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Beyond Teacher Knowledge: Connections Between Content Knowledge, Mindset For Learning, And Teaching In Two-Year College Mathematics Faculty

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BEYOND TEACHER KNOWLEDGE: CONNECTIONS BETWEEN CONTENT KNOWLEDGE, MINDSET FOR LEARNING, AND TEACHING IN TWO-YEAR COLLEGE MATHEMATICS FACULTY

LUCY HERNANDEZ MICHAL
Doctoral Program in Teaching, Learning and Culture

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Dedication

I dedicate this work to my beautiful son, Emil V. J. Michal, and my late husband, Emil J. Michal, Jr. My son inspires and encourages me every day. He is a firm believer of taking care of the world around us and of helping his students learn and appreciate mathematics. I learn something from him every day. My husband is a source of energy I can harvest by looking up at the night sky. His dedication to teaching is at the core of this work. They are the two people I love most and who make me who I am.

I also dedicate this to my parents Vicente and Conception Hernandez and my siblings, Laura, Patty, Susie, Mackett, Rosie, Pera, Gordo, Elva, Terry, and Nena and their beautiful families. Very soon, it will be possible for me to spend more time with them!

This work is also dedicated to my students. I hope they still believe they can learn and use mathematics.
BEYOND TEACHER KNOWLEDGE: CONNECTIONS BETWEEN CONTENT KNOWLEDGE, MINDSET FOR LEARNING, AND TEACHING IN TWO-YEAR COLLEGE MATHEMATICS FACULTY

by

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DISSERTATION

Presented to the Faculty of the Graduate School of
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Abstract

BEYOND TEACHER KNOWLEDGE: CONNECTIONS BETWEEN CONTENT KNOWLEDGE, MINDSET FOR LEARNING, AND TEACHING IN TWO-YEAR COLLEGE MATHEMATICS FACULTY

Addressing thousands of students entering unprepared to enroll in college freshman mathematics courses is a challenge two-year college mathematics faculty face every semester. The study addresses this challenge through a mixed methods sequential nested multi-case design to investigate what faculty need, beyond teacher knowledge, to teach mathematics effectively to students. The three-part study measured the mindset learning profile and content knowledge of a sample of mathematics faculty at a two-year college. The quantitative and qualitative data provided references to select four faculty typologies to interview and observe. Interviews reflected faculty perceptions of teaching and learning. Class observations captured images of faculty teaching. Narratives on lesson objectives measured faculty lesson objectivization. Open, expert, and meaning coding was employed to identify emerging themes in the analysis of data for the multi-case study. Analysis of both quantitative and qualitative data facilitated the integration of methods to investigate connections between faculty content knowledge, a mindset for learning, and teaching.

A key finding from the study was that faculty with a knowledge of mathematics as a discipline, coupled with a growth mindset, engaged students in activities that develop meaningful and productive learning of mathematics. Data also reported a statistically significant relationship between content knowledge and lesson objectivization (Pearson’s $r = 0.6148$, $p < 0.05$). Other emerging themes included the following: two-year college mathematics faculty knowledge of content varies by domain; two-year college mathematics faculty demonstrated limited awareness of the guiding principles of learning sciences including but not limited to principles of effective learning in mathematics classrooms.
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Chapter 1: Introduction

1.1 Statement of the Problem

The January 2014 data report from the Texas Higher Education Coordinating Board indicated that more than 80 percent of the latest Texas fall cohort of students enter college needing developmental mathematics courses. Many of these students may never complete a college mathematics course with a grade of “C” or better (Rutschow & Diamond, 2015). Developmental mathematics courses were designed to prepare these large numbers of underprepared students to enroll and succeed in college-level mathematics courses. However, these courses have become barriers for students (Bonham & Boylan, 2012) and have ended students’ aspirations for a college education (Bryk & Treisman, 2010).

While studies have shown how curricula and innovative programs have positive effects on student learning, widespread implementation has yet to occur (Gutstein, 2003) and those implemented do not show significant gains past the time of treatment (Yeager & Dweck, 2012). Reasons may include logistical, financial, or human resource limitations inherent in the infrastructure of educational institutions. In particular, in mathematics, the added nature of people’s belief and mindset (Dweck, 2006) of their ability to learn mathematics, may complicate implementation of any curriculum and pedagogy. The difficulty in understanding the complex structure and substructures inherent in topics such as fractions (Charalambous & Pitta-Pantazi, 2007) introduced early in elementary school, remains for some students in their work with mathematics in K-12 schools and continues into their college studies. In looking at teacher knowledge and their mindset, an added construct to consider is humans’ two levels of mental
activity (Uznadze, 1966). Uznadze (1966) argued that these levels of mental activity lead to two different ways of constructing understanding of the world around us. As teachers construct understanding, these levels of thought may come into play when using lessons objectives.

As we look at the large numbers of students entering college underprepared for college-level mathematics courses, it is critical to ask how two-year college faculty address the needs of these students in ways that will get them on track to succeed in courses and programs of study. New curriculum that uses research-based pedagogy and course design are beginning to replace older curriculum. However, what other teacher characteristics might make faculty more effective classroom teachers? Studies tell us that as faculty members prepare to teach these lessons, identifying and understanding lesson objectives can determine the effectiveness of a lesson (Shulman, 1986). Mathematical content is another determining factor that makes teachers effective (Hill, Ball, & Schilling, 2008). What teachers believe about learning mathematics may also play a role in teacher effectiveness. As teachers identify objectives, how might mathematical content together with mindset for learning mathematics produce teaching that engages students in enhanced learning experiences? How might identifying more detail in the knowledge base for teachers in two-year colleges assist that critical mass of students?

1.2 Purpose and Research Questions

This study investigates teacher content knowledge, their mindset for learning, lesson objectivization and connections between these three variables. Teachers’ beliefs of what they need to be effective classroom teachers may provide a great source of other knowledge believed to make classrooms more effective learning places. Participants in the study are faculty teaching mathematics in a two-year college where the only credential required for teaching mathematics is
a master’s degree with eighteen graduate credit hours in mathematics. Because the purpose for faculty in two-year colleges is teaching, the terms teachers, instructors, and faculty will be used interchangeably in this study. While faculty may know content knowledge for their subject matter and pedagogy for effective teaching and student learning, Shulman (1986, 1987) introduced the notion of pedagogical content knowledge as another knowledge needed for teaching. Since Shulman (1986) introduced this concept, it magnified and influenced this knowledge type of research in education.

Hill, Ball, & Schilling (2008) added that conceptualizing teacher knowledge would move us closer to providing effective teaching in classrooms. In recent studies on student learning, another teacher characteristic, a mindset for learning (Dweck, 2006), may also address ways to create compelling teaching in classrooms. Mindset as an incremental growth model and mindset as a fixed model (Dweck, 2006) in teachers may also contribute to teachers’ knowledge base and impact how teachers see the learning of mathematics in students. As a belief about learning, Yeager and Dweck (2012) claim that one’s mindset has an impact on one’s belief in whether or not one has the capacity to learn content in any subject, not just mathematics. In particular, Yeager and Dweck (2012) posit that having a growth mindset may significantly impact student experiences in learning. The idea of a growth mindset is explored mostly in K-12 classrooms where participants include students, teachers, or both. There is a need for adding the concept of mindset and its effect on teaching and learning in postsecondary mathematics classrooms. This study looks at knowledge two-year mathematics faculty may need, to make teaching more effective for the 80% of students enrolling in college who are underprepared to take and complete college-level mathematics courses. The research questions for the study are:
1) What is the relationship between the content knowledge of mathematics faculty in two-year colleges and their mindset for learning mathematics?

2) How does lesson objectivization in two-year college mathematics faculty relate to their content knowledge?

3) How are content knowledge, mindset, and lesson objectivization connected to teaching in two-year college mathematics faculty?

1.3 **DATA COLLECTION AND METHOD**

The study is a mixed methods study with both quantitative and qualitative data. The study is a sequential nested explanatory multi-case study (Creswell, 2013). Quantitative and qualitative data were collected and analyzed consecutively in three parts of the study (Creswell, 2013) to gain a better understanding of teacher knowledge needed to engage students in lesson objectives in ways that will help define their mathematical concepts and identities to learn mathematics. The study is nested in its design and has three parts. In Part 1, participants include thirty-three (33) mathematics faculty from a two-year college in the southwest. In Part 2, participants include fourteen (14) faculty drawn from the sample of participants in Part 1. In Part 3, four participants are selected using purposive sampling (Babbie, 1970; 2014) from those who participated in Part 2 for a multi-case qualitative study. The selection of participants is described in Chapter 3. The study provided additional understanding of the knowledge base identified by faculty for effective teaching in mathematics classrooms in two-year colleges. In Part 1 of the study, quantitative data from two surveys provided data to address Research Question 1.

What is the relationship between the content knowledge of mathematics faculty in two-year colleges and their mindset for learning mathematics?
Survey 1 measured mindset (Yeager et al., 2013; Dweck, 2008) and collected demographic data from the participants. Survey 2 measured content knowledge using items from an item bank written to prepare teachers for state certification exams in mathematics for middle school teaching. More survey details appear in Chapter 3.

Part 2 of the study was qualitative and started with the collection of written narrative data from 14 participants. Participants viewed a video of an expert lesson and wrote a narrative description of the lesson objectives. The data provided information on the ability of faculty to identify and match expert objectives in a lesson. Qualitative methods were used to analyze written narratives (Babbie, 2014) of the lesson objectives. Data analyzing the matches to the lesson objectives were used to answer the second Research Question.

How does lesson objectivization in two-year college mathematics faculty relate to their content knowledge?

Part 3 of the study involves a purposive sample (Babbie, 1970; Babbie, 2014, Gravetter, 2013) of four participants for case studies who are a subset of the sample used in Part 2 of the study. Participants were selected based on measures falling in the highest and lowest quartiles in content scores and on measures falling in the highest and lowest quartiles in mindset scores. To interview participants, a three-part individual interview process (Seidman, 1998) was used. For the classroom observations, a section on agency, ownership, and identity from The Teaching for Robust Understanding (TRU) Observation Guide (Schoenfeld, 2013) was used. Their mindset assessment profiles from the mindset surveys data were analyzed along with transcribed interviews and activities observed while teaching. Class observations were scheduled after the second interview to observe how faculty engaged students with mathematical content in lessons. Measures from classroom observations were collected using the classroom observation protocol.
Thus, data were collected on content knowledge, mindset, and lesson objectivization. Three ways to collect mindset profiles occurred: in surveys, in individual interviews on faculty beliefs about learning mathematics, and in class observation. Three ways to collect data on lesson objectivization occurred: in narratives of lesson objectives during focus groups, in individual interviews, and in class observations. Data from Part 3 of the study will answer the third Research Question:

How are content knowledge, mindset, and lesson objectivization connected to teaching in two-year college mathematics faculty?

Findings surfaced out of connections between the quantitative measures and qualitative measures. Quantitative measures collected came from the following variables: content knowledge, mindset, and quantified data from lesson objectives. Qualitative measures were drawn from analysis of faculty teaching as beliefs and practices as they surfaced in the qualitative analysis of lesson objective narratives, individual interviews, and classroom observations. Analysis of data was approximately 40% quantitative and 60% qualitative.

1.4 DEFINITION OF TERMS

1.4.1 Teacher Knowledge

Teacher knowledge is the knowledge, and skills teachers need to know and be able to use to create dynamic experiences for student learning. Shulman (1986) posited teacher knowledge comes from four sources: scholarship in content disciplines, educational materials and structures, formal educational scholarship, and wisdom of practice.

1.4.2 Mathematical knowledge for teaching

Mathematical knowledge for teaching is the knowledge that teachers need to activate and use in recurrent tasks of teaching mathematics to students (Ball, Thames, & Phelps, 2008).
1.4.3 **Teacher content knowledge**

Teacher content knowledge is knowledge of the content taught to students; for faculty or a teacher of mathematics, the content would be mathematics (Shulman, 1986).

1.4.4 **Lesson objectivization**

Lesson objectivization refers to the ability to identify objectives for a lesson (Michal & Tchoshanov, 2017).

1.4.5 **Uznadze’s levels of mental activity**

*Level of set*

Level of set is the level of mental activity used when thinking about routine things that are learned once and placed as a set for future reference.

*Level of objectivization*

Level of objectivization is the level of mental activity used when one encounters an obstacle or problem, turns the obstacle/problem into an object, unpacks it, uses steps to understand it, and draws on known processes to overcome the obstacle/problem (Uznadze, 1966).

1.4.6 **Mindset**

*An entity or fixed mindset*

Individuals with a fixed mindset believe they were born with a certain amount of intelligence and cannot expand it (Dweck, 2006).

*Incremental or growth mindset*

Individuals with a growth mindset believe they can grow their intelligence incrementally (Dweck, 2006).
1.4.7 Effective Teaching

Effective teaching is teaching that engages students in activities that develop agency for learning mathematics and promote the learning of a larger community of learners. Effective teaching uses mindset to address broader issues of inequities (Gutierrez, 2018).

1.5 Past Literature

There are three groups of studies on mindset in the literature. One group of studies provided background on mindset and productive persistence in teaching and learning and helped define fixed or growth mindset (Dweck, 1989; Dweck, 2006; Shively, 2013; Matthews, 2005; Dweck & Leggert, 1988; Molden & Dweck, 2006; Furnham, 2014). Background reading on mindset and productive persistence (Querishi, 1981, 1987), revealed that this construct of two mindsets might map to Uznadze’s psychology of set and his levels of mental activity (Uznadze, 1966; Uznadze, 2009). A second group of studies on mindset (Blackwell, 2007; Chan, 2012; Haager, Kuhbandner, & Pekrun 2014; Howard & Whitaker, 2011; Mihai, 2014; Olson & Knott, 2013; Rattan, 2012; Subramaniam, 2007; Yeager et. al, 2013) reveal how mindset influences teachers as they pose problems to students, and how students develop positive and negative effort beliefs based on mindset type. A third type of studies on mindset dealt with psychological and social contexts (Chan, 2012; Furnham, 2014; Haager et al., 2014; Johnson & Stapel, 2010; Karoly & Newton, 2006; Kleinfeld, 2009; Miller et al, 2012; Rattan et al, 2012; Storek & Furnham, 2013). In those studies evidence of mindset changing because of interventions applied to participants or how mindset beliefs impacted intelligence of others as well as self-estimated intelligence.

Deficiencies in Past Literature

Findings provide a rationale for exploring mindset further and the extent to which mindset and other teacher knowledge may provide more effective teaching in classrooms as part of the
knowledge base (Shulman, 1986). The literature has many current studies conducted in K-12 settings with the most current example in the dissertation by Menanix (2015). Only a small number of studies were at two-year colleges, and are now beginning to appear. Examples include dissertations from Best (2016) and Kiser (2016) who examined two-year college mathematics settings with faculty as participants of the study. This study attempted to add to the knowledge base of mindset as part of the knowledge base for teachers. It also tried to show what mindset in combination with teacher knowledge has an association with teachers’ ability to provide lessons that lead to the effective teaching of mathematics in postsecondary classrooms.

The main finding is that there is a paucity of empirical studies of two-year colleges. There are even less of these studies on teaching at two-year colleges.

1.6. **SIGNIFICANCE**

If we can identify teacher knowledge needed to objectivize lessons that lead to effective teaching in classrooms, the study may significantly impact teaching and learning in mathematics classrooms at all levels K-16. Both teacher preparation programs and in-service teachers in K-12 will stand to benefit, as well as postsecondary faculty. In particular, it will impact students like those in the 80% fall 2013 cohort reported by the Texas Higher Education Coordinating Board (2014). As they enter mathematics classrooms underprepared to succeed in their undergraduate classes, the impact of identifying additional teacher knowledge may enhance effective classroom teaching and learning address thousands of students beginning their programs of study. It will inform two-year colleges on how best to prepare faculty to address and face these students.
1.7 LIMITATIONS AND DELIMITATIONS

1.7.1 Limitations

Limitations of the study include that while opening the study to a random sample of 80 mathematics faculty in a two-year college, participation was voluntary and not everyone accepted the invitation to participate. The data from Parts 1 and 2 of the study was limited by those who gave consent to participate in the study. To see how the sample participants in Part 1 of the survey compared to the total population of faculty, a review of characteristics of the population by full-time vs. part-time status, gender, primary teaching course(s), and degrees held was conducted. The population sample at the study site had similar demographics, characteristics, and teaching experience as the sample who participated in Part 1. All participants of Part 1 of the study were invited to the focus groups and represent a nested subset of participants in Part 1. To see how the sample participants in Part 2 represented scores in all four quartiles for each survey, analysis of the participant scores of the 14 faculty was done to ensure the scores in the participants were from all quartiles for each survey. That is, the participants in Part 2 included faculty who scored in all four quartiles of the mindset survey and content survey. For Part 3 of the study, a purposive sample nested within the sample in Part 2 of the study was selected for the case studies and is described in Chapter 3.

1.7.2 Delimitations

The study takes place in a border community in a large international metropolitan area. Cohorts of entering students mirror the general population; the student population is about 85% Hispanic. While Hispanics make up most of the student population, there is diversity within the group of Hispanics. Students may be first generation U. S. citizens,
transnationals, may come from military families who are very mobile, may come from families living on both sides of the border, may have more than one job, and may have a variety of other attributes. Because of this, delimitations of the study may include characteristics unique to a border community. However, the study is only focusing on content knowledge, a mindset for learning, and the ability to objectivize lessons. Results may apply to faculty teaching in communities with similar characteristics. There will be future opportunities for the researcher to talk about other factors, such as language, that impact teaching and learning. These other factors are essential to teaching and learning and may form future studies.

1.8 Summary

Adding to the knowledge base of what faculty must know and be able to use in two-year college mathematics classrooms may provide another structure for preparing mathematics faculty. More clarity to the knowledge base required for teachers to impact student success in courses and completion rates in programs of study will further the equity agenda. It may address how to connect with the large numbers of underprepared students in freshman mathematics courses. A few other steps are needed for students to progress through programs, but this first step may address successful entry and completion of college-level mathematics courses. Once past their first freshman mathematics courses, students will need additional support as they progress through programs of study. However, completing mathematics courses for their programs of study will make students better equipped for transferring to a four-year university or for seeking gainful employment in the workforce.
Chapter 2: Literature Review

In this chapter, teacher knowledge as a broad theory is discussed followed by a discussion of other types of knowledge needed for teaching such as content knowledge, levels of mental activity, mindset for learning, and lesson objectivization. The focus of the study is how these constructs are connected to classroom teaching.

2.1 Theoretical Framework

Introduction

As part of a funded project to align mathematics curriculum, instruction, and assessment, K-16, I was asked to observe teachers in classrooms in elementary, middle, and high schools in a city in the southwestern part of the United States. These observations took place more than fifteen years ago. Many teachers provided excellent examples of classroom teaching. Students in these classrooms were learning mathematics in ways that would lead to successful careers. And yet, years later, many of these same students entered college classrooms underprepared to enroll in and complete college-level mathematics courses. In January 2014, the Texas Higher Education Coordinating Board reported that for the latest Texas fall college cohort, more than 80 percent of students who enter college need developmental mathematics courses before entering college-level mathematics courses. In fact, many may never complete a college mathematics course with a grade of “C” or better (Rutschow & Diamond, 2015).

We can ask ourselves then, what teacher knowledge will nurture student learning in elementary school mathematics in ways that allow students to continue to learn mathematics in middle schools, high schools, and continue learning mathematics in college classrooms? What teacher knowledge is needed to prepare students to enroll and complete college-level mathematics courses required in programs of study? And, once students enter college, what knowledge for
teaching must two-year college mathematics faculty have to continue to engage students successfully throughout college mathematics courses?

Much research is related to content, pedagogy, and pedagogical content knowledge needed for teaching, however, what may go beyond teacher knowledge? The theoretical framework for the study begins with teacher knowledge. Once teacher knowledge is measured and identified, do faculty use it to clarify lesson objectives that can make students more aware of their agency to learn mathematics and utilize this learning to achieve other goals? Teacher knowledge may further be enhanced by mindset for learning mathematics and by identifying lesson objectives as a means for knowing what to do when encountering difficult concepts that may create obstacles. When used as a goal to create more dynamics between teaching and learning, a growth mindset may socialize learning and make classrooms more equitable for students as they form identities for learning mathematics (Menanix, 2015). Getting the concept of a growth mindset introduced in the preparation of teachers may increase the number of students that complete mathematics courses and programs of study.

2.1.1 Teacher Knowledge

Teacher knowledge may be the most influential factor for providing our nation’s schools with effective classroom teachers. When Shulman (1986, 1987) built a foundation for teaching reform thirty years ago, he emphasized comprehension and reasoning along with transformation and reflection. He justified this emphasis because he believed research and policy up to that point had ignored those aspects in what teachers should know and be able to do to be effective teachers (Shulman, 1986, 1987). He prioritized the sources of knowledge for teaching; how to conceptualize these sources; what teachers believe, understand, and know how to teach; and what allows teachers to be effective teachers (Shulman, 1986, 1987).
In a world where students spend eight hours per day in classrooms, teachers may be the most influential factor for student learning. When addressing what teachers must know to teach, Shulman (1986, 1987) introduced four concepts that form a knowledge base for teachers. All four concepts connect to teachers and teaching. Hill, Ball, and Schilling (1988) argued that conceptualizing what teachers need to know forms the critical piece in our search for creating productive learning environments. In their research, they claimed that conceptualizing what teachers must know would get us closer to providing effective classroom teaching. As researchers uncover findings of teaching and learning, teachers will need to know how to interpret and apply these findings (Shulman, 1987). He describes this knowledge base as elaborate and beginning with an understanding of what students need to know and be able to do, what one must do to get students to understand the content, and how to use this understanding as a vehicle for other goals (Shulman 1987).

Shulman (1987) begins by listing four sources within the knowledge base for teaching: 1) scholarship in content disciplines, 2) educational materials and structures, 3) formal educational scholarship, and 4) the wisdom of practice. He posits that it is in the third source where teachers learn the philosophical, critical, and empirical literature that may clarify the goals, visions, and dreams of teachers. The third source represents the most significant part of the scholarly base of teaching (Shulman, 1987). These influences from formal educational scholarship give teachers images of what is possible in their classrooms when students receive opportunities to learn (Shulman, 1987) and where this study bases its theoretical framework. Teacher knowledge and instructional quality continue to be of great interest to scholars. This study investigates growth mindset in teachers and lesson objectivization as teacher characteristics that go beyond teacher knowledge.
2.1.2 Teacher Content Knowledge

The first of Shulman’s (1987) sources in the knowledge base for teaching is a scholarship in content disciplines. It is there where teacher content knowledge resides. In teacher preparations programs, the specialized methods courses embed specific content into methods for learning. In this way, teachers are prepared to create effective classroom teaching experiences. It is here where students experience and learn subject matter woven with pedagogy, technology, knowledge of students, and curriculum. It is here where they experience the dynamic of teaching and learning. Important to note is that faculty teaching in two-year colleges are credentialed to teach based solely on the attainment of at least a master of arts or a master of science degree with eighteen (18) graduate credit hours in their content teaching field. Scholars agree that content knowledge is necessary for teaching, however, it is not sufficient for providing effective classroom teaching (Shulman, 1987). Many other scholars have added to the three types of knowledge first posited by Shulman (1987): content, pedagogy, and pedagogical content. In a recent study, Tchoshanov (2011) found that of the three content knowledge types, knowledge of facts and procedures, knowledge of models and generalizations, and knowledge of concepts and connections, it was the third type that positively and significantly related to student achievement and instructional quality. With a more detailed description of content knowledge types and its impact on student achievement and instructional quality, one sees the immediate connection to two things. One is that teacher knowledge must be studied further to get at what other determining factors may increase student achievement levels. The second is that research findings can inform interventions designed to give faculty more focused professional development.
2.1.3 Lesson Objectivization

Uznadze (1966) argues that there are two planes of reality affecting behavior: the external reality in the world around us and the verbal reality represented by words. Both planes contribute to two levels of mental activity, the level of set and the level of objectivization (Uznadze, 1966). One uses the level of set when thinking about routine things. As routines, they are learned once and then placed as a set for future reference, like learning how to place settings on a table for dinner. One uses the level of objectivization when one encounters an obstacle or problem in understanding or in doing something (Uznadze, 1966). In using the level of objectivization to deal with an obstacle, Uznadze (1966) believes one turns the obstacle or problem into an object, unpacks the obstacle/problem, uses steps to understand the obstacle/problem, and completes the act that was delayed by the obstacle/problem. In this way, teachers can use the level of objectivization for teaching practices and for thought to develop intellectually as effective teachers. How teachers handle an obstacle may depend on their level of mental activity and may result in effective or ineffective teaching.

Using the level of objectivization, when teachers reach an obstacle, they may turn it into an object and use a non-routine strategy in their lesson to address the unanticipated obstacle or problem. Taking students through this progression of confronting and working out non-routine solutions may motivate students to believe they can learn and use mathematics. In the level of set, when teachers encounter an unanticipated obstacle, they are unprepared and may dismiss the obstacle and disengage their thinking of it because it does not fit into their known routine practices. When taking this route, teachers may not know how to engage students in learning when an obstacle arises. Shulman (1987) sees knowledge for teaching beginning with a teacher’s understanding of what is to be learned and how to engage students in learning. When teachers
know the objectives of a lesson and inform their students of these clear lesson objective(s), students achieve more learning (Shulman, 1987). What do teachers need to know to use this level of thought to be able to objectivize lessons and to use this thinking level efficiently when an unanticipated obstacle or problem confronts them? What characteristic may assist teachers when this occurs?

2.1.4 Mindset for Learning

Dweck’s (2006, 2008) mindset theory involves two constructs: a fixed mindset for learning also called an entity mindset, and a growth mindset for learning also called a malleable or incremental mindset. Growth mindset manifests itself in humans who believe their learning is incremental and can grow; they actively engage in understanding (Dweck, 2006). Students with a growth mindset believe they can improve their intelligence and skills through training and practice (Sparks, 2015). A fixed mindset manifests itself in individuals with passive mental activity drawing only on the level of set and paying little attention to things that seem complicated and non-routine. When an unforeseen obstacle or problem occurs, these individuals will not welcome the challenge to deal with it when a catalyst creates the challenge but instead draw only on what they already know, and may even develop anxiety about how much fixed intelligence they have (Dweck, 2008). A mindset manifests itself in individuals based on how they see themselves. It is also based on what they can and cannot do, what they can and cannot succeed in, what they can and cannot learn, and what they can and cannot become. All these are significant implications for education. In mathematics classrooms where teachers have a fixed mindset, their thinking may always be routine. Teachers may not be able to get students engaged in learning using only routines. Teachers with growth mindset may use the level of objectivization to treat obstacles or
problems as challenges. They may then use them as catalysts for students to actively engage in learning, drawing on what students know, and build on to create new models for learning. They may use non-routine tasks and their ongoing learning of language and thought to develop identities for learning mathematics in students.

It is important to note that more must be studied about a person’s mindset for learning in general and how it may be different from a person’s mindset for learning a specific content. As an example, one may use a growth mindset belief in learning when learning a new language and use this incremental growth belief while learning the language. However, when it comes to mathematics, a person may believe they cannot learn mathematics and believe their learning of mathematics is fixed. For this study, we are using the Uznadze’s levels of mental activities as ways teachers identify and use objectives in a lesson to create activities for effective teaching to investigate connections in lesson objectivization and mindset to teaching.

2.1.5 Uznadze’s Levels of Objectivization and Dweck’s Growth Mindset

Uznadze’s two levels of thought (1966) and Dweck’s mindset (2006) have a parallel construction. Faculty with a fixed mindset may see the development of mathematics as being routine and engage in students in activities that draw only procedural skills when needed and may use mental activities that employ only the level of set. Faculty with a growth mindset may see the development of mathematics differently and recognize the importance of engaging students in higher cognitive demands needed to nurture productive learning of mathematics and use mental activities that employ the level of objectivization. Recall that the higher cognitive demands call on students to make connections to understand concepts and the highest cognitive demands call on students to generalize and form conjectures with the mathematics they are learning. Teaching with
activities that engage students in applying concepts, making general statements, and analyzing concepts to form conjectures can be observed as lesson objectives that nurture a growth mindset by using demands that use the level of objectivization. These may be observed in the activities teachers use in classrooms.

Teachers may use lesson objectives to get students to use higher orders of cognitive demand and require that students use the level of objectivization. Describing lesson objectives that use activities that engage students in the level of objectivization may lead teachers to create promising teaching experiences for students. To see this, teachers were given the task to describe the lesson objectives for a lesson they viewed. This task created an obstacle for teachers that called for solving a non-routine problem. To investigate how teachers use the level of objectivization to identify lesson objectives, teachers viewed a lesson in mathematics outside of their teaching realm with lesson objectives not explicitly stated and wrote a description of the lesson objectives. The task provided a view of how teachers use their thinking, their level of objectivization, to describe lessons that engage students in activities that nurture a growth mindset.

In this way, faculty use lesson objectivization as a means to make lessons more active and engage students in non-routine problems. A non-routine problem will inherently pose obstacles and demand that students unpack the obstacle and use scaffolding steps to solve it. The very nature of working a non-routine problem does not allow the student to use the level of set; instead, the student will use the level of objectivization. Used strategically, the ability for teachers to use lesson objectives can engage students in non-routine problems and guide students to use the level of objectivization. The level of objectivization as a mental activity is how someone with a growth mindset would work through a problem. As class observations are recorded, teaching activities that utilize the level of objectivization can provide data on teaching.
2.1.6 Summary

Infusing the teacher knowledge base with lesson objectivization and growth mindset may provide a framework for studying what teachers must know to engage students in learning experiences that nurture incremental understanding. Relying on beliefs about learning mathematics, where obstacles are likely to occur, may be connected to Uznadze’s level of objectivization (Uznadze, 1966) leading to a growth mindset (Dweck, 2006; 2008). This may connect to how teachers create lesson objectives with activities that nurture non-routine thinking.

Examining teaching through a variety of constructs may lead to some interesting connections. In this study, knowledge for teaching is investigated by looking at content knowledge, mindset, and lesson objectivization. As stated earlier, there may be other variables that influence mindset or teaching, the three mentioned above are used for this study. Investigating content alongside some psychological ideas of how teachers use mental activities for learning and their beliefs about learning is represented in Figure 1. It is an attempt to visually model how teacher knowledge may be shaped by the two levels of mental activity described by Uznadze (1966).
2.2 LITERATURE REVIEW

Over the past years, the number of students entering colleges underprepared to enroll and complete college mathematics courses has been a concern to colleges, universities and legislative bodies that provide funding for education. Success rates in mathematics courses remain unchanged with many students not completing their program of study. A review of peer-
reviewed articles was conducted to see teacher characteristics needed for effective teaching other than content, pedagogy, technology, classroom management, and the intersection of these types of knowledge. The review of twenty peer-reviewed articles related to teacher knowledge provided an elaborate landscape of knowledge teachers need to know and be able to use to create effective classrooms. The review showed that knowing the content was necessary but not sufficient to produce effective teaching in classrooms. To understand the impact that implicit theories of intelligence may have on the success of students in mathematics classes, twenty-four peer-reviewed articles related to mindset were analyzed. The review showed publications with positive results for participants with growth mindset in a variety of empirically designed studies. One article provided some answers to what may be done to remedy practice in classrooms. A search for lesson objectives produced few items. However, related terms like lesson goals were used to shed light on how teachers set goals for lessons. Added to these articles are three dissertations selected for review based on their topics related to mindset for teaching and learning mathematics.

2.2.1 Problem

For years, research studies revealed how curricula and innovative pedagogy have positive effects on student learning and student success in various content areas including mathematics. However, the widespread implementation of the curricula and innovative pedagogy has yet to occur and those implemented and evaluated in recent years by the Institute of Education Sciences do not show significant gains in achievement past the time of treatment (Yeager & Dweck, 2012). Reasons may include logistical, financial, or human resource limitations inherent in the infrastructure of educational institutions. In particular, in mathematics, the added nature of people’s belief in their ability to learn mathematics may complicate implementation of any curriculum and pedagogy.
The ability to objectivize lessons is also part of the formal educational scholarship of teachers. The ability to objectivize lessons as a mental activity may fall within the level of objectivization (Uznadze, 1966) running parallel to a growth mindset for learning (Dweck, 2006; 2008).

Scholars have written that the time people are willing to devote to learning depends on their motivation to learn (National Research Council, 2000). As human beings, we have the motivation to develop competence and to solve problems White (1959). However, while other things may affect one’s motivation to learn when challenges and difficulties arise, the tendency to persist is strongly affected by whether one is “performance oriented” or “learning oriented” (Dweck, 1989). The notion of people having a fixed mindset or a growth mindset for learning (Dweck, 2006) is something emerging educators exploring are now exploring. Dweck defines having a fixed mindset as the belief that all that one may learn is already fixed and is determined at birth (Dweck, 2006). A study conducted on whether or not learners believed that intelligence was a set entity that is determined at birth or an incremental one that describes a malleability of intelligence revealed that it might, in part, also be based on cultural beliefs (Shively, 2013). Nieto argues that it is based on cultural beliefs (1999, cited in Matthews, 2005). A person with a growth mindset believes that one learns incrementally and continues to learn as time goes by (Dweck, 2006). Important to note is that in the articles reviewed, mindset theory is studied as an implicit theory. As an implicit theory, it is defined along with other theories that “are defined as core assumptions about the malleability of personal qualities” (Dweck, et al., 1995; Dweck & Leggett, 1988; Molden & Dweck, 2006; Yeager & Dweck, 2012). Background reading on mindset and productive persistence revealed that this construct of two mindsets may be mapped to Uznadze’s psychology of set and his levels of mental activity (Uznadze, 1966, 2009). Along with answering
the research questions below, this review will also explore how to tie what has been published on mindset and productive persistence to see if it may reveal ways to run parallel structures on these concepts with the levels of mental activity (Uznadze, 1966).

2.2.2 Guiding Thoughts for Literature Review

The literature review focused on existing research on how mindset and productive persistence have been applied to teaching and learning mathematics and to see what the landscape of teacher knowledge looks like for effective classroom teaching.

2.2.3 Need

Research on mindset was conducted on K-12 teachers, University faculty, and their students. Few empirical studies exist for teacher and students in two-year college classrooms and in particular, developmental (DE) mathematics classrooms. DE mathematics courses are courses that do not count towards college credits; instead, they are courses with topics that are covered in K-12 mathematics courses. They are offered to prepare students who enter college but are not ready to enroll and complete college mathematics courses. In fall 2012, there were 7.2 million undergraduate students enrolled in the Nation’s two-year postsecondary institutions representing 40% of postsecondary enrollment (National Center for Education Statistics, NCES, 2014). In four-year institutions, 10.6 million undergraduate students represented 60% of postsecondary enrollment (NCES, 2014). Also, 75% of students entering community colleges needed at least one developmental mathematics course before starting their first college-level mathematics course. While developmental subject areas include reading, mathematics, and writing, it is in mathematics where the highest rates of non-completion occur (Bonham & Boylan, 2012).
Currently, community colleges are identified as the entry point for high-risk students entering colleges and universities (Hagedorn, et al., 2010). Colleges have restructured and redesigned courses for students who need remediation in mathematics to address this urgent need to remediate large numbers of students. Along with redesigning and restructuring mathematics courses, colleges are also creating new pathways for students underprepared for their first college mathematics course. This literature review is devoted to researching how employing mindset together with productive persistence may impact teaching and learning to address the need for redesigning successful mathematics courses for students in community colleges. Its effect on teaching and learning mathematics in community college developmental classrooms is important. Mindset, like content knowledge, may provide connections to teaching needed for two-year college mathematics faculty.

2.2.4 Methods for Search

A literature review of available empirical studies in English from 1970 to 2014 was conducted on the use of mindset and productive persistence in educational research and how mindset and productive persistence fit into the knowledge teachers need to be effective teachers and on teacher knowledge in general. Search engines used were ACADEMIC SEARCH COMPLETE, EBSCO, JSTOR, Education Full Text, and Google Scholar, each assessed through the library at a university and the Internet. Keywords used for the search included: teacher knowledge, mindset, motivation, will, productive persistence, learning, developmental mathematics, competence motivation, and objectivization. Searches were conducted in mathematics education, psychology, psychology education, and social science education. Articles selected for analysis were all from academic peer-reviewed journals; some chapters in published
books were chosen to provide background and connections. Reverse citation searches yielded additional references as well as publications cited in articles found in the original search. In particular, articles and books of interest selected for review were those on the application of mindset in classrooms, educational structures, settings related to learning, and a bibliography on Uznadze and his writings.

Searches were also conducted on selected authors cited in the original searches; these included Boaler, Dweck, Shulman, Uznadze, and Yeager. Other references and articles from other sources and books are included to provide background and current views of mindset as an implicit theory. Additional non-peer reviewed articles are included to provide background on mindset and productive persistence.

Of approximately forty-six articles and resources on keywords that were found to be relevant at first inspection, only twenty-seven were analyzed; twenty-five of the twenty-seven are summarized including those providing background. Results included four sections: 1) a summary highlighting the diverse application of mindset and productive persistence; 2) a summary of mindset and productive persistence in teaching and learning; 3) a summary relating mindset to topics in psychology and social practice; and 4) a review of the direction discussed in one of the later articles suggesting directions for research on this topic. A fifth area included three current dissertations related to a mindset for learning in mathematics classrooms were reviewed.

Table 2.1 provides a summary of the categories of articles in the literature review and how they are used to inform the study. Some articles provided background on productive persistence and mindset (Dweck, 2008; Boaler, 2013) and were not studies per se. The search did not reveal a large number of studies using mindset related to learning mathematics. However, it did produce studies revealing promising results in a variety of psychological and social studies settings. These
are summarized as the third category; mindset and its study as an implicit theory of intelligence. They are the third type of resource in Table 1. One article discussed at the end proved instrumental in providing insight for future directions. It is the fourth resource type. Three dissertations were reviewed, all based on studies on mindset and is the fifth resource type. Teacher knowledge is another category and is the sixth resource type. Objectivization is the last category and is the seventh resource listed in the table.

Table 2.1: Resource Categories

<table>
<thead>
<tr>
<th>Resource type</th>
<th>Number</th>
<th>Dates</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meta-analysis/background/foundational</td>
<td>10</td>
<td>1966 – 2012</td>
<td>Background</td>
</tr>
<tr>
<td>Mindset in Teaching and learning</td>
<td>9</td>
<td>2005 – 2014</td>
<td>Results</td>
</tr>
<tr>
<td>Mindset in Psychological/social contexts</td>
<td>8</td>
<td>1970 – 2014</td>
<td>Results</td>
</tr>
<tr>
<td>Future directions</td>
<td>1</td>
<td>2005</td>
<td>Discussion</td>
</tr>
<tr>
<td>Current studies in dissertations</td>
<td>3</td>
<td>2015 – 2016</td>
<td>Current findings</td>
</tr>
<tr>
<td>Studies on Teacher knowledge</td>
<td>9</td>
<td>2006 – 2016</td>
<td>Results</td>
</tr>
<tr>
<td>Studies on Objectivization</td>
<td>3</td>
<td>2009 – 2013</td>
<td>Results</td>
</tr>
</tbody>
</table>

2.3 MINDSET STUDIES

2.3.1 Application of Mindset and Background for Productive Persistence

Important to note is that most of the articles on mindset, as applied in studies related to teaching and learning, were studies published within the last decade, from 2005 to 2014. The earlier publication included a study conducted in 1956 by M. Y. Querishi and published in 1970 and is based on the Michill Adjective Rating Scale (MARS) investigated data gathered from
Marquette University undergraduates (Querishi, 1970). The MARS study included an instrument that contained one hundred adjectives given to participants to self-rate themselves. In the study, participants used a scale from one to five where one is very untypical of themselves to describe themselves with this adjective and five is very typical of themselves to describe themselves with this adjective. Participants were also asked to rate someone they knew well, like a relative or friend.

The Quereshi study cross-validated factors identified by MARS to facilitate the more widespread use of MARS by other researchers. Using data from 441 students taking a 100-item adjective rating scale to identify factors correlated by the 100 adjectives, Quereshi utilized statistical tests to extract these factors. Then, ten factors were identified and used to select four factors from those showing significant correlations: unhappiness, extraversion, self-assertiveness and productive persistence. This 1970 publication of the Quereshi MARS study is where the term productive persistence first appears in the literature. Quereshi used the adjectives correlated to this fourth factor to come up with the term, productive persistence. In fact, productive persistence appeared in the review of studies and publications by Quereshi (1970, 1981, & 1987) until 2012. It appeared again in the work of David Yeager, senior scholar at the Carnegie Foundation for the Advancement of Teaching and Learning in the Carnegie publications site (2012). Quereshi (1970) labeled productive persistence from the adjectives correlated to it:

Factor 4. The high positive loadings on this factor of such adjectives as determined, studious, ambitious, serious, logical, deliberate, dependable, hard-working, productive, persistent, energetic, and enterprising indicate that persons scoring high on this factor would possess a great deal of resilience, ego strength, and persistence and are also characterized by a high degree of productivity. Factor 4, therefore represents not only such
motivational characteristics as determination and persistence but also presumes the presence of abilities to direct them into certain desired channels. Since to the author’s knowledge no single term in the English language adequately describes both persistence and productivity, factor 4 is tentatively labeled as Productive Persistence (pp. 194, 196).

Productive persistence, as identified by Querishi (1970), states the many attributes we see in students who are not only learning but learning while struggling. When one thinks of students struggling to learn difficult concepts in mathematics or any content area, this definition of productive persistence is what we want students to do when they are using thinking levels that will produce learning. It is this kind of persistence that forms positive identities for learning.

In a second paper, data indicated that the four MARS factors represent characteristics that have remained stable for over 40 years when the data are based on self-ratings on measures of self-esteem (Quereshi, 1981). A more recent definition of productive persistence was found in the Carnegie Foundation for the Advancement of Teaching’s (2012) definition as:

The set of behaviors that involves the tenacity and good strategies students need to be academically successful (p. 18).

As with Querishi’s (1970) definition of productive persistence, Carnegie’s (2012) definition describes the general sense of attributes academically successful students have.

Table 2.2 lists articles used to analyze and establish background and working knowledge of earlier articles on the MARS studies by Quereshi (1970) where productive persistence first
appears in the literature. The author, title, and year appear in column one; column two contains a brief description/rationale of the study, and column three contains results.

Table 2.2: Meta-analysis/background/foundational resources

<table>
<thead>
<tr>
<th>Author/year</th>
<th>Purpose</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Querishi 1970</td>
<td>Cross validate MARS factors, facilitate wider use of MARS</td>
<td>Four factors identified: unhappiness, extraversion, self-assertiveness, productive persistence</td>
</tr>
<tr>
<td>Querishi 1981</td>
<td>Write consistent procedures to implement, score, use MARS</td>
<td>Data separated by sex in this study</td>
</tr>
<tr>
<td>Querishi 1987</td>
<td>Measure differential social desirability of four factors</td>
<td>Difference not seen in productive persistence, Desire more productive persistence in oneself than in others</td>
</tr>
<tr>
<td>Dweck, Leggert 1988</td>
<td>To identify and explain behavior patterns based on implicit theories of intelligence and link patterns to psychological processes</td>
<td>Conceptualizes implicit theories, intelligence as fixed or malleable entities, cognition-affect behavior patterns impact adaptive functioning</td>
</tr>
<tr>
<td>Levy et al. 2001</td>
<td>How human fixed or dynamic theories foster different meanings when responding to information</td>
<td>Theories influence susceptibility to stereotypes and discriminatory behavior</td>
</tr>
<tr>
<td>Matthews 2005</td>
<td>To illuminate problematic features of equity articulation over 20 years</td>
<td>Seven constraints identified; promotion of teaching as social justice and critical pedagogical perspectives into teacher development</td>
</tr>
<tr>
<td>Schoenfeld 1986</td>
<td>To identify another teacher knowledge</td>
<td>Acknowledged teachers for needing to know other knowledge types besides content and pedagogy</td>
</tr>
<tr>
<td>Schoenfeld 1987</td>
<td>To identify teacher knowledge</td>
<td>Brought research in education to the attention of others</td>
</tr>
<tr>
<td>Yeager et al. 2013</td>
<td>Create practical measures to measure productive persistence and use in studies</td>
<td>Identifies ways educators may address change, predict at-risk students and conduct improvement research</td>
</tr>
</tbody>
</table>
Yeager, et. al 2012  

| Study impact of mindset on academic and social resilience | Students taught malleability of intellect to show higher achievement across challenging transitions and completion rates in math courses, lower adolescent aggression and stress, enhanced school performance |

2.3.2 Mindset in Teaching and Learning

In Table 2.3, articles on studies related to teaching and learning in classrooms, K-16, are listed and summarized. An important observation to make is that articles applying mindset to teaching and learning appear in more current publications during the years 2005 – 2014 and are from psychological and behavioral science journals. Participants in the studies included students, teachers, or students and teachers. The first column states the authors and publication year. In the second column, there is a rationale for the study. In the third column, a brief description of results appears.

Articles revealed several findings; one significant finding is Olson and Knott’s (2013) study of college instructors using problem-posing to develop students’ mathematical reasoning. This study shows teachers’ mindsets influence how they pose problems for their students to solve. Teachers with growth mindsets focus on the process of understanding mathematics; teachers with fixed mindsets focus on products of mathematical activity and emphasize answers. Olson and Knott (2013) showed that students in classrooms of teachers with growth mindsets view mathematics as a dynamic entity having multiple representations and perspectives where reason replaces magic and memorization.

Studies also show incremental (growth) theory related to positive effort beliefs and fixed mindset theory was related to negative effects (Blackwell, 2007; Howard & Whitaker, 2011; Mihai, 2014; Rattan, 2012, Subramaniam, 2007). Adolescents with more of an incremental theory...
of malleable intelligence also have stronger learning goals, positive beliefs about effort, and choose more positive effort-based strategies as they move on to high school. It is noteworthy that these critical beliefs occur during students’ divergent and transition period from middle schools to high schools (Blackwell, 2007). Other studies revealed developmental mathematics students had a change in mindset when they were successful (Howard & Whitaker, 2011). An example of a study showing negative effects of students with a fixed mindset involved music students. In this study, students with fixed mindset interacted less with people and proposed fewer solutions to problems posed than those with growth mindsets (Mihai, 2014). Studies focusing on teachers revealed the importance of teacher roles as they may attribute more positive student feelings towards their capabilities and intelligence (Rattan, 2012). In another study, teacher insights affected by their mindset played an essential role in mediating student learning within a zone of proximal development (Subramaniam, 2007).

Table 2.3: Mindset in teaching and learning resources

<table>
<thead>
<tr>
<th>Author</th>
<th>Rationale</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackwell</td>
<td>Study the role of implicit theories in adolescent math achievement</td>
<td>Incremental theory related to positive effort beliefs, stronger learning goals, positive effort-based strategies</td>
</tr>
<tr>
<td>2007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chan</td>
<td>Classify Chinese gifted student’s mindset based on varying views of perfectionism</td>
<td>High standards and order correlated substantially and significantly with incremental, growth mindset and discrepancy with a fixed mindset</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fox &amp; Hein</td>
<td>Impact of information technology on the mindset to change teaching</td>
<td>Mindsets remained unchanged because of existing constant assessments</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Haagger</td>
<td>How affective state may interfere with processes used for problem-solving</td>
<td>The positive affective condition is optimum tool for changing the fixed mindset</td>
</tr>
<tr>
<td>2013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Author(s) &amp; Year</td>
<td>Research Overview</td>
<td>Findings</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Howard &amp; Whitaker 2011</td>
<td>To understand students’ perceptions about learning math to inform practice and initiate future research</td>
<td>Successful students had changed in mindset; experiences engender shifts in student perceptions; motivation and experiences closely linked</td>
</tr>
<tr>
<td>Mihai 2014</td>
<td>Compare people’s interaction style with fixed and growth mindset in teamwork</td>
<td>Participants primed with deliberative mindsets interacted more and expressed more solutions to tasks</td>
</tr>
<tr>
<td>Olson &amp; Knott 2013</td>
<td>How college instructors use problem posing to develop students’ math reasoning</td>
<td>Instructors’ mindset influenced problem posing</td>
</tr>
<tr>
<td>Rattan et al. 2012</td>
<td>How comforting struggling students demotivates pool of students pursuing math-related fields</td>
<td>Comforting related to less encouraged and less motivation; learning strategies related to being more encouraged, motivated, improved grades</td>
</tr>
<tr>
<td>Subramaniam</td>
<td>Understand mediator role of a teacher using technology and a particular mindset</td>
<td>Participants natural metaphoric language revealed four mindset insights that mediated learning within the zone of proximal development</td>
</tr>
<tr>
<td>Yeager et al. 2013</td>
<td>A methodological review of practical measurement and case study to create practical measures to measure productive persistence and use in studies</td>
<td>Productive persistence defined; identifies ways educators may address change by using research-based practice, predict at-risk students and conduct improvement research</td>
</tr>
</tbody>
</table>

2.3.3 Mindset in Psychological and Social Contexts

A second theme in the articles on mindset included studies where mindset was used to see how it was related to happiness, multiple intelligences, positive and negative effects, social comparison information, gender, willpower, universal intelligence, and self-estimated intelligence. While these studies were not done to measure student achievement in classrooms, their findings can contribute to student success levels in any academic area and specifically in mathematics. As long as we see the total individual and the human condition he or she is in, we may safely say that
psychological and social contexts play a critical role in learning and classroom performance. Table 2.4 has a list of studies with findings summarized.

Some studies expand original studies including Dweck’s studies on the ideas of mindset. One such article is Chan’s (2012) study on the happiness of gifted students. This study is included in the section on teaching and learning. In this study, high standards and order correlated substantially and significantly with a growth mindset, while low standards correlated with a fixed mindset (Chan, 2012). In another study, personality correlated with incremental mindset and entity attitudes are related not just to intelligence but also to multiple intelligences (Furnham, 2014). Furnham (2014) showed that people do distinguish between different types of intelligence; intelligence thought to be easier to change were verbal, intra-personal and sexual and less likely to change were creative and musical intelligence.

In almost all studies, some participants were given a condition that a resource was limited and some were led to a condition that a resource was not limited. In all cases, the fixed/limited/entity condition yielded results that were not as fruitful as those of the growth/unlimited/incremental condition that yielded more positive results. In a study related to mathematics, participants received positive and negative affects before solving problems. Subjects assigned to a positive affect condition were more likely to use readily available simple solutions than those receiving a negative affect condition (Haager, Kuhbandner, & Pekrun, 2013). In a similar study on willpower, participants receiving the condition that willpower was limited did not improve during the second half of their tasks after taking a break. Participants that were led to view willpower as a non-limited resource continued to increase in their accuracy during the second half of their given tasks (Miller, Walton, Dweck, Job, Trzesniewski, & McClure, 2012).
Studies, based on gender and college enrollment and the mindset of students, showed that more females had well-developed plans to attend college than males. Males possessed two different mindsets about college; they saw it as something expected of them, or they had no knowledge of the job market and did not see college as part of their success or social standing (Kleinfeld, 2009). In another study, there was no significant relationship between mindset beliefs and gender. However, gender and general intelligence were the best predictors of participants’ estimated measure of mathematics, verbal, and spatial intelligence (Storek & Furnham, 2013).

And finally, there is Rattan’s empirical study of university students’ beliefs about universal intelligence verses non-universal intelligence (2012). Universal intelligence is the belief that all people have the potential to become highly intelligent. Non-universal intelligence is the belief that only some people have the potential to become highly intelligent. In a multi-part study, results showed that university students in India were more likely than students in an American university to believe that everyone has the potential for high intelligence (Rattan, 2012). As more and more studies reveal these type of student beliefs, this may change the culture of learning in classrooms throughout the United States. Table 2.4 has a summary of these resources.

Table 2.4: Mindset in Psychological and Social Context Resources

<table>
<thead>
<tr>
<th>Author/year</th>
<th>Purpose/Rationale</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chan 2012</td>
<td>Classify Chinese gifted student’s mindset based on varying views of perfectionism</td>
<td>High standards and order correlated substantially and significantly with incremental, growth mindset and discrepancy with a fixed mindset</td>
</tr>
<tr>
<td>Furnham 2014</td>
<td>Looked at personality correlates of incremental/entity attitudes towards the growth/increase of multiple intelligences</td>
<td>Intelligence easy to change were verbal, intrapersonal and sexual; intelligence least easy to change were creative and musical</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Study Title</td>
<td>Summary</td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td>---------</td>
</tr>
<tr>
<td>Haager et al. 2012</td>
<td>How affective state may interfere with processes used for problem-solving</td>
<td>Positive affective condition is optimum tool for changing the fixed mindset</td>
</tr>
<tr>
<td>Johnson &amp; Stapel 2010</td>
<td>To extend previous research by examining the relationship between mindsets and responses to social comparison</td>
<td>Mindset determines how people use and respond to information</td>
</tr>
<tr>
<td>Karoly &amp; Newton 2006</td>
<td>Real-time study of approach and avoid mindsets using Intentional Mindsets (IMSs)</td>
<td>ANOVA showed significant mindset effect on ability/willingness to monitor attention symbols while performing other tasks</td>
</tr>
<tr>
<td>Kleinfeld 2009</td>
<td>To understand the gender gap in college; understand male mindset to develop interventions</td>
<td>Females had better developed plans for college; males did not view college as part of their success and social standing</td>
</tr>
<tr>
<td>Miller et al. 2012</td>
<td>To highlight interactive nature of motivation and cognitive processes</td>
<td>During first part of the study, participants in both conditions were equally accurate; During the second part, participants led to view willpower as limited did not improve during the second half; participants to view willpower as non-limited continued to increase in accuracy</td>
</tr>
<tr>
<td>Rattan et al. 2012</td>
<td>Study people’s belief about intelligence to learn about people’s intelligent capabilities to use to leverage support for policies for underrepresented groups to redistribute resources</td>
<td>Indians significantly more likely than Americans to believe everyone has the potential for higher intelligence; participants exposed to ideas that people may reach higher intelligence predict support for measures that redistribute resources</td>
</tr>
<tr>
<td>Storek &amp; Furnham 2013</td>
<td>Study of self, estimated intelligence, SEI, measured types of intelligence and beliefs of intelligence</td>
<td>Males believe themselves more intelligent than females; Mindset beliefs were not significantly related to gender; Gender and general intelligence scores were best predictors of participants’ self-estimated measures of mathematical, verbal, and spatial intelligence</td>
</tr>
</tbody>
</table>
2.3.4 Mindset in Selected Dissertations

Growth mindset positively impacts learning as seen in three selected studies. Focused on its impact on decisions teachers make in classrooms (Menanix, 2015) found that just teaching for mindset alone does not give students success in learning mathematics; students must experience challenging activities while engaged with the content to form identities for learning mathematics. Teachers may support and develop learners in classrooms; however, mindset matters when students make decisions and they examine their motivation for learning because of experiences in their past (Kiser, 2016). When teachers are learning in virtual settings, Best (2016) found they preferred having face-to-face meetings with others to engage in sharing learning experiences. As with any factor that presents findings that will impact learning, the need for more studies on how mindset enters into the dynamic of teaching and learning is apparent. Table 2.5 contains these studies.

Table 2.5: Current Studies in Selected Dissertations

<table>
<thead>
<tr>
<th>Author/year</th>
<th>Purpose/Rationale</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best 2016</td>
<td>Apply How People Learn lens to investigate how knowledge-centered learning in a virtual classroom project functioned and impacted teachers' engagement, beliefs, and classroom practices</td>
<td>Teachers motivated to learn and encouraged by leaders, preference is given to face-to-face meetings rather than virtual meetings, online learning best when supported by face-to-face interaction with other teachers</td>
</tr>
<tr>
<td>Kiser 2016</td>
<td>To explore student and instructor's perspectives on learning in developmental (DE) mathematics courses to understand the role DE has in helping students persist through mathematics courses</td>
<td>Students' mindset matters, past experiences will determine students' decisions and motivations</td>
</tr>
<tr>
<td>Menanix 2015</td>
<td>To compare a teacher's instruction of the same mathematical content in two different courses</td>
<td>Teaching for growth mindset alone does not produce productive growth mindsets in students, what must also be included are opportunities for students to experience learning that gives them</td>
</tr>
</tbody>
</table>
2.4 **Teacher Knowledge**

A product of identifying the knowledge base for teaching is effective teaching and having effective teaching will result in student learning. While much interest has pointed to the knowledge teachers must have to make them effective teachers, I can state that after the literature review of teacher knowledge, the list of knowledge needed for teaching has not been exhausted by the studies appearing in journal articles. In 1986, Shulman introduced pedagogical content knowledge to the community of research educators and sparked much research on the types of teacher knowledge that have been identified and that can be measured. Hill, Ball, and Schilling (2008) are among many scholars who see Shulman’s (1986) introduction of pedagogical content knowledge a critical contribution to effective teaching and learning. Why? One reason may be that mathematics is a subject many people believe they cannot learn. For example, the understanding of fractions is associated with being among the most complex and problematic, and one that students encounter early in education (Charalambous & Pitta-Pantazi, 2007). Learning fractions and operations with fractions may be one of the reasons a student forms unpleasant beliefs about learning that last all through their studies in mathematics. The benefits of uncovering the base knowledge for teaching the sub-constructs of fractions (Charalambous & Pitta-Pantazi, 2007) will go far in the work of mathematics education and will help in making more students experiences in learning mathematics more fruitful in getting them ready for college-level mathematics courses. Understanding the knowledge base for teaching rationalized the review of nine articles.
Scholars have identified, studied, and discussed an elaborate landscape of knowledge that includes more than subject matter knowledge and pedagogy (Shulman, 1986). A partial list includes: pedagogical content knowledge (Shulman, 1986; Alonzo, Kobarg, & Seidel, 2012; Park & Chen, 2012), knowledge of content and students (Hill, Ball, & Shilling, 2008) technological pedagogical content knowledge (Mishra & Koehler, 2006), and mathematical knowledge for teaching (Charalambous & Hill, 2012; Charalambous, 2016). Shulman (1987) argued that understanding the knowledge for teaching, meant that it was essential to identify the sources that provide teacher knowledge: 1) scholarship in content disciplines, 2) educational materials and structures, 3) formal educational scholarship, and 4) the wisdom of practice. It is the third source of teacher knowledge, formal educational scholarship (Shulman, 1987) where knowledge of mindset resides. Charalambous & Hill (2012) looked at mathematical knowledge needed for teaching together with curriculum used for teaching and found that teaching, as a profession, was a complex construct even when supported by a research-based curriculum.

Starting with the assumption that more than just content knowledge is needed for teaching (McCrary, Floden, Ferrini-Mundy, Reckase, & Senk, 2012), Charalambous (2016) looked at four teaching practices. These four teaching practices were: providing and evaluating explanations; selecting and using representations; analyzing student errors, misconceptions and non-conventional solutions; and selecting tasks, and found that while good knowledge of pure knowledge of mathematics was important, it was not enough to carry out the important work of teaching mathematics. Looking further at teacher knowledge, Charalambous & Hill (2012) characterized that formal educational scholarship forms a complex relationship when coupled with curriculum and quality instruction. Tchoshanov (2011) provided a deeper look into the knowledge needed for teaching. He studied how three cognitive types of mathematical content knowledge
impacted teaching. These cognitive types were: knowledge of facts and procedures, knowledge of concepts and connections, and knowledge of models and generalizations. Tchoshanov (2011) found that the second type, knowledge of concepts and connections, were positively and significantly related to instructional quality.

To bring in students’ culture, language, and student lives when looking at pedagogy, Leonard (2009) reminded us that culturally relevant pedagogy is complex but is an excellent way to draw from the culture, language, and experience students bring to classrooms to impact learning. Teacher beliefs about culture, like their belief about learning, influence identity development and practice in culturally relevant pedagogy (Leonard, 2009). As a type of teacher knowledge, this kind of teacher knowledge takes more than one training and more than one course to use expertly. Thus, there are other things besides content to consider when examining teacher knowledge. Table 2.6 has the results of fifteen of the studies summarized.

<table>
<thead>
<tr>
<th>Author/year</th>
<th>Purpose/Rationale</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charalambous, 2016</td>
<td>To investigate the validity of the mathematical knowledge for teaching construct by focusing on four teaching practices and tasks teachers engage in to support student learning</td>
<td>Pure knowledge of mathematics is important for teachers however it is not sufficient knowledge for carrying out the work of teaching</td>
</tr>
<tr>
<td>Charalambous, &amp; Hill, 2012</td>
<td>Set up the case studies conducted to simultaneously look at mathematical knowledge for teaching and curriculum materials teachers contribute to the quality of instruction</td>
<td>Curriculum materials used are a key contributor to the quality of instruction Note: Analysis of study seen as a view of teaching and not of teachers as a complex process improving instruction</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Title</td>
<td>Findings</td>
</tr>
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<td>-----------------------------------</td>
<td>----------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Charalambous &amp; Pitta-Pantazi, 2007</td>
<td>To investigate students’ constructions of the different sub-constructs of fractions</td>
<td>Association of fractions constructed as five sub-constructs was supported by the study. Note: Study adds teacher knowledge of conceptually understanding fraction to get at good teaching.</td>
</tr>
<tr>
<td>Hill, Ball, &amp; Schilling, 2008</td>
<td>To conceptualize, define, and measure teachers’ knowledge of content and students (KCS) as part of pedagogical content knowledge</td>
<td>Created new understanding of teacher knowledge of content and students (KCS) yet recommended that other studies were needed to quantify it.</td>
</tr>
<tr>
<td>Hill &amp; Charalambous, 2012</td>
<td>To develop further the contribution mathematics knowledge for teaching and curriculum materials have to the quality of instruction</td>
<td>Mathematical knowledge for teaching and curriculum materials each contribute to the quality of instruction. It is unclear if jointly, they contributed to the quality of instruction based on the level of MKT and the materials used.</td>
</tr>
<tr>
<td>Leonard, 2009</td>
<td>To examine how culture and language entered into student learning</td>
<td>Although complex, using culture in the context of teaching mathematics and the vibrant nature of students’ language went far into student learning in classrooms.</td>
</tr>
<tr>
<td>McCrory, Floden, Ferrini-Mundy, Reckase, &amp; Senk, 2012</td>
<td>To examine the knowledge needed for teaching algebra</td>
<td>Showed that rather than needing higher mathematics to teach algebra, teachers instead needed pedagogical content knowledge for teaching algebra.</td>
</tr>
</tbody>
</table>
Mishra & Koehler, 2006  To integrate technological pedagogical content knowledge within the pedagogical content knowledge

While technology had changed practice and routine in workplaces, by the year 2006, technology has advanced very little in classroom use. As a result, technological pedagogical content knowledge (TCPK) was developed around content, pedagogy, and technology.

Park & Chen, 2012  To explore pedagogical content knowledge

As teacher knowledge is examined, there are multiple other types of knowledge that are not based only on content nor around pedagogy, but rather situate teacher knowledge along with an intersection that is formed by content and pedagogy.

Tchoshanov, 2011  To explore how the cognitive type of teacher knowledge is related to instructional quality

The knowledge of concepts and connections (a second cognitive type of content knowledge) was positively and significantly related to instructional quality.

2.5  OBJECTIVIZATION

In this study, objectivization is used in the term level of objectivization and is defined as one of two levels of mental activity identified by Uznadze (1966). It refers to a level of mental activity that one uses when encountering a difficult obstacle to understand or comprehend. Level of objectivization is not a variable that is measured for this study. As a mental activity, it describes how teachers may engage students in activities in classrooms. Objectivization is also used in the term lesson objectivization and refers to teachers’ ability to identify the objectives for a lesson. Lesson objectivization is a variable that is measured in Part 2 of the study. When searching for
articles on the ability of teachers to identify objectives for a lesson, it was necessary to use the terms goal setting or the ability for teachers to achieve lesson goals. Additionally, when looking at objectivization and mindset as variables, an essential note to make is that other variables may influence the mindset of teachers and are not controlled in this study. The study only looked at content knowledge, mindset, and lesson objectivization.

In a study of future teachers and how they reposition themselves to teach with learning objects, Mgombelo and Buteau (2009) make the distinction between knowing and knowledge. For Gattegno (1970), education is explained as a state of being that consists of knowing rather than knowledge. Once this view of education is accepted, knowledge can be seen existing outside of us and knowing can be seen as what is occurring within us and is identified with us (p. 125). In looking at goal attainment expectancy, Senko and Hulleman (2013) showed students see mastery-approach goals as those that are closer to attain and goals that appear harder are less likely to be pursued by students. For teachers, descriptions that identify lesson objectives to students will matter significantly. They will influence how students view and pursue them. Teachers can define goals or objectives for lessons regarding improving one’s prior performances, making the goals for the lesson more attainable (Senko & Hulleman, 2013). More specifically, mastery goals or performance goals when stated in an approachable manner promote educational benefits (Barron & Harackiewicz, 2001). When the teaching practice of preservice teachers was studied, Paparistodemou, Potari, and Pitta Pantazi (2014) found teacher reflections of their teaching focused mainly on class management issues. When looking further on their attention to class tasks for geometry, for example, these may develop through reflections on their teaching (Paparistodemou, Potari, & Pantazi, 2014). Table 2.7 serves to organize the studies reviewed for the study.
Table 2.7: Studies Related to Objectivization

<table>
<thead>
<tr>
<th>Author/year</th>
<th>Purpose/Rationale</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mgombelo &amp; Buteau, 2009</td>
<td>To identify a framework for how prospective teacher’s learning is shaped</td>
<td>Grounds a forthcoming framework for using teaching objectives in classroom activities to reposition teachers and gain knowledge for teaching</td>
</tr>
<tr>
<td>Paparistodeou, Potari, &amp; Pitta-Pantazi, 2013</td>
<td>To investigate preservice teachers’ attention to classroom tasks</td>
<td>Actual teaching and after class reflection is mostly focused on management issues. Attention to tasks can be developed through reflection of teaching</td>
</tr>
<tr>
<td>Senko &amp; Hulleman, 2013</td>
<td>To compare competence goal attainment expectancy separately for mastery-based goals and performance-approach goals</td>
<td>Mastery-approach goals are closer to attain, as goals appear harder they are less likely they will be pursued. Mastery-approach predicted situational interest and performance-approach goals predicted achievement</td>
</tr>
</tbody>
</table>

2.6 DISCUSSION

The review was also conducted to uncover possible studies that might map and connect Uznadze’s two levels of thinking (Uznadze, 1966; Uznadze, 2009) to Dweck’s two mindsets and possibly lead to studies connecting with Yeager’s (2012) productive persistence. Additional questions have surfaced after the review: How may growth mindset prevail in success for students enrolled in two-year college mathematics classrooms? Can classroom educators use mindset to lead and engage students in activities that nurture productive persistence, a behavior needed for learning mathematics in two-year college classrooms? With teachers as the closest and most influential factor in teaching and learning, would teachers’ clear view of objectives in lessons nurture productive persistence and promote growth mindsets for learning mathematics? Can this
clear view of lesson objectives be likened to Uznadze’s (1966) level of objectivization? With a clear view of lesson objectives, teachers are using a level of thinking based on objectivization, where reality is objectivized and where intellectual acts rise and intelligence forms (Uznadze, 1966).

Some articles, suggest that teachers can influence the orientation of these beliefs (Howard & Whitaker, 2011). Howard & Whitaker (2011) showed students experiencing one type of mindset limited their motivation, and another type fueled their motivation. Findings such as these have huge implications for practice in classrooms everywhere, especially in community college mathematics classrooms, the very places where many students struggle through mathematics courses.

Each study, whether directly related to mathematics teaching and learning or a person’s psychological or social being, had a direct impact on what students as humans and individuals face daily. In mathematics classrooms, it may prove fruitful for teachers and students to understand how students perceive their success and their success capability since it is perceptions of success capability that influence the performance of students (Howard & Whitaker, 2011). Because the number of students needing developmental mathematics courses is increasing by the year (Hodges, Kennedy, & Krzemien cited in Howard 2011; Hagedorn, Lester, & Cypress, 2010), whether students believe they will be able to do mathematics or not is an urgent and critical issue. If teachers understand how these perspectives are formed and how these perspectives may be altered, more effective and successful experiences for students in mathematics courses and programs of study may be included in pre-service teacher preparation programs and in-service professional development programs. Studies on mindset are beginning to clarify how to identify students’ beliefs about their intelligence and how we might shift their beliefs towards healthier more
productive beliefs that nurture growth mindsets. This addresses the equity agenda in granting each student “… an unquestioned entitlement … a rich and inalienable right to learn” (Darling-Hammond, 2009, p. 8). When growth mindset and its impact on student learning is included in a teacher’s knowledge base, then professional accountability may create practices that are oriented to students and are knowledge-based (Darling-Hammond, 2009). Hand (2012) argues that this will allow students to see themselves as part of a community of learners that take up space and are becoming the community. This may lead to ways that conceptualize equity in classrooms.

Mindset may give future direction to assist students in learning mathematics. However, it is not that simple. Another variable to consider in the equation for student learning is the cultural and social nature of learning. Vygotsky’s arguments originated with the “inseparability of consciousness and human (socially and culturally mediated) behavior” (Moll, 1992). Nieto reminds us of this when she states, “In practice, the mistake teachers and other educators often make is that they consider learning an ‘individual psychological process’ rather than cultural and perhaps even social” (Nieto, 1999 cited in Matthews, 2005). This connects us to Uznadze’s meaning of existence as something that would be found in culture (Ketchuashvili, 1994). Ketchuashvili wrote that for Uznadze, the most important goal of education in schools was teaching students to take independent, creative approaches to anything they studied (1994). This is a reminder that it is not just looking at how students think about intelligence theories that are important, but equally critical is the need to examine how students experience learning in the classroom and the experiential environment around their time in the classroom. Accomplishing this requires that efforts must concentrate on teaching. How will teachers deliberately connect mindset and productive persistence to what we know they must understand about cultural and social learning? In teaching, this perspective is directly related to teacher knowledge; in learning,
it is directly related to nurturing a growth mindset in students that may nurture productive persistence. Both serve as useful enrichments to the dynamic of teaching and learning.

Connected to the importance of the experiential environment in teaching reminds me of the importance Vygotsky (1986) saw when defining the zone of proximal development. He saw the zone of proximal development as a place where functions were not yet mature but where functions were still in an embryonic state, getting ready for being mature (Vygotsky, 1978). Activities that may bring in growing mindsets and productive persistence may allow teachers to see the distance between a student’s actual developmental level and the level of potential development in the same way Vygotsky (1978) saw it. When beginning with implicit theories of intelligence, no harm will occur as long as the work includes and continues with the social and cultural nature of learning mathematics alongside these theories.

Exploring pathways that connect teaching and learning to mindset and productive persistence may perhaps give educators a way to gain positive effects on the dynamic of teaching and learning. Infusing teacher knowledge with mindset theory and its implications will allow teachers to nurture a growth mindset in students. This allows them to use broader horizons of how students can and are willing to learn and learn effectively, how they ask for more challenges, and how they display resilience when failing (Boaler, 2013). Infusing more knowledge in teacher programs so that messages about fixed ability are thrown out and replaced with messages about growing intelligence and activities that create opportunities for students to improve their knowledge while engaged in learning will lead to student success (Boaler & Humphreys, 2005).

The review has shown some limitations in the literature. Only one of the more recent studies focused on how mindset and productive persistence impacted mathematics teaching and learning; also, not many studies have taken place in two-year colleges. Work at the Carnegie
Foundation for the Advancement of Teaching provided us with a view in two-year colleges (Yeager et al., 2013). Measures on the impact on mindset and productive persistence of students in learning mathematics are focused mainly on K-12 and not in two-year colleges. This limitation critically points to the need for more research in two-year college developmental mathematics classrooms. Further research is also needed to study how teacher knowledge for two-year college instructors allows them to measure and nurture a growth mindset in students who enter college underprepared to enroll and complete college-level mathematics courses.

2.7 CONCLUSION

Every study ends with the recommendation of the need for more studies. Each article presents a challenging next step for education practitioners and researchers. Because little in published research uses implicit theories about mindset in mathematics classrooms, and in particular in two-year college mathematics classrooms, there is an opportunity for more research in this area and how it may inform the base of teacher knowledge. Next steps may include beginning to ask teachers in community colleges to rethink any remaining traditional and antiquated processes and replace them with processes utilizing activities nurturing growth mindset, productive persistence in students, and active learning. Revisiting beliefs about learning mathematics and how implicit theories of learning and intelligence may change students’ mindset is next on the agenda for reform. This reform may require a more flexible pedagogy and range of ways to reach more students. Difficult and costly as it may seem, the benefits for students struggling in mathematics classrooms will provide an enormous payoff. In altering students’ mindsets about learning mathematics, we may reach many more students than those we now reach. It is a necessary next step as we redesign mathematics courses and classrooms.
Including the ideas of a growth mindset may begin to address postsecondary issues for student success. Our problem is that many students come to college not ready to successfully enroll and complete their first college-level mathematics course. We have modified, revised, created and tried many curricula and still need further work to address this problem. However, a major constraint we have not dealt with in two-year postsecondary institution is the mindset of both students and faculty. This review of literature gives us a starting place to move to the next step when addressing students entering colleges underprepared in mathematics. Mindset as a term and concept is in the educational K-12 news almost every day, however, rarely is it in postsecondary news or research. Because few studies appeared on implicit theories about mindset in college mathematics classrooms, the community of educators at two-year colleges may use this time to begin to explore mindset on their campuses.
Chapter 3: Method

This chapter describes the study design and the process to conduct the study. It describes the study site, sample, and resources for data. Included is a context for the study to better situate the educational environment. Additionally, the chapter includes the analyses of data collected for the three variables in the study: content knowledge, a mindset for learning, and lesson objectivization. All three variable measures will be used to investigate any connections they have to teach in the next chapter.

3.1 Introduction

This is a mixed methods study designed to explore teacher characteristics in two-year college mathematics faculty that lead to effective teaching. Effective teaching is seen to include the dynamic of students' learning content. In this study, knowledge needed for effective teaching in classrooms will focus on the ability of faculty to identify lesson objectives that engage students in mathematics that develop students as learners of mathematics and develop a larger community of learners of mathematics. It will attempt to answer the following research questions:

1) What is the relationship between content knowledge of mathematics faculty in two-year colleges and their mindset for learning mathematics?

2) How does lesson objectivization in two-year college mathematics faculty relate to their content knowledge?

3) How are content knowledge, mindset, and lesson objectivization connected to teaching in two-year college mathematics faculty?
3.2 STUDY SITE, PARTICIPANTS, AND CONTEXT

3.2.1 Study Site

The study takes place in a large multi-campus two-year college located in a city along the southwest border of the United States. Student population at the college is predominantly Hispanic (85%). Other two-year colleges in the area are proprietary schools. The population studied is the full-time and part-time mathematics faculty at a two-year college. The sample for the study came from one-hundred twenty-two (122) two-year mathematics faculty; thirty-five (35) are full-time faculty, and eighty-seven (87) are adjunct faculty. It is the college where the researcher taught mathematics for many years. It was selected as a study site because of the rich diversity of the population of the region and because of the efforts that were taken to prepare K-12 students to enter freshman college mathematics courses when they graduate from high schools.

The region has a higher high school graduation rate compared to the rest of the state. However the number of entering students who were not prepared to enter college-level mathematics was above 98% over 14 years ago. These rates have since lowered to 65%. While many successful students enter college-ready, most have to take courses that will prepare them to enroll and succeed in freshman mathematics courses required for programs of study. In Texas, most college four-year programs require at least one college-level mathematics course. The college has participated in many state and local initiatives to address student success, and many of the interventions the college offers are working well. However, there are still many students who take developmental courses more than once before completing them with a grade of A, B, or C. The added nature of students' beliefs in learning mathematics is something that concerns the researcher.
3.2.2 Participants

Participants in the study are mathematics faculty at a two-year multi-campus community college in the southwest with a student population that is predominantly Hispanic. Approximately 80 mathematics full-time and adjunct faculty representing all campuses were invited to participate.

Analysis of Random Sample

A random sample of 40 faculty members generated by the Institutional Research area of the college identified the participants for the study. The random sample generated and assigned random values to all mathematics faculty. Once each faculty member was associated with a random value, faculty members listed in descending order according to their value. Keeping the same approximate full-time, part-time ratio, 1:3, as the entire population, the first ten full-time faculty listed and the first thirty part-time faculty listed were identified as potential participants. In this way, the random faculty represented the same proportion of full-time to part-time as was represented in the population of the mathematics faculty. The initial invited faculty included forty mathematics faculty in the random sample with ten full-time faculty members and thirty part-time faculty members. Ten out of thirty-six full-time faculty represented 28% of all full-time faculty; the thirty out of one-hundred and eight (108) part-time faculty represented 28% of all part-time faculty. Only nineteen of the faculty took both surveys. A second list of forty additional mathematics faculty members were invited to participate in the study to increase the number of participants. The same process as above was used to invite the second group of participants.

From these two iterations of 80 randomly selected faculty, thirty-three (33) submitted both Surveys 1 and 2. Participants for Part 2 of the Study were drawn from the 33 participants who submitted both surveys in Part 1. Descriptive data from the participants shows the following
summary of the participants in the study compared to the entire faculty population at the college and the faculty in the random sample. Fourteen of the thirty-three participating in both surveys were female (42%) and 19 of the 33 participating were male (58%). The makeup of gender represented by the sample compares well with the entire population of the College’s mathematics faculty where 61 of 144 were female (42%), and 83 of 144 were male (58%). When looking at the full-time and part-time status in the study and the entire mathematics faculty, 12 of the 33 participants in the study were full-time faculty (37%), and 21 of the 33 participants in the study were part-time faculty (63%). In the entire population of the College’s mathematics faculty, 36 out of 144 faculty members are full-time (25%), and 108 out of 144 faculty members are part-time (75%). These comparisons appear in Table 3.1.

<table>
<thead>
<tr>
<th>Category</th>
<th>Sub-categories</th>
<th>Math Faculty N = 144</th>
<th>Study Participants n = 33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>Female</td>
<td>61 (61/144 = 42%)</td>
<td>14 (14/33 = 42%)</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>83 (83/144 = 58%)</td>
<td>19 (19/33 = 58%)</td>
</tr>
<tr>
<td>Status</td>
<td>Full-time</td>
<td>36 (36/144 = 25%)</td>
<td>12 (12/33 = 37%)</td>
</tr>
<tr>
<td></td>
<td>Part-time</td>
<td>108 (108/144 = 75%)</td>
<td>21 (21/33 = 63%)</td>
</tr>
</tbody>
</table>

Faculty participating in the study included 23 (70%) Hispanic, 7 (21%) White, and 3 (9%) African-American, Asian/Pacific Islander or Other. Two thirds, 22 of 33 (67%) of the participating faculty are credentialed to teach both developmental mathematics courses and college-level courses; one third, 11 of 33 (33%) are credentialed to teach only developmental mathematics courses. When looking at the credentials of participating faculty, 39% had degrees in mathematics, 39% in mathematics education, 9% in engineering, and 12% in other areas. Thirty-six (36) is the
number of hours/credits in graduate mathematics required by the accrediting agency of the College to teach college-level mathematics courses. There were 52% (17 in 33) with 18 to 36 hours in mathematics and 24% (8 in 33) with more than 36 graduate hours in mathematics. The rest, 24% (8 in 33) had only the required number of hours needed for teaching developmental education mathematics courses in college. Developmental education courses are courses with high school mathematics content and are offered to prepare to enter students for college-level mathematics classes. Less than one third, 30%, were certified to teach in K-12. Some of these characteristics are stated in Table 3.2.

Table 3.2: Participant Characteristics

<table>
<thead>
<tr>
<th>Categories</th>
<th>Sub-categories</th>
<th>n = 33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credentials to teach</td>
<td>College-level and developmental math courses</td>
<td>22 (67%)</td>
</tr>
<tr>
<td></td>
<td>Developmental math courses only</td>
<td>11 (33%)</td>
</tr>
<tr>
<td>Degree</td>
<td>Mathematics</td>
<td>13 (39%)</td>
</tr>
<tr>
<td></td>
<td>Mathematics Education</td>
<td>13 (39%)</td>
</tr>
<tr>
<td></td>
<td>Engineering</td>
<td>3 (9%)</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>4 (12%)</td>
</tr>
<tr>
<td>Graduate math hours</td>
<td>More than 36</td>
<td>8 (24%)</td>
</tr>
<tr>
<td></td>
<td>18 - 36</td>
<td>17 (52%)</td>
</tr>
<tr>
<td></td>
<td>0 - 17</td>
<td>8 (24%)</td>
</tr>
<tr>
<td>Race/Ethnicity</td>
<td>Hispanic</td>
<td>23 (70%)</td>
</tr>
<tr>
<td></td>
<td>White/Non-Hispanic</td>
<td>7 (21%)</td>
</tr>
<tr>
<td></td>
<td>Black</td>
<td>1 (3%)</td>
</tr>
<tr>
<td></td>
<td>Asian/Pacific Islander</td>
<td>1 (3%)</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>1 (3%)</td>
</tr>
</tbody>
</table>
3.2.3 Context

At the two-year college selected for the study, most faculty teach developmental (DE) mathematics courses within their course load. DE courses are defined as courses with content normally taught in K-12; college-level courses are courses with content taught at the postsecondary level. As with most two-year colleges nationwide, a large percentage of students entering are not prepared to enroll in and complete a college-level mathematics course. Because of this, efforts to accelerate student experiences in DE mathematics courses is prioritized by colleges to get students to successful completion of their college-level mathematics course and eventual completion of their programs. Faculty have been asked to use new curricula with content more aligned with their programs of study and include active learning strategies that support student learning and result in higher success rates for students in mathematics courses. The college is participating in nationwide reform efforts to create Mathematics Pathways (Kazis & Cullinane, 2015) with courses where the mathematics being taught and learned is more in line with students’ programs of study. The pedagogy and curricula of the newly designed courses are more active-learning based and require a shift from traditional methods of teaching. As with all other reform efforts, mathematics faculty have being asked to participate in professional development to prepare them for teaching these new courses. This may be a factor that goes beyond the scope of this study and may be studied further as we look at teachers’ needs when asked to change their mode of instruction.

Many instructors at two-year colleges teach developmental courses, that is, courses normally taught in K-12 that do not award college-level credits. Because many students need developmental courses, efforts to accelerate student experiences in mathematics courses is prioritized to get students through successful completion of their freshman level mathematics course leading to successful completion their programs of study. With a new curriculum designed
to make courses more relevant to programs of study and more active learning strategies, faculty will begin to teach newly revised courses.

3.3 RESEARCHER’S ROLE AND POSITIONING

As a tenured mathematics faculty member at the study site, the researcher saw large numbers of students coming in as first-time-in-college students each fall who were unprepared for their first college-level mathematics course. Involvement by the college in a variety of initiatives and programmatic offerings to create higher levels of student placement for entering students has been fruitful and a variety of programmatic changes to impact college readiness and student placement have been in practice for over 12 years. While the College has seen higher levels of mathematics placement scores for students and the number of students entering with dual credit for college mathematics courses, there are still large numbers of students entering at the developmental course level. There is a vested interest in getting more students to accelerate experiences in developmental mathematics courses to enroll in college-level courses required for their programs of study. This research will carry great significance for students’ first year experiences in college. As a researcher, I will remain in a separate space not taking into account the “geographic space of shared experience” (Fine, Weis, Weseen, & Wong, 2003, p. 173) I have with some of the faculty.

Many students experience success as they enter college-ready, enroll in STEM or liberal arts college-level courses, finish their programs of study, earn associate degrees, and transfer to four year-university partners or gain employment. Far too many, however, remain in developmental mathematics courses and experience them as barriers to degree attainment (Bryk & Treisman, 2010). The importance of what we may learn will position me in ways where my
responsibility will be to form “safe spaces” (Fine, Weis, Weseen, & Wong, 2003, p. 193) for faculty to tell their stories.

Although I know about 30% of the faculty, I have not been teaching for some semesters. It will be important that my positioning is understood to be only to uncover what faculty may share about teacher knowledge needed to make their classrooms effective places for learning. As a qualitative researcher, I am positioned where the researcher (me) and the researched (faculty) joined in an ongoing moral dialogue (Denzin & Lincoln, 2003). For the study, I positioned myself as a researcher with the purpose of identifying knowledge needed for faculty who bring effective teaching to classrooms, allowing for more students to complete mathematics courses and programs of study.

It is essential to recognize the potential personal bias of the researcher. My educational journey starts with my earliest memory, to my memories of the first days at elementary school, intermediate school, high school, college, graduate school for my master’s degree, to the graduate program for my doctoral degree. All memories are that I have always enjoyed school. My parents did not finish high school. However, they pushed all their children to graduate from high school. Enrolling in college was left up to those who were willing to work to save and pay for college expenses. Teaching mathematics has been my only career except when taking on a directorship to align mathematics and science for a National Science Foundation-funded project. As a classroom instructor of mathematics, I have experience in teaching students who are very prepared for freshman mathematics courses as well as teaching students who may not be as prepared. However, I understand my role as researcher and have built in the study at least three ways verify data in its analysis. In addition to this, my experiences in national panels with mathematics
educators from both state and local entities, help me recognize the importance of how studies can be used to inform the education community.

3.4 RESEARCH DESIGN AND DATA COLLECTION

The study is a mixed methods study and will have both quantitative data and qualitative data. It is an explanatory sequential nested study (Creswell, 2013). It had a nested sample selection process and was conducted using a three-part sequence: In Part 1, participants included 33 two-year college mathematics faculty. In Part 2, participants were drawn from the sample of participants in Part 1. In Part 3, four participants were selected to form a purposive sample (Babbie 2014; Tchoshanov et al., 2017) from those who participated in Part 2 for a multi-case qualitative study. The study provided additional understanding of the knowledge base identified by faculty for effective teaching in two-year college mathematics classrooms. The study had three parts. Approximately 40% of the data will be quantitative and was collected during Part 1 of the study. Approximately 60% of the data is qualitative and was collected during Parts 2 and 3 of the study. Table 3.3 shows the number of participants, description, and data sources for each part.

Table 3.3: Three-part research design with sample size, description, and data sources

<table>
<thead>
<tr>
<th>Study Part Sample size</th>
<th>Description</th>
<th>Data Sources/Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1 n = 33</td>
<td>A quantitative study on the association between teacher content knowledge and mindset to answer research question 1</td>
<td>Teacher Content Knowledge (TCK) Mindset Assessment Profile (MAP)</td>
</tr>
<tr>
<td>Part 2 n = 14</td>
<td>A qualitative study on the lesson objectivization of an expert lesson to answer research question 2</td>
<td>Lesson objectivization (LO) in lesson objective(s) narratives</td>
</tr>
<tr>
<td>Part 3 n = 4</td>
<td>Case studies on the connections between teacher content knowledge, the mindset for learning</td>
<td>Teacher content knowledge survey</td>
</tr>
</tbody>
</table>
mathematics, and lesson objectivization and teaching to answer research question 3

3.4.1 Instruments

Part 1

Part 1 of the study provided data for to select the participants for the second and third parts of the study. It also provided data to answer research question 1:

What is the relationship between the content knowledge of mathematics faculty in two-year colleges and their mindset for learning mathematics?

In Part 1, eighty (80) faculty were invited to participate in two close-ended surveys. Only the thirty-three (33) faculty that participated in both Survey 1 and Survey 2 were included in the study sample. The first set of items in Survey 1 collected demographic and faculty credential data. Faculty demographic and credentialing data provided ways to compare the faculty participating in the study to the actual population of the faculty at the study site. A descriptive comparison of the study sample and the actual population appeared in Tables 3.1 and 3.2. The second set of items in Survey 1 included eight (8) items that measured faculty mindset assessment profiles for learning (Dweck, 2008; Yeager & Dweck, 2012; Yeager, D., Bryk, A., Muhich, J., Hausman, H., & Morales, L., 2013). Scores were placed along a mindset continuum to allow varied measures based on the discipline one may be learning. Using a continuum allows participants to be placed on a continuum that showed mindset profiles to be between measures closer to a fixed mindset or closer to a growth mindset.
Survey 2 in Part 1 measured teacher content knowledge. The content of instruction is an essential variable in research on factors affecting student achievement (Porter, 2002). In this study, determining a measure of the content knowledge of participating faculty played an essential role in investigating the knowledge needed for teaching. As an intervening variable, the knowledge of content can be used to test how well particular reforms of educational content, impact student achievement gains (Gamoran et al., 2007).

Selection of the items was strategic as it was essential to measure content knowledge in within a broad range of mathematical areas. Twenty-three (23) items from an item bank written to prepare teachers for state certification exams in mathematics produced Survey 2. Items selected came from six areas of mathematics including number and operations; patterns and algebraic reasoning; geometry and measure; functions and graphs; probability and statistics; and mathematical processes. These six areas form foundational concepts in mathematics that are essential for students entering college prepared to complete entry-level mathematics courses. Items focused on these six areas in mathematics and were also coded using three levels of cognitive demand: 1) memorizing and performing procedures, 2) conceptualizing and making connections, and 3) generalizing and making conjectures (Porter, 2002; Tchoshanov, 2011). Cognitive demands describe how one engages thinking with the topic and what one must know and be able to do with the topic (Porter, 2002; Tchoshanov, 2011). Thus, in the content survey, each item represents one of the six areas in mathematics and is associated with one of the three levels of cognitive demand. In this way, the content knowledge of faculty is measured and described in language that has two dimensions, the topic and the cognitive demand (Porter, 2002). Data from the surveys included data only from faculty completing both surveys and all items in the survey.
At the study site, there will be added need to provide statistics courses in postsecondary programs of study, items selected for the content knowledge survey emphasized three of the six areas: patterns and algebraic thinking; functions and graphs; and probability and statistics. These areas are consistent with two types of course content at the study site. They are also consistent with areas foundational to the mathematics standards for the first two years of college (AMATYC, 1995). These areas lead to the content in mathematics courses leading to Science Technology, Engineering, and Mathematics, STEM, programs of study and the content in statistics or quantitative reasoning courses leading to non-STEM programs of study. Content in statistics and quantitative reasoning courses is strategically important in the first college-level freshman mathematics courses offered to students who pursue non-STEM related fields. Statistics and probability courses and quantitative reasoning courses are becoming more prominent choices as the mathematics course for students in non-STEM programs and fields of study. This is in contrast to the courses offered for freshmen students in the past, where the emphasis of mathematical content in courses was more representative of content leading and preparing students for STEM fields of study. The other areas emphasized in the Survey 1 items are patterns and algebraic reasoning, and functions and graphs. The survey item structure provided data on mathematical content knowledge in these broad areas representing two types of courses. These two types of freshman-level courses represent content faculty have taught historically and what they will be teaching in the very near future.

Constructing Survey 2 provided data with measures of mathematical content knowledge not necessarily based on the number of topics in graduate courses beyond a four-year degree, but rather on the content faculty currently teach, algebraic functions and graphs, and added content faculty would be expected to teach in the future, statistics and quantitative reasoning. Measures
from Survey 2 include data recording overall scores from all items and data recording scores by cognitive demands for each faculty member collecting a two-dimensional measure of knowledge (Porter, 2002). The analysis included the relationship between each of these pairings 1) mindset profile to scores and mathematical content knowledge, 2) mindset for learning and Level 1 content scores, 3) mindset for learning and Level 2 content scores, and 4) mindset for learning and Level 3 content scores.

To see how the six areas in Survey 2 cover the foundation that underlies topics in gateway mathematics courses offered at the two-year college site, a review of courses and the content areas was undertaken. The review of the objectives of all six entering freshman gateway mathematics courses, indicated that all require a basic understanding of the mathematical areas within the items of the content survey. Table 3.4 summarizes the mathematics as it appears in the foundational topics of the six gateway mathematics courses.

Table 3.4: Course Content Mapped to Content Survey Items

<table>
<thead>
<tr>
<th>Course</th>
<th>Number &amp; Operation</th>
<th>Patterns &amp; Algebraic Reasoning</th>
<th>Functions &amp; Graphs</th>
<th>Geometry &amp; Measure</th>
<th>Probability &amp; Statistics</th>
<th>Math Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1314 College Algebra</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>1324 Business Algebra</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>1332 College Math</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>1333 Math in Modern World</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>1342 Fund. of Statistics</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>
A Cronbach (1951) alpha as a measure internal consistency for the items in each survey was calculated. The mindset items in Survey 1 had a Cronbach $\alpha = 0.6291$, adequate. The knowledge content items in Survey 2 had a Cronbach $\alpha = 0.7489$, acceptable. This value of alpha for the content items is considered acceptable since it is higher than 0.70 (UCLA, 2016). Alpha values between 0.62 and 0.86 are considered adequate to good (Creswell, 2013).

**Part 2**

Data collected in Part 2 was used to answer Research Question 2:

How does lesson objectivization in two-year college mathematics faculty relate to their content knowledge?

In Part 2 of the study, every faculty member participating in Part 1 was invited to participate in focus groups. Focus groups were scheduled at each campus to facilitate faculty attendance. Fifteen faculty members met for the focus groups scheduled in part 2. Faculty used the same pseudonym they created to take the surveys in Part 1 of the study. Because only 14 of the 15 faculty were matched to pseudonyms collected from the surveys, data from only 14 were analyzed for Part 2 of the study.

The focus groups provided a venue for participants to view a middle school lesson on pattern building (Boaler & Humphreys, 2005). A middle school classroom video was selected for the lesson to ensure that participants had not seen or used the lesson before. It was a lesson with
foundational content that students in college mathematics classrooms were expected to know and be able to use. After viewing the lesson, participants were asked to write a narrative describing the lesson objectives. Faculty members wrote narratives on a page with the following directions:

Write what you believe to be the objectives of the lesson you just viewed. Please write in complete sentences and include details, if needed, to clarify these objectives.

A matrix was used to record the faculty descriptions that matched lesson objectives identified by participants. This process allowed a record of the ability of faculty to match the expert lesson objectives. The matrix listed content and process standards in rows and faculty participants in columns to record each match to the expert’s lesson objective by content or process. Open coding was used to analyze narratives written by faculty (Babbie, 2014) describing the expert lesson objectives. Participants were not given directions on what they were going to write until after they viewed the lesson. This allowed the faculty to view the lesson as a whole before writing anything and to prevent the introduction of any prior thoughts while viewing the lesson. Data were recorded every time faculty matched objectives identified by objectives to mathematics content standards from six mathematical areas: number and operations; patterns and algebraic reasoning; geometry and measure; functions and graphs; probability and statistics; and mathematical processes. Expert coding (Boaler & Humphreys, 2005) from the lesson objective narratives were used to match all objectives and to identify those who matched the expert objectives. Collecting emerging themes was made possible with linguistic analysis and meaning coding (Kvale & Brinkmann, 2009).
Data collected in Part 3 was used to answer research question 3:

How are content knowledge, mindset, and lesson objectivization connected to teaching in two-year college mathematics faculty?

For Part 3 of the study, the sample was a purposive sample (Babbie, 2014; Tchoshanov et al., 2017) of four participants and was a subset of the focus group participants in Part 2. The Seidman (2006) three-part interview process was used to collect brief life histories, current teaching experiences, and how their life may form their work. Selection of four faculty members provided participants with combinations of high and low content scores and with growth and fixed mindset profiles (Dweck, 2008) measured along a continuum and additional information on their match to expert lesson objectives. This allowed for the selection of four participants in Part 3 of the study that had scores reflecting: 1) high content and growth mindset scores, 2) low math content and growth mindset scores, 3) high content and fixed mindset scores, and 4) low math content and fixed mindset scores. Qualitative methods were used to tally the type of activities and student experiences teachers used in their lessons. Frequencies of the number of times the activities were recorded to give a measure of the type of activities observed that promote learning mathematics and produce resilience while learning (Yeager & Dweck, 2012). Part 3 of the study allowed for the integration of 3 variables: teacher content knowledge, their mindset for learning, and lesson objectivization. These variables were used along with the qualitative interview data (Seidman, 2006) and class observations from a protocol adapted from the Teaching for Robust Understanding Observation Guide (Schoenfeld, 2007). Interviews used during the case studies allowed the collection of faculty perspectives about beliefs about learning and teaching (Seidman,
1985). Class observations added faculty images. Thus, both quantitative and qualitative data helped describe the four faculty portraits in the case studies.

A diagram showing the sequence of Parts 1, 2, and 3 and how the samples are nested in the previous sample appears in Figure 3.1.

![Figure 3.1: Mixed methods sequential nested study](image)

Figure 3.1: Mixed methods sequential nested study
3.5 **DATA**

Quantitative data were collected from two surveys in Part 1. Randomly selected faculty received the invitation to participate by email. The directions to fill out the surveys and provide consent were included in the email and were sent by hard copy campus mail. Participants could elect to take the surveys online or at scheduled times for each campus. After the first invitation to the first set of randomly selected faculty, a second email went out to another forty randomly selected faculty members. Over ninety percent (90%) of participants filled out the surveys online. This yielded 33 participants in Part 1 of the study. All participants of Part 1 were invited to participate in Part 2.

Qualitative data were collected in Part 2 and Part 3 of the study. Data in Part 2 were collected through focus groups. During the focus group meetings, participants viewed a fifteen-minute video of an expert lesson. After viewing the lesson, participants were asked to write a narrative describing the objectives of the using the prompt: Write a paragraph that describes the lesson objective or lesson objectives of the lesson you viewed.

Data collected from the narratives served to measure the ability to identify expert lesson objectives termed by this researcher as lesson objectivization and is connected to using the level of objectivization as a level of thinking (Uznadze, 1966; Uznadze, 2009). It is important to note that participants received no other directions, nor was there any discussion before or after participants were asked to write their narratives to identify the lesson objectives. The absence of directions insured that there was nothing introduced in participants’ mindsets (Uznadze, 1966; Uznadze 2009) while viewing the video. Expert coding (Boaler, J. & Humphreys, 2005) was used to match narrative descriptions of the lesson objectives written to the lesson objectives used by the expert for the lesson.
For part 2, open coding (Charmaz, 2006) and expert coding (Boaler, J. & Humphreys, 2005) were used to see how the narrative descriptions connect with the objectives of the lesson viewed. To provide consistency and to calibrate the coding, the researcher analyzed text separately using open coding first and then coded emerging themes from the narratives and interview transcripts using focused coding (Emerson, Fretz, & Shaw, 2011).

Question two asked:

How does lesson objectivization in two-year college mathematics faculty relate to their content knowledge?

When analyzing the written narratives describing lesson objectives, the analysis included open coding first and then expert coding of the objectives and included whether objectives chosen came from content standards or process standards. The lesson objectives identified and used by the expert who authored and videotaped the lesson were twofold (Boaler & Humphreys, 2005). These objectives were to use patterns, expressions, equations, and relationships to generalize. Patterns, expressions, and relationships is a mathematical content standard, and generalization is a mathematical process standard.

The lesson objective narratives were quantified using a matrix showing participants that matched expert objectives with a one (1) and those not matching expert objectives with a zero (0). Of the fourteen participants, thirteen matched the lesson objectives to the expert’s objectives addressing process standards. Of the fourteen participants, four matched the lesson objectives to expert objectives that address content objectives. Only four of the fourteen participants matched those that addressed both content and process standards. Only one participant did not match any of the expert objectives.
In Part 3 of the study, four participants were interviewed and observed in one class. Data from part three came from interviews and class observations. Open coding (Charmaz, 2006; Emerson, Fretz, & Shaw, 2011) and expert coding were used to identify pattern generalization content that develops algebraic thinking (NCTM 2000; Boaler & Humphreys, 2005). This allowed the researcher to identify objectives used to bring understanding to generalization. Coding interviews in this same way in Part 3 of the study provided another measure of teachers’ classroom beliefs in learning as did coding of classroom observations. Each faculty was observed to quantify measures of student engagement in activities that allow students to gain agency for their learning. Discussion of the variation in each participant’s lesson using the observation guide shows how the collective experiences of students in classrooms may or may not be designed to reinforce the development of agency for the class as a group (Kemmis & McTaggart, 2005).

Analysis of the interviews and class observation helped to connect content knowledge, beliefs about learning, and lesson objectives to teaching. Questions related to their mindset for learning analyzed from data collected in interviews and class observations. An observation protocol measured how faculty engaged students during the lesson. Thus, three types of data were collected on mindset for learning: the mindset items in Survey 1, the individual interviews on faculty beliefs about learning mathematics, and the actual class observations. Three types of data were also collected to identify lesson objectivization, the ability to identify lesson objectives: the narratives on lesson objectives, the individual interviews, and the class observations.

Research questions for the study guided data collection and were used to analyze data. Question one asks, what is the relationship between content knowledge of mathematics faculty in two-year colleges and their mindset for learning? Measures of content knowledge from survey 1 and of their mindset profile from survey 2 provided data to measure if teacher content knowledge
and mindset for learning were related. Data did not show sufficient relationship. However, data measures from these two surveys facilitated the selection of participant typologies for Part 3 of the study. Using these measures allowed for the inclusion of combinations of high to low content knowledge together with fixed or growth mindset in the participants. The demographics and interest survey provided a view of the population in general, however, also provided insights for the four participants selected for the case studies.

Using Seidman’s (2006) interview process, teachers provided insights on how mindset and math content played major or minor roles, if any, in a teacher’s classroom when paired together with classroom observations and field notes. As with all class observations, one can observe a variety of events; in this study, the analysis was focused on whether students are engaged in activities that develop mathematical concepts and ideas (Gutierrez, 2009; Gutierrez, Sengupta-Irving, & Dieckmann, 2010). The class observation after the second interview using an observation guide (Schoenfeld, 2016) provided some insight on how faculty engaged the student with the content. The guide contains five areas that one may concentrate on 1) Content; 2) cognitive demand; 3) equitable access to content; 4) agency, ownership, and identity; and 5) formative assessment (Schoenfeld, 2016). For this study, the observations focused on the fourth area, agency, ownership, and identity to indicate if students develop their mathematical learning. In particular, observations focused on the number of opportunities students had to explain their ideas, how their ideas built knowledge and contributed to learning, and how students were recognized as capable and able to participate (Schoenfeld, 2016).

**Faculty Case Studies.** Four faculty were selected from quantitative data collected from Surveys 1 and 2 in Part 1 of the study for case studies. Using case studies helped unpack the overall knowledge base each of the teachers described for themselves. Doing this made the case studies
descriptive. Recall that mindset was not related to content knowledge, nor to any of the levels of content knowledge taken separately. Because of this, purposive sample using content knowledge and mindset scores identified participants representing all four combinations of high and low content and high and low growth mindset scores to gain further insight on how content knowledge and mindset profile score played a role in faculty’s lesson objectivization. Faculty scores were listed in one column from the highest content knowledge measures to the lowest. On the second column of the list, their mindset profile scores were listed from highest to lowest. Participants selected represented faculty that had a high content score with a growth mindset, high content score with fixed (low growth) mindset, low content score with a growth mindset, and low content with a fixed mindset. Table 3.5 represents the type of faculty selected to be interviewed and observed in classrooms. One from each cell was selected.

<table>
<thead>
<tr>
<th>Growth mindset</th>
<th>Fixed mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>High content</td>
<td>High content with a growth mindset</td>
</tr>
<tr>
<td>Low content</td>
<td>Low content with a growth mindset</td>
</tr>
</tbody>
</table>

Using this matrix, the purposive sample (Babbie, 2014; Tchoshanov et al., 2017) of participants selected represents all four combinations of content knowledge and mindset measure. The faculty sample included full-time and part-faculty; tenured and non-tenured faculty; male and female; and faculty from different campuses. Faculty names are pseudonyms and are each discussed separately as well as using a cross-case analysis (Tchoshanov et al., 2017). Faculty selected were in the highest quartile score in both content knowledge and mindset, lowest quartile
scores in both content knowledge and mindset, highest quartile in content and lowest quartile in mindset, and lowest quartile in content and highest quartile in mindset. Tables 3.6 and 3.7 shows the distribution of content knowledge scores and mindset profile criteria used to select the faculty.

Table 3.6: Faculty content knowledge scores and mindset profile scores by quartiles

<table>
<thead>
<tr>
<th>Quartile</th>
<th>Content Score Range</th>
<th>Mindset Profile Score</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q4</td>
<td>20 – 18</td>
<td>Q4</td>
<td>42 – 39</td>
</tr>
<tr>
<td>Q3</td>
<td>17 – 16</td>
<td>Q3</td>
<td>38 – 35</td>
</tr>
<tr>
<td>Q2</td>
<td>15 – 14</td>
<td>Q2</td>
<td>34 – 31</td>
</tr>
<tr>
<td>Q1</td>
<td>13 – 10</td>
<td>Q1</td>
<td>30 – 27</td>
</tr>
</tbody>
</table>

The four faculty who met the criteria for the purposive sample were Daniela, Frankie, Gracie, and Silver (pseudonyms). Table 3.7 presents faculty selected.

Table 3.7: Selective faculty for purposive sample

<table>
<thead>
<tr>
<th>Faculty</th>
<th>Content Quartile</th>
<th>Mindset Quartile</th>
<th>Content, Mindset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daniela</td>
<td>18, Q4</td>
<td>41, Q4</td>
<td>Q4, Q4</td>
</tr>
<tr>
<td>Frankie</td>
<td>19, Q4</td>
<td>30, Q1</td>
<td>Q4, Q1</td>
</tr>
<tr>
<td>Gracie</td>
<td>10, Q1</td>
<td>39, Q4</td>
<td>Q1, Q4</td>
</tr>
<tr>
<td>Silver</td>
<td>13, Q1</td>
<td>30, Q1</td>
<td>Q1, Q1</td>
</tr>
</tbody>
</table>

Three faculty selected have taught developmental mathematics courses and college-level mathematics courses. One faculty has only taught developmental mathematics courses. For this part of the study, data sources included three faculty interviews, one class observation, and field notes. This part provided insight into other types of knowledge faculty believed they used for classroom teaching along with the quantitative data already collected in Parts 1 and 2. All interviews were audio recorded and transcribed by the researcher. In interview one, teachers
reflected on how their families and early experiences influenced their decisions to become teachers and their experiences while learning mathematics. During the second interview, faculty reflected on their current experiences in teaching, what they observed in an expert lesson, and how they viewed student beliefs about learning mathematics. For the third interview, teachers reflected on their work as a reflection of their lives, their beliefs about learning and how those two things might connect to the knowledge they need and rely on for teaching.

Class observations following the second interview provided a view of current teaching practices observed through the lens of how they engaged students in the lesson. They were processed and tabulated with a protocol to see if faculty engaged students in the six activities. These were: clarified lesson objectives; provided students with time to develop and express ideas; gave students opportunities to be heard; encouraged student interactions; assigned tasks to analyze, synthesize, and explain reasoning; and attributed ideas to students to build student ownership and identity (Schoenfeld, 2016). The interviews and observations served to verify faculty mindset about learning and how it manifested itself in classroom lessons.

3.6 INTEGRATION AND VERIFICATION OF THE DATA

3.6.1 Integration of the Data through Mixed Methods

Figure 3.2 shows the flow for gathering data for the study. Approximately one-fourth of the data came from the two surveys given in part one from a sample of thirty-three faculty members. The rest of the data was collected in Part 2 of the study with 14 participants and Part 3 of the study of 4 participants.
During the qualitative part of the study, a measure of faculty ability to identify lesson objectives, lesson objectivization was based on the number of expert objectives they matched. A calculated Pearson coefficient of $r = 0.614892403$ showed a significant correlation, with $\alpha = .05$, between teacher mathematical knowledge and the measure they receive from the narrative of objectives they observed for the video lesson. Thus, their lesson objectivization, ability to identify objectives to expert objectives, and scores of their knowledge content mastery were positively related.

### 3.6.2 Verification of data

To gain trustworthiness and address validity of the data collected there were two facets to consider. The first is the collection of the quantitative data from two surveys in Part 1. The faculty had ample time to complete the surveys once requested to take them. Faculty participating in Part 1 of the study were assured their responses were anonymous. Using pseudonyms created anonymity for all parts of the study. Faculty had the choice of completing surveys using an online link or during survey taking sessions at each of the campuses. They could elect to complete one
survey and then return to complete the second survey to give flexibility of time needed to finish responses.

The second is for the collection of the qualitative data in parts 2 and 3, where individual narratives were collected and individual interviews followed by class observations were scheduled. Questions grouped by three parts for the interviews were scheduled to allowed time to investigate patterns, themes, or categories emerging from the data (Charmaz, 2006). Audio recordings were used for the interviews and transcriptions were reviewed by the researcher as many times as were needed to ensure all that was audible was recorded in the written transcriptions. The researcher used member checking of all faculty who were interviewed. Faculty interviewed had the opportunity to read the narratives of transcriptions from interviews to allow for accurate stories and voices recorded by the researcher. In one instance, the faculty member sent a revised transcription correcting a few typographical errors and adding a clarification for one of the inaudible statements. These revisions were incorporated in the transcribed interview.

In research studies where qualitative data is collected, class observations and field notes may not always capture everything that occurs in the classroom. To ensure that data recorded was not biased, triangulation of the data was made possible through review of several places where the data addressed the variables. Triangulation of data (Charmaz, 2006) collected on mindset and lesson objectives was made possible through review of the narratives in focus groups, responses to questions on interviews, and class observations. The mindset survey, individual interviews, and classroom observations allowed for triangulation of data on mindset assessment profiles. In this way, the process of collecting data supported any convergence or divergence appearing from the different data sources. In one instance triangulation did not lead in one direction and instead
produced an alternate view for one of the participants. This divergence is discussed in chapter four for one of the case studies to make sense of it (Mathison 1988).

### 3.7 Summary

As a mixed methods study, the collection of data and analysis of both the quantitative data and qualitative data were carefully designed to address the research questions. In parts one and two of the study, the collection of data was defined. Part three of the study allowed for the integration of all variables being measured, content knowledge, mindset, and lesson objectivization as they played a role in classroom teaching. Beliefs of what teachers need to know to create effective classroom teaching are not something that is learned in a course; however, what teachers believe, what they actually do, and the connections with their teaching and the knowledge they have to do this can be observed and recorded. The analysis that gets to results, findings, and implications are discussed in chapters four and five that follow.
Chapter 4: Results

The study explores one type of teacher knowledge and two teacher characteristics. These are mathematical content knowledge, a mindset for learning, and lesson objectivization. Data analyzed for Parts 1 and 2 of the study, produced results to answer Research Questions 1 and 2. Analyzed data from Parts 1 and 2 helped in selecting four faculty participants for the case studies in Part 3 of the study to address Research Question 3. This chapter presents results and key findings.

4.1 Research Question 1

What is the relationship between content knowledge of mathematics faculty in two-year colleges and their mindset for learning mathematics?

There were two surveys in Part 1. Each participant completed two surveys. Survey one measured faculty content knowledge, survey two measured mindset profiles. A Cronbach alpha for items in both surveys, reported in Chapter 3, showed each set as reliable and internally consistent in what they measured. Analysis of the two surveys showed no statistically significant relationship between content knowledge and mindset profile scores with p < 0.05. All but two of the mindset assessment profile scores identified participants in the growth mindset range. Scores from the other two participants fell within the range of measures identifying a person with a fixed mindset. Using a mindset continuum allows varying measures of mindset for any one person based on a variety of disciplines they may be learning. As an example, a person may have a fixed mindset about learning mathematics, but a growth mindset for learning a new language. That is, they may believe that they cannot learn mathematics, that all the mathematics they know is contained as an
entity and in their brain at birth. They may also believe that they can learn a new language incrementally with guided practice. The continuum allows the concept that one is not placed into fixed or growth mindset but that their mindset profile for learning can be found along a continuum of measures. Forty faculty through a random sample process were invited to complete the survey. When this produced less than thirty participants, a second request to another random sample of forty more faculty was sent; these two invitations yielded thirty-three faculty members who completed both surveys in Part 1 of the study.

4.1.1 Results on a Mindset for Learning

Eight items in Survey 1 assessed the mindset profile for learning for each participant. These measures determined mindset profile scores for learning using the concepts of a growth mindset and fixed mindset (Dweck, 2006; Yeager & Dweck, 2012). Profile measures placed participants along a continuum with growth mindset on one end and fixed mindset on the other end. Mindset profile score can range between 8 and 48. A profile of 8 represents the lowest fixed mindset measure while 48 represents the highest growth mindset measure. Most of the distribution of scores fell within growth mindset measures. Thirty-one of the thirty-three participants fell in a growth mindset range as seen in Figure 4.1. The F2 and F2 buckets with lowest score 21 and highest score 28 show two participants scores in the fixed mindset range, that is, towards the belief that intelligence does not change. The G1 through G5 buckets with range lowest score 29 and highest score 48 show thirty-one of the participants scores in the growth mindset range. These are participants with profiles in the growth mindset range, the belief that you can increase your intelligence by learning. Mindset scores along with content scores will help determine four faculty members for Part 3 of the study. Figure 4.1 shows the distribution of mindset profile scores.
4.1.2 **Results in Content Knowledge**

There were 23 content items in Survey 2. The survey awarded one point for each item answered correctly. Descriptive measures include a mean raw score of 15 correct (65%) with standard deviation four correct (17%). Scores ranged from the lowest raw score of 5 correct (22%) to the highest raw score of 22 correct (96%).

An analysis of the ranking for the difficulty of the twenty-three items showed four of the top five most difficult items were from probability and statistics. Most faculty members at the study site do not teach courses focused on probability and statistics. Four items that were the least difficult for participants were from two areas: 1) functions and graphs and 2) geometry and
measure. Instead, most faculty members teach courses focused on these two areas of mathematics. If the college site follows state and national trends and implements Mathematics Pathways (Kazis & Cullinane, 2015), there will be a greater need and demand for more faculty prepared to teach statistic or quantitative reasoning courses. Students in these non-STEM areas will be taking more freshman statistics courses.

Results on item difficulty identify where the needs lie in mathematical content areas for faculty teaching the freshman mathematics courses. The two most difficult items were items 12 and 22, from probability and statistics and mathematical processes areas respectively. Less than one-fourth of the participants answered these items correctly. The items tested concepts in cognitive levels 1 and two respectively. Items 9, 11, 5, and 8 were the items that were the least difficult for participants. These items tested concepts in the mathematical areas of functions and graphs; and geometry and measure and tested concepts at the second, first, second, and second cognitive levels respectively. Of the thirty-three participants, 88% to 91% of them answered those problems correctly. A summary of these results is represented in Table 4.1.

<table>
<thead>
<tr>
<th>Item Difficulty</th>
<th>Item numbers</th>
<th>Mathematical Area</th>
<th>Percent answering correctly</th>
<th>Item Cognitive Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most difficult</td>
<td>12, 22</td>
<td>Probability and Statistics, Mathematical Processes</td>
<td>24%</td>
<td>1, 2</td>
</tr>
<tr>
<td>Least difficult</td>
<td>9, 11, 5, 8</td>
<td>Functions &amp; Graphs, Geometry &amp; Measurement</td>
<td>88 – 91%</td>
<td>2, 1, 2, 2</td>
</tr>
</tbody>
</table>

To answer Research Question 1, a Pearson correlation test was calculated to see if the data on participant content measures and data on mindset measures are related. The Pearson correlation
was calculated to see if content knowledge was positively related to mindset profile scores. The correlation of content knowledge scores to mindset assessment profile scores for 33 participants and \( \alpha = .05 \), the calculation for Pearson was \( r = 0.1057 \), a value not sufficient to say the two scores were positively related. The same analysis of the correlation between teacher content knowledge was calculated to see if there was a correlation between mindset scores and scores for items in cognitive level 1 (L1), cognitive level 2 (L2), and cognitive level (L3). Each Pearson \( r \) value was not sufficient to say that any of the pairs of scores were positively related. A summary of these calculations is presented in Table 4.2. The accompanying graphs for each are in Figure 4.1.

<table>
<thead>
<tr>
<th>Relationships</th>
<th>Pearson correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mindset with all items in TCK</td>
<td>0.1057</td>
</tr>
<tr>
<td>Mindset with L1 Cognitive Items</td>
<td>0.1249</td>
</tr>
<tr>
<td>Mindset with L2 Cognitive Items</td>
<td>0.0165</td>
</tr>
<tr>
<td>Mindset with L3 Cognitive Items</td>
<td>0.1407</td>
</tr>
</tbody>
</table>

Table 4.2: Correlation Coefficients for Mindset and Content Knowledge Scores
Figure 4.2: Relationship between Mindset and TCK, L1, L2, and L3

The Pearson coefficients do not show sufficient evidence of mindset and content, mindset and items of level 1, mindset and items of level 2, and mindset and items of level 3 scores having any relationship.

4.2 RESEARCH QUESTION 2

How does lesson objectivization in two-year college mathematics faculty relate to their content knowledge?

All faculty participating in Part 1 of the study were invited to participate in focus groups for Part 2 of the study. Fourteen faculty members attended the focus groups and to view a video of an expert lesson. They wrote narratives identifying the lesson objectives. In the first analysis, narratives were coded using open coding. In the second coding identified objectives stating content standards and process standards. They were coded a third time using expert coding (Boaler & Humphreys, 2005). A match was recorded if faculty identified objectives used by the expert by content standard or by process standard. This produced data for our third variable, lesson
objectivization, the ability of faculty to identify lesson objectives in an expert lesson (Michal & Tchoshanov, 2017).

Lesson objectives identified were coded into six content areas and four process areas. The objective narratives were quantified using a matrix showing participants that matched expert objectives with a “1” and those not matching expert objectives with a “0”. Of the fourteen participants, 13 matched the lesson objectives to the expert’s objectives addressing process standards. Of the fourteen participants, 9 matched the lesson objectives to the expert’s objectives addressing content and process standards. Only one of the fourteen faculty did not match of any of the expert objectives. Four matched only process standards. Chapter 3 contains these results. The cross-case review of pairs of faculty members uses these results.

Ability to identify objectives for a lesson, defined as lesson objectivization in this study, is important as it adds another facet to the Shulman (1986) model for knowledge needed by for teaching: content knowledge, pedagogical knowledge, and pedagogical knowledge.

Table 4.3 is the data recorded from the narratives of the lesson objectives. A zero indicates the faculty member did not match the expert lesson objective. A one indicated the faculty member matched the expert lesson objectives. Matches are indicated by content and process.

<table>
<thead>
<tr>
<th>Participants</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>Process</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>13</td>
</tr>
</tbody>
</table>

A Pearson test was used to see the content knowledge of faculty was related to how they matched narratives of the lesson objectives to those of the expert video lesson. This produced a
A statistically significant correlation with Pearson $r = 0.6148$, for $\alpha = 0.05$. A Pearson test was calculated to see if mindset profile scores were related to lesson objectivization were related to lesson objective narrative match scores and produced a correlation of $r = -0.2546$, not statistically significant for $\alpha = 0.05$. Table 4.4 presents these relationships.

Table 4.4: Relationship between content knowledge and mindset to lesson objectivization

<table>
<thead>
<tr>
<th>Comparison between</th>
<th>Pearson r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content knowledge and Lesson Objectives</td>
<td>0.6148*</td>
</tr>
<tr>
<td>Mindset and Lesson Objectives</td>
<td>-0.2546</td>
</tr>
</tbody>
</table>

*p < 0.05

4.3 Research Question 3

How are content knowledge, mindset, and lesson objectivization connected to teaching in two-year college mathematics faculty?

There are four case studies in this next section. Each of the four case studies provides the portrait of a faculty member. Their content knowledge, mindset, and lesson objectivization are explored through the lens of their beliefs about the learning and the knowledge they need to teach mathematics. Faculty members selected for the cases represent four typologies from combinations of the upper quartiles and lower quartiles of content and mindset scores. These four types represent faculty with: high content knowledge and growth mindset scores; high content knowledge and fixed (low growth) mindset scores; low content knowledge and growth mindset scores; and low content knowledge and fixed mindset scores. First, individual faculty are described. Then, the four faculty members are discussed collectively through data from the three variables in the study: content knowledge, mindset for learning, and lesson objectivization. Next, a comparison between
faculty member types connected to teaching is presented using cross-case examinations (Tchoshanov et al., 2018) and the data from classroom observations. After going through emerging themes for each variable, an overall discussion presents the data from class observations. The summary focuses on how the three variables connect to activities recorded during class observations. In particular, the chapter ends with a summary of the observed differences in the activity frequencies each faculty uses to develop students as agents of their learning within a community of learners.

4.3.1 Faculty Case Studies

In this subsection, each faculty is described using data collected on content knowledge, mindset, and lesson objectivization. Themes emerging from interviews with each of the faculty and class observations present a window where one may see how content knowledge and mindset unfold and may be associated with lesson objectivization. After that, pairs of faculty are compared using a cross-case examination. Pseudonyms are used for anonymity.

Four faculty descriptions

**Daniela**

Daniela is a faculty member with content knowledge score and mindset score both in the highest quartiles. Her mindset score indicated she believes her intelligence can increase, cares about learning, and is willing to work hard (Dweck, 2008). People with her profile score want to do well and may give more importance to learning than to performing well (Dweck, 2008). She is credentialed to teach both developmental and college-level mathematics. Daniela matched the content and the process standards in the expert lesson objectives although was not the highest match in lesson objectivization. During class observations, Daniela did not have many instances
where students are in activities that develop them as agents for learning mathematics. She used three instances that engaged students in tasks with the highest level of cognitive demand. These tasks called for students to analyze and synthesize evidence and explain their reasoning. In only one instance did she attribute ideas to students, to build student ownership and identity. Student-to-student interactions were not encouraged in her class. However, Daniela used techniques to build student ownership and identity for learning mathematics, and recognized ideas as student ideas. These last two activities connect her teaching to a high mindset score and to building more growth mindset activities in her lesson objectives.

**Frankie**

Frankie had a content knowledge score in the highest quartile and mindset profile score in the lowest quartile. Her mindset score was low enough to be in the low quartile of measures for mindset profile, however; it was high enough to measure a beginning growth mindset. People with her mindset score are unsure about changing their intelligence. They care about performing and want to learn, but are not willing to work to perform well (Dweck, 2017). Credentialed to teach both developmental courses and college-level courses, Frankie initially did not match the lesson objectives for the expert lesson. However, when interviewed, she mentioned going back to her office and rethinking the objectives. With reflecting on the lesson objectives, she was the faculty who best identified the process standard of the expert lesson objectives, exhibiting lesson objectivization well. Her class observation showed her as an exemplary faculty member at encouraging student beliefs about learning mathematics.

In the second interview, the response to the question, “What do you believe the students in the lesson believed about learning mathematics?” She stated, “The math for them was explaining the question that was on the board, [it was] what they wanted to say. They believed their answer
but were not 100% sure. But, once they were explaining, it [the answer] was justified. They were also contrasting, or they were comparing their answers”. In the interview when responding to the question, “What do you do to prepare a lesson?” she placed importance on the stated course objectives on the syllabus first and on posting guided notes for students to use. For Frankie, adding guided notes for her students served two purposes, to guide students through new concepts and to assist the English language learner. Assisting the language learner indicates her awareness of addressing equity in her classrooms. She also worked problems before using them in a lesson to make sure there was time to show and justify every step.

Frankie included the importance of using lesson objectives to prepare a lesson and re-evaluated the expert lesson objective from her narrative and revised it during her interview. In the class observation, Frankie modeled the importance of engaging students in constructing the mathematics during the lesson and mirrored how students were engaged in the expert lesson. During the class observation, she used a variety of ways to credit students with ideas and solutions as ways to build student ownership and identity for learning mathematics. She praised and encouraged students as mediators of their learning and called them experts of the mathematics the class was learning. In class, she asked students to break down and explain their solutions, a process that parallels Uznadze’s level of objectivization as a level of mental activity. She also ended each student presentation on the board by saying, “so, Alberto (pseudonym for the student), is the expert on compound interest. If you have any questions, you can ask him. Alberto is the expert on annuities.”

With a growth mindset score, even though in the first quartile, Frankie models the level of mental activity described by Uznadze (1966 & 2009), the level of objectivization. She continued to encourage the level of objectivization as students presented solutions to classmates. Continually
referring to students as experts of mathematical concepts they were presenting, she reinforced their identities for learning mathematics. During her interview, she supported this level of mental activity when stating, “I ask them to write solutions step by step and ask them to justify their steps.”

Grace

Grace had a content score in the lower quartile and mindset score in the highest quartile. Like Daniela, her score indicated she believes her intelligence can be increased, cares about learning, and is willing to work hard. She also believes it is more important to learn than to perform well (Dweck, 2008). She is credentialed to teach both developmental and college-level courses. Originally studying to be an engineer, she did not plan on becoming a teacher. During her class observation, one could see how her engineering background influences her lesson objectives and her teaching. Grace encouraged student-to-student discussion and productive dialogue. She assigned a project to be worked in groups that developed analyzing and synthesizing skills. The expectation that students explain their reasoning in their work was evidence that she wanted students to develop these skills. In her interview, she states that learning mathematics is a lot like learning how to deal with life. She tells students, “Learning mathematics is the most beautiful thing in your learning because learning mathematics is learning the logic, learning the reason, learning the problem solving, and those are skills, [that are] part of your whole life, they are life skills.” She engaged students in tasks that encouraged them to apply the mathematics they were learning through projects. Grace did not identify the lesson objectives for the expert lesson. She used the board work displaying student work to justify the lesson objective as learning and using the distributive property of the real numbers. If you isolate only the student work on the board, identifying this objective might be justified for the expert lesson. However, it does not identify the other activities of the lesson and misses identifying the content objective. How she talks to
students about mathematics connects her high growth mindset score to her teaching. She connects learning mathematics to learning life skills. Tasks for students during her class observation, included that that they develop ideas and promoted productive student exchanges. She expected students to analyze, synthesize, and reason while learning mathematics with group projects.

Silver

Silver is credentialed to teach developmental courses and represents content and mindset profile scores in the first quartiles. His mindset profile describes people who believe their intelligence does not change, do not like to make mistakes, are not willing to put in a lot of work, and believe learning should be easy (Dweck, 2008). In several responses to the interview questions, he attributes successful learning of mathematics to the idea that mathematics may be best taught by using memorization and conjuring clever ways to remember content and processes, a skill seemingly devoid of understanding. This runs parallel to the thinking level of set Uznadze (1966) describes as thinking routinely and without much thought, using only sets of knowledge that are learned once, stored, and are called on routinely when needed. Class observations show few times where students were engaged in developing and expressing ideas, had their voices heard, exchanged student-to-student conversations, or developed student ownership of learning the mathematics. His fixed mindset profile score coincides with the level of mental activity Uznadze terms as the level of set. The use of tricks to do mathematics causes concern. When students learn only tricks to work routine problems, the preparation for how to think when a non-routine problem occurs is not nurtured or learned and may leave students unprepared for the next problem, chapter, course, or program of study.
A qualitative review of the faculty interviews, lesson narratives for the four cases, and class observations, show how some of the previous data collected on mindset profiles reflected faculty practices and teaching in the classroom.

4.3.2 Faculty beliefs about what they need for teaching

Two themes emerged from coding faculty beliefs about the knowledge they need for teaching. One is that there is overwhelming agreement that mathematical content knowledge is what is needed most for teaching. The second was the need for knowledge of the social sciences. Important to note was that none of the faculty expressed the need for knowing anything about the cognitive and developmental sciences. As part of the learning sciences (National Research Council, 2000; Sawyer, 2009) knowing about these provide insight on how teachers may pay attention to how students learn (National Research Council, 2000). One mentioned the need for knowing pedagogy. She pointed out that knowing the pedagogy to use to help students understand fractions was useful when teaching some of the developmental mathematics courses.

Mathematical content knowledge is needed for teaching.

As expected, faculty place major importance on knowing mathematical content for teaching. Daniela acknowledged that 75% of what she needs to know is mathematical content knowledge and added that most of what she teaches she already knows. She added that knowing more pedagogy would help her teach basic fundamental skills like fractions but did not place a strong need for knowing pedagogy. When asked what part content played in the knowledge she needed for teaching, Frankie stated, “a huge percentage, the majority I guess”. Grace mentions, “… knowing the mathematics, and to me is almost like 100% of teaching the content”. She was
not certain about how much and stated, “I’m not sure that I can measure it.” Silver did not have a specific quantified answer but stated, “Most of it, most of it comes from my knowledge of math”. None of the faculty qualified whether their knowledge of mathematics was limited to their knowing the mathematical content in the courses they teach or knowing all specific areas of mathematics. The mathematical content survey showed the items at the top of the list of being answered incorrectly were from the area of statistics. Most faculty answered items from algebraic areas correctly. This coincides with the practice that most faculty at the study site teach courses that are algebraic, and less faculty teach courses that are focused more on statistics.

Knowledge of the social sciences is needed for teaching.

The second theme seen to emerge is what faculty believe they need for teaching is knowledge of the social sciences and communication skills. Social science, one of the major disciplines of study that include the study of relationships among individuals. Included in this major discipline are psychology and sociology. Psychology was mentioned by two faculty and sociology was indirectly stated by faculty as other types of knowledge needed to teach. Silver stated, “Well, you need, like, have like psychology… like you don’t just teach, uh, help them a lot…” Grace mentioned that while mathematics was the number one type of knowledge she needed to teach, she added:

Of course, the communication skills, and I think, uh, psychology. You know when you teach math, the instructors, most of the instructors majored in math, right and so the courses they have been taking are just math courses. But suddenly we are put in situations assuming we know how to analyze students, how to grade, and how to handle the conflict and how to balance life, and your professional life, and relationships, professional
relationships with your colleagues or your students. So suddenly, we are just thrown in, and as instructors we still need other skills. So, there are many other skills, I don’t know how to call it, maybe academic skills, and maybe coaching skills. Sometimes the teachers are doing counseling, and we need counseling skills.

In an earlier part of the interview, Grace also mentioned that knowing your content area was important, however, she added, “There are some people who know math very well and can solve the problems very well, but they cannot explain to the students, so that doesn’t help you to be a good educator. And some people have good communication skills, but they don’t have the mathematical knowledge, and they cannot teach either.” She also mentioned that as faculty, “you realize that you have a lot of gaps, and you need to know why, and you need to know how to explain to students deeper… and also the mentoring skills need to be there because you need to know how to motivate your students.” Her discourse painted a more comprehensive view of the knowledge needed for teaching, more so than that of the other faculty.

Frankie alluded to needing communication skills when she stated, “…well, I guess I need to know how to read the students. Um, for example, if they are getting stuck, if they look like they are trying to say something, if they have doubt in their face, um, or even some other body language, you can tell they need more explanation, or they have doubts. So, knowing how to read their body language… Knowing how to work with a lot of different personalities, because there are a lot of older students, there a lot of younger students, fresh out of high school, so each class is different. For example, in the next class starting at noon, they just graduated from high school, and require a lot more energy in the classroom from me.”

Silver stated the need for knowing psychology and social skills. He stated:
Well you need, uh, to have like, psychology, uh, be friendly with the students. You know what I mean, have a nice personality, be personable with the students. Like you know, you don’t just teach, uh, [you] help them a lot. In other words, you need to be involved with your students, not just show them; you know what I mean? Like if they have a problem, help them out. I like to help them out.”

Knowing good communication skills and knowledge in social skills surfaced as the second most important skill needed for teaching from three of the four faculty members. Only one mentioned pedagogy. In coding and re-coding the transcripts of knowledge for teaching, pedagogy was missing in all but one of the faculty interviews.

**There was no mention of issues related to learning**

As important as the cognitive, developmental, and the learning sciences (National Research Council, 2000; Sawyer, 2009) have contributed to teaching and learning, it is important to note that knowledge of these sciences was not mentioned by the faculty. As a fairly new body of science, it has appeared in many articles over the past 10 to 15 years. And yet, none of the faculty spoke of this as contributing to their knowledge for teaching. Much research over the past thirty years has emphasized learning how to understand better what processes, both cognitive and social, create effective teaching (Sawyer, 2009). In many cases, researchers from different disciplines have begun to work together bringing broad approaches to reform education. Bringing in more to what faculty may know about learning can potentially turn the page for many students who have been on the same page when it comes to learning mathematics.
4.3.3 Faculty beliefs about a mindset for learning mathematics

In Part 1 of the study, Survey 1 collected quantitative data to measure faculty mindset assessment profiles. In Part 3 of the study, we see how growth and fixed mindset beliefs appear in faculty responses as they talk about their experiences when learning mathematics, how they view learning, and how they view student learning. Themes around struggling to learn, the usefulness of mathematics, and ways to learn mathematics emerged when faculty talked about their learning and student learning of mathematics.

Faculty members with a growth mindset and that use lesson objectives to engage students in productive learning activities can practice effective teaching. However, faculty must recognize broader issues that may prevail. Effective teachers include activities where students are included in a community of learners that add to their knowledge and to the knowledge of a community of learners. Effective teachers also recognize that constructs alone, like growth mindset, do not address broader institutional and systemic inequalities that may exist.

Struggles, problems, issues, and negative thoughts about learning mathematics

Three of the four faculty fell within the range of growth mindsets on the mindset continuum scale, with one of those three falling almost at the fixed mindset range. However, all faculty used negative descriptors when describing student experiences when learning mathematics. Although all described good experiences about their learning of mathematics, not all started with good experiences. However, when talking about students, they all stated that many students do not have positive experiences about learning mathematics. While three of four faculty members use their own experiences to assist students who do not believe they can learn math, one just tells them they can do it. Table 4.5 has a summary of faculty experiences while learning mathematics, what they
do to help students of their students, and what they think about students in the lesson they viewed on video.

Table 4.5: Faculty Descriptions and Understandings of Learning Mathematics

<table>
<thead>
<tr>
<th>Faculty</th>
<th>Faculty experiences</th>
<th>How do faculty help students learn mathematics?</th>
<th>Students in the video lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daniela</td>
<td>I struggled…, was not proficient, I had trouble … until more caring teachers said it was like a puzzle… it did not matter that it took more time to work the puzzle.</td>
<td>I try to remember myself when I struggled…, just – it takes them longer.</td>
<td>The students in the video looked like they were enjoying themselves.</td>
</tr>
<tr>
<td>Frankie</td>
<td>We had a group of nerds in high school, we talked math. I tutored math in high school.</td>
<td>I have several … show up in my office with negative thoughts. I have to keep a positive mind and work to have students change their perspective … if they have issues that block their learning, I have an open-door policy to address these issues.</td>
<td>If it is from their reaction, the math was explaining the problem. They had to believe in their answer; they believed; they did do the math correctly.</td>
</tr>
<tr>
<td>Grace</td>
<td>I never thought math was boring. I knew I needed it for engineering, so I learned my math. As soon as my teacher showed me the reason why I began learning and understanding mathematics.</td>
<td>I tell them practice is the key thing for any successful skill. I was not always an A student because I did not see the application. As my teacher showed me the reason why I began learning. I share how I learned math to convince them.</td>
<td>In that lesson, [there] was active learning. The students did not know they were doing mathematics … so the students started reasoning.</td>
</tr>
<tr>
<td>Silver</td>
<td>I found it as a hobby; I always liked it. In college, I was not bored at all.</td>
<td>Most students do not like math; I found that out. You just tell them that it can be done. I just tell them, if you practice, you can do it.</td>
<td>That there are different ways to do the math, not just one way. There are different ways to solve a problem.</td>
</tr>
</tbody>
</table>
Daniela and Grace scored in the highest quartile on growth mindset. There is evidence that each has a strong belief in growing mindset in students. Daniela states, “I struggled with mathematics as a young child… But as I got older and started algebra, I had teachers who were more patient. It was like working a puzzle. I had several teachers who were more caring, math teachers who said it was like working a puzzle and it did not matter that it took time to work the puzzle. When asked how she worked with students who believed they could learn mathematics, one sees how she uses her learning experiences. She states, “I try to remember myself when I was struggling and think sometimes it takes a little longer and it’s not necessarily that they can’t learn it, just that it takes them longer.” When asked how her beliefs about learning informed her teaching she states, “I think it influences a lot. Because some of the articles I’ve been reading are about how the brain works”. Daniela included the idea of growth mindset in her classroom from these articles, she states, “… in my algebra class, we were looking at how the brain works, regarding how to get some ideas across, and the activity helped them [the students] for problem-solving.” Grace stated this to her students in more direct ways, she states, “I always tell my students to think about what they are doing in life, if they are playing basketball, or baseball or playing an instrument, they have to practice, but they have to combine those skills as well, those other skills, together… practicing mathematics is similar to the way you need to learn that it’s your brain that puts those skills together.” She later added “… you can modify your learning.”

Although Frankie and Silver had mindset profiles in the first quartile, Frankie has a score along the continuum that was in the growth mindset range and Silver has a profile in the fixed mindset range. Frankie has tried to change student perspectives and negative attitudes, she states, “I know that I have to keep a positive mind and have the students change their perspective as well. … I do positive reinforcement.” Little evidence pointed to student growth mindset in the interview
with Frankie. However, during the class observation, Frankie had the highest number of times where she engaged students in activities to create their identities for learning mathematics. Her profile score shows a low growth mindset profile score. However, her classroom activities give students options to stimulate growth mindset. This does not appear as strong on her responses to the interview questions related to mindset.

Silver also scored in Quartile 1 for mindset profile and showed no evidence from his interview of knowing or using the concept of mindset for learning. This is further evidenced by his classroom observation. During the class observation, he worked all the problems on the board while students copied his work. He asked for questions after each problem, however; students did not have questions nor comments. While stating several times that he enjoys teaching and learning mathematics and enjoys helping students, these qualities seemed not to nurture classroom objectives that led to learning.

As a formal type of knowledge, faculty members did not talk about using activities that nurture a growth mindset in students. However, each believes their way of helping students may work. Daniela tries trying to make “math fun”; Frankie gives them “giving them positive reinforcement,” Grace connects learning math skills to “learning life skills,” and Silver by working all the problems himself and “showing them tricks.” There is no common thread to their strategies for teaching. In unpacking the data more closely in the cross-case analysis, we see how content knowledge and mindset connect to their teaching.

4.3.4 Faculty and Lesson Objectivization

During individual interviews, faculty members were asked to reiterate the lesson objectives for the expert lesson they viewed in Part 2. Like an ability to identify lesson objectives, results
showed for the most part; faculty tended to identify mathematical process standards and not content standards. This was a similar finding in a pilot study with preservice teachers (Michal & Tchoshanov, 2017). Only one of the four faculty identified the expert lesson objectives. In the earlier sample of 14 faulty in the focus group, there were four identified both the content and the process objectives for the lesson. In this smaller nested group of four faculty, none identified the content standard. Table 4.6 summarizes their lesson objectivization ability as seen from the interviews and the earlier lesson objective narratives.

Table 4.6: Faculty Narratives and Interview Responses for the Expert Lesson Objectives

<table>
<thead>
<tr>
<th>Faculty</th>
<th>What were the lesson objectives for the lesson?</th>
<th>Narratives</th>
<th>Interviews</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daniela</td>
<td>To discover the properties of the distributive property, associative property, etc.</td>
<td>Like I mentioned earlier, I think it was looking at the properties of numbers, um, also like dealing with maybe factoring. Cause of the way that they were breaking it down.</td>
<td></td>
</tr>
<tr>
<td>Frankie</td>
<td>[It] had several. One was problem-solving, critical thinking skills, to listen and repeat others opinions, the order of operations</td>
<td>Well, I think, when I came back to the office, I was thinking this could lead to deductive reasoning and generalizing a formula for an n by n cube.</td>
<td></td>
</tr>
<tr>
<td>Grace</td>
<td>The distributive property</td>
<td>The distributive property and I forgot if I put down the associative property or not, but definitely I put down the distributive property. I think it is the grouping of the distributive property.</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>Using different ways to solve a problem, using different operations to solve a problem</td>
<td>To show them the different ways that something can be solved. And I had another one, but I forgot. Let’s see, and to have the interaction between students, to have the students so they can be involved, involving the students in the lesson.</td>
<td></td>
</tr>
</tbody>
</table>
Frankie had content knowledge in the fourth quartile and a mindset profile in the first quartile, however, measured a growth mindset. It was clearly very implicit to her when she viewed the video lesson but only after going back to her office. Along with stating and matching the expert objective during the interview, she also saw the connection in the process used by the teacher in the video. She made the same assumptions as the teacher in the lesson. The most important of these was that she attributed her students as the experts in mathematical content. As each presented a problem, she called the student the expert for that content and made them responsible for receiving questions from students. In doing so, she created in her students their identities for learning mathematics.

4.4 CROSS-CASE FACULTY COMPARISONS

In this section, we see how the three variables may connect to what was observed in faculty classrooms using cross-case comparisons for some pairs of the faculty. Table 4.8 has a summary of faculty classroom observations. The observation protocol collected activities and tasks that led to students developing identities for learning and become agents of their learning in mathematics. Table 4.7 shows frequencies of how faculty used the objectives for their lessons to develop student identities for learning mathematics. While the class observations were used to triangulate some of the data from the interviews, it has provided both a mirror of what is occurring in classrooms and a window of what we may be studied next. Along with the data collected from the protocol, some faculty shared handouts, adding artifacts to the data sources for some faculty. Faculty names are pseudonyms to preserve anonymity.

Table 4.7: Faculty* Class Observations of Activities and Tasks

<table>
<thead>
<tr>
<th>Class activity</th>
<th>Daniela</th>
<th>Frankie</th>
<th>Grace</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Clarifies lesson objectives</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
2. Provides time for students to develop and express ideas

3. Works to ensure all students have opportunities to be heard

4. Encourages student to student discussions, promotes productive exchanges

5. Uses tasks and questions calling for marshaling, analyzing, synthesizing, reasoning

6. Uses techniques attributing ideas to students, build ownership, identity

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Daniela</th>
<th>Frankie</th>
<th>Grace</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Help for students</td>
<td>Offer general help to students</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Responsibility for student learning</td>
<td>Offers specific help to students, mentor, understand, encourage, cheer on, contact students; provide open door policy, positive thinking activities, motivation, tutoring</td>
<td>1</td>
<td>8</td>
<td>14</td>
<td>1</td>
</tr>
</tbody>
</table>

Faculty Totals: 15 37 27 13

*Pseudonyms*

Table 4.8 records frequencies of faculty responses when talking about their teaching.

Tables 4.7 and 4.8 record observations and narratives about their teaching and show connections between mindset, lesson objectives, and their teaching in the cross-case examinations. Faculty members with strong differences in activities were selected to discuss in the cross-case analyses. The cross-case analysis involved fixing content scores first and then fixing mindset profile scores. A key finding from analysis of the four cross-case analyses added another cross case between two faculty members with similar frequencies in some activities.

Table 4.8: Faculty Responses about their Teaching
<table>
<thead>
<tr>
<th>Faculty engaged in mathematics</th>
<th>Show them, tell them, do it, work problems, explain</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students engaged in mathematics</td>
<td>Ask students to show, explain work, write paragraphs about results, teach to someone, compare answers and write comparisons, involves students</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>Addresses equity</td>
<td>Attributes decision to teach to an equity agenda</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Mathematics as fun</td>
<td>See mathematics as a puzzle, using tricks</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Mathematics as a discipline</td>
<td>See mathematics as a science, a tool for careers, logical, making meaning, connecting skills</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Connections</td>
<td>Connect mathematics to the real world, uses applications</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Cognitive</td>
<td>Connect brain to learning and modifying learning</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>Lesson planning</td>
<td>Use lesson objectives, resources, projects, guided notes, current events, and applications; develop vocabulary; prepare examples</td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Value continued knowledge</td>
<td>Continue learning, develop professionally, develop skills, share knowledge</td>
<td>2</td>
<td>2</td>
<td>13</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4.4.1 Cross-Case Examination #1

**Two faculty with low content scores, one with high and one with low mindset scores**

In this section, we look at Grace and Silver. Both had low content scores. Grace was in the highest quartile of mindset scores, and Silver was in the lowest quartile of mindset scores. Examining faculty data across cases, we see there is a contrast in two faculty members, Grace and
Silver in two activities. In activity #2, providing time for students to develop and express ideas Grace has a four while Silver has a zero. In activity #4, encouraging the student to student discussions that may promote productive exchanges, Grace has six while, and Silver has zero. They are both in the first quartile of content scores, however, differ in their mindset scores. For this difference in the two, we see how mindset connects to this type of activity to faculty with low content scores.

Table 4.9 shows the frequencies for Grace and Silver previously recorded.

**Table 4.9: Faculty Class Observations of Activities and Tasks, Grace and Silver**

<table>
<thead>
<tr>
<th>Class Activity</th>
<th>Grace</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Provides time for students to develop and express ideas</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>4. Encourage student to student interactions promotes productive exchanges</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Grace used the activities sometimes, while Silver was not observed using them. She used a project for students to apply the use of linear systems of equations to solve a problem using Kirchhoff’s Rules to find currents in a circuit. In her interview, Grace stated that she learned mathematics only when she saw it in applications and she was able to see how it was used. Using applications to learn is of major influence in her choice of class activities and in how her growth mindset connect with her teaching. With a growth mindset, she adds a difficult topic, physics, into her lessons to allow the student to see connections to mathematics. Her students worked in groups to apply systems of equations, to solve the application problem. She is the one who started out as
an engineering major and understands the applications students need to connect mathematics to fields of study.

The table shows no tallies for Silver under these two activities. While students were given the opportunity to ask questions, he worked all the problems on the board. His students were engaged in copying solutions to the problems he was working on the board. During his interviews, he stated his main reason for teaching was for his enjoyment. Open coding of his interview reveals the number of times where it is he who is working the mathematics with the phrases “I show them” and “I tell them” appearing 17 and 12 times respectively. Table 4.10 shows a comparison of statements as Grace and Silver talk about students who believe they cannot learn mathematics and how they viewed student learning during the expert lesson.

Table 4.10: Contrast of two Faculty with lower content scores and high, low mindset scores

<table>
<thead>
<tr>
<th>How do you work with students who believe they cannot learn mathematics?</th>
<th>Grace – assigned a project for students to apply the mathematics</th>
<th>Silver – worked all the problems on the board</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Usually, it will take me some time to discover who they are.</td>
<td>I just tell them that it can be done, I try to tutor them. I just tell them. I tell them, it can be done. It can be done. You just got to learn how to do it.</td>
</tr>
<tr>
<td>2</td>
<td>I will try to meet with them individually, and I like to sit down with them</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>I encourage them to learn more math</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>It was very active learning. I think the students did not think they were doing mathematics.</td>
<td>That there are different ways to do the math, there is not just one way that you can do it.</td>
</tr>
<tr>
<td>5</td>
<td>The students can express their opinion; they can use a different kind of thinking.</td>
<td></td>
</tr>
</tbody>
</table>

When we see the interview data for Grace and Silver, in Table 4.8, Grace commented 14 times of the responsibility for student learning. Silver stated, that “he likes to helps students”
however did not state specific ways of how he might support learning other than showing students “other ways to do mathematics.” Grace gives specific examples of what she wants students to learn and how she helps them, stating, “I want them to be independent learners, not just mathematicians. … How to start the mathematics when they don’t have the resources. Or when they don’t have good instructors, to learn mathematics in a meaningful way. To combine skill… and to know that your brain puts those skills together.”

4.4.2 Cross-Case Examination #2

Two faculty with high content scores, one with high and one with low mindset scores

In this section, we look at Daniela and Frankie. Here both faculty membersss are in the highest quartile in content scores but at different quartiles in mindset profile scores. Daniela is in the highest mindset quartile and Frankie in the lowest mindset quartile. Table 4.11 shows the frequencies for Daniela and Frankie previously recorded. Frankie has very high frequencies in both activities.

Table 4.11: Faculty Class Observations of Activities and Tasks, Daniela and Frankie

<table>
<thead>
<tr>
<th>Class Activity</th>
<th>Daniela</th>
<th>Frankie</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Uses tasks and questions calling for marshaling and analyzing, synthesizing, reasoning</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>6. Uses techniques attributing ideas to students, build ownership, identity</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>

In Table 4.8, Frankie has eight times where she talks about her responsibility for student learning where Daniela has one. When looking at how they view mathematics, we see Frankie sees mathematics for students as a discipline while Daniela views making mathematics fun for
students. An observed contrast also appears in Table 4.11 between Daniela and Frankie in activities #5 and #6 that call for students marshaling, analyzing, synthesizing, reasoning and the use of techniques attributing ideas to students, building ownership, and identity for learning. The high use of activities for Frankie may be due to her lesson objectivization. Frankie directly matched the expert process objectives in her narrative and then when asked in her interview, matched both the content and process objectives. Daniela did not directly identify the objectives but did connect and identify concepts leading to both content and the expert objectives. Statements from their interviews of several questions reveal information on how they engaged students in the class. The frequencies of activities during class observation shows specific descriptions of activities Frankie describes direct ways to create student identities for learning mathematics compared to the general applications stated by Daniela and the idea that math is like puzzles. Neither of the faculty has changes that redesign the lesson in major ways. Table 4.12 shows this contrast in responses from interview questions.

<table>
<thead>
<tr>
<th>Question</th>
<th>Daniela</th>
<th>Frankie</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do you need you know to teach?</td>
<td>To make the links to the real world, applications to see the links to the real world.</td>
<td>You need to know what their previous knowledge is, not just to do the work but get them to explain the work. [I] try to get them to know their mistakes and to learn the right way to do it.</td>
</tr>
<tr>
<td>What would you have done differently to teach the lesson in the video?</td>
<td>I would have been starting with showing them with numbers too.</td>
<td>I would have them, go up to the board, and while they were up at the board, see how they did it differently.</td>
</tr>
<tr>
<td>What did students in the video believe about their learning?</td>
<td>They looked like they were enjoying themselves, and exploring, talking about it so they could learn it.</td>
<td>In the beginning, I think they were shy. They believed their answer, but [were] not 100% sure. … Once they were</td>
</tr>
</tbody>
</table>
explaining, it was justified. They believed they did do the math correctly.\\n\[297\times53]106 \\
[193\times709]16 \\
[193\times695]17 \\
How does your life inform your teaching? \\
18 I like to do math puzzles and stuff ... so I try and incorporate those in so when I teach, to make the math a little more fun. \\
19 I focus on allowing the students to show their work. I guess current events. Each class has several projects. I showed them how to use the Central Appraisal District website and to compute a 4% property tax increase for the next year...and provide me a paragraph of the results and who they taught it to. \\
20 \\
21 \\
22 \\
23 \\
24 \\
25 \\
26 \\
27 \\
\[72\times476]4.4.3 Cross-Case Examination #3 \\
Two faculty with high mindset scores, one with low and one with high content scores \\
In this section, we look at a cross-case examination of Daniela and Grace. Both scored high in mindset scores but were in high and low content quartile scores. Daniela scored high in content and Grace scored low in content. Table 4.13 shows the frequencies for Daniela and Frankie previously recorded. Daniela seems not to give students activities that build ownership of learning mathematics. \\
Table 4.13: Faculty Class Observations of Activities and Tasks, Daniela and Grace \\
<table>
<thead>
<tr>
<th>Class Activity</th>
<th>Daniela</th>
<th>Grace</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Uses tasks and questions calling for marshaling and analyzing, synthesizing, reasoning</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>6. Uses techniques attributing ideas to students, build ownership, identity</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
In Table 4.8, there is a noticeable contrast in faculty talking about their responsibility for student learning. Grace has a fourteen, and Daniela had a one for this category. Grace sees mathematics as a discipline impacted by what she would like anyone to learn. She says, “… my background was engineering, mechanical engineer and I see the occasional math. …I never though mathematics was boring, I knew I needed it for my engineering. So, I learned my math because it was a tool for me, because of what I was trying to accomplish for my engineering field … to me it was like logic … it just made sense, and I love that it is as black and white, there is no grey area.” Daniela understands student’s struggles and tries to remedy that by looking at mathematics like solving puzzles. The contrast in how they view mathematic occurs in some responses in Table 4.14. They talk about their experiences as teachers, student beliefs in the lesson they viewed, and how their lives impact their teaching.

Table 4.14: Two faculty with high mindset score but different content scores

<table>
<thead>
<tr>
<th>Questions</th>
<th>Daniela</th>
<th>Grace</th>
</tr>
</thead>
<tbody>
<tr>
<td>What can you tell me about your experiences in being a teacher? How is teaching different from other careers?</td>
<td>Originally, I had chosen actuarial science. But as an actuary, I spent my time in a little cubicle. So, I have a little more understanding.</td>
<td>It is a huge responsibility, my experience being an engineer was from 8 to 5; I found that I could tune myself out. But being a teacher, everything is connected to the classroom. And, this is [a] huge responsibility because it is the future of your students.</td>
</tr>
<tr>
<td>What did students in the video believe about their learning?</td>
<td>They looked like they were enjoying themselves, and exploring, talking about it so they could learn it.</td>
<td>It was very active learning. I think the students did not think they were doing mathematics. The students can express their opinion; they can use a different kind of thinking. It was what the mathematics is supposed to be. The students started reasoning; they came up with their formula or expressions for that, not given by the teacher.</td>
</tr>
</tbody>
</table>
How does your life inform your teaching?

I like to do math puzzles and stuff. So, I try and incorporate those in so when I teach, to make the math a little more fun.

I discovered that many students did not know mathematics that I learned in middle school, so I was a little shocked with that. At that time, I was carrying my older one, and I did not want my child to grow up in this kind of environment, so I wanted to contribute to the society to make a little bit of change, change that view that people had, to change the view about learning mathematics. That is my starting point in teaching and I still believe that. I want to teach to learn mathematics in a meaningful way. I want them to learn to be independent learners ...

4.4.4 Cross-Case Examination #4

Two faculty members with low mindset scores, one with high and one with low content scores

In this section, we look at a cross-case examination of Frankie and Silver. Both scored low in mindset scores but were in high and low content quartile scores. Frankie scored high in content and Silver scored low in content knowledge. Table 4.15 shows the frequencies for Frankie and Silver previously recorded.

Table 4.15: Faculty Class Observations of Activities and Tasks, Frankie and Silver

<table>
<thead>
<tr>
<th>Class Activity</th>
<th>Frankie</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Provides time for students to develop and express ideas</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>3. Works to ensure all students have opportunities to be heard</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6. Uses techniques attributing ideas to students, build ownership, identity</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 4.8 reports a noticeable difference in the frequencies that where faculty are doing the mathematics and where students are doing the mathematics. Frankie has no instances where the faculty is the one engaged in the content and six times where students are doing the work. Silver shows one response where students are doing the mathematics and 29 instances where the responses are coded as the faculty is the one engaged in and is doing the mathematics. As reported in an earlier cross-case analysis, there are 12 times documenting him saying “I show them”, and 17 times saying “I tell them” when referring to classroom teaching. Table 4.16 documents frequencies of class activities during class observations and shows the contrast in three class activities. In activities #2 #3, and #6 during class observations where faculty are providing time for students to develop and express ideas, ensuring students are heard, and attributing ideas to students to give them ownership and identity, Frankie scores 7, 5, 8 and Silver scores 0, 1, 3, respectively. We also see evidence of this in their interview responses in Table 4.16.

Table 4.16: Cross-case of two faculty with low mindset score and different content scores

<table>
<thead>
<tr>
<th>Questions</th>
<th>Frankie</th>
<th>Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td>How did your experiences in learning mathematics enter in you selecting teaching as a career?</td>
<td>1 As I moved to college, tutoring and helping others in learning others came very easy. There was a time when I was just taking math classes, I was taking physics, all of K-12, and all of a sudden, I ended with a lot of math courses. A professor told me to apply for a teaching assistantship and I was involved in a lot of projects. I did a lot of summer research programs, and that gave me exposure with that I could do with math.</td>
<td>I wanted people to learn what I knew. And I just started teaching, doing just that. When I was working, I used a lot of math. I always liked to teach.</td>
</tr>
</tbody>
</table>
What would you have done differently to teach the same lesson in the video?

16

Oh, wow! I don’t know where

13

… the only different thing is

17

[to] have them determine what

18

was multiplied and added. And

19

instead of having the teacher

20

write the operations for each of

21

them to compare 8 and 6 …

22

Writing those steps down and

23

then, I guess, I don’t know

24

maybe to compare the 8 and the

25

6 and have all the students

26

repeat that. And while they

27

were up at the board, see how

28

they did it differently.

29

30

31

32

What would I have done
differently? That’s different,
because I probably would have
done the same thing. I
probably would have, well first
of all show them the different
ways that it could be done.
And then, no pos [no well] I
would just show them the
different ways that is could be
done. Well probably would
have, well first of all would
show the different ways that it
could be done. And then, okay
now you do it, show me a way
that you would do it. Show me
the best way.

In table 4.6, we see some interesting statements by the faculty starting with line 18 where Frankie suggests that instead of having the teacher write the operations, she would have students each of the students compare the differences they see in using an 8 by 8 square and using an 6 by 6 square. In her responses we see her referring to having students do the work and showing each step of the work. This may come from here high match of the lesson objectives and her growth mindset of how things may be done incrementally, step by step, to grow understanding and intelligence incrementally. Silver would show students the different ways himself.

The cross-case analyses tell us that looking at teacher knowledge together with beliefs and practice makes these things difficult to assess. We do see differences when faculty have growth mindset and a good sense of lessons that may create productive lessons, but our further study may be needed to see if these differences may also be due to other things.
4.5 FINDINGS

#1 Faculty with the knowledge of mathematics as a discipline to study, coupled with a growth mindset, engage students in activities that develop meaningful and productive learning of mathematics for themselves and for the community of learners around them.

Combining mindset, lesson objectivization with applications from other fields and the daily world of students may help students develop agency for learning mathematics in classrooms. The study did not identify the knowledge faculty need that will lead to developing agency for learning in students. However, in this study, faculty used strategies leading to activities that create agency for learning and develop identities for learning. Faculty stated beliefs about learning and gave us images of classroom activities of what they are doing and not doing while teaching. Effective classrooms were defined for this study to be classrooms where students develop agency for learning and for the learning of others. Two faculty, one with high content knowledge and one with low content knowledge but both with a well-defined sense of mathematics used activities that have been shown to develop students as productive learners of mathematics. The two faculty that understand mathematics as a discipline for learning, are faculty that embrace the equity agenda in their classrooms, and use lesson objectives with well-defined activities for learning. Analyses of the cross-cases showed that high content knowledge is needed for effective teaching. The cross-cases also showed that when content is low, a growth mindset will make a difference in effective teaching.

#2 Two-year college mathematics faculty knowledge of content varies by domain. Knowledge of statistics and probability are not as comprehensive as knowledge in the areas of functions and graphs and of geometry and measure. After analysis of results from the two surveys it was found that faculty content knowledge is strongest in functions and graphs and in geometry
and measure. Faculty content knowledge is the weakest in statistics and probability. The four items from the area of functions and graphs and the area of geometry and measure were the ones most participants answered correctly these items. Four out of the top five items that were most often missed were from the area of statistics and geometry. More detailed analysis showed items participants most missed were from the highest level of cognitive demand. For the study site, this is relevant because more faculty will be asked to teach statistics and quantitative reasoning courses.

#3 Knowledge of effective learning in research from cognitive, developmental, and effective learning strategies were not included in what faculty identified as knowledge needed for teaching.

Faculty in Part 3 of the study agree that their content knowledge, mathematics, plays a major and extremely important part in the knowledge they need to know to be able to teach. In fact, it is what accrediting agencies specifically and clearly place at the top of the list when credentialing faculty to teach at postsecondary institutions. When asked about other knowledge needed, faculty included the some of the social sciences and pedagogy (pedagogical content knowledge did not surface as a type). However, faculty did not mention the need for knowledge of the cognitive sciences or developmental sciences. While this knowledge may have been alluded to, it did not appear as a major piece of knowledge needed for teaching by any of the faculty. This was unusual since studies on many of the learning sciences are not new. As part of the studies on learning, the learning sciences is a field that has studied teaching and learning since the 1970’s (Sawyer, 2006). Specifically, current studies in the cognitive and developmental learning sciences have clarified much of what may positively impact the dynamic of teaching and learning in classrooms today.
4.6 Summary

The qualitative data added faculty voices and classroom images of four faculty. The faculty represented combinations of high and low content knowledge and mindset profile scores. The two faculty that showed effective classroom strategies are the faculty with clear visions of an agenda for equity in classrooms. As faculty, they understand how knowledge in mathematics creates knowledge for better lives, something Gutierrez calls political knowledge for teaching (Gutierrez, 2009 & 2018). As effective teachers, they recognize the broader institutional issues students face and find ways to include students with applications and projects that stimulate their thinking. In this way, these faculty know how to use classrooms as platforms for advancing equity.

It is clear that all faculty expressed heartfelt desires to help students succeed in their courses. Daniela with high content and high mindset measure has the right idea when wanting to learn more pedagogy to use in her classroom and when using studies connecting the brain to learning. However, her interview and classroom observation do not show a lot of student engagement in their own learning nor any specific description of applications she used to nurture mental activities in ways other than making mathematics like puzzles and fun. She stressed the importance of the role of pedagogy for knowledge needed for teaching showed no signs of using pedagogies or pedagogical content knowledge.

Frankie, with high content and low growth mindset, uses applications that engage students in activities. She sees mathematics as a discipline leading to self-learning and better lives for students. Through a clear view of the objectives of her lessons, she demands her students to use the level of objectivization as they work through projects, explain them, and justify their steps. Her teaching reflects a strong count in the activities she used to build agency in her students. Grace, with low content and high growth mindset, uses her interest in continued learning and engages
students in project based learning. A firm believer in modifying one’s knowledge, she engages students and guides students to make connections between the mathematics they are learning and projects in other fields. Additionally, of the four studied, Grace has the most generous equity agenda, an agenda that provides her purpose and motivation for teaching. Silver, with low content and low mindset profile scores, truly has his heart in wanting to help students successfully complete mathematics courses. In faculty-centered classrooms, like his, faculty explain and show every problem that will be used to assess student learning. However, as a model for teaching, learning that is faculty centered does not nurture agency for learning in students. His strong desire to help students will benefit from added professional development in teaching and learning.

Content knowledge alone may not give faculty what they need to have classrooms that engage students in learning mathematics. Adding knowledge in pedagogy and pedagogical content will get us closer to effective classroom teaching. However, this study focused on content, mindset, and lesson objectivization. Although mindset was measured for every participant, the study was not designed to offer professional development to develop the ideas of a mindset for learning to students. However, adding mindset and lesson objectivization to professional development may assist faculty in using growth mindset in teaching. It may allow more strategic ways of using activities that nurture a growth mindset and bring in the level of objectivization into classrooms. Doing so may lead to ways to grow identities for learning mathematics in students in communities of learners. Understanding how a mindset for learning mathematics can run parallel to levels of mental activity that make students objectivize obstacles, may get faculty closer to providing students what they need to become their own agents for learning.

Teaching and learning as a dynamic using levels of mental activity goes beyond using the level of set; it demands the use of the level of objectivization. This may get faculty closer to
creating effective classrooms. Knowledge of a mindset for learning, closely parallels Uznadze’s levels of mental activity. Professional development programs can focus on connecting faculty beliefs in growing intelligence incrementally to engage students in nurture this level of mental activity in students.

So, what are the connections? Data from all three variables connect to activities recorded during class observations while faculty were teaching. Some major differences in frequencies were observed. In particular, the differences in faculty frequencies of activities that address equity in classrooms are pulled out of Table 4.7 for two faculty Grace and Frankie. They are the two faculty that appeared ahead in effective classroom activities in the four cross-case analyses. This analysis helped to unpack other similarities in Frankie and Grace and possible connections to teacher characteristics and effective teaching. Their observed frequencies for two, activities, #2 and #6, are recorded again in Table 4.17 for easy reference.

<table>
<thead>
<tr>
<th>Class Activity</th>
<th>Grace</th>
<th>Frankie</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Provides time for students to develop and express ideas</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>6. Uses techniques attributing ideas to students, builds ownership, identity</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>Faculty totals</td>
<td>7</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 4.17 shows the frequencies for Grace and Frankie previously recorded in Table 4.8.

This table will be discussed again in Chapter 5 as it presents an interesting cross-case analysis for discussion.

Results indicate that content knowledge alone may not ensure faculty have the ability to identify lesson objectives in ways that nurture students as learners of mathematics. Faculty
engaging students in activities that promote students as their own learners of mathematics are not necessarily faculty with the highest measure of content knowledge, they are faculty with a strong sense of mathematics as a discipline or field of study. They are faculty who have embraced equity in their classrooms as part of their agenda on equity. Students in the faculty observed in Part 3 of the study were engaged in their learning and in the learning of the larger community of learners in their groups.

Grace and Frankie, two faculty creating students as agents of their learning, offer effective classroom teaching where students work projects or applications connecting mathematics to other fields of study or to the world they live in. The classroom experienced by their students engaged students with the mathematics while learning the mathematics. In each cross-case analysis where Grace and Frankie appeared, we see student engagement with mathematics more prominent in classroom observations and in interview responses. Both used applications and projects to get students through learning activities that connect their learning to mathematics.
Chapter 5: Discussion and Conclusion

Chapter 5 discusses the results and findings from the data and overall investigations in the study and discuss how they inform the purpose and objectives of the study. It includes implications of the study and recommendations stemming from the results and key findings. The discussion starts with the objectives of the study and how the results and key findings inform each objective. The discussion includes implications that specific findings may have on teaching and learning at two-year colleges. Following the implications are some recommendations the community of educators and researchers of education may consider for action.

Introduction

The study was designed to identify the connections between the content knowledge, a mindset for learning, lesson objectivization, and teaching of two-year college mathematics faculty. It investigated connections between content knowledge, mindset, and lesson objectivization and teaching of mathematics faculty in two-year colleges. Designed to look beyond content knowledge for teaching, the study focused on teacher characteristics of mathematics faculty in two-year colleges. The study looked at four faculty who face large numbers of students entering underprepared to succeed in first-year college-level mathematics courses. If the study helps determine and identify the kinds of knowledge and characteristics that mathematics faculty need to create effective classrooms, it will address the needs of thousands of students. The study stands to give two-year colleges more insight on how best to prepare mathematics faculty for teaching students as they enroll in the first two years of college. Presently, the result of having so many
students who are taking courses in college with content that was covered in high school courses is impacting funding for those courses in many states.

5.1 Objectives of the Study in Light of the Results and Findings

The first objective was to review studies in the literature on growth and fixed mindsets for learning, productive persistence, and knowledge needed for teaching in two-year colleges. A key finding from this objective is that there are few empirical studies in the published literature on two-year colleges. The second objective was to collect measures from faculty on three variables: content knowledge, a mindset for learning, and lesson objectivization and to explore the relationships between them. One key finding from the second objective was that faculty know very little about cognitive learning, developmental learning, and the learning sciences in general. Another key finding from the second objective is that faculty are stronger in their content knowledge on algebraic topics than they are on statistics and probability. This may be very local because of the requirements of programs of study. The third objective was to explore connections between the three variables and teaching using case studies of four mathematics faculty. A key finding was faculty who viewed mathematics as a discipline to study not just a course that prepares one for another course, or a course that has to be checked off before going on to other courses of study. These faculty engaged students in activities to nurture their learning of mathematics and the learning of the community of students around them. In viewing mathematics as a discipline, they provided students with not just content but also immediate application of the content as they learned the content.
5.1.1 Objective 1

To review the literature

The literature review was based on empirical studies available from 1970 to 2014 on the use of mindset and productive persistence in research, and its place within the body of knowledge that mathematics faculty in two-year colleges need for teaching. Key terms used for the search included teacher knowledge, mindset, productive persistence, objectivization, developmental mathematics, learning, motivation, competence motivation, and will. Searches were also conducted in mathematics education, psychology, psychology education, and social science education. Articles were from academic peer-reviewed journals. Searches on authors that were cited in original articles produced by the initial key-word searches were also conducted. These included Boaler, Dweck, Schulman, Uznadze, and Yeager. In addition to empirical studies, the review also included chapters in published books to provide background, connections, and current views of the concept of a mindset as an implicit theory. Reverse citation searches provided additional references as well as publications cited in articles in the original search results. Articles and books of interest were selected for review if they provided clear applications of mindset in classrooms, educational structures, and settings related to learning. The literature review includes only those articles that addressed various types of knowledge needed for teaching.

There were four major themes of resource types in the final list of included resources. The first type of resource provided background and foundational information. These resources were a source for meta-analysis of how the concept of a mindset for learning and productive persistence evolved in education. Productive persistence as a knowledge type is not included in the study, however both ideas in classrooms add to knowledge for teaching. It was also interesting to see how the term “productive persistence” first surfaced in the literature. The second type of resource
revealed articles of a growth or fixed mindset studied in teaching and learning. Resources falling in this category were studies about using a mindset for learning and its impact on the dynamic of teaching and learning. Major findings for this type show incremental or growth mindset theory related to positive effort beliefs and entity or fixed mindset related to negative effects. The third type of resource included studies of a growth or fixed mindset in psychological and social contexts. These studies were more about how a mindset may impact psychological attributed like happiness, multiple intelligences, positive and negative effects, social comparison, willpower, and self-estimated intelligence. However, findings in these studies impact student success levels in all the academic areas. The fourth type offered a perspective on future directions for the concept of mindset.

The fifth type of resource included current studies in selected dissertations. These studies found that teaching for mindset alone positively impacts student success in mathematics. Studies showed that going through the experiences found in challenging activities assisted students in forming identities for learning mathematics. The sixth type of resource covered studies in teacher knowledge. The literature here resulted in a plethora of types of knowledge needed for teaching. In summary, the articles contain evidence that there are other things besides knowledge of teaching content to consider when examining teacher knowledge. And finally, the seventh type of resource was studies on objectivization. To find studies on identifying objectives for a lesson, it was necessary to look for terms related to goal setting or teacher ability to achieve goals in a lesson as there were little results when searching for lesson objectives. Using goal setting or teacher ability to achieve goals yielded three articles with sufficient information to tie to our literature review. A major finding in the articles included that choice of goals for a lesson will impact how students view and pursue them. In addition to this, how a teacher states goals can make them more
attainable and may promote educational benefits for students. Studies also revealed that it is teachers who mold content knowledge into classroom activities for a lesson and may cause varying levels of implementation. It would stand to reason how faculty view lessons and interpret them into classroom activities may be a contributing factor that impacts student learning.

The literature suggests that what faculty know from other types of knowledge, other than content, will cause differences in how students are engaged in classrooms. Specifically, not knowing how a mindset for learning impacts how students view learning. This may make it harder to engage students with any content. Studies suggest that it is possible to provide students, especially those who believe they cannot learn mathematics, with the identities they need to learn mathematics. However, if faculty do not know how to nurture a mindset for learning when engaging students in classroom lessons, nurturing agency to learn mathematics will be more difficult and may not produce positive results.

The overall finding for this objective is that there is a paucity of studies conducted on two-year colleges, faculty, or students regarding teacher knowledge. This is in contrast to the many studies that exist for K-12 institutions or four-year colleges and universities. After compiling the initial literature review, various studies on the effects of growth and fixed mindset in classrooms began to appear. Additional studies were reviewed and added to the study.

5.1.2 Objective 2

To collect faculty measures on three variables: a mindset for learning, content knowledge, and lesson objectivization, and to explore the relationship between them

Measures of a mindset for learning. To collect the mindset profile scores, a survey of eight items was administered during Part 1 of the study. Faculty mindset profile scores were placed on
a continuum measuring a fixed mindset on one end and a growth mindset on the other end. Thirty-one of the thirty-three faculty had mindset profile scores on the growth mindset range. One had mindset score right on the edge of growth and fixed scores. Some faculty had higher scores than others. Two faculty had mindset profile scores on the fixed mindset range.

Mindset for learning mathematics served as an important construct for this study. In a study of college instructors from the literature review, Olson & Knott (2013) found that using problem-posing to develop mathematical reasoning shows that instructor mindsets influence how they pose problems for their students to solve. Teachers with growth mindsets focus on the process of understanding mathematics; teachers with fixed mindset focus on products of mathematical activity and emphasize answers. They observed that teachers with growth mindset have classrooms where students view mathematics as a changing, dynamic entity having multiple representations and perspectives, and takes place where reason reigns not magic and memorization.

Studies by Howard & Whitaker (2011) revealed developmental mathematics students had a change in their mindset when successful. In a study involving music students, data indicated negative effects of students with a fixed mindset where students with fixed mindset interacted less with people and proposed fewer solutions to problems posed than those with growth mindsets (Mihai, 2014). Studies focusing on teacher mindsets revealed the importance of the roles teachers play in classrooms. Those with growth mindset may attribute more positive student feelings towards their capabilities and intelligence (Rattan, 2012). In another study, Subramanian (2007) found that teacher insights affected by their mindset played a critical role in mediating student learning within a zone of proximal development (Vygotsky, 1978). Changes in faculty professional development are needed if faculty members do not to know of the effect a growth
mindset can have on student learning. Professional development programs that model teacher knowledge as an evolving concept are needed. These programs may help faculty develop lessons objectives using applications that engage students. While engaging students is not a new concept, engaging students as producers of their learning and the learning of others allows them to feel they contribute to the learning of other students.

As profiles for learning, the two levels of mental activity (Uznadze, 1966), the level of set and the level of objectivization, and two mindset profiles describing a growth mindset and a fixed mindset (Dweck, 2006 & 2008) were used in this study as parallel constructs. When observing faculty, Uznadze’s levels of mental activity distinguished how faculty viewed their lessons. Faculty that encouraged students in higher cognitive demands through their lesson objectives were faculty that moved students towards using the level of objectivization. Faculty with activities and group work that encouraged non-routine tasks give students experiences that force them to use the level of objectivization.

Faculty mindset profile measures and measures of content knowledge create a typology for the selection of four faculty for case studies in Part 3. Content mindset measures identified a typology of faculty to participate in the case studies together with measures of lesson objectivization, their ability to identify lesson objectives. Observations and interviews provided a way to connect classroom images to measures of mindset.

*Content knowledge.* A survey using quantitative measures was administered during Part 1 of the study to collect measures of content knowledge. Twenty-three items measured content knowledge in six areas of mathematics and were written using three levels of cognitive demands. Collecting measures of content from the six basic areas in mathematics helped identify faculty knowledge in the basic areas of mathematics that form foundations for courses taught in two-year
colleges. The six areas in mathematics were taken from the content and process standards in the curriculum of middle school mathematics in the state where the study took place. The rationale for this was to measure content knowledge in the mathematical areas that form the foundation of all both developmental and college-level courses in the first two years of college. The distribution of scores had a mean percentage score of 65%, a standard deviation of 17%, and a median at 67%. Cronbach (1951) alpha of 0.7489 was computed to measure reliability and internal consistency for the items and considered acceptable (UCLA, 2016; Creswell, 2013).

Recall that as a variable, content knowledge in the area of instruction is an essential variable in research that impacts student achievement (Gamoran et al., 1997; Porter, 2002) and played a major role in this study. All faculty participating in the case studies agreed that knowing their content knowledge was extremely important. While they placed importance in knowing their content, an item analysis of the items answered incorrectly uncovered some areas that may not be remembered well by faculty. The item analysis indicated four of the top five items answered incorrectly were from the area of probability and statistics. Items from the area of functions and graphs and the area of geometry and measure were answered correctly by more participants. Most faculty at the study site teach courses with foundation content coming from functions and graphs or geometry and measure. There are fewer faculty who teach courses with content in statistics and probability. If the college follows the current national trend of providing mathematics pathways designed to offer students courses that are more relevant to their programs of study, then there will be a demand for teaching more statistics and quantitative reasoning courses. Why? As students enter two-year programs at the study site, they will be guided into pathways designed for STEM programs of study or non-STEM programs of study. Currently, most of the non-STEM programs of study require a STEM-based freshman mathematics course, but with pathways will soon require
mathematics courses in statistics or quantitative reasoning. This will create a shift in the preparation of faculty teaching at two-year colleges; more faculty will be required to teach statistics or quantitative reasoning courses. These results will be discussed further in the implications section below.

The relationship between overall content knowledge scores and the mindset profile scores for faculty was not statistically significant. Pearson tests were also calculated between mindset profile scores and content scores by cognitive demand. These tests did not show any statistically significant relationship between the faculty mindset profile scores and content scores by cognitive levels.

Using the content scores and the mindset scores for faculty, the sample selected for Part 3 of the study included four faculty with high and low scores on content knowledge scores and mindset profile scores.

Lesson objectivization. In this study, lesson objectivization is the ability of faculty to identify expert objectives for a lesson. To measure lesson objectivization, faculty attended focus groups where they viewed a lesson and wrote narratives of the objectives of the lesson they viewed. Lesson objectivization was measured using qualitative data from narratives on lesson objectives to explore it as a mental activity that involves the level of objectivization. Faculty with clear beliefs about learning, teaching, assessment, all within the content of mathematics are more apt to present lessons with objectives that match their beliefs. Interviews and class observations helped in identifying connections between the three variables measured and faculty beliefs and practices observed while teaching. A mindset for learning, content knowledge, and lesson objectivization were used to triangulate data with what faculty said about their teaching and what they did while teaching. This allowed identifying connections to teaching. The narratives were coded three times: once to code all objectives identified; a second time to code objectives focusing on content
standards or on process standards; and a third time to record if they matched the expert objectives for the lesson. Most of the faculty matched the objectives process standards. Of the fourteen participating in focus groups, all but one had a match to the either a content or a process objective in the expert lesson. Nine matched the process standards, four had a match to both the process and the content objectives in the lesson. In the literature, faculty as teachers play an important role in learning that occurs in classrooms. For example, the way faculty write lesson objectives may determine how students view their own learning. As a variable in our study, lesson objectivization connected to how faculty structured activities for their lessons. Clear lesson objectives that engage students in mathematics create a dynamic in teaching and learning that lets faculty and students depend on each other when learning. This may also nurture good habits for learning in students. In the study, faculty measures on lesson objectivization were taken from qualitative analysis of narratives of lesson objectives and on how these narratives matched lessons for an expert lesson. For a future study, it may be more strategic to collect lesson objectives to analyze as an artifact or another source of data. Because there were more matches to the process objectives and less to the content objectives, further study may be conducted to see how faculty write their lesson objectives.

5.1.3 Objective 3

To explore connections between the three variables, content knowledge, mindset, and lesson objective, and teaching in the case studies of four faculty

Key finding

Faculty members in the case study who understood mathematics as a discipline for learning provided students with activities that engaged them in learning.

Understanding mathematics as a discipline to study is the notion that faculty are not teaching mathematics to students to get them to the next course, or to have students learn enough mathematics to get them to finish a course. Instead, when they understand mathematics as a
discipline, they teach students mathematics as a discipline with concepts that have applications and meaningful uses.

Four faculty were selected to study possible connections between content knowledge, mindset, and lesson objectivization. They represented four typologies coming from high and low measures in content knowledge and mindset. These are high content, high mindset; high content, low mindset; low content, high mindset; and low content, low mindset. For convenience, these will be labeled using ordered pairs. The first letter will designate a high or low content quartile score and the second letter will designate a high or low mindset quartile score. For example (H, H) designates a faculty in the high content quartile and high mindset quartile; (H, L) will designate a faculty in the high content profile and low mindset quartile and so on.

Most two-year colleges credential faculty using information on awarded degrees with evidence of graduate course work in their content area. This is based on the assumption that faculty only need to know their content knowledge to be effective teachers. Once in service, faculty are expected to develop their knowledge for becoming effective teachers by adding knowledge in pedagogy, pedagogical content knowledge, knowledge of technology, and, more recently, any other knowledge that brings equity to the college classroom. In this study, we had three variable measures for faculty. Adding the interviews and class observations provided data sources that brought in beliefs about teaching and learning. Interviews and observations also added visual images of what classroom teaching looked like for faculty participating in the case studies. Using cross-case examinations, cases were paired for analysis. Large differences in frequencies were looked at when examining and selecting the pairs of faculty for analysis. Analysis of connections to content knowledge involved pairing two faculty with the similar mindset scores but different content scores. To isolate mindset, two faculty with similar content scores but different mindset
scores were paired. These cross-case examinations are summarized in Table 5.1. The last column represents the faculty in the pair that exhibited effective classroom teaching from each of the cross-case examinations. Recall that effective teaching was defined as teaching that engaged students in activities promoting students as agents of their learning and participating in the learning of others.

Table 5.1: Cross-case examinations

<table>
<thead>
<tr>
<th>Cross-case</th>
<th>Faculty pairs</th>
<th>Faculty with more beliefs and practices leading to effective classroom teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Varied content, same mindset (L)</td>
<td>(H, L) and (L, L)</td>
<td>(H, L) more effective</td>
</tr>
<tr>
<td>Varied content, same growth mindset (H)</td>
<td>(H, H) and (L, H)</td>
<td>(L, H) more effective</td>
</tr>
<tr>
<td>Varied mindset, same high content (H)</td>
<td>(H, H) and (H, L)</td>
<td>(H, L) more effective</td>
</tr>
<tr>
<td>Varied mindset, same low content (L)</td>
<td>(L, H) and (L, L)</td>
<td>(L, H) more effective</td>
</tr>
</tbody>
</table>

The four cross-case analyses resulted in (H, L) and (L, H) having more beliefs and practices identified with effective teaching. To determine connections between (H, L) and (L, H), an additional cross-case examination to review interviews and class observation activities for this pair was conducted for (H, L) and (L, H). This review found the following results. Both faculty had high frequencies for the categories in the class observation and in the analysis of transcribed interviews that are summarized in the list below. They were both found to use a broader view of mathematics as a discipline to:
a. use applications and projects from fields of study their students were pursuing and connected projects used during class and for homework to their daily lives
b. create more opportunities where students created their own understanding of mathematics
c. articulate a broader view of mathematics as a discipline to students
d. articulate a desire to helping students as they would help their families
e. use moral purpose/responsibility to describe teaching differently from other careers
f. have a strong equity agenda reflected in their teaching and beliefs about learning
g. offer specific (not general) help to their students

Tables 4.8 and 4.9 in chapter 4 provided the data used to identify the similarities observed in the two faculty members. Table 5.2 contains frequencies for Grace (L, H) and Frankie (H, L) previously recorded in Chapter 4.

Table 5.2: Faculty Class Observations of Activities and Tasks, Grace and Frankie

<table>
<thead>
<tr>
<th>Class Activity</th>
<th>Grace</th>
<th>Frankie</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Provides time for students to develop and express ideas</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>6. Uses techniques attributing ideas to students, build ownership, identity</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

The two faculty members incorporated a combination of beliefs and experiences to make sense of what students experience in mathematics classrooms and what they need to do to change things. Their clear lesson objectives in projects and applications allow them to stand alongside their students in ways that make the faculty and students depend on each other for learning. Usually in mathematics, faculty see teaching as being free of anything but the content itself. For
the most part, faculty in two-year colleges have not had professional development in methodology, pedagogy, critical pedagogy, pedagogy related to their content, or any theories related to curriculum/instruction/assessment that would address challenges when providing equitable classroom experiences. Many may not see how critical it is to give students opportunities to become agents of their own learning. This is complicated by the fact by the time students enter college, many have struggled in mathematics and are still connected to memories of when they first discovered a topic that did not make sense. For many, the topic was fractions as they first saw them in elementary school and for many, fractions still remain unclear. For those who have gone past the fractions, it was algebra, and the list continues with the next concept in mathematics that will break the next student. Seen as obstacles in learning mathematics, teachers can unpack difficult concepts in mathematics, form objectives to get students through each step, and continue with their learning. Effective teaching involves engaging students are engaged in these activities alongside the faculty member.

Results from this last cross-case examination convey that the faculty with effective teaching are those who use applications and projects while students are learning mathematics. They are faculty who have students interacting with other students as they learn in a community. They are faculty who view mathematics as a discipline to study, mathematics as a tool for learning, and mathematics as an avenue giving students better lives. Their own agenda for equity is reflected in their lessons. This has major implications for the national equity agenda for education.

5.2 IMPLICATIONS AND RECOMMENDATIONS

Doing research for a study offers the opportunity to use results and findings to present some implications and make recommendations. Implications are discussed first, followed by recommendations for each of the findings.
5.2.1 Implications for teaching

**Key finding**

*Faculty in the case study who understood mathematics as a discipline for learning were faculty who provided students with activities that engaged them in learning.*

**Implications**

*Faculty who promote equitable teaching can create classrooms where students become agents of their own learning and become part of the community of learners of mathematics.*

During classroom observations, faculty used knowledge and mindset to create and objectivize lessons for student learning. The level of objectivization (Uznadze, 1966; Uznadze 2009) allows one the ability to take an obstacle, turn it to an object, break it into manageable parts, address each part with what one knows, and overcome the obstacle. Faculty who understood and identified objectives engaged students in activities that used the mathematics in projects and applications and connected the mathematics to learning. For the most part, faculty entered the profession of teaching with a moral purpose for teaching. As faculty, they participate in a service-oriented profession. Their efforts prepare students to continue academic programs of study that lead to social mobility. Post-secondary mathematics faculty in colleges, may not have had opportunities to add to their scholarship portfolio. Staying current in knowledge needed for teaching and connections to mindset for learning, faculty will need to allot time to read manuscripts, articles, journals, books, and other publications. For many faculty, this may not be possible; for a few faculty, this allotment of time is already routine. Change in practices in mentorships,
Recommendations

To keep faculty current in ways to make the learning of mathematics as a discipline to study and to make the learning of mathematics within a community of learners by:

- Participating in scholarly sessions scheduled at least twice a year to include formal presentations of current studies, monographs, books, or selected journals in followed by a discussion and dialogue led by faculty. Topics for presentation may include but now be limited to effective teaching strategies related to mindset, learning theory, and critical pedagogy.

- Creating activities that nurture students as part of a community of learners that nurtures their learning and the learning of others.

5.2.2 Implications and recommendations for faculty professional development

Key Finding

Faculty had limited knowledge of effective teaching strategies that address cognitive learning or developmental learning. There was also no mention of the theories in critical pedagogy associated with providing students with equitable learning experiences in classrooms.

Implications

Professional development for mathematics faculty in two-year colleges may lack topics that address effective teaching strategies, cognitive or developmental learning, or mindset for learning.
This study included faculty voices about the knowledge they believed they needed for teaching. Investigating teacher beliefs about learning and what they believe they need for teaching revealed a need to reform professional development for two-year college mathematics faculty. The four portraits of teachers in the case studies provided insights into how their content knowledge, mindset profiles, and lesson objectivization were reflected in the knowledge they believed they needed for teaching.

The study proposed to determine the content knowledge, a mindset for learning mathematics, and ability to identify lesson objectives faculty have. It was also designed to determine if a combination or intersection of these types of knowledge help faculty identify and use objectives for effective lessons. Faculty that identified content knowledge weighed heavily in what they believed they needed most to teach. It was also clear that they knew very little about a mindset for learning and how it may lead to more effective classroom teaching. For the sample of faculty in this study, results show quantitative measures of mathematical content knowledge and mindset profiles were not related. However, content knowledge and lesson objectivization was related. With carefully designed professional development programs this relationship can be strengthened to include to enrich the activities for students in mathematics classrooms.

As colleges and universities were formed in the United States, very little had been studied about cognitive and developmental sciences or, more generally, the learning sciences. The learning sciences were defined as a new science of learning in an interdisciplinary field devoted to the study of teaching and learning. Included in the sciences of learning are cognitive science, educational psychology, computer science, anthropology, sociology, information sciences, neurosciences, education, design studies, instructional design, and other fields (Sawyer, 2009). As studies in the learning sciences evolved, knowledge needed for teaching also evolved. Faculty interviews
revealed the need for knowing some sociology and psychology at best; they also revealed that faculty have not really focused on studying the cognitive and developmental sciences, educational psychology, or the neurosciences. This research study suggests that professional development for faculty would be enhanced by including studies in the learning sciences.

Much has been added to the field about knowledge of cognitive learning, developmental learning, and other learning theories that faculty may not consider in their list of things they need to know and be able to use when teaching. The literature on learning forms a composite of nearly four decades of research on the learning sciences. Currently, two-year mathematics faculty face large numbers of students who would benefit from innovative experiences in mathematics classrooms as they enter college. By giving attention to the new science of learning in faculty professional development programs, two-year college faculty will be positioned well to create changes in teaching. These changes would impact student success at the local, state, and national level.

Topics from courses in critical pedagogy may also rejuvenate the moral purpose that faculty may have used when selecting teaching as their career. Measures of student success have much to gain by adding sessions and topics to faculty professional development programs in critical pedagogy, critical race theory, literacy and bi-literacy, and learning theory. These may be offered at two-year colleges as part of their regularly scheduled professional development at the start of every semester. Faculty may also be better prepared to incorporate equitable teaching when they understand the effects a growth mindset can have on teaching and learning.

This means that regular professional development at the start of every academic year and every semester for faculty is needed to keep the idea alive that every student who walks into
classrooms will learn mathematics. This type of change does not happen overnight. However, newly designed professional development programs that incorporate the learning sciences and critical pedagogy may initiate, provide, and sustain change. As professional development is planned, it may include theories in cognitive science, developmental learning of mathematics, and focus on active learning. As a requirement for faculty, professional development may also include a review of student achievement data and at least one empirical study related to challenges two-year faculty may face. For two-year colleges, professional development is already included in the contract for faculty and scheduled at the start of every academic year and semester. All that is needed is a well thought out plan.

**Recommendations**

Professional development programs for two-year college mathematics faculty must be reviewed as processes that evolve and include:

- presentations on local, state, and national data on student achievement;
- presentations on data identifying populations who may not be well-served;
- levels of mental activity (Uznadze, 1966 & 2009) and their connections to mindset (Dweck, 2006 & 2009)
- cognitive and developmental learning strategies, using growth mindsets for learning, critical pedagogy, literacy and bi-literacy, technology based learning, and new studies that impact teaching and learning; and
- faculty teams to unpack the content they are teaching, use current instructional materials adopted by the discipline, and provide regular updates after the initial session.
5.2.3 Implications and recommendations about research

Key Finding

Teacher knowledge, teacher practice, and teacher beliefs make the research on teacher knowledge needed for teaching limited and complex.

Implications

Two year colleges have few empirical studies to inform them of reforms needed to address knowledge teachers need to address issues, current trends, and needs of students entering colleges.

Literature based on two-year colleges is relatively small. The review for this study went from 1970 to 2014. Current studies and dissertations were added after the initial literature review. However, searches for research in education yielded very few empirical studies related to mathematics teaching and learning in two-year colleges. As more popularity with articles on growth mindset developed, more articles were added to the study.

This study was designed because of the many students entering programs in two-year colleges underprepared to enroll in college level mathematics. There are data available at the state and local level that report trends in student achievement and completion of two-year courses and programs of study. Once trends begin to show higher levels of underprepared students, studies designed to give us information on how this might be affecting students may help address current issues.

As a practice, colleges change textbooks every two years for a variety of reasons. This practice brings in newer, more current use of technology and supporting resources with the newer
textbooks. However, professional development has not kept up with the changing curriculum and ancillaries connected with new textbooks.

Research in two-year colleges will help guide undergraduate education and findings from research may help guide the practice of teaching. Publication of research will help disseminate results for others to replicate and use but the research has to come first. At two-year colleges, research departments can easily target populations of students who may need assistance. Once a population has been identified for study and questions about the population are formed, study can be designed. Studies can be conducted within an institution or studied in collaboration with other institutions to design and provide studies to answer questions regarding student achievement levels. If every two-year college would launch its own study and submit for publication, it would add valuable information to two-year colleges and education entities in general. It is important to note that research can be conducted through internal or external centers in any college depending on resources of the college. The implication from the finding is that whether it is internal or external, research in teaching and learning may initiate in an area of a college or in a larger external entity that conducts research with larger populations. The college site for this study has been involved with many externally funded research studies. However, the studies have not targeted knowledge needed for teaching at two-year colleges.

New research about equity is finally emerging with themes that include inclusivity by removing barriers and the needs of groups that have been historically marginalized. This study included a class observation tool that measured frequency of certain activities that engage students in creating their learning while including them in a community of learners within the classroom. These types of activities are now being utilized in the mathematics pathways courses with many activities categorized as active learning. Unless other strategies are used to engage students in
active learning, not using these types of activities may not be removing barriers in student learning. In addition to this, studies have clarified the distinction between equality and equity. Equality in teaching implies that everyone gets the same kind of support without regard to what they might need individually. Equity in teaching is based on the understanding that some students require different support. In our study, the two faculty members observed to have effective classroom activities exhibited equitable teaching. It would be beneficial to students to see more faculty doing the same.

**Recommendations**

*Two-year colleges and research centers can support more research in undergraduate mathematics education by:*

- Seeking funding to create research collaboration centers between two-year colleges and four-year university partners
- Designing research studies using existing data to uncover trends in student enrollment, progress, and completion of freshman mathematics courses in local two-year colleges
- Designing research studies to explore instructional strategies that promote equitable experiences for students in two-year college mathematics courses

**5.4 LIMITATIONS REVISITED**

The study had limitations. The sample size of participants for the study is a limitation. Sites wishing to duplicate the study are advised to pursue a larger sample. For Part 1 of the study
a random sample of faculty was generated by the institutional research department of the study site. The first 40 faculty identified as the random sample were invited to participate. Because participation was voluntary, only nineteen filled out both surveys in the allotted time. Another random sample of forty faculty was invited to attend and resulted in thirty-three faculty participating in Part 1. The sample was analyzed by gender, ethnicity, credentials, and campus site. Results show the sample was closely related in these factors as the entire population of faculty it represented. For Part 2, all participants from Part 1 were invited to the focus groups and resulted in 14 participants; they were a nested sample and included faculty who scored in all four quartiles of both surveys. Part 3 is was a purposive sample and was nested within the sample in Part 2 of the study.

Another limitation is that the content knowledge survey was a multiple-choice survey (Schoenfeld, 2007). Because of this, it was important to interview and observe faculty in their classrooms to provide their perspective on teaching, learning, and expressing their beliefs about the knowledge they believe is need for teaching. It is important to note that teachers observed during the study were observed while teaching only one type of course, a developmental course or a freshman level course. For this study, two faculty were observed in developmental education course classes and two were observed in freshman-level course classes.

A delimitation of the study is that the study site is in a border community within a large international metropolitan area with unique characteristics.

5.5 Conclusion

Investigating content knowledge, mindset for learning, and lesson objectivization unfolded connections with teacher beliefs and their levels of mental activity. In Uznadze’s levels of mental activities, using the level of set allows one to draw only upon something that was learned before
and can be used only to deal with a routine occurrence. It is a way of thinking that does not depend on anything except the ability for to remember what was utilized before. This way of thinking is useful, however, it is not enough when looking at learning rich content like mathematics. This is in line with the belief that one’s intelligence is fixed to deal with only routine things. Anything that may involve a new problem or obstacle may not be solvable by a person who believes they have fixed mindset.

In Uznadze’s second level of mental activity, the level of objectivization, one can deal with non-routine problems or obstacles. It is a way of thinking that forms the obstacle or problem into an object, proceeds to unravel it, draws upon several known units, and uses steps to connect the units and solve it. In this way, the level of objectivization is a mental activity that nurtures an incremental growth mindset for learning. It combines several skills into conceptual ways for solving or learning how to deal with non-routine problems. Activities that piece together units of known sets and processes to solve something or learn something new have the potential to add intelligence each time an obstacle is encountered. Teachers who use the level of objectivization have added resources that are needed to nurture students as they engage in activities and experiences that grow their intelligence. Mathematics faculty can be made aware of the power this type of mental activity. Participating in a service oriented occupation and teaching content that most individuals believe they cannot learn, mathematics teachers and faculty stand to make a significant impact while facing many students each fall semester.

While it is not clear if mindset for learning in general is the same as mindset in learning specific content, there are some connections that are apparent in teaching mathematics. It will take further studies to look at this with varying content. It is critical for teachers to view knowledge for teaching as an evolving concept that uses more than just content knowledge. Because beliefs and values are part of any career, faculty may enter teaching with beliefs about learning that may not lead to effective teaching. Professional development is one way to bring knowledge for teaching closer to what is needed for effective teaching. All teachers will encounter challenges; unpacking what they need to know to be effective teachers will allow the dynamic of teaching and
learning to address the needs of thousands of students. It is important to study what teachers need to be effective teachers, to provide development for teachers, and to support them as teachers of students and not just teachers of content. Grace reminded us of this when she stated:

… I started and I thought I was ready to teach math with the courses I had in college and because of the grade[s] I had in high school. But when you start teaching, then you realize that you still have a lot of gaps. And you need to know why and you need to know how to explain to students deeper, especially engineering students. You need to know how to use mathematics and how to use the logic behind it (Faculty interviews, Dec. 2107)

The study looked at two other variables besides content knowledge. It looked at mindset for learning and lesson objectivization. Neither of these are topics in the formal scholarship for teaching; faculty preparing to teach at two-colleges are required to take only mathematics courses. As faculty, they are prepared with only content and a desire to teach. The desire to teach for three of the four faculty was a decision that presented itself after their first career was interrupted. While the interruptions were situated differently, the decision to teach was not their first choice. Therefore, it is important to investigate what makes teachers like them effective. Exploring knowledge needed for teaching may help understand the dynamic of teaching and learning. Looking only at content knowledge may not give us an indication of who may be ready to teach mathematics in two-year colleges and who may not. For two-year college mathematics faculty, other kinds of knowledge impact their teaching. For many college faculty, knowledge needed for teaching forms when teaching begins. It was good to hear two of the faculty expressing beliefs that connected teaching their own agenda for equity (Gutierrez, 2018). For them it was an agenda that evolved from teaching and from life experiences. They are teaching in ways that enrich
learning experiences for students and nurture beliefs about their contribution to learning. How can we get more faculty to do the same?

Current national events have recently unfolded and revitalized an equity agenda for education. These current events have begun a national discussion and dialogue in networks of colleges and researchers in education. National debates have also fueled discussions for an equity agenda to broaden the participation to students that have stood in the margins. Within these conversations, equality implies that everyone is given the same support. Equity is distinguished as giving support that is uniquely identified for the needs of a subpopulation or individual. Faculty will need professional development that address practices that both Frankie and Grace have embraced in teaching.

An important first step is to have institutions identify and remove barriers to reach full participation in colleges and to recognize support systems for those who have been kept from participating. For this study, equity may begin by recognizing that faculty professional development must be seen as a continual and evolving process. It must also be seen as a process that may need periodic adjustments. Recommendations listed in this chapter include to revise faculty development and to include research in mathematics education in two-year colleges. These recommendations may allow professional development to keep up with the needs of entering students. They may also serve to broaden the lens of how two-year colleges have served populations who have been traditionally underserved.

Renewed faculty development may impact student achievement through improved knowledge for effective teaching. In examining the knowledge needed for teaching, faculty may be better able to assist students in identifying themselves among those who understand, utilize,
and question mathematics. Effective classrooms can create students that see themselves as a viable part of a larger community of learners who are forming their lives and improving the lives of larger communities. I hope the study begins conversations to ensure that as mathematics adjusts its focus to innovative fields of study, professional development for faculty also adjusts. In the end students stand to be the beneficiaries.
References


Charalambous, C. Y. & Hill, H. C. (2012). Teacher knowledge, curriculum materials, and


El Paso Community College.


Psychology, 49(4), 703-723. Doi: 10.1348/014466609X476827.


Theories and the perception of groups: Different routes to different destinations. 


Yeager, D. S. (2012). *Productive persistence: A practical theory of community college student*
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**Appendix**

**Observation Guide** *

Date ___________________ Campus ___________________ Pseudonym ____________________

<table>
<thead>
<tr>
<th>Teachers</th>
<th>n</th>
<th>comments</th>
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<tbody>
<tr>
<td>Clarify lesson objectives</td>
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<tr>
<td>Provide time for students to develop and express ideas</td>
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<tr>
<td>Work to make sure all students have opportunities to have their voices heard</td>
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<tr>
<td>Encourage student-to-student discussions and promote productive exchanges</td>
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<tr>
<td>Assign tasks and pose questions that call for marshaling, analyzing and synthesizing evidence, and for students to explain their reasoning</td>
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<tr>
<td>Employ a range of techniques that attribute ideas to students, to build student ownership and identity</td>
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</tbody>
</table>

*Adapted from the Teaching for Robust Understanding Observation Guide (Schoenfeld, 2013)
**MINDSET ASSESSMENT PROFILE SCORE SHEET**

Pseudonym _______________________

<table>
<thead>
<tr>
<th>Do you agree or disagree?</th>
<th>Disagree a lot</th>
<th>Disagree a little</th>
<th>Agree a little</th>
<th>Agree a lot</th>
<th>Profile Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No matter how much intelligence you have, you can always change it a good deal</td>
<td></td>
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<tr>
<td>2. You can learn new things, but you cannot really change your basic level of intelligence.</td>
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<tr>
<td>3. I like my work best when it makes me think hard.</td>
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<td></td>
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<tr>
<td>4. I like my work best when I can do it really well without too much trouble.</td>
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<tr>
<td>5. I like work that I’ll learn from even if I make a lot of mistakes.</td>
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<tr>
<td>6. I like my work best when I can do it perfectly without any mistakes.</td>
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<tr>
<td>7. When something is hard, it just makes me want to work more on it, not less.</td>
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<tr>
<td>8. To tell the truth, when I work hard, it makes me feel as though I’m not very smart.</td>
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</tbody>
</table>

Mindset Assessment Profile Number

Adapted from Mind Works (Dweck, 2008)
INTERVIEW PROTOCOL

Individual Interview 1, Life History - 30 to 90 minutes (whatever is needed by the interviewee)

Purpose: To explore biographical and historical life history data that may have led to making a career choice

Questions
1. What is your family background?
2. What role did your family play in selecting teaching as a career?
3. What role did your schools play in selecting teaching as a career?
4. What experiences did you have learning mathematics?
5. How did these experiences enter into selecting teaching as a career?

Individual Interview 2, Current Experience in teaching - 30 to 90 minutes (whatever is needed)

Purpose: To explore current work experiences in teaching

Questions
1. What can you tell me about being a classroom teacher? How is being a teacher different from other careers?
2. What do you believe you need to know to teach?
3. How do you prepare for teaching a lesson?
4. What would you have done differently to teach the same lesson (on the video we watched)?
5. How do you deal with students who believe they cannot learn mathematics?
6. Let’s go to the lesson you viewed during the focus group.
   What were the lesson objectives for the lesson?
7. What do you believe the students in the lesson believed about learning mathematics?

Individual Interview 3, Meaning of their work as part of their life – 30 to 90 minutes if needed

Purpose: To make meaning of their teaching as they reflect on how their work teaching forms from their beliefs and from part of their lives

Questions
1. What part of your knowledge for teaching comes from knowing mathematical content?
2. What other kinds of knowledge for teaching do you need for teaching?
3. How does your life inform your teaching?
4. How do your beliefs about learning mathematics inform your teaching?
NARRATIVES FOR THE LESSON OBJECTIVES

Prompt used in the faculty focus groups

Write what you believe to be the objectives of the lesson you just viewed.

Please write in complete sentences and include details, if needed, to clarify these objectives.
1. Write a pseudonym starting with a two-digit number that does not have any zeros and then adding a name. An example would be: 48Apple.

2. What courses do you normally teach? Select all that apply.
   ____ a. DE MATH Courses
   ____ b. College level MATH Courses
   ____ c. Both DE and College Level MATH Courses
   ____ d. Precalculus and Calculus Courses
   ____ e. MATH Courses and Science Courses

3. Gender
   ____ a. Female
   ____ b. Male

4. Race/Ethnicity
   ____ a. White, Non-Hispanic
   ____ b. Black
   ____ c. Hispanic
   ____ d. Asian/Pacific Islander
   ____ e. American Indian/Alaskan Native
   ____ f. Other, specify __________________

5. What is your degree in?
   ____ a. Mathematics
   ____ b. Mathematics Education
   ____ c. Other, Specify __________________

6. Are you certified to teach in K-12?
   ____ a. Yes
   ____ b. No

7. How many graduate hours of mathematics have you taken?
   ____ a. 0 – 17 hours
   ____ b. 18 – 36 hours
   ____ c. More than 36 hours
Survey #1 Page 2 – Mindset Assessment Profile

This is an opinion survey about beliefs and goals regarding ability and performance. It is very important that you give your honest opinion, not what you believe someone else would think best. It is a gauge of your thinking right now. Read each statement, decide how much you agree or disagree with the statement. Indicate how you agree or disagree with each statement by selecting a response.

1. Disagree a lot
2. Disagree
3. Disagree a little
4. Agree a little
5. Agree
6. Agree a lot

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>10. I like my work best when it makes me think hard.</td>
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<td>11. I like my work best when I can do it really well without too much trouble.</td>
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<tr>
<td>13. I like my work best when I can do it perfectly without any mistakes.</td>
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<tr>
<td>14. When something is hard, it just makes me want to work more on it, not less.</td>
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<td>15. To tell the truth, when I work hard, it makes me feel as though I’m not very smart.</td>
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</tbody>
</table>
Survey #2

Thank you for participating in the study. Survey #2 will take between 50 – 75 minutes to complete. You should have already filled out the questions in Survey #1 and have written your own pseudonym. Use this same pseudonym for question 1 of this survey. Directions: Work all problems and select the answer that best matches your work for each problem. You may use a scientific calculator. Once you have taken the survey, please do not share its content with others as they may not have taken the survey. The first question is to write your own pseudonym for the survey. To remain anonymous, a link was given to you to respond to the survey and there will be no record of your name connected to your responses. Please use the pseudonym that you used for Survey #1.

1. Write a pseudonym you will use for the study by writing any two digit number that does not have a zero followed by a name of your choice. For example 92Apple. Write this down so you will remember this pseudonym for other surveys related to the study.

2. You give students a proof of the binomial formula $(a + b)^2 = a^2 + 2ab + b^2$ in class. Which of the properties of real numbers do you use to derive the formula?

Answers:

A. Distributivity only.
B. Commutativity and Associativity.
C. Commutativity and Distributivity.
D. None of the answers is correct.

3. A shoe store raises the price of a pair of shoes by 20%, then a week later offers the shoes at a 20% discount. After the two price changes, the shoes are now . . .

A. cheaper than before.
B. more expensive than before.
C. selling at the same price as before.
D. the other three choices are wrong.
4. Which of the following questions leads to the division problem \( 3 \div \frac{1}{4} \)?

A. How many \( \frac{1}{4} \) liter containers can Joanne fill from a 3-liter jug?

B. How much sugar does John need to bake 3 dozen muffins if the recipe calls for \( \frac{1}{4} \) pound of sugar per dozen?

C. How much money does Alicia make from selling 3 glasses of lemonade at a price of 25 cents per glass?

D. How much time does it take Martin to travel a quarter of a mile when he walks at a speed of 3 miles per hour?

5. If the side length of a cube is doubled, its volume increases by a factor of:

A. 2

B. 3

C. 4

D. 8

6. What happens to the greatest common divisor of two numbers, if both numbers are multiplied by a factor of 5?

A. Their greatest common divisor is multiplied by a factor of 5.

B. Their greatest common divisor is divided by a factor of 5.

C. Their greatest common divisor remains unchanged.

D. None of the other three answers is correct.

7. Which of the following statements about the graphs of linear functions is true?

A. All graphs of linear equations pass through the origin.

B. All graphs of linear functions cross the x-axis, but some do not pass through the y-axis.

C. All graphs of linear functions cross the y-axis, but some do not pass through the x-axis.

D. Some graphs of linear functions cross neither the x-axis nor the y-axis.
8. The vertex of the graph of the quadratic function \( q(x) \) is the point \((3,1)\), and the graph of \( q(x) \) also passes through the point \((4, 6)\). Which of the following points must the graph of \( q(x) \) also contain?

A. \((2, 6)\)
B. \((3, 11)\)
C. \((4, -10)\)
D. \((5, 1)\)

9. The vertex of a quadratic function \( g(x) \) is \((3,1)\), and the graph also passes through the point \((4, 6)\). The equation of the function is:

A. \( f(x) = 5(x+3)^2 - 1 \)
B. \( f(x) = 5(x - 3)^2 + 1 \)
C. \( f(x) = 5(x - 3)^2 - 1 \)
D. Cannot be determined given only the above information

10. Which is the best explanation of the correct value of the infinite repeating decimal 0.777777...?

A. Its value is a rational number because decimals are special cases of fractions.
B. It is not well-defined because the 7’s never end
C. It is not well-defined because each additional 7 increases the value of the number.
D. Its value is well-defined because as the number of digits increases, the value gets closer and closer to the limit.

11. Which of the following answers can be the lengths of the sides of a right triangle?

A. \( 3, \sqrt{4}, \sqrt{5} \)
B. \( 3, 4, 7 \)
C. \( 3/4, 4/4, 7/4 \)
D. \( 2, 3, \sqrt{13} \)

12. How is a histogram different from a bar graph?

A. You use a histogram when there are outliers.
B. You use a histogram when there are a lot of values close together.
C. You use a histogram when the underlying variable is a measurement variable.

D. You use a histogram when you want to show part-to-whole relationships.

13. If a distribution has one mode and its mean is greater than its median, then the distribution is probably:

A. approximately symmetric

B. positively skewed (i.e., skewed to the right)

C. negatively skewed (i.e., skewed to the left)

D. continuous

14. At a certain college, the last election showed that 42% of the students voted for party A. If 160 students are selected at random, what is the expected number of students who did not vote for party A?

A. 67 or 68

B. About 98

C. 80

D. None of the above.

15. Consider the experiment of tossing a fair coin three times. In the following, HTT, means that the first toss comes up heads and the last two tails. Let $A = \text{event that at least two tosses come up tails}$. Which of the following is a correct description of $A$?

A. \{TTH\}

B. \{TTH, THT, HTT\}

C. \{TTH, THT, HTT, HHH\}

D. \{TTH, THT, HTT, TTT\}
16. The scores of a certain population on the Wechsler Intelligence Scale for Children (WISC) are thought to be normally distributed with mean $\mu$ and standard deviation $\sigma = 10$. A simple random sample of 25 children from this population is taken and each is given the WISC. The mean of the 25 scores is $X$ = 104.32. Based on these data, a 95% confidence interval for $\mu$ is

A. $104.32 \pm 0.78$
B. $104.32 \pm 3.29$
C. $104.32 \pm 3.92$
D. $104.32 \pm 19.60$

17. A market research company wishes to find out whether the population of students at a university prefers brand A or brand B of instant coffee. A random sample of students is selected, and each one is asked to try brand A and then brand B, or vice versa (with the order determined at random). They then indicate which brand they prefer. This is an example of

A. an experiment.
B. an observational study, not an experiment.
C. stratified sampling design.
D. block design.

18. You are conducting a unit designed to teach students about the three types of central measures: mean, median, and mode. While investigating the weight of an object, a student takes 3 measurements and records them. The recorded weights (in grams) are 6.2, 6.0, and 6.3. The student concludes that the object weighs 6.0 grams and justifies her answer by saying that all the numbers are close to 6.0. Which of the following would be the most reasonable NEXT step for the student to take to evaluate the validity of this conjecture?

A. Take more measurements
B. Nothing further needs to be done.
C. Weigh another object and take measurements.
D. Use a different scale

19. The main advantage of using arrays as opposed to set models for multiplication is that:
A. Arrays are more beneficial when teaching the commutative property.
B. Children have extreme difficulty drawing set models.
C. Arrays are more beneficial in teaching multiplication as repeated addition.
D. Arrays are more beneficial in teaching why division by zero is not possible.

20. As you look at addressing probability and statistics strand, you see that students must know how to find measures of central tendency from a dataset. You have time to do only one of the following to start a unit on statistics. Which is the best activity to select for your class?

A. Students measure their height in inches and determine the mean, median and mode from these measurements
B. Students write the definition of mean, median, and mode
C. Students take a bar graph and determine the range of the dataset
D. Students work 10 multiple choice questions on mean, median, and mode

21. You are beginning the study of statistics with students in your class. You set your students main learning goal for the unit to be:

A. To be able to collect and organize data and use it to describe world phenomenon
B. To be able to determine mean, median, mode, and range for any dataset
C. To be able to show any dataset in different kinds of graphs
D. To be able to find frequencies for given datasets

22. As you are setting up an activity to start a unit on probability. You place 10 red marbles, 3 black marbles, and one blue marble in a box to demonstrate what it means to count the total number of favorable outcomes in an event. What should you have students do?

A. Each student should draw a marble, record the event as being a red marble or not a red marble, and place the marble in a jar outside of the box so it will not be drawn again. Repeat this until all the marbles are drawn.
B. Each student should draw a marble, record the color, and place the marble back in the box to be possibly drawn again. Repeat this until each student draws.

C. Each student should draw three marbles and record the colors. Repeat this until each student draws.

D. Each student should draw marbles until a red is drawn, the number of marbles drawn should be recorded. Repeat until each student draws.

23. You are starting a lesson on measures of central tendency. As students are finding mean, median, and mode for different sets of data, the teacher notices that students know how to find each but do not know what each measures. In designing a formative assessment for students, the teacher must include which of the following in her assessment?

A. Problems that have students find all three of the measures for given sets of data.

B. Problems that have students find only mean for some problems, only median for another problem, and only mode for another problem for given sets of data.

C. Three problems with given sets of data where the student is asked to mean for one problem, median for another problem and mode for another problem and for each problem the student must explain what information that gives about the data.

D. Any of the above.

24. As a way to start a unit on exponential growth models, what activity would allow for students to see data on exponential growth?

A. Draw five circles where their diameter double in size and measure their circumference.

B. Draw five circles with different diameters and calculate their area.

C. Draw five circles where the diameter doubles in size and calculate their circumference.

D. Draw five circles where the diameters double in size and calculate their areas.

(Note) This question did not include a choice that included more than one correct answer and was left out of the raw scores.
Vita

Lucy Hernández Michal was born in El Paso, Texas to her parents, Vicente and Conception Hernández. She is one of thirteen children born to Vicente and Concha. Lucy and Dr. Emil J. Michal, Jr., her late husband, have a beautiful son Emil V. J. Michal.

Lucy is a tenured mathematics professor at El Paso Community College (EPCC). She received her B. S. in Mathematics from The University of Texas at El Paso and her M. S. in Mathematics from Michigan State University. She is a National Leadership Fellow for the Dana Center Mathematics Pathways (DCMP) at the University of Texas at Austin.

Her memberships in professional societies and organizations include: the Mathematics Association of America, the American Mathematics Association, the American Education Research Association, the American Association for the Advancement of Science, the Association of Women in Community Colleges, the National Education Association, the Texas Faculty Association, and the Texas State Teachers Association.

Lucy has dedicated her life as an advocate for equity in education and the central challenges faced during education reform.

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