R: -4, 4 at the power 2, aha now we need to get the calculations.

S29: aah you did not tell me that. (ktA)

Rogelio said in loud voice “we need to do the calculations when number are substituted”.

Then Rogelio continued walking around the groups helping students who asked for help.

One student called Rogelio

S30: teacher, please come here (he showed his work)

R: this information (a, b y c) needs to be substituted, now we need to make operations, this is at the power 4x4=16 it would be 16… then the result would be… (ktA)

Then Rogelio attended other student who asked for help. The student said:

S31: eh teacher I got 398.

R: why 398 (he is looking at student’s work). Ah ok no, this should be separated in two parts no, look, add and subtract, -4-30 divided by 2. (ktA)

Several times students asked Rogelio about their work and he said that it is right or wrong and explained them what they need to do or what is wrong and until he made the calculations. (ktA)

Other students asked Rogelio if it is right and he repeated the instructions and he almost solve the problem to the student. (ktA)
Then Rogelio continued revising students’ work.

Other student asked him: is it right?

R: no. (ktA)

Just one student out of 31 completed the work that Rogelio assigned for that class.

He did not have a good classroom management. Students are talking all the time. many students did not pay attention to Rogelio’s teaching. most of the students did not work. Rogelio usually said what students need to do until he almost solve the problem. Thus, students did not spend time trying to figure out the problem.

The class was over.
Maria’s Observation 1 May 7, 2014

I visited the technical middle school #1 during the morning session at 8:05 am on May 7, 2014. I went to the principal office to ask for permission to access to Maria’s class. I was taken to the Maria’s class at 8:15am. At this time, the mathematics class of Maria (participating teacher) was going to start. She teaches mathematics only to one group per academic year because she teaches mainly science to several classes. This math class was an 8 grade level. There were 40 students. Students already know where they have to be sat. Students are assigned to work in groups of 5-6 students. The teacher keeps the students’ textbooks with her. She gave students their books back to start working.

She started the class assigning equations to solve from the textbook; page 186, 4 equations. Maria said that this topic was also taught the previous day. She said that students continued working on that part of the book and when they finish it, one student per group will go to the front to write the equation on the whiteboard. After that she said “first students need to work individually for 10 minutes.”

Maria walked around the groups and she mentioned in loud voice “remember we have been working on it since Tuesday, and yesterday, also we did some activities”. Then one student asked:
S1: teacher can we use our notebook to recall what we did?
M: no, first try it like this, alone, remember what we did last Friday, Tuesday and yesterday.
(OC: KtA)

S2: teacher we are going to do the same?
M: yes, it is the same, remember that we are working on this, what is you unknown? What term do we have? (then she left that group.)

Maria asked for the procedure completed and the result written in the same paper.

One student asked her about the problem, she started to check it and she asked the student “what happen with the sign” and the student responded something but the teacher said “no” and then Maria started to explaining the problem and each step, but she was asking what’s going on there, why the student did that, she did part of the procedure to get the solution but without doing all the work or providing the solution. (ktA: she helped him it is like a hint or tip to solve it) then she asked this student to do step by step, do not skip steps. And she said “continue doing this until solve for x.”

While students are working on their assignment, Maria started to name students one by one to see if they are present in the class. After that she walked around the groups and stopped with one student. She was observing the student’s work and asked why he did this, then student explained her, but she asked the student to do first certain steps and she explained what he needed to do. (KtA)

Maria continued walking around the groups to supervise students’ work. She asked students for organization such as “circle the result”, “write it on certain part” among other things.

One student is explaining her what he did, and the teacher listen to him and checked his work. Then she said:
M: ahaa, but remember this is already known. What it means. That this 4 what is it going to happen to it? (she really did not provide him time to answer her questions, she was explaining step by step and asking the student why that step, and she told him what else he need to do. She left that group) OC: KtA

Then she stopped with other student and asked “ what happen with this -7?" and she also answered herself checking student’s work “ oh, you moved it to the other side ?
S3: yes
M: ok because you moved it, what happen with it? Aaah (she asked and explained that to help the student to identify his mistake) then what did you do with it?
S3: oh it should be positive (he realize what he had a wrong solution)
M: aha (then she paid attention to another student that was to review her work.)
S4: teacher is it correct? (Maria checked it)
M: yes, just you need to circle the result. (OC: KtA)
Maria continued supervising students’ work. One student asked the teacher to review her work. And Maria was checking it, she started to explain it to her but at the same time, she was asking the student about her work; she asked questions such as “how much? What else? To subtract what? Why is this part like this? (She allowed the student to tell her why) (OC: KtA she did not provide the answer)
She went to other group. She revised students’ work and identified a mistake, she started to explain the problem, and asked him questions about his procedure. Then she tried to make him recall it. She continued asking him, and he answered her, but he answer wrong one question and Maria told him “no carlos, the number 4, the 4” (KtA she did not tell him the correct answer, then she went with other students)
Other student took his paper to be revised by the teacher. Then she started to check it and she said:
M: why -2?
S5: it is -12.
M: here I see this -12, from where this number come up?
S5: from this side
M: and where did you leave the other term?... 3x-x? (then the student is justifying his response and Maria continued asking questions) and where in which side these coefficients should be? (then the student answered her all her questions and said why he did that. Maria asked him to do all in order, to do the procedure in order.) KtA
Other student asked to check her solution. She checked it and started asking where this number is. The student explained the teacher what she did, but Maria said “ok but remember here you need to remove this” then the student continued asking Maria about her solution and Maria was checking part by part and asking her calculations, she spent some time revising this student’s problem until she identified a problem and told the student “pay attention here this number is negative and how did you move to the other side?” (OC: ktA)
S6: negative
M: and it should be positive (then the student saw her mistake and went to her seat) (OC: ktA)
Several students asked her whether their answers or procedures are right. Most of the time she guides the students step by step of what is needed to do, she explained with a lot of details. Maria realized that 10 minutes that she provided for the activity are almost done.
One student revised his work, she checked it and Maria explain him why he has a mistake:
M: this coefficient -10 you moved it to the second term then it should be positive, remember that it moves to the other side with the opposing sign. (ktA) (then she left the student and attend another student).
Several students went to the teacher’s desk to ask for checking their solutions. But she checked it and explained them with details, and then she told them in few moments you are going to write on the whiteboard.
One student went to the desk to ask: “I have to move this number to the other side like this?”
M: yes, remember you are going to multiply only what you have in the parenthesis. Students asked her about the step where they were and she explained them what else needs to be done. (ktA)

One student asked the teacher if her work is right and Maria explained to the student that “you need to remember that coefficients are in one side and these in the other side”

S7: no no these numbers no.
M: what? What variables do you have?
S7: mmm
M: what are your variables?
S7: this one.
M: but it is only this x?
S7: mmm
M: this is one will be left here and then…

Other student asked her to check her work. Maria checked it and started to explain and asking the student about her procedure part by part. (ktA)

Like that several students went to the teacher’s desk to ask for revision of their work, Maria checked it one by one and she started explaining in loud voice step by step that the students did until they are able to see their mistakes in order to make the corrections. (ktA)

S8: this teacher
M: from where did you get the -20?
S8: from here
M: No but this 20 it does not say that it is 20x
S8: no
M:12x+4
S8:16
M: ah but you cannot add or subtract them, because this is a variable and this is a coefficient. (ktA) Then she continued checking the rest and explain him one more step and asked him to correct all the rest that was wrong.

One student took her problem to be revised. The teacher checked it and explained her what was needed to do and at the end she said this was already cover. (KtA already taught-4).

She went to check the work of one group. She was checking and explaining the procedure. Then she started to ask questions about calculations of numbers and signs, and she said what else they need to do. (ktA)

One student asked her if his work is correct and he explained the teacher what he did. Maria asked him “why did you do this?” and the students answered her “this” and she explained hi why he is wrong. (ktA)

One student is checking his work, and the teacher saw a mistake and she told him: ok they are added but what happen with the sign? There are the same signs. What happen with the signs in an addition or subtraction?

Questions like that she was doing to her students and then she explained them what they need to do step by step.

One student was asking several questions during the class. This student went to ask the teacher again and she said “no, you should know how to do it by yourself and go to sit down”.

She continued checking students’ work and she was explaining what they did and needed to do with many details. She realized the class was almost over, and she said that tomorrow continue working on it. She asked them to leave their books.
At the beginning she said that students have 10 minutes to solve the equations. However she provided the whole class to work on the equations and most of the students did not finish them. One student revised his work the teacher checked it and said “yes Juan just write your name to the paper”.

Maria usually asked students: what is it said here? From where did you get this? Remember this …? If students do not know she starts to explain it and she is providing information about how to solve it step by step. (ktA)

All the students are working. However, many of them have doubts or questions. Most of the time of the class she is explaining the students the equations the procedures, students are doing pretty much the same mistakes for example: they do not change the sign when they moved a number or variable to the other side. And she continued walking around and explaining with a lot of details the work of the students. She usually said where the mistakes are.

Maria’s Observation 2

On May 12, 2014, I visited the technical middle school #1 at 9:30am to observe Maria’s class. Maria’s class of mathematics started at 9:45. I was in her classroom before her mathematics class started, it was a science class. Students arrived to the classroom. There were 35 students. Maria asked students to be in groups. It is a 8 grade class. She asked students to do a worksheet (consigna). She said the topic that was going to be seen that day was “solution to first grade equations”. Maria asked students to work individually even though they were sitting in groups. She mentioned that the following day they were going to go to the whiteboard to present their solutions.

Maria provided the instructions to work on their worksheets; she said that the worksheet was going to be the last activity for that topic.

M: read carefully the problems. It is the same topic. You need to answer it using a pencil instead of a pen.

One student stood up to ask:
S1: is it right?
Maria replied him saying what his mistake was and she explained what to do with a lot of details. (ktA)
S2: is it correct like this?
M: this is your procedure, you only need what? To calculate the value of what? (ktA)
Other student asked Maria:
S3: is it like this?
Maria told her what is her mistake and she helped the student to identify it in order to be solved. (ktA)
Other student called Maria to show her work. And Maria said:
M: put it in order as the previous activity, look at here is asking for perimeters. (She continued explaining the student until the student understood it.) (ktA)
Other student asked Maria:
S4: how is it?
M: try to remember it. (then the student explained what he thought). Do it but in order to avoid confusions. (ktA)
Other student asked Maria:
S5: how to do this?
M: how do you get the perimeter of a shape? (Then the student pointed it something in the worksheet.)
M: ah ok, you only need this, it is the same. (ktA)
Other student asked the teacher.
S6: like this?
M: here is an equality, you have to represent it. Then x= . (Then the student answered her correctly)
M: then you do the other problem like this one by yourself. You know how to do it. Also you know how to do it the rest of the problems. (ktA)
Maria harried up the students to finish the problems. Other student called her to show her work.
M: this number should not be here, look, (she continued asking for corrections and checking her work. Maria explained her with a lot of details in order to provide awareness of her mistake and be able to solve it) (ktA).
Other student stood up to revise his work.
M: but from where did you get this 6, ah ok when you put it there, this x was positive and was moved as? ah ok then 2x...(subtraction) equal to (the coefficients). (ktA)
One student told her:
S7: I do not know how to do it
M: but you need to know the perimeter. How do you calculate the perimeter? (the student understood her)
Other student revised his problem and Maria said that it was right. (ktA)
Then other student asked Maria if his work was correct, then Maria checked it and said: “yes it is right” but then she identified a mistake and told the student “recall this, how it should be done” (ktA)
Maria was walking around the groups. Then she stopped in one group and she was observing their work. After that she started to ask one student of that group:
M: why this mija? (then the students started to explain it. Then Maria explained her how it should be done) (ktA)
Other student asked for her revision. Maria checked it and said “you are doing well just you need to write this instead of this and represent the equation.”(ktA)
Maria was taking care of the students’ work. Then one student asked her:
S8: should it be done as this?
M: you need to calculate the perimeters and that’s it (ktA)
After 22 minutes one student completed the activity. Maria stopped in other group and she asked one student:
M: why did you do this? Then the student justified his solution and the Maria told him:
M: no recall you should do this. (ktA)
Other student asked Maria if it is right and Maria revised it and explained how it should be done. (ktA)
One student asked her to check his work. Maria revised it and showed the student in which part the student’s mistake was. (ktA) then Maria continued revising students’ work because students asked for that revision. After she revised a student’s work where a student’s mistake was about the law of signs, she said in loud voice “recall how the management of signs is”
Other student was checking her work with Maria, then Maria was revising it and asking the student “who by who? (then the student replied her) sign by sign + by +? 3*4?” (the student was responding correctly)

M ok like that you are going to do the rest and do not forget how to use the signs. (ktA)
M: the beginning of the problem you need to get the equation of Sandra, then, you will make an equality with Bertha’s equation, then you will move the variables to one side and the coefficient to the other to solve it. (Then Maria continued explaining the procedure to the student but at the same time she asked him why, and what it is next.) (ktA)
A student asked Maria if her work was right. Then Maria checked it and started to asked the student questions such as “plus what? What is next? And how are you going t do it? And if I will move it what is is going on with the sign?” And the student responded all these questions correctly and went to her seat to continue doing her work. (ktA)
Maria checked one student’s work and asked the student why did you do this? In several parts of the problem’s solution until Maria told him where his mistake was. (ktA)
Several students asked Maria for revision and she was checking one by one and where she saw a mistake or something wrong she said to the students checked it again without saying where the mistake was. (ktA)
M: ah ok, how did you represent the perimeter here? (the student explained it) How many sides?
M:x+x?
S9: 2x
M: then? 2x+?
S9: 6
M:2x+6 is equal, now how do you represent the perimeter of this shape?
The student responded her and Maria said:
M: now that you have the equation… (the student was writing part of the solution in his notebook) ah ha until you find x. (ktA) (Maria did not tell him the mistake neither the solution she explained him how it should be done with details.)
Other student asked Maria if her work was right
M: “do not forget the law of sign minus by minus, then make the procedure first and then show it to me”. (ktA)
Other student took his work to be revised by Maria, and she checked it and said “it is right, leave it on my desk” (ktA)
Three students asked her questions about their work and Maria explained them asking “what it is happening here” and more questions about their work. (ktA)
Student was explaining his work to the teacher to see if was right.
M: let’s see the perimeter of the rectangle how did you represent it? He was explaining her but she interrupted and said:
M: no, what is here 12 or 2
S10:12
M: but where did you get it?
S10: from here
M: ah ok the coefficients 6+6 , that it is right but you have this, (the student explained her)equal to the perimeter, perimeter is what we are working, when you have this part you will get the solution. (ktA)

Maria scolded one student for helping other and the student said “because he does not know” and Maria said ”yes he knows”
One student asked Maria to check his work. Maria was checking it and said:
M: 54...+16? - by -? This what? (The student was answering her questions at the same time, then Maria indicated where the mistake was) (ktA but it was after the questions and student’s reasoning)
She harried up the students because 3 minutes were remaining.
Maria was walking and stopped with one student and asked “why -11? 8-6?”
S11:-2
M: -2 , no (Maria explained it and said) but also here, then 160-60 how much is it?
S11: 100
M: Ah ha and what did you write there, (ktA)
Three students submit their work but Maria asked them where did you get this, how did you get it? Then she allowed students to respond, but Maria said “no”. Then she explained and asked them questions at the same time (ktA).
Then students were leaving to their recess because the class was over.
Maria likes to explain the problems with a lot of details to help students to grasp how to get the solution. She was supervising students’ work, and asking and responding students’ questions.

Maria’s Observation 3
On May 3, 2014, I visited the technical middle school #1 during the morning session. I went to the third observation of Maria. I arrived at 7:30am. It is an 8 grade class. There were 37 students. Maria only teaches mathematics to this class, hence these students are the same than the students in the other observation.
Maria asked students to be in groups. There were 8 groups. Maria gave students’ work revised back. She asked the class not do any correction yet. She said that all groups were going to go the whiteboard to solve a problem. She named it as “common exposition” (puesta en comun). After students solved the problem on the whiteboard, students were allowed to make corrections in one side of the worksheet and mark their mistake in order to be aware what your little mistakes were. She said the most of the mistakes were about signs. .
Maria divided the problems of the worksheets to the groups.
She asked one student to read in loud voice a problem and solve it on the whiteboard.
S1: considering that next shapes, they have the same perimeter (the addition of their sides is equal to the same number) write an equation and solve it for x:

The student read the problem and solved the problem on the whiteboard.
M: Esteban, can you rad again the problem please and the student read it.
M: now we pay attention to hear the explanation of the group #7.
The student who wrote it on the whiteboard explained her procedure and solution. Then the teacher asked the class:
M: do you agree? Any question?
Students: no.
M: are you sure, is there any doubt? (Then she asked other student to explained the same problem.) Clara can you repeat the explanation of your classmate.
S2: 2x is obtained multiplying the side by the perimeter.
M: right students? Do you agree?.
Students: no.
M: let’s see Uziel.
S2: I moved the x’s and the 12 because it is two sides, then x is the other shape, then this x is representing +16, that is 8+8 and then we subtract them, ah no, we join the x’s…
And then Maria provided him a hint she said:
M: and then, the x’s in one side and in the other what do we have? To whom
Students: coefficients.
Then Maria completed to solve the problem and she explained at the same time because this last student struggled to do it. (ktA)
Then Maria asked another student to read and solve the problem #2 on the whiteboard. When the student was solving the problem, she interrupted to the student to ask him “what is that?” the student responded it and the teacher asked students for questions. The class said “no”. Then the teacher allowed the student to complete the problem’s solution and then the teacher asked the class “questions? Look did you realize where your mistake was? Yes?” then most of teh students in the class realized where their mistake was. (ktA)

Then Maria asked other student to do the problem #3. the student read it and solve one out of the three equations.
S6: here we start with 3(x+4)=−5x−36, we need to multiply 3 by (x+4) we get 3x +12, and then we will move -5x to the other side like it is because it does not have parenthesis and it is going to be
M: let’s see Escamilla, look at what your classmate is doing, group 4, is it ok? Nobody has responded if is it right until this point?
Students: yes.
M: look at.
S7: teacher we got 8x but I do not understand how he got a positive sign, it is supposed that the ‘s are going to be joined or the coefficients, no?
M: let’s see Escamilla, yes it is right but form where did you get the positive sign?
S6: ah when I did this, it is multiplied by -36, yes or no?
Students: no.
Maria asked if somebody wanted to help Escamilla. One student went to the front and they were discussing about the solution. The Maria asked them:
M: Where are you struggling?
S6: we do not remember from where we got this and how it become to be positive, if we change it the sign changes right or no?
Then Maria sked to the whole class
M: what your classmates did is right, 3x +5x?
Students: no.
M: until here we were right. Look at, what did you do with 5x, it was moved to where, and then Escamilla?
S6: the addition of 3x+5x=8x, is equal…
M: Escamilla, Paola wanted to make a comment.
S8: then you moved it wrong, because it should be negative.
M: that coefficient 12, Is moved to the other side?
Students: negative.
M: then what happen with that -36 we moved it? Paola.
S8: it stayed in the same side.
M: then it stayed as it is.
S6: sorry, then it is 36-12.
M: you moved the -36, or you left it in the same place, it stayed in the same side as it is as -36.
S6: -36-12 is equal -48, no? 8x=-48, x=-48 divided by 8x then x will be -6.
M: look at the solution, are you sure there is a not any question?
S9: when you move something to the other side, - by – = +, then we should have a negative sign -.
M: let’s see mija, we have -36-12.
S9: yes but the signs are multiplied.
M: but this is a subtraction and addition. When it is like this the same sign stayed. (OC: ktA in the whole sis and also in this last part because Maria allowed the student to explain why the student thought that)
In this previous situation there was a student’s mistake and the teacher took advantage of it to make students to think and be involved on the procedure and solution of the problem. Students were engaged to figure out if everything was right.
Then Maria asked other student to go to the whiteboard to solve other equation. When she finished it Maria said:
M: is it right? Are you sure? (ktA she promote critical thinking with this type of questions)
Then Maria asked the student to explain it.
S10: well here 5 multiply by the parenthesis and also here, below, we are developing each side, then we add them, then this provides the result.
M: let’s see guys, observations.
S11: I got a different answer, because when i was solving for r i got 30-20 then it is 10.
M: look at it, you got to the results but the coefficients, you need to look at the previous step. That 30 where is it? 5r+30, what did you do with that positive 30 Laura?
S11: I moved to this side.
M: aha, now -30+20. What else is it missing? r is equal to?
Then, the student corrected her equation. (ktA)
M: now. we agree?
Students: yes.
Then she asked other student to solve the last equation of the worksheet. She solved on the whiteboard. Maria asked if the student is doing right the problem? Student said yes others no.
Maria asked one student why not and he said because a number should be positive. Then Paola who was writing the solution on the whiteboard realized about her mistake and made the correction. Then the teacher asked her to complete it (ktA).
Maria asked students if they have any question however, nobody had questions. Then Maria allowed students to make corrections in their worksheets taking notes about what they did in the worksheet and what was written on the whiteboard. She said
M: observe what you have and what it is on the whiteboard, identify your mistake, I know that there were students who have mistaken just in signs. Then circle your mistake.
One student said: ah yes, I know where my mistake was.
M: ok then make the correction by yourself (ktA)
Maria was walking around the students to help them with questions.
She said that “with this worksheet we finished the unit the only thing missing is the evaluation, hence tomorrow we will evaluate it “.
One student asked her if it is correct and she asked him to make corrections without her assistance. Then she continued walking around the groups but she did not help the students anymore.
She asked student to paste the worksheet on their notebooks.
One student asked her how to make correction in certain problem if she did not find her mistake, then Maria explained her about the problem’s solution but she did not provide the solution (ktA).
Other student asked her about one problem and Maria said “do it by yourself, read it again and compare to the solution on the whiteboard” (ktA)
Then she called each student to see if they were present in the class.
Maria has 7 years teaching math but not every year she taught the same grade level. However, she usually just one class of math and several science classes.
Appendix: J Observation Protocol

Date: _______ Time: Start: _______ End: _______ Teacher: ______________________
Observer: ______________________ # of students: ______ Class: ______

Content: ______________________

1. When a student solves a particular mathematical problem and asks if the answer or steps are correct:

2. When a student does not recognize the same pattern in a different situation:

3. When a student provides a nonsensical solution to a given problem:

4. When a student makes a mistake:

5. When a student is unable to see an obvious pattern in the problem:
4 When you assign an activity about a previously learned topic, and some students express that they do not know how to do it:

6 When you make a mistake and a student notices the mistake:

7 When a student asks you a question from a different perspective that was not previously considered by you:

10 When students are having hard time to complete an assigned activity/task:

11 When you are teaching a topic that you are not knowledgeable at and a student asks you to elaborate more on that topic:

8 When a student continuously responds with wrong answers to questions posed by you:
Vita

Maria D. Cruz Quinones holds a bachelor’s degree in Computer Science Engineering from the University of Juarez City (Universidad Autónoma de Ciudad Juárez). Maria also received her Master’s degree in Mathematics Education from the same university (Universidad Autónoma de Ciudad Juárez).

EDUCATION

Doctor of Philosophy, Teaching, Learning and Culture, University of Texas at El Paso, 2014, Dissertation title “Relationship Between Cognitive Types of Teacher Content Knowledge and “Knowing-to act”: A Mixed Methods Study of Mexican Borderland Middle School Teachers”.

Master of Science, Teaching Mathematics, University Autónoma de ciudad Juarez, 2010, thesis title “Caracterización de las prácticas sociales asociadas a la génesis de la inducción matemática: Un estudio socioepistemológico”.

Bachelor of Science, Computation systems Engineering, University Autonoma de ciudad Juarez, 2007, thesis title “Sistema de information para un evento deportivo (caso Basket Ball)”.

English Diplom, eleven levels of English, Centro de Lenguas, University Autónoma de ciudad Juárez, 2007.

ACADEMIC/TEACHING EXPERIENCE

Research Assistant, University of Texas, El Paso, Advising, Recruitment, Career Center, El Paso, TX, U.S., June 2014- Present.

Graduate Assistant, University of Texas, El Paso, West Great Center, El Paso, TX, U.S., September 2012- May 2014.

Graduate Assistant, University of Texas, El Paso, Teacher Education Department, El Paso, TX, U.S., January 2012-August 2012.


- Teach undergraduate students.

Particular professor, 7042 Rosaleda st, cd. Juarez, Chih, Mex, August 2008-Present.

- Teach undergraduate and high School students.
COURSES TAUGHT

- Discrete Mathematics II
- Computer Assisted Draw (AUTOCAD)
- Numerical Analysis
- Introductory Course for Engineering CUNA

ACADEMIC HONORS AND AWARDS

Award, Teacher Education Department Travel Grant, 2012, 2013, 2014

Award, Student Government Association Travel Grant, 2013, 2014.


Scholarship, University Autónoma de ciudad Juárez, 2002-2010.

Second Place of excellent grades, Colegio de Bachilleres 5, 2001.

CONFERENCE PRESENTATIONS

Una visión a la investigación en Matematica Educativa: experiencia de maestria y estudios de doctorado, Conference, VIII Jornadas de Física y Matemáticas, University Autónoma de ciudad Juárez, UACJ, April, 2014.


Correlational Study of Middle School Mathematics Teachers’ Content Knowledge and Student Achievement in Russia. The 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Chicago, IL, November, 2013.


Same or Different? Taking a look at the ABE System in Mexico. The annual Literacy Texas Conference, San Marcos, TX, August, 2013.
Mexican Adult Basic Education: A Comprehensive Overview. Annual meeting Texas Association for Literacy and Adult Education, TALAE, Austin, TX, February, 2013.


Caracterización de las prácticas sociales asociadas a la génesis de la inducción matemática: Un estudio socioepistemológico, Seminario de la construcción del conocimiento online, Nov. 29, 2010.


Caracterización de las prácticas sociales asociadas a la génesis de la inducción matemática: Un estudio socioepistemológico researching advances, Seminario de Matemática Educativa, University Autónoma de ciudad Juarez, May. 28, 2010.

RESEARCH INTERESTS

Education
Mathematics Education
Teaching, learning and culture
Teacher knowledge: Teacher content knowledge and knowing-to

RESEARCH/SCHOLARLY ACTIVITIES


Superfine, A (Eds.). *Proceedings of the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (p. 915).* Chicago, IL: University of Illinois at Chicago.


**FOREIGN LANGUAGE ABILITIES/SKILLS**

English and Spanish writings and readings abilities.

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