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When Revolutions Happen: Algebraic Explanation

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Abstract

At first glance, it may seem that revolutions happen when life becomes really intolerable. However, historical analysis shows a different story: that revolutions happen not when life becomes intolerable, but when a reasonably prosperous level of living suddenly worsens. This empirical observation seems to contradict traditional decision theory ideas, according to which, in general, people's happiness monotonically depends on their level of living. A more detailed model of human behavior, however, takes into account not only the current level of living, but also future expectations. In this paper, we show that if we properly take these future expectations into account, then we get a natural explanation of the revolution phenomenon.

1 Formulation of the Problem

When revolutions happen: usual understanding. People usually believe that revolutions happen when the situation worsens to such extent that life under the old regime becomes practically intolerable. Paraphrasing the famous saying attributed to Marie-Antoinette, people start a revolution when they do not even have enough bread to eat.

When revolutions actually happen. However, a historical analysis shows that the usual understanding is wrong; see, e.g., [1, 2, 3]. Most revolutions happen *not* when the situation is at its worst, they usually happen when the situation has been improving for some time and then suddenly gets worse – although, by the way, never as bad as it was before the improvement started.

How can we explain this? This is an interesting observation, but it leaves one puzzled: why? There are well-designed theories of human decision making, and experiments show that in most situations, people act rationally: the more their needs are satisfied, in general, the happier they are.

So how come that right before the revolution, when the level of living is higher (often much higher) than in the recent past, people are so much less happy that they start a revolution – while in the past, when their living conditions were much worse, they were sufficiently satisfied – at least so as to remain obedient.

How can we explain this unexpected (and somewhat counterintuitive) behavior?

What we do in this paper. In this paper, we show that this seemingly counterintuitive revolution phenomenon can actually be well explained within the standard decision theory.

2 Analysis of the Problem

Traditional decision theory: a brief reminder. According to traditional decision theory, people's preferences are described by numerical values called *utilities*; see, e.g., [4, 5, 6, 7, 9].

The actions of a person are determined not just by this person's current level of satisfaction – as described by the current utility value u_0 – but also by the expected future utility values u_1 at the next moment of time, u_2 in the second next moment of time, etc. The future utility values come with a discount; e.g., the possibility to buy a new car a few years in the future does not bring as much happiness as buying a car right away.

This is similar to the value of future money: future money is less valuable than the same amount right now, since if we have the same amount – say \$1000 – now, we can place it in a bank and, due to accumulating interest, get a larger amount in the future. If we denote the annual interest rate by α, then after year t, each invested dollar will turn into $(1+\alpha)^t$ dollars. Thus, \$1 at time t is equivalent to q^t dollars now, where we denoted $q \stackrel{\text{def}}{=} \dfrac{1}{1+\alpha}$. So, if we get the amount a_0 now, the amount a_1 in the next year, the amount a_2 in 2 years, etc., this is equivalent to getting the following amount now:

$$a_0 + q \cdot a_1 + q^2 \cdot a_2 + \ldots$$

A similar formula can be used to describe the overall utility based on the current utility u_0 and expected future utilities

$$u_0 + q \cdot u_1 + q^2 \cdot u_2 + \ldots$$

This general approach requires extrapolation. The future amounts are based on extrapolation. So, to apply this theory to our situation, we need to understand how exactly people extrapolate.

In general, extrapolations means that:

- we select a family of functions characterized by a few parameters

$$u_t = f(p_1, \dots, p_n, t),$$

- then we find the values $\widehat{p}_1, \dots, \widehat{p}_n$ of the parameters that best fit the observed data u_0, u_{-1}, u_{-2}, etc., i.e., for which

$$f(p_1, \dots, p_n, 0) \approx u_0, \quad f(p_1, \dots, p_n, -1) \approx u_{-1}, \dots,$$

- and then we use these values to predict future values as $f(\widehat{p}_1, \dots, \widehat{p}_n, t)$.

It is reasonable to use models which are linear in the its parameters. A reasonable idea is to use models that linearly depend on the corresponding parameters: for such models, matching parameters to data means solving systems of linear equations, which is very feasible and much easier than solving systems of nonlinear equations – which are, in general, NP-hard.

Thus, we consider models of the type $u_t = \sum\limits_{i=1}^{n} p_i \cdot f_i(t)$, where $f_i(t)$ are given functions, and p_i are appropriate parameters.

Which basis functions $f_i(t)$ should we choose? Most transitions are smooth, so it is reasonable to require that all the functions $f_i(t)$ used to extrapolation are smooth.

Another reasonable requirement is related to the fact that the numerical value of time depends on the choice of a measuring unit – years or months – and on the choice of a starting time. For example, during the French revolution, the year of storming the Bastille was considered Year 1.

If we change a measuring unit by a new one which is a times smaller, then each original value t is replaced by the new value $a \cdot t$. Similarly, if we change the original starting point with the new starting point which is b units in the past, then the original value t is replaced by the new value $t + b$.

The general formulas for extrapolation should not depend on such an arbitrary things as selecting a unit of time or selecting a starting point. It is therefore reasonable to assume that the approximating family $\left\{\sum\limits_{i=1}^{n} p_i \cdot f_i(t)\right\}$ will not change if we simply re-scale time to $t \to a \cdot t$ or to $t \to t + b$.

In other words, we require that for every $a > 0$ and for every b, we have

$$\left\{\sum_{i=1}^{n} p_i \cdot f_i(a \cdot t)\right\}_{p_1,\dots,p_n} = \left\{\sum_{i=1}^{n} p_i \cdot f_i(t + b)\right\}_{p_1,\dots,p_n} = \left\{\sum_{i=1}^{n} p_i \cdot f_i(t)\right\}_{p_1,\dots,p_n}.$$

It turns out that under these conditions, all the basic functions – and thus, all their linear combinations – are polynomials; see, e.g., [8].

Thus, it is reasonable to approximate the actual history by a polynomial. Let us show, on a simple example, that this indeed explains the empirical revolution phenomenon.

Two simple situations. Specifically, we will compare two simple situations:

- a situation in which the level of living is consistently bad, i.e.,

$$u_0 = u_{-1} = \ldots = u_{-k} = \ldots = c_1$$

  for some small value c_1, and

- a situation in which the level of living used to be much better, but now somewhat decreased, i.e., in which

$$u_{-1} = u_{-2} = \ldots = c_+$$

  but $u_0 = c_- < c_+$ – although this decreased value $u_0 = c_-$ is still better than the value c_1 from the first situation.

If people did not take their future happiness into account when making decision, the situation would have been very straightforward – and in full accordance with the commonsense understanding of the revolutions: people in the first situation would be much less happy than people in the second situation and therefore, more prone to start a revolution.

What will happen if we take future expectations into account? In the first situation, of course, a reasonable extrapolation should lead to the exact same small value $u_0 = c$; thus, the overall utility is equal to

$$u_0 + q \cdot u_1 + \ldots = c \cdot (1 + q + q^2 + \ldots) = \frac{c}{1-q}.$$

But what to expect in the second situation?

Let us start with the simplest possible extrapolation. Let us start our analysis with the simplest possible extrapolation, when we make our future predictions based only on two utility values: the current utility value u_0 and the previous utility value u_{-1}.

Which degree polynomials should we use? In this case, we have two values u_0 and u_{-1} to fit the model, so it is reasonable to select the degree of the approximating polynomial for which the corresponding family of polynomials depends on exactly two parameters. Polynomials of a general degree d have the form

$$a_0 + a_1 \cdot t + \ldots + a_d \cdot t^d.$$

This family depend on $d + 1$ parameters a_i, so in our case, we should have $d + 1 = 2$ and $d = 1$ – i.e., we should use linear functions for extrapolation.

Since $u_0 < u_{-1}$, we thus get a linear decreasing function. Its values tend to $-\infty$ as the time t increases. So, when q is close to 1, the corresponding value

$$u_0 + q \cdot u_1 + \ldots \approx u_0 + u_1 + u_2 + \ldots$$

becomes very negative – and this explains why in the second situation, the revolution is much more probable.

What about more realistic approximation schemes? One may think that the above explanation is caused by our oversimplification of the extrapolation model. Of course, linear extrapolation is a very crude and oversimplified idea. What happens if we use higher degree polynomials for extrapolation?

Let us assume that for extrapolation, we use polynomials of order d. The corresponding family of polynomials have d_1 parameters, so we can fit $d+1$ values. Thus, if we use these polynomials, then, in our extrapolation, we can use not only the two values u_0 and u_{-1}, we can use $d+1$ values

$$u_0, u_{-1}, \ldots, u_{-d}.$$

Let us find the polynomial $P(t)$ of degree d that fits all these values, i.e., for which $P(-i) = c_+$ for all i from 1 to d, and $P(0) = c_-$. These conditions become even easier if we consider an auxiliary polynomial $Q(t) \stackrel{\text{def}}{=} P(t) - c_+$. For this auxiliary polynomial, we have $Q(-d) = \ldots = Q(-1) = 0$ and $Q(0) = c_- - c_+$. This polynomial of degree d has d roots $t = -1, \ldots, t = -d$, thus, it is divisible by the monomials $t - (-i) = t + i$ for all i from 1 to d, and therefore, it has the form $Q(t) = C \cdot (t+1) \cdot (t+2) \cdot \ldots \cdot (t+d)$, for some constant C. This constant can be determined from the condition that $Q(0) = c_- - c_+$, so $C \cdot 1 \cdot 2 \cdot \ldots \cdot d = c_- - c_+$ and thus,

$$C = \frac{c_- - c_+}{1 \cdot 2 \cdot \ldots \cdot d}.$$

Therefore, for any $t > 0$, the extrapolated value of $P(t) = c_+ + Q(t)$ has the form

$$Q(t) = c_+ + (c_- - c_+) \cdot \frac{(t+1) \cdot (t+2) \cdot \ldots \cdot (t+d)}{1 \cdot 2 \cdot \ldots \cdot d}.$$

Since $c_- < c_+$, this value is negative – and tends to $-\infty$ as the time t increases. In comparison with the linear extrapolation case, it tends to $-\infty$ even faster than in the case of linear extrapolation – as t^d.

So, *the revolution phenomenon can be explained* no matter what degree of extrapolation we use.

Discussion. Based on our analysis, in addition to our main conclusion (that we have explained the seemingly counterintuitive revolution phenomenon), we can make two auxiliary conclusions (which also fit perfectly well with common sense):

- revolutions only happen if people care about the future; if they don't, if $q \approx 0$, people are happy with their present-day level of living.
- the more into the past the people go in their analysis, the more probable it is that they will revolt; people who do not know their history are less prone to revolutions than people who do.

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