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Abstract

In several of their papers, Victor Raskin and coauthors proposed to use fuzzy techniques to make ontological semantics techniques more adequate in dealing with natural language. Specifically, they showed that the most adequate results appear when we use min as an "and"-operation and max as an "or"-operation. It is interesting that in other applications of fuzzy techniques, such as intelligent control, other versions of fuzzy techniques are the most adequate. In this chapter, we explain why the above techniques are empirically the best in the semantics case.

1 Formulation of the Problem

Use of fuzzy degrees in ontological semantics: a brief (and simplified) reminder. In several of his papers, V. Raskin at al. proposed to use fuzzy techniques to make ontological semantics a more adequate description of natural language; see, e.g., [14, 15, 16].

For example, according to Raskin et al., from the purely logical viewpoint, there are many correct answers to a query "Who can drive a car?". For example, "a man", "a woman", "an engineer" are all logically correct answers. However, from the natural language viewpoint, the above answers sound weird. From the natural language viewpoint, the most natural answer is "an adult."

This answer is the most natural because it is the most general. It is desirable to utilize this idea in computer-based natural language processing algorithms. One of the main difficulties in such use of computers is that computers have been originally designed to process numbers, not words, and computers are still much better in processing numbers than in processing textual information. Thus, to utilize the above idea in a computer-based system, it is desirable to describe different degrees of generality by numbers.

Notions from the a natural language form a natural hierarchy, ranging from the most general notions to the most specific ones. The most general notions have the highest level of generality. Let us denote this level by an integer L. Slightly less general ones – barely distinguishable from the most general ones – fill the next, $(L-1)$-st level of generality; then comes the $(L-2)$-nd level, etc., until we get to the lowest level of generality. It is reasonable to select the integer L so that the lowest level of generality is level 0 – in other words, it is reasonable to select, as L, the number of levels in the corresponding hierarchy minus one.

To each notion, we can thus assign a value v from 0 to L that describes the degree to which this particular notion is general. The idea of a degree to which some property is satisfied is well known in computer science. This idea can be traced to the 1960s pioneering works of Lotfi Zadeh who realized that many imprecise ("fuzzy") natural-language concepts like "general" (or "small") do not represent well-described dichotomy into general–not-general, small–not-small. Instead, each object can be described by a *degree* to which this object satisfies the given property – e.g., a degree to which a notion is general, or a degree to which an object is small. This realization led to a development of efficient technique for processing such degrees, technique known as *fuzzy logic*; see, e.g., [1, 3, 7, 11, 12, 18].

These degrees can be viewed as a natural generalization of simple dichotomic notions (i.e., notions which can be either true or false). For such simple notions:

- "true" is usually represented in a computer as the number 1, and
- "false" is usually represented as the number 0.

From this viewpoint, it is natural to re-scale the degrees so that:

- the largest possible degree – corresponding to maximum confidence – becomes 1, while
- the smallest possible degree becomes 0,

so that intermediate degrees corresponds to numbers from the intervals $(0,1)$.

For original degrees ranging from 0 to some number L, the most natural way for achieve such a re-scaling is to divide the corresponding degree by L. This is exactly how degrees are usually assigned in fuzzy logic: To describe to what extent some value of a quantity of interest is small, we ask the expert to mark its degree of slowness of a Likert-type scale – e.g., on a 0 to 10 scale. If the expert marks his/her degree by a value 7 on a 0 to 10 scale, we take $7/10 = 0.7$ as the degree to which, to this expert, the corresponding object is small.

Need for "and"- and "or"-operations with fuzzy degrees. One of the main applications of fuzzy logic is utilizing imprecise (fuzzy) expert knowledge in automated computer systems, e.g., in systems designed for control. To achieve this objective, we need to translate expert knowledge into computer-understandable numerical terms.

The main ideas of fuzzy logic enable us to provide a numerical value for simple properties like "pressure is low". However, expert rules often use more complex conditions, e.g., "and"- and "or"-combinations of simple properties. For example, an expert can recommend a certain control strategy for situations in which the pressure in a chemical reservoir is low *and* the temperature is low, and another strategy for the opposite situation which the pressure is high *or* the temperature is high.

In the ideal world, we should do, for each such combination, what we did for individual properties: ask the expert, for each pair of pressure and temperature, to indicate to which extend the complex property "pressure is low and temperature is low" is satisfied for the two given values. However, realistically, this is not possible: there are too many such combinations, and it is not feasible to ask the expert's opinion about all of them. Because of this impossibility to explicitly *elicit* the corresponding degree from the experts, we need to *estimate* these degrees based on whatever information we have – i.e., based on the known degrees to which individual statements hold.

In other words, we need to be able, given our degrees of confidence a and b in statements A and B, to provide a degree to which the "and"-combination "A and B" is satisfied. The algorithm that computes this estimate is known as an *"and"-operation* (or, for historical reasons, a *t-norm*); see, e.g., [1, 3, 7, 11, 12, 18]. We will denote such an algorithm by $f_\&(a,b)$.

Similarly, we must be able, given our degrees of confidence a and b in statements A and B, to provide a degree to which the "or"-combination "A or B" is satisfied. The algorithm that computes this estimate is known as an *"or"-operation* (or, for historical reasons, a *t-conorm*) [1, 3, 7, 11, 12, 18]. We will denote such an algorithm by $f_\vee(a,b)$.

Which "and"- and "or"-operations should we choose: general idea. There are many possible "and"- and "or"-operations. Depending on the objective, in different situations, different pairs of "and"- and "or"-operations are optimal; see, e.g., [4, 5, 6, 8, 9, 10, 11, 13, 17]; for example:

- if we want to select the most stable control, then it is best to select $f_\&(a,b)=\min(a,b)$ and $f_\vee(a,b)=\min(a+b,1)$;
- if we want to select the smoothest control, then it is best to select $f_\&(a,b)=a\cdot b$ (or $f_\&(a,b)=\max(a+b-1,0)$) and $f_\vee(a,b)=\max(a,b)$;
- if we want to select a control which is, on average, the least sensitive to measurement uncertainty with which we know inputs, then we should select $f_\&(a,b)=a\cdot b$ and $f_\vee(a,b)=a+b-a\cdot b$;
- if we want a control which is the least sensitive in the worst case, then we should select $f_\&(a,b)=\min(a,b)$ and $f_\vee(a,b)=\max(a,b)$; the same pair should be selected if we want a control which is the easiest (and fastest) to compute, etc.

In different application areas, different pairs of operations most adequately describe expert reasoning: e.g., different pairs are most appropriate in describing expertise of medical doctors and expertise of geoscientists; see, e.g., [2].

Which "and"- and "or"-operations are most adequate for ontological semantics: an empirical fact that needs explaining. According to [14, 15, 16], for ontological semantics, the most adequate selection is $f_\&(a,b) = \min(a,b)$ and $f_\vee(a,b) = \max(a,b)$.

Why? How can we explain this empirical fact? The main objective of this paper is to provide such an explanation.

2 Analysis of the Problem

Possible degrees. As we have mentioned earlier, in the ontological semantics, the corresponding degrees $d \in [0,1]$ are obtained after we divide an integer – the ordinal number of the corresponding level of generality – by the ordinal number L of the level containing the most general notions. As a result, we have the following set S of possible values of the degree of generality:

$$S = \left\{0, \frac{1}{L}, \frac{2}{L}, \ldots, 1 - \frac{1}{L}, 1\right\}.$$

Thus, we are interested in operations $f_\&(a,b)$ and $f_\vee(a,b)$ that transform values $a, b \in S$ into a new value $c \in S$. By using mathematical notations, we can say that we need to find functions $f_\& : S \times S \to S$ and $f_\vee : S \times S \to S$.

What properties should these functions satisfy?

First property: "and" and "or"-operations must be conservative. As we have mentioned earlier, degree 0 usually corresponds to "false" and degree 1 usually corresponds to "true". Thus, it is reasonable to require that when each of a and b is either 0 or 1, our "and"- and 'or"-operations become the usual "and"- and "or"-operations of the 2-valued propositional logic:

$$f_\&(0,0) = f_\&(0,1) = f_\&(1,0) = 0, \quad f_\&(1,1),$$

$$f_\vee(0,0) = 0, \quad f_\vee(0,1) = f_\vee(1,0) = f_\vee(1,1) = 1.$$

Second property: monotonicity. Our degree of belief in $A \,\&\, B$ cannot be larger than our degree of belief in each of the original statements. So, we must have $f_\&(a,b) \le a$ and $f_\&(a,b) \le b$.

Similarly, our degree of belief in $A \vee B$ cannot be smaller than our degree of belief in each of the original statements. So, we must have $a \le f_\vee(a,b)$ and $b \le f_\vee(a,b)$.

Third property: "and"- and "or"-operations must be consistent with distinguishability. According to the original definition of degree, the neighboring degree $\dfrac{i}{L}$ and $\dfrac{i+1}{L}$ are barely distinguishable from each other. To be more precise, there is no intermediate degree between then. So, if we slightly increase the lower degree of slightly decrease the higher degree, then will become truly indistinguishable. In this sense, pairs neighboring values are "barely distinguishable" in the sense that they are limits of indistinguishable pairs. If we denote indistinguishability by $\approx$ and "barely distinguishable" by $\sim$, then we can say that $a \sim a'$ if and only if there exist values $a_n \to a$ and $a'_n \to a'$ for which $a_n \approx a'_n$ for all n.

Intuitively, if a is indistinguishable from a' and b is indistinguishable from b', then, e.g., $f_\&(a, b)$ should be indistinguishable from $f_\&(a', b')$. So:

- if $a \sim a'$ are limits of indistinguishable pairs $a_n \approx a'_n$, $b \sim b'$ are limits of indistinguishable pairs $b_n \approx b'_n$,
- then, since a_n is indistinguishable from a'_n and b_n is indistinguishable from b'_n, we conclude that $f_\&(a_n, b_n)$ should be indistinguishable from $f_\&(a'_n, b'_n)$.

In the limit $n \to \infty$, we conclude that the limit values $f_\&(a, b)$ and $f_\&(a', b')$ should be barely distinguishable: $f_\&(a, b) \sim f_\&(a', b')$.

Now, we are ready to formulate our main result.

3 Definitions and the Main Results

Definition 1. *Let L be a positive integer, and let S be the set*

$$\left\{0, \frac{1}{L}, \frac{2}{L}, \ldots, 1 - \frac{1}{L}, 1\right\}.$$

We say that values $a, b \in S$ are barely distinguishable *and denote it by $a \sim b$ if they either coincide or are neighbors in the above sequence.*

Comment. One can easily check that

$$a \sim b \Leftrightarrow |a - b| \leq \frac{1}{L}.$$

Definition 2. *By an* OS-"and"-operation*, we mean a function $f_\& : S \times S \to S$ that satisfies the following properties:*

- $f_\&(1, 1) = 1$*;*
- $f_\&(a, b) \leq a$ *and* $f_\&(a, b) \leq b$ *for all a and b;*
- *if $a \sim a'$ and $b \sim b'$, then $f_\&(a, b) \sim f_\&(a', b')$.*

Comment. Note that we did not use all the above properties – e.g., we did not use the fact that $f_\&(0, 1) = 0$, neither we require commutativity or associativity, as is usually done with "and"-operations in fuzzy literature. We did not use these additional properties because, as the following result shows, even without these properties, we can uniquely determine the corresponding "and"-operation. The three properties that we did use are necessary: as we show a few lines later, without even one of these properties, the following result will not hold.

Proposition 1. *The only OS-"and"-operation is* $\min(a, b)$.

Comments.

- For readers' convenience, all the proofs are placed in a special (last) Proofs section.
- This result explains the empirical fact – that minimum is the most adequate "and"-operation for operational semantics.
- All three properties of an OS-"and"-operation are needed for Proposition 1 to hold:
- if we do not require that $f_{\&}(1, 1) = 1$, then we can have $f_{\&}(a, b) = 0$ for all a and b;
- if we do not require monotonicity, then we can have $f_{\&}(a, b) = 1$ for all a and b; and
- if we do not require consistency with distinguishability, then we can have $f_{\&}(1, 1) = 1$ and $f_{\&}(a, b) = 0$ for all other pairs (a, b).
- A similar result explains why maximum turned out to be the most adequate "or"-operation.

Definition 3. *By an* OS-"or"-operation, *we mean a function* $f_\vee : S \times S \to S$ *that satisfies the following properties:*

- $f_\vee(0, 0) = 0$;
- $a \le f_\vee(a, b)$ *and* $b \le f_\vee(a, b)$ *for all* a *and* b;
- *if* $a \sim a'$ *and* $b \sim b'$, *then* $f_\vee(a, b) \sim f_\vee(a', b')$.

Proposition 2. *The only OS-"or"-operation is* $\max(a, b)$.

Comment. All three properties of an OS-"or"-operation are needed for Proposition 2 to hold:

- if we do not require that $f_\vee(0, 0) = 1$, then we can have $f_\vee(a, b) = 1$ for all a and b;
- if we do not require monotonicity, then we can have $f_\vee(a, b) = 0$ for all a and b; and
- if we do not require consistency with distinguishability, then we can have $f_\vee(0, 0) = 0$ and $f_\vee(a, b) = 1$ for all other pairs (a, b).

4 Auxiliary Result: From Binary to n-ary Operations

In practice, we often need to combine more than two statements. In fuzzy logic, it is usually assumed that we combine them one by one: for example, if we need to "and"-combine a, b, and c, then:

- first, we combine a and b into $f_\&(a, b)$, and then
- we combine the result of a-and-b combination with c, resulting in

$$f_\&(f_\&(a, b), c).$$

Instead, we can explicitly define n-ary "and"- and "or"-operations for every n.

Definition 4. *Let $n \geq 2$ be an integer. By an n-ary* OS-"and"-operation, *we mean a function $f_\& : S^n \to S$ that satisfies the following properties:*

- $f_\&(1, \ldots, 1) = 1$*;*
- $f_\&(a_1, \ldots, a_n) \leq a_i$ *for all i;*
- *if $a_i \sim a'_i$ for all i, then $f_\&(a_1, \ldots, a_n) \sim f_\&(a'_1, \ldots, a'_n)$.*

Proposition 3. *The only n-ary OS-"and"-operation is* $\min(a_1, \ldots, a_n)$.

Definition 5. *Let $n \geq 2$ be an integer. By an n-ary* OS-"or"-operation, *we mean a function $f_\vee : S^n \to S$ that satisfies the following properties:*

- $f_\vee(0, \ldots, 0) = 0$*;*
- $a_i \leq f_\vee(a_1, \ldots, a_n)$ *for all i;*
- *if $a_i \sim a'_i$ for all i, then $f_\vee(a_1, \ldots, a_n) \sim f_\vee(a'_1, \ldots, a'_n)$.*

Proposition 4. *The only n-ary OS-"or"-operation is* $\max(a_1, \ldots, a_n)$.

5 Proofs

Proof of Proposition 1.

1°. Let us first prove that for every $a \in S$, we have $f_\&(a, a) = a$. Specifically, we will prove, by induction over k, that for all k, we have

$$f_\&\left(1 - \frac{k}{L}, 1 - \frac{k}{L}\right) = 1 - \frac{k}{L}. \tag{1}$$

Indeed, due to conservativeness, this property holds for $k = 0$, so we have the induction base. Suppose now that the property (1) holds for k, let us prove that it holds for $k + 1$ as well. Due to monotonicity, we have

$$f_\&\left(1 - \frac{k+1}{L}, 1 - \frac{k+1}{L}\right) \leq 1 - \frac{k+1}{L}. \tag{2}$$

On the other hand, we have

$$\left(1-\frac{k+1}{L}\right) \sim \left(1-\frac{k}{L}\right).$$

Thus, due to consistency with distinguishability, we have

$$f_{\&}\left(1-\frac{k+1}{L}, 1-\frac{k+1}{L}\right) \sim f_{\&}\left(1-\frac{k}{L}, 1-\frac{k}{L}\right) = 1-\frac{k}{L}. \tag{3}$$

We know that $a \sim b$ means $|a-b| \leq \dfrac{1}{L}$, thus

$$a \geq b - \frac{1}{L}.$$

Therefore, the property (3) implies that

$$f_{\&}\left(1-\frac{k+1}{L}, 1-\frac{k+1}{L}\right) \geq \left(1-\frac{k}{L}\right) - \frac{1}{L} = 1-\frac{k+1}{L}. \tag{4}$$

From (2) and (4), we conclude that

$$f_{\&}\left(1-\frac{k+1}{L}, 1-\frac{k+1}{L}\right) = 1-\frac{k+1}{L}.$$

The induction step is proven. Thus, indeed, $f_{\&}(a,a) = a$ for all a.

2°. Let us now prove that $f_{\&}(a,b) = \min(a,b)$ for all a and b. In Part 1, we have already proved it for the case when $a = b$, so to complete the proof, it is sufficient to consider the case when $a \neq b$.

We will prove it for the case when $a > b$; the case $a < b$ is similar. Specifically, for each k, we will prove, by induction over $\ell \geq 0$, that we have

$$f_{\&}\left(\frac{k}{L}, \frac{k-\ell}{L}\right) = \frac{k-\ell}{L}. \tag{5}$$

For $\ell = 0$, we have already proved it in Part 1 of this proof. Let us now assume that we have already proved the equality (5) for a given value ℓ; let us prove that it is true for $\ell + 1$ as well. Indeed, from monotonicity, it follows that

$$f_{\&}\left(\frac{k}{L}, \frac{k-(\ell+1)}{L}\right) \leq \frac{k-(\ell+1)}{L}. \tag{6}$$

On the other hand, since

$$\frac{k-(\ell+1)}{L} \sim \frac{k-\ell}{L},$$

the consistency with distinguishability implies that

$$f_{\&}\left(\frac{k}{L}, \frac{k-(\ell+1)}{L}\right) \sim f_{\&}\left(\frac{k}{L}, \frac{k-\ell}{L}\right) = \frac{k-\ell}{L}. \tag{7}$$

As we have mentioned in Part 1 of this proof, $a \sim b$ implies

$$a \geq b - \frac{1}{L}.$$

Therefore, the property (7) implies that

$$f_{\&}\left(\frac{k}{L}, \frac{k-(\ell+1)}{L}\right) \geq \frac{k-\ell}{L} - \frac{1}{L} = \frac{k-(\ell+1)}{L}. \tag{8}$$

From (6) and (8), we conclude that

$$f_{\&}\left(\frac{k}{L}, \frac{k-(\ell+1)}{L}\right) = \frac{k-(\ell+1)}{L}.$$

The induction step is proven. Thus, indeed, $f_{\&}(a, b) = \min(a, b)$ for all a and b.

Proof of Proposition 2. One can easily check that a function $f_{\vee}(a, b)$ is an OS-"or"-operation if and only if its "dual"

$$f_{\&}(a, b) \stackrel{\text{def}}{=} 1 - f_{\vee}(1-a, 1-b)$$

is an OS-"and"-operation. Since min and max are duals to each other, Proposition 2 follows from Proposition 1.

Proof of Proposition 3 is similar to the proof of Proposition 1:

- first, similarly to Part 1 of that proof, we prove, by induction, that $f_{\&}(a, \ldots, a) = a$ for all a;
- then, to prove that $f_{\&}(a_1, \ldots, a_n) = \min(a_1, \ldots, a_n)$, similarly to Part 2 of that proof, we start with $a = \max(a_1, \ldots, a_n)$ and then step-by-step decrease each input by $\dfrac{1}{L}$, proving each time that the desired equality if preserved.

Proof of Proposition 4. One can easily check that a function $f_{\vee}(a, b)$ is an n-ary OS-"or"-operation if and only if its "dual"

$$f_{\&}(a_1, \ldots, a_n) \stackrel{\text{def}}{=} 1 - f_{\vee}(1-a_1, \ldots, 1-a_n)$$

is an n-ary OS-"and"-operation. Thus, Proposition 4 follows from Proposition 3.

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