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Scalar Polarization Operator In A Superfluid Medium

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to my

FAMILY

with love

SCALAR POLARIZATION OPERATOR IN A SUPERFLUID MEDIUM

by

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THESIS

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Abstract

A QCD-inspired effective theory of fermions is considered to study superfluidity. We calculate the quark propagator in a theory with fermion-fermion condensate at finite density. In this theory color degrees of freedom are absent and the quark-quark interaction is modeled through a Yukawa interaction term. Since the diquark condensate is neutral, the system is a superfluid. To study the response of this superfluid to the scalar field propagation, we find the scalar polarization operator of the system.

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Chapter 1

Introduction

The fundamental building blocks that constitute matter are quarks and leptons. The nucleus of an atom which consists of neutrons and protons, is made of quarks. Nuclear constituents, the proton and the neutron, are approximately 1800 times heavier than the electron due to the larger masses of quarks, and occupy an extraordinarily small volume compared to that of an atom. Compact stars, which are the second-densest objects in nature after black holes, having the masses of the order of the mass of the sun, $M \sim 1.4M_{\odot}$, and radii of only about ten kilometers, $R \sim 10$km are mostly made of nuclear matter. Traditionally, *compact* stars are known as *neutron* stars even though these can either be made of nuclear matter (*neutron star* having a quark matter core) or exclusively of quark matter (*quark star* or *strange star*). Quark matter in compact stars may exist in many phases depending on different temperature and densities. The existence of different phases depends on the interaction between quarks which is studied in the frame of quantum chromodynamics (QCD). Quarks are fermions but when a pair of quarks is created at the Fermi surface, and there is an attractive interaction, they can pair into the so-called Cooper pairs. The mechanism for the formation of the pair, known as Cooper mechanism, leads to superfluidity or superconductivity depending on whether the pair is neutral or charged with respect to the group of symmetry considered. A system of quarks can be subject to both, superconductivity and/or superfluidity, depending on the physical conditions and the model used to describe it.

In this thesis, we are motivated to study the superfluid state of quark matter. Specifically, we will see how a scalar field is affected while propagating through the superfluid medium. Although we will consider a simplified model to describe the quark interactions, we hope it will help us to get some insight of the physics that may occur in a more restricted model.

Compact stars are relatively cold in the context of QCD for two reasons: The temperature right after a compact star is born in a supernova explosion decreases down to temperature in KeV range during the evolution of the star which is small compared to the QCD scale and the temperature in compact stars are small compared to the baryon chemical potential. Hence it makes sense to consider zero temperature (T = 0) in the studies of dense quark matter that are applicable to compact stars conditions.

In this chapter, we shall discuss some of the underlying theories, basic ideas of color superconductivity and superfluidity, and the model associated with this work. In chapter 2, we review the background methods and give the detail calculation for the polarization operator to study the superfluid medium. In chapter 3, we discuss the results.

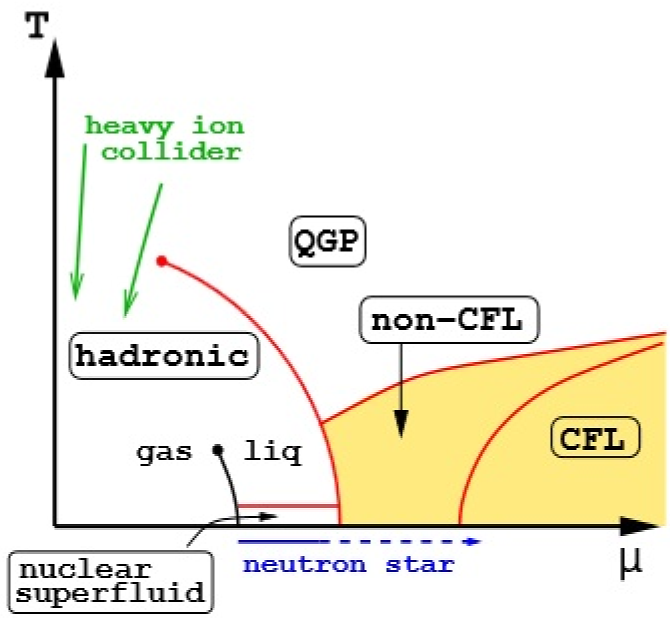


Figure 1.1: Conjectured phase diagram of QCD in $\mu - T$ plane

1.1 Yukawa Theory

In 1934, a Japanese theoretical physicist, Hideki Yukawa, made the earliest attempt to explain the nature of nuclear force and predicted the existence of the meson. He proposed in his theory that massive bosons (mesons) mediate the interaction between two nucleons. According to this theory, the interaction between a Dirac fermion of mass m and a real scalar field of mass μ is given by

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2\right) + \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi - \lambda \phi \bar{\psi}\psi \tag{1.1}$$

where the interaction Lagrangian is

$$\mathcal{L}_{\text{Yukawa}} = -\lambda \phi \bar{\psi}\psi \tag{1.2}$$

In four-space dimensions $[\phi] = 1$, $[\psi] = \frac{3}{2}$ and the coupling is dimensionless: $\lambda = 0$. Thus, we expect the theory to be renormalizable. This theory is also invariant under $\psi = e^{-i\alpha\psi}$ transformation, which shows a global U(1) symmetry. In the case of pseudoscalars, the Yukawa interaction term is

$$\mathcal{L}_{\text{Yukawa}} = -\lambda \phi \bar{\psi}\gamma^5 \psi \tag{1.3}$$

In our work, the Lagrangian for fermions of mass m interacting via the exchange of scalar bosons of mass M_S, reads as [1]

$$\mathcal{L} = \bar{\psi}\left(i\gamma.\partial - m\right)\psi - g\phi\bar{\psi}\psi + \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - M_S^2 \phi^2\right) \tag{1.4}$$

where g is the coupling constant, which determines the strength of the force exerted in the interaction. In the interaction, a potential arises from the exchange of a massive scalar field of the following form [2]:

$$V_{\text{Yukawa}}(r) \sim \frac{1}{r} e^{-M_S r} \tag{1.5}$$

The force of the corresponding potential is always attractive. Since the field mediator is massive, the force has a certain range, which is inversely proportional to the mass. An analogy of Yukawa potential can be found with Coulomb potential in electromagnetism when $M_S = 0$. The Yukawa theory can also be used in the standard model to describe the coupling between the Higgs field and massless quark and lepton fields.

1.2 Color-Superconductivity and Superfluidity

In this section, we give a brief description of color superconductivity and superfluidity. The study of quark matter and underlying theories of QCD have made significant progress in recent years. Since the phenomenon of color superconductivity is predicted to occur only in quark matter under certain conditions, detailed studies were necessary and many researchers have been doing this. Despite limitations of findings, the current understanding of the color superconductive state of quark matter guides us to believe that it may occur naturally in compact stars often called "neutron" stars. In order to have an overall idea, we briefly discuss the quarks, Cooper pairs and superfluidity in this section.

We know that baryons are composite subatomic particles made of quarks and quarks are the fundamental building blocks of nuclear matter. So far it is known that color superconductivity occurs when the baryon density is sufficiently high (above nuclear density) and the temperature is much lower than the density scale.

Quarks have six flavors: up(u), down(d), charm(c), strange(s), top(p), bottom(b). These are grouped into three generations, in which the first and second members of each generation have an electric charge of $\frac{2}{3}$ and $-\frac{1}{3}$ respectively. Also, quarks have color charges such as: red, green and blue. The interaction between two quarks is mediated by a gluon, which is defined as the strong interaction and described by QCD. From a *theoretical* point of view, it can be stated that a color superconducting phase is a state of sufficiently dense and cold quark matter. One gluon exchange interaction is attractive at the Fermi surface leading to the formation of Cooper pairs [3]. These Cooper pairs form a Bose condensate. The *phenomenological* view states that this phase breaks some of the symmetries of the associated theory [4] leading to a different spectrum of excitations and different transport properties.

In QCD, a one-gluon exchange interaction is attractive in the color anti-triplet channel. One would expect to have quarks condensate into Cooper pairs at a sufficiently large quark chemical potential μ and a sufficiently low temperature T. These pairs are color anti-triplet breaking the $SU(3)$ color symmetry of the ground state[5], [6], [7]. This gives rise to color superconductivity.

Different flavors and color charges lead to different varieties of quark matter, each of which is a separate phase. We can expect to have three (up, down, strange) flavors in the core of a compact star. From these different flavors, a 9x9 color-flavor matrix of pairing patterns is possible and each of the patterns breaks different symmetries. Here, we briefly discuss a 2-flavor and 3-flavor color superconductor.

1.2.1 Two Flavor Color Superconductor

It is the simplest color-superconducting phase, namely 2SC phase with spin-0 Cooper pairing in quark matter. In 2SC ground state, the vector-like $SU(3)_C$ is broken down to $SU(2)_C$ subgroup[9], where the original symmetry

$$G = SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \tag{1.6}$$

is broken to

$$H_{2SC} = SU(2)_C \otimes SU(2)_L \otimes SU(2)_R \tag{1.7}$$

In this case, Cooper pairs are formed with two out of the three quarks and two (conventionally red and green) out of the three colors. In most studies of color superconductivity, it was assumed that Cooper pairs were formed by the two lightest quark flavors u and d. Due to the larger strange quark mass (which is approximately 100 MeV), the Fermi surfaces of nonstrange and strange quarks do not match[6]. As a result, no us or ds Cooper pairs form and in addition, blue quarks remain unpaired. The presence of unpaired blue quarks is also connected with the absence of baryon superfluidity in 2SC phase. The generator[9] of the baryon number conservation symmetry is as follows:

$$\tilde{B} = B - \frac{2}{\sqrt{3}} T_8 = \text{diag}_{\text{color}}(0, 0, 1) \tag{1.8}$$

where B is the generator of $U(1)_B$ symmetry in vacuum, which mixes with the color generator T_8 to produce the generator $\tilde{B}$ of the $\tilde{U}(1)_B$ symmetry in the medium.The color-flavor structure of the condensate of 2SC color superconductor is given by [9]

$$\left\langle \left(\bar{\psi}^C\right)_i^a \gamma^5 \psi_j^b \right\rangle \sim \epsilon_{ij} \epsilon^{ab3} \tag{1.9}$$

where ϵ_{ij} and ϵ^{ab3} are antisymmetric tensors in flavor and color spaces respectively. This condensate points in the blue color direction, and blue doesn't participate in the pairing. Also, it denotes a singlet representation of the global $SU(2)_L \times SU(2)_R$ chiral group, which means that the chiral symmetry is not broken. In addition, the isospin symmetry remains unbroken. In 2SC phase, the $\tilde{U}(1)_{em}$ gauge symmetry is unbroken, and its corresponding generator is

$$\tilde{Q} = Q - \frac{1}{\sqrt{3}} T_8 \tag{1.10}$$

here, $Q = \text{diag}_{flavor}(\frac{2}{3}, -\frac{1}{3})$ is the generator of the $U(1)_{em}$ symmetry in vacuum.

1.2.2 Three Flavor Color Superconductor

While converting non-strange quarks into strange quarks, a reduction in free energy might be possible[10], which leads to a speculation that strange quark matter is the ground state of baryonic matter. This fact may give a reason to state that dense quark matter can be composed of not only up and down quarks but also strange quarks. In this case, we consider a three flavor ($N_f = 3$) quark matter where all quarks are assumed to be massless and quark model possesses global $SU(3)_L \times SU(3)_R$ chiral symmetry and a global $U(1)_B$ symmetry connected with the baryon number conservation.

In the ground state, the chiral symmetry is broken down to its vector-like subgroup by the formation of a condensate of Cooper pairs. The symmetry breaking pattern[11] is

$$SU(3)_{color} \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{color+L+R}$$

The spin-0 condensate that determines the ground state is

$$\left\langle \left(\bar{\psi}^C\right)^a_i \gamma^5 \psi^b_j \right\rangle \sim \sum_{I,J=1}^{3} C^I_J \epsilon_{ijI} \epsilon^{abJ} + ... \tag{1.11}$$

Though the mechanism of chiral symmetry breaking is quite unusual, since it occurs by locking color and flavor symmetries, it can be described in terms of two separate condensates. These are made of left-handed and of right-handed fields and the corresponding phase is called color-flavor-locked (CFL) phase[11]. In the three-flavor structure, there are nine original quark states, which in CFL give rise to a singlet Goldstone boson, an octet of Goldstone bosons associated with chiral symmetry breaking, an octet of vector mesons, and an octet and singlet of baryons[2]. Except the U(1) goldstone boson, other states match the quantum numbers of the low lying multiplets in QCD at low density. This $U(1)_B$ baryon number symmetry breaking in the ground state makes the CFL phase superfluid. The CFL phase is also not an electromagnetic superconductor like the 2SC phase because of an unbroken $U(1)_{em}$ gauge symmetry remains in the ground state.

In the CFL phase, there are no gapless quasiparticles, as with the 2SC phase, in the low energy spectrum. $exp(-\Delta/T)$ suppresses the contribution of quark quasiparticles to all transport and many thermodynamic properties at $T << \Delta$. Rather, Nambu-Goldstone bosons play an important role in many transport properties[12], [13].

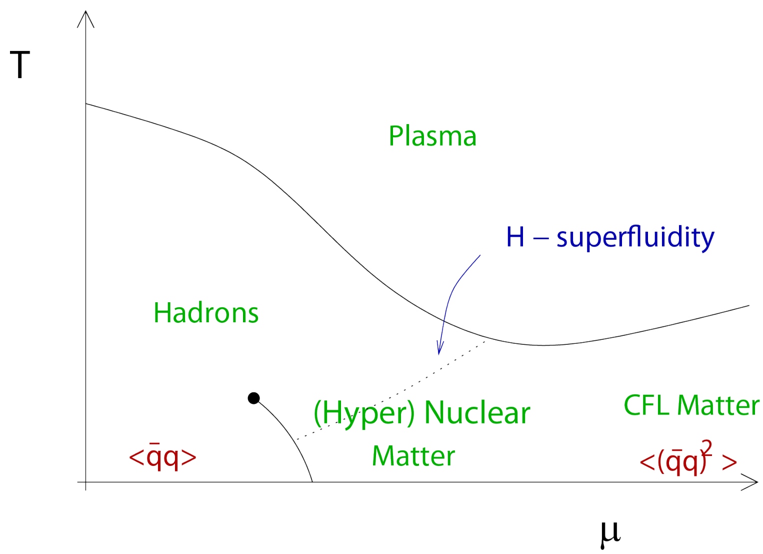


Figure 1.2: Conjectured phase diagram of $N_f = 3$ hadronic matter in the limit of exact flavor symmetry

1.3 Polarization Operator in Field Theories

In quantum field theory, the vacuum state is the quantum state with the lowest possible energy. This state contains no physical particles but it contains virtual particle-antiparticle pairs which are spontaneously created out of the vacuum. Sometimes an intuitive picture of these virtual particles can be provided from the Heisenberg energy-time uncertainty principle:

$$\Delta E \Delta t \geq \hbar \tag{1.12}$$

following the argument that the short lifetime of the virtual particles allows the borrowing of large energies from the vacuum. Also, these pairs contain various kinds of charges. In QCD, they are color and electrically charged quarks and in QED, they are electrically charged leptons. In quantum electrodynamics, the lowest energy state, or the ground state of the electromagnetic field when the fields are quantized, is called the QED vacuum, in which these pairs are virtual electron-positron pairs. In the electromagnetic field, around an electron, an electron-positron pair acts as an electric dipole and changes the original distribution of the charge and current. This effect is known as the screening effect or the dielectric effect, and the process is referred to as the vacuum polarization or the photon self-energy which is quantified by the vacuum polarization tensor ($\Pi^{\mu\nu}$). $\Pi^{\mu\nu}$ is also known as polarization operator. The photon polarization tensor is the central building block of an effective theory used to describe photon propagation in the quantum vacuum and contains essential information about the renormalization properties of QED. Also it gives the quantum correction to the law of Coulomb force. A general effective theory [17] for soft electromagnetic fields in quantum vacuum reads as

$$\mathcal{L}[\mathcal{A}] = -\frac{1}{4}\mathcal{F}\mu\nu(x)\mathcal{F}^{\mu\nu}(x) - \frac{1}{2}\int_{x'} a_\mu(x)\Pi^{\mu\nu}(x,x')a_\nu(x') \tag{1.13}$$

here, $\mathcal{F}_{\mu\nu}$ is the field strength tensor and $\mathcal{A}_\mu = A_\mu + a_\mu$ where A_μ is an external magnetic field and a_μ is a propagating photon field. In particular, an expression for the photon self energy, known as the Schwinger-Dyson equation of Fig.(1.3), can be given as

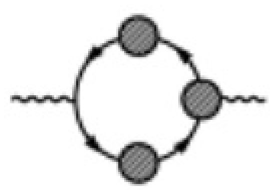


Figure 1.3: One loop contribution of a fermion−antifermion pair to the vacuum polarization

$$\Pi^{\mu\nu}(k) = e^2 T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left[\gamma^\mu \mathcal{G}(p)\Gamma^\nu(p, p-k)\mathcal{G}(p-k)\right] \tag{1.14}$$

The main source of knowledge about properties of matter is the response of matter to an external perturbation. To calculate such a response, the linear response theory can be used as the simplest framework in QCD. In the study of dense quark matter, it is found that the one-gluon exchange interaction between quarks is dominant when quarks are weakly interacting and the interaction is partially screened by a surrounding dense medium. In QCD, screening effects play an important role at length scales larger than the average distance between quarks. For example, Debye screening and Landau damping are the main effects in normal phase. The polarization tensor in dense quark matter can help to extract these properties. The inverse propagator of the medium modified gluon can be written in the following form:

$$\left(\mathcal{D}^{-1}\right)^{\mu\nu}_{AB} = \left(\mathcal{D}_0^{-1}\right)^{\mu\nu}_{AB} + \Pi^{\mu\nu}_{AB} \tag{1.15}$$

Gluons with soft momenta play the key role in Cooper pairing and the corresponding soft gluon polarization tensor is calculated in the hard dense loop (HDL) approximation, where $\Pi^{\mu\nu}_{AB} \sim \alpha_s \mu^2$. In this approximation, the dominant one-loop contribution, in which the internal quark momenta are hard, $p \sim \mu$, is taken into account since this contribution is large compared to other contributions such as ghost and gluon loops. Polarization tensor in the 2SC phase [18] is given by the following one-loop expression:

$$\Pi^{\mu\nu}_{AB}(P) = \frac{1}{2}\frac{T}{V}\sum_{K} \mathrm{Tr}_{\mathrm{D,c,f,NG}} \left[\hat{\Gamma}^{\mu}_{A} S(k) \hat{\Gamma}^{\nu}_{B} S(K-P)\right] \tag{1.16}$$

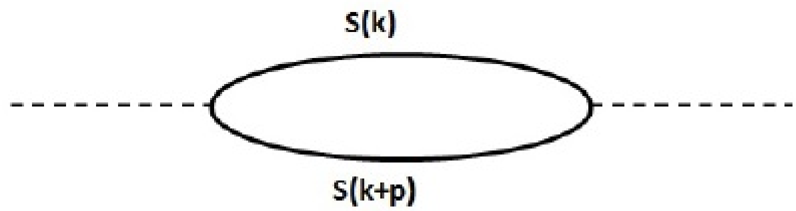
where the trace is over the Dirac, color, flavor and Nambu-Gorkov indices. In the static limit ($p_0 = 0$), it reduces to

$$\Pi^{\mu\nu,ab}_{\mathrm{HDL}} = -\frac{4\alpha_s \mu^2}{\pi} \delta^{ab} u^{\mu} u^{\nu} \tag{1.17}$$

which describes the static screening of quark color charges at large distances in the normal phase of dense quark matter. Eq.(1.17) can be used to get Debye screening mass and Meissner screening mass in normal phase.

In our work, we are going to study how a scalar field is affected by a superfluid medium when the field propagates through it. In this case the polarization operator of the following diagram reads as

$$\Pi = \sum_{k_4} \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}\left[\Gamma S(k) \Gamma S(k+p)\right] \tag{1.18}$$



1.4 NJL Model

Y. Nambu and G. Jona-Lasino developed a dynamical theory [14] of elementary particles motivated by the observation of an analogy between the properties of Dirac particles, and the quasi-particle excitations that appear in the theory of superconductivity (BCS theory). That model was introduced for the description of nucleons with dynamically generated masses. Before QCD arrived, there were indications for the existence of chiral symmetry, but the challenge was to find a mechanism which could explain the large nucleon mass without destroying the symmetry. The pioneering idea, which was inspired from the BCS theory could give a way to generate the mass gap in the Dirac spectrum of the nucleon. They also introduced a Lagrangian for a nucleon field ψ of bare mass m, with a point like, chirally symmetric four fermion interaction, which is

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\left\{\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right\} \tag{1.19}$$

where $\vec{\tau}$ is a Pauli matrix acting in isospin space, and G is a dimensionful coupling constant. After the development of QCD, NJL model is adopted and commonly used for the description of quarks. As for example, the NJL model can be used for studying color superconducting phases. In this case, ψ of Eq. (1.19) is a quark field with two flavor and three color degrees of freedom. The Lagrangian can be modified by adding other chirally symmetric interaction terms. For example, a simple NJL model that respects $SU(2)_L \times SU(2)_R$ global chiral symmetry, in the limit $m_i^{(0)} \to 0$, is given by the following Lagrangian density[15] :

$$\begin{aligned}\mathcal{L}_{NJL} \quad &= \bar{\psi}_i^a\left(i\gamma^\mu\partial_\mu + \gamma^0\mu - m_i^{(0)}\right)\psi_i^a + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(i\bar{\psi}\gamma_5\vec{\tau}\psi\right)^2\right] \\ &+G_D\left(i\bar{\gamma}^C\varepsilon\epsilon^a\gamma_5\psi\right)\left(i\bar{\psi}\varepsilon\epsilon^a\gamma_5\psi^C\right)\end{aligned} \tag{1.20}$$

Here, $\psi^C = C\bar{\psi}^T$ is the charge-conjugate spinor, $C = i\gamma^2\gamma^0$ is the charge conjugate matrix in the Dirac structure, $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are Pauli matrices in flavor space and $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$ and $(\epsilon^a)^{bc} \equiv \epsilon^{abc}$ are the antisymmetric tensors in flavor and color spaces respectively.

In the case of the existence of deconfined quark matter in the core of compact stars, Eq.(1.20) is one of the various effective models of QCD that can be used to study such strongly interacting regime of quark matter, where quark chemical potential $\mu \simeq 500MeV$.

In our model, Eq.(1.4), we have a single-flavor Lagrangian and the fermions are not charged. As we explore a homogeneous condensate with gap matrix $\Delta(x, y) = \gamma_5\Delta = \Delta^+$, the interaction can be simulated by a point-like four fermion interaction as in NJL-type models . The reduced gap equations [1] in this case are

$$\begin{aligned} \phi^+_{R+} \equiv \phi^-_{R-} &= \phi^+_{l-}F(\phi^+_{l-}) \\ \phi^+_{l-} \equiv \phi^-_{l+} &= \phi^+_{r+}F(\phi^+_{r+}) \end{aligned} \tag{1.21}$$

The number of gap equations reduces to two as the gaps for the left- and right-handed quasiparticles equal those of the corresponding quasiparticles. In the limit $M_S \to \infty$, the model represented by Eq.(1.4) reduces to NJL-type model, in which quasiparticle and quasi-antiparticle gaps become degenerate. But the scalar boson exchange over a finite range, $M_S < \infty$, lifts the degeneracy and produces two independent gap functions.

Chapter 2

Methods and Calculation

2.1 Fermion-Fermion Condensate

A quark-quark condensate $\Delta^{ij}_{fg,\alpha\beta}$ is a $N_c \times N_c$ matrix in color space $(i, j = 1, 2, ..., N_c)$, a $N_f \times N_f$ matrix in flavor space $(f, g = 1, 2, ...N_f)$, and a 4×4 matrix in Dirac space $(\alpha, \beta = 1, ..., 4)$. Here we focus on Dirac structure only ignoring color or flavor structure of the gap matrix. The system of fermions that we have considered is color- and flavor-less. This assumption make the model simple.

In principle, quark pair condensation can occur in channels with arbitrary total spin J. But it has been indicated from most of the previous studies that the condensation in the channel with total spin $J = 0$ is favored most for two or more flavors. In our case we can expect the formation of Cooper pairs at finite density and sufficiently low temperature in $J = 0$ channel as scalar one-boson exchange is attractive. The fermions in our Lagrangian (1.4) are not charged, so the condensate shows superfluidity, not superconductivity. However, the Dirac structure of the superfluid condensate is similar to that of the color superconducting condensate in QCD.

2.1.1 Gap Equation and Gap Matrix

Relativistic gap equation [6] may be derived using a generalization of a method developed for a non-relativistic case by Nambu [16]. The gap equation for a system of fermions where we have fermion-fermion condensate and shows superfluidity has the following form:

$$\Delta(k) = g^2 \frac{T}{V} \sum_q D(k - q, M_s) G_0^-(q) \Delta(q) G^+(q) \tag{2.1}$$

Here

$$\left[G_0^{\pm}\right]^{-1}(k) = \gamma.k \pm \mu\gamma_0 - m \tag{2.2}$$

is the free fermion propagator and

$$G^{\pm} = \left\{\left[G_0^{\pm}\right]^{-1} - \Sigma^{\pm}\right\}^{-1} \tag{2.3}$$

is the fermion propagator. $\Sigma^{\pm} = \Delta^{\mp} G_0^{\mp} \Delta^{\pm}$. $D(p, M_s)$ is the scalar boson propagator. For a translationally invariant system, $\Delta(x, y) \equiv \Delta(x - y)$. The Fourier transform of gap matrix is

$$\Delta(k) = \int_x e^{ik.x} \Delta(x) \tag{2.4}$$

where $\int_x \equiv \int_0^{\frac{1}{T}} d(it)$.

In J =0 channel massive fermions have eight possible gaps, but they reduce to four in ultra-relativistic limit, which have the following form:

$$\Delta(k) = \phi_{r+}^{+}(k)\mathcal{P}_{r+}^{+}(\mathbf{k}) + \phi_{l-}^{+}(k)\mathcal{P}_{l-}^{+}(\mathbf{k}) + \phi_{r-}^{-}(k)\mathcal{P}_{r-}^{-}(\mathbf{k}) + \phi_{l+}^{-}(k)\mathcal{P}_{l+}^{-}(\mathbf{k}) \tag{2.5}$$

Here, $\phi_{r,l\pm}^{\pm}$ are individual gap functions and $\mathcal{P}_{r,l\pm}^{\pm}$ are projectors onto states with given chirality and helicity. Fermions of different helicity do not form a condensate in J = 0 channel. For both massive and massless fermions, they need to have the same helicity in the center of momentum frame of the pair. The same helicity means that the projectors of the spin along the direction of motion are anti-parallel.

Since the full fermion propagator (2.3) determines the excitation spectrum in a superfluid, it is necessary to derive them first in the ultra-relativistic limit. Then the gap equation (2.5) gets the form

$$\begin{aligned}\Delta(k) = \quad & g^2 \frac{T}{V} \sum_q D(k-q, M_s) \left[\frac{\phi_{r+}^{+}(q)}{q_0^2 - \left[\epsilon^{+}\left(\phi_{r+}^{+}\right)\right]^2} \mathcal{P}_{l+}^{-}(\mathbf{q}) + \frac{\phi_{l-}^{+}(q)}{q_0^2 - \left[\epsilon^{+}\left(\phi_{l-}^{+}\right)\right]^2} \mathcal{P}_{r-}^{-}(\mathbf{q}) \right. \\ & \left. + \frac{\phi_{r-}^{-}(q)}{q_0^2 - \left[\epsilon^{-}\left(\phi_{r-}^{-}\right)\right]^2} \mathcal{P}_{l-}^{+}(\mathbf{q}) + \frac{\phi_{l+}^{-}(q)}{q_0^2 - \left[\epsilon^{-}\left(\phi_{l+}^{-}\right)\right]^2} \mathcal{P}_{r+}^{+}(\mathbf{q}) \right]\end{aligned} \tag{2.6}$$

Taking into account the following projectors

$$\begin{aligned}\mathcal{P}^{+}_{r+}(\mathbf{k}) &\equiv \mathcal{P}_r\mathcal{P}_+(\mathbf{k})\\ \mathcal{P}^{+}_{l-}(\mathbf{k}) &\equiv \mathcal{P}_l\mathcal{P}_-(\mathbf{k})\\ \mathcal{P}^{-}_{r-}(\mathbf{k}) &\equiv \mathcal{P}_r\mathcal{P}_-(\mathbf{k})\\ \mathcal{P}^{-}_{l+}(\mathbf{k}) &\equiv \mathcal{P}_l\mathcal{P}_+(\mathbf{k})\end{aligned} \tag{2.7}$$

the gap equations for individual gap functions can be derived. But for a point-like four-fermion interaction as in NJL models, $D(k-q, M_s) \to \frac{\delta(k-q)}{M_s^2}$. This is the case we will consider when we calculate the polarization operator. After defining

$$F(\phi) \equiv \frac{g^2}{2M_s^2}\frac{T}{V}\sum_q \left[\frac{1}{q_0^2 - [\epsilon^+(\phi)]^2} + \frac{1}{q_0^2 - [\epsilon^-(\phi)]^2}\right] \tag{2.8}$$

the gap equations reduce to (1.21)

$$\begin{aligned}\phi^{+}_{R+} \equiv \phi^{-}_{R-} &= \phi^{+}_{l-}F(\phi^{+}_{l-})\\ \phi^{+}_{l-} \equiv \phi^{-}_{l+} &= \phi^{+}_{r+}F(\phi^{+}_{r+})\end{aligned}$$

which are mentioned in NJL-model section. The Dirac structure of our quark-quark condensate is

$$\left\langle \bar{\psi}_C(x)\psi(y)\right\rangle \equiv \Delta(x,y) \tag{2.9}$$

If we assume a homogeneous condensate of magnitude Δ, we get $\Delta(x,y) = \gamma_5\Delta = \Delta^+$ and $\Delta^- = \gamma_0\left(\Delta^+\right)^\dagger\gamma_0$. We will derive the gap equations in next section.

2.2 Mean-Field Approximation

The grand partition function of a statistical mechanical system associated with the Lagrangian (1.4) at finite temperature T, chemical potential μ, and where fermions interact via N-component bosonic field is

$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\phi exp \left\{ I \left[\bar{\psi}, \psi, \phi \right] \right\} \tag{2.10}$$

The action in the grand partition function reads as

$$\begin{aligned} I \left[\bar{\psi}, \psi, \phi \right] &= \int_x \left\{ \bar{\psi}(x) \left[G_0^+ \right]^{-1} (x) \psi(x) - \frac{1}{2} \phi(x) D^{-1}(x) \phi(x) \right\} \\ &- g \int \bar{\psi}(x) \Gamma \psi(x) \phi(x) \end{aligned} \tag{2.11}$$

where $D^{-1} = \partial^2 - M_s^2$ is the inverse scalar propagator. After integrating in the scalar field $\phi(x)$, we get

$$\mathcal{Z} = \mathcal{N}' \left(det D^{-1} \right)^{-\frac{1}{2}} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi exp \left\{ I \left[\bar{\psi}, \psi \right] \right\} \tag{2.12}$$

and

$$I \left[\bar{\psi}, \psi \right] = \int_x \left\{ \bar{\psi}(x) \left[G_0^+ \right]^{-1} (x) \psi(x) + \frac{g^2}{2} \bar{\psi}(x) \Gamma \psi(x) D(x) \bar{\psi}(x) \Gamma \psi(x) \right\} \tag{2.13}$$

The last term in the action corresponds to current-current interaction and it is biquadratic in fermion fields which don't allow the integration to be carried out over $\bar{\psi}, \psi$. But if we expand the product of two fields into a diquark condensate and the fluctuation term and only keep terms up to quadratic order in the fields, the integration over $\bar{\psi}, \psi$ can be carried out. This idea is called mean-field approximation method. In this case we contract either $\bar{\psi}$ and $\bar{\psi}$ or ψ and ψ.

Introducing the charge conjugate spinor $\psi_C(x) = C\bar{\psi}^T(x)$ and defining $\bar{\Gamma}_a$ as $\bar{\Gamma}_a = C\Gamma_a^T C^{-1}$, we get the grand partition function in the mean-field approximation for a fermion-fermion condensate as

$$\mathcal{Z}_{<FF>} = \mathcal{N}' \left(det D^{-1} \right)^{-\frac{1}{2}} exp \left(\frac{1}{2} g^2 \Delta^2 \right) \int \mathcal{D}\bar{\psi}\mathcal{D}\psi exp \left\{ I \left[\bar{\psi}, \psi \right] \right\} \tag{2.14}$$

where the mean field action is

$$I \left[\bar{\psi}, \psi \right] = \int_x \bar{\psi}(x) \left[i\gamma.\partial + \mu\gamma_0 - m \right] \psi(x) + \frac{1}{2} \int_x \left[\bar{\psi}_C(x) \Delta^+ \psi(x) + H.C. \right] \tag{2.15}$$

and the gap equation after the approximation reads as

$$\Delta^+(x) = g^2\bar{\Gamma}\left\langle \psi_C(x)\bar{\psi}(x)\right\rangle \Gamma D(x) \tag{2.16}$$

After solving this equation and taking the Fourier transform, Eq.(2.16) becomes

$$\Delta^+(k) = g^2\frac{T}{V}\sum_q \bar{\Gamma}D(k-q)G_0^-(q)\Delta^+(q)G^+(q)\Gamma \tag{2.17}$$

2.3 Nambu-Gorkov Propagator

It is more convenient to study diquark condensates in dense quark matter using Nambu-Gorkov spinors which are formed by two Dirac spinors and allow us to use the known formulas to perform the functional integrals in the fermions. To do this, we introduce:

$$\Psi = \begin{pmatrix} \psi \\ \psi_c \end{pmatrix} \tag{2.18}$$

$$\bar{\Psi} = \begin{pmatrix} \bar{\psi} & \bar{\psi}_c \end{pmatrix} \tag{2.19}$$

where the charge-conjugate spinor ψ_C is defined earlier. The structure of inverse quark propagator in this basis reads

$$S_0^{-1} = \begin{pmatrix} [G_0^+]^{-1} & 0 \\ 0 & [G_0^-]^{-1} \end{pmatrix} \tag{2.20}$$

while in the ground state with Cooper pairing, it becomes

$$S^{-1} = \begin{pmatrix} [G_0^+]^{-1} & \Delta^- \\ \Delta^- & [G_0^-]^{-1} \end{pmatrix} \tag{2.21}$$

Here

$$\left[G_0^{\pm}\right]^{-1} = \gamma^\mu k_\mu \pm \mu\gamma_0 \tag{2.22}$$

are inverse Dirac propagator for massless quarks G_0^+ and charge-conjugate quarks G_0^-. Since charge conjugate spinors appear in the mean field action (2.15) of the previous section, it is not yet in the usual form of integration. After introducing Nambu-Gorkov spinors, it becomes easy to integrate. Now its compact matrix notation form is

$$I\left[\bar{\Psi}, \Psi\right] = \frac{1}{2}\int_x \bar{\Psi}(x) S^{-1}(x)\Psi(x) \tag{2.23}$$

Taking the Fourier transformation of the fields and $\left[G_0^{\pm}\right]^{-1}$, $\Delta^{\pm}$ and using the following identities

$$\begin{aligned} \psi_C(k) &= C\bar{\psi}^T(-k) \\ \bar{\psi}_C(k) &= \psi^T(-k)C \\ \Delta^-(k) &= \gamma_0\left[\Delta^+(k)\right]^\dagger \gamma_0 \end{aligned} \tag{2.24}$$

the action (2.23) can be transformed into momentum space as

$$I\left[\bar{\Psi}, \Psi\right] = \frac{1}{2}\sum_k \bar{\Psi}(k)\frac{S^{-1}(k)}{T}\Psi(k) \tag{2.25}$$

To perform the Grassman integration over $\bar{\psi}$, ψ in the grand partition function (2.14), it is necessary to rewrite the following:

$$\begin{aligned} \mathcal{D}\bar{\psi}\mathcal{D}\psi &\equiv \prod_k d\bar{\psi}(k)d\psi(k) \\ &= \tilde{\mathcal{N}}\prod_{k>0} d\bar{\psi}(k)d\psi_C(k)d\psi(k)d\bar{\psi}_C(k) \end{aligned} \tag{2.26}$$

and

$$\frac{1}{2}\sum_k \bar{\Psi}(k)\frac{S^{-1}(k)}{T}\Psi(k) \equiv \sum_{k>0}\bar{\Psi}(k)\frac{S^{-1}(k)}{T}\Psi(k) \tag{2.27}$$

which allow to get finally

$$\int \mathcal{D}\bar{\psi}\mathcal{D}\psi exp\left\{I\left[\bar{\psi}, \psi\right]\right\} \equiv \tilde{\mathcal{N}}\left(det\left[\frac{S^{-1}}{T}\right]\right)^{\frac{1}{2}} \tag{2.28}$$

Solving $1 = S^{-1}S$, the Nambu-Gorkov propagator can be found as

$$S = \begin{pmatrix} G^+ & -G_0^+\Delta^- G^- \\ -G_0^-\Delta^+ G^+ & G^- \end{pmatrix} \tag{2.29}$$

2.4 Calculation of the Polarization Operator

The polarization operator of Fig.(2.1) is

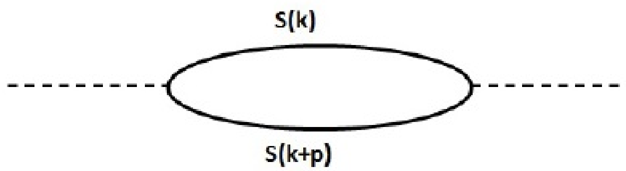


Figure 2.1: Diagram for the polarization operator

$$\Pi = \mathrm{Tr} \int \Gamma S(k) \Gamma S(k+p) d^4k \tag{2.30}$$

At finite temperature the integration in k_4 is replaced by the Matsubara sum, the above expression can be rewritten as

$$\Pi = G \sum_{k_4} \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}\left[\Gamma S(k) \Gamma S(k+p)\right] \tag{2.31}$$

In the following sections we find the elements of the Nambu-Gorkov propagator, take the trace, do the Matsubara sum over momentum and finally the integration.

2.4.1 Calculation of $G^{\pm}$ and $\Sigma^{\pm}$

In this section we use the following relation:

$$G^{\pm} \equiv \left\{ \left[G_0^{\pm}\right]^{-1} - \Sigma^{\pm} \right\}^{-1} \tag{2.32}$$

$$\Sigma^{\pm} \equiv \Delta^{\mp} G_0^{\mp} \Delta^{\pm} \tag{2.33}$$

We have already defined the free propagator for massless quarks as $\left[G_0^{\pm}\right]^{-1}(k) = \gamma^\mu k_\mu \pm \mu\gamma_0$ and $\Delta^{\pm} = \pm\Delta\gamma_5$.

Proposing a structure for $G_0^+(k)$ as

$$G_0^+ = \gamma^\mu \alpha_\mu \pm \mu\gamma_0 \tag{2.34}$$

and using the condition that $[G_0^+][G_0^+]^{-1} = 1$, we get

$$G_0^+ = \frac{\gamma^\mu k_\mu^+}{(k_0+\mu)^2 - |\vec{k}|^2} = \frac{\gamma^\mu k_\mu + \mu\gamma_0}{(k_0+\mu)^2 - |\vec{k}|^2} \tag{2.35}$$

Similarly,

$$G_0^- = \frac{\gamma^\mu k_\mu^-}{(k_0-\mu)^2 - |\vec{k}|^2} = \frac{\gamma^\mu k_\mu - \mu\gamma_0}{(k_0-\mu)^2 - |\vec{k}|^2} \tag{2.36}$$

Substituting (2.35) and (2.36) into (2.33) , the expressions for $\Sigma^{\pm}$ are found as

$$\Sigma^+ = \Delta^2 \frac{\gamma^\mu k_\mu^-}{(k_0-\mu)^2 - |\vec{k}|^2} = \Delta^2 \frac{\gamma^\mu k_\mu - \mu\gamma_0}{(k_0-\mu)^2 - |\vec{k}|^2} \tag{2.37}$$

$$\Sigma^- = \Delta^2 \frac{\gamma^\mu k_\mu^+}{(k_0+\mu)^2 - |\vec{k}|^2} = \Delta^2 \frac{\gamma^\mu k_\mu + \mu\gamma_0}{(k_0+\mu)^2 - |\vec{k}|^2} \tag{2.38}$$

Substituting (2.35), (2.36), (2.37) into (2.38), it is possible to derive

$$\begin{aligned} [G^+]^{-1} &= \frac{\gamma_0}{(k_0-\mu)^2 - \epsilon^2} \left\{ \left[(k_0-\mu)^2 - \epsilon^2\right](k_0+\mu) - (k_0-\mu)\Delta^2 \right. \\ &- \left. \gamma_0 |\vec{k}| \hat{k} \left[(k_0-\mu)^2 - \epsilon^2\right] \right\} \end{aligned} \tag{2.39}$$

Introducing energy projectors

$$\Lambda^{\pm} = \frac{1 \pm \gamma_0 \hat{k}}{2} \tag{2.40}$$

it becomes easier to rewrite the above expression in the following form:

$$[G^+]^{-1} = \frac{k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2}{k_0^- + |\vec{k}|} \gamma_0 \Lambda^+ + \frac{k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2}{k_0^- - |\vec{k}|} \gamma_0 \Lambda^- \tag{2.41}$$

In a similar way, we may also get

$$[G^-]^{-1} = \frac{k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2}{k_0^- + |\vec{k}|}\gamma_0\Lambda^+ + \frac{k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2}{k_0^- - |\vec{k}|}\gamma_0\Lambda^- \tag{2.42}$$

Here $\Lambda^\pm$ satisfy

$$\begin{aligned} \Lambda^+\Lambda^- &= 0 \\ \Lambda^+\Lambda^+ &= \Lambda^+ \\ \Lambda^-\Lambda^- &= \Lambda^- \end{aligned}$$

To calculate $G^\pm$ using (2.41) and (2.42), we propose a structure for $G^\pm$ as

$$G^\pm = \alpha_\pm\gamma_0\Lambda^+ + \beta_\pm\gamma_0\Lambda^- \tag{2.43}$$

and using the condition that $G^\pm\left(G^\pm\right)^{-1} = 1$, we get

$$G^+ = \frac{k_0^- - |\vec{k}|}{k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2}\gamma_0\Lambda^+ + \frac{k_0^- + |\vec{k}|}{k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2}\gamma_0\Lambda^- \tag{2.44}$$

$$G^- = \frac{k_0^+ - |\vec{k}|}{k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2}\gamma_0\Lambda^+ + \frac{k_0^+ + |\vec{k}|}{k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2}\gamma_0\Lambda^- \tag{2.45}$$

The off-diagonal elements of Nambu-Gorkov propagator can be found using (2.35), (2.36), (2.44) and (2.45), which are

$$\begin{aligned} -G_0^+\Delta^-G^- &= \Delta\frac{\gamma^\mu k_\mu^+\gamma_5(k_0^+ - |\vec{k}|)}{[\left(k_0^+\right)^2 - |\vec{k}|^2][k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2]}\gamma_0\Lambda^+ \\ &+ \Delta\frac{\gamma^\mu k_\mu^+\gamma_5(k_0^+ + |\vec{k}|)}{[\left(k_0^+\right)^2 - |\vec{k}|^2][k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2]}\gamma_0\Lambda^- \end{aligned} \tag{2.46}$$

$$\begin{aligned} -G_0^+\Delta^-G^- &= -\Delta\frac{\gamma^\mu k_\mu^-\gamma_5(k_0^- - |\vec{k}|)}{[\left(k_0^-\right)^2 - |\vec{k}|^2][k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2]}\gamma_0\Lambda^+ \\ &-\Delta\frac{\gamma^\mu k_\mu^-\gamma_5(k_0^- + |\vec{k}|)}{[\left(k_0^-\right)^2 - |\vec{k}|^2][k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2]}\gamma_0\Lambda^- \end{aligned} \tag{2.47}$$

Now we have all the elements of S(k). In this step we take S(k) into (k+p)-space and take the product of them to get the following:

$$S(k)S(k+p) = \begin{pmatrix} G_k^+ G_{k+p}^+ + (G_0^+ \Delta^- G^-)_k (G_0^- \Delta^+ G^+)_{k+p} & -G_k^+ (G_0^+ \Delta^- G^-)_{k+p} - (G_0^+ \Delta^- G^-)_k G_{k+p}^- \\ \\ -(G_0^- \Delta^+ G^+)_k G_{k+p}^+ - G_k^- (G_0^- \Delta^+ G^+)_{k+p} & (G_0^- \Delta^+ G^+)_k (G_0^+ \Delta^- G^-)_{k+p} + G_k^- G_{k+p}^- \end{pmatrix} \tag{2.48}$$

In next section we will take the trace. But before we proceed, we summarize all necessary expressions and their structure here.

In k-space

1.

$$G^+ = \alpha_+ \gamma_0 \Lambda^+ + \beta_+ \gamma_0 \Lambda^+ \tag{2.49}$$

where

$$\alpha_+ = \frac{k_0^- - |\vec{k}|}{k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2} \tag{2.50}$$

$$\beta_+ = \frac{k_0^- + |\vec{k}|}{k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2} \tag{2.51}$$

2.

$$G^- = \alpha_- \gamma_0 \Lambda^+ + \beta_- \gamma_0 \Lambda^+ \tag{2.52}$$

where

$$\alpha_- = \frac{k_0^- - |\vec{k}|}{k_0^2 - (|\vec{k}| - \mu)^2 - \Delta^2} \tag{2.53}$$

$$\beta_- = \frac{k_0^- + |\vec{k}|}{k_0^2 - (|\vec{k}| + \mu)^2 - \Delta^2} \tag{2.54}$$

3.

$$G_0^{\pm} = a_{\pm}\gamma^{\mu}k_{\mu}^{\pm} \tag{2.55}$$

where

$$a_{\pm} = \frac{1}{(k_0^{\pm})^2 - |\vec{k}|^2} \tag{2.56}$$

In (k+p)-space

4.

$$G^{+} = \alpha'_{+}\gamma_0\Lambda^{+} + \beta'_{+}\gamma_0\Lambda^{+} \tag{2.57}$$

5.

$$G^{-} = \alpha'_{-}\gamma_0\Lambda^{+} + \beta'_{-}\gamma_0\Lambda^{+} \tag{2.58}$$

6.

$$G_0^{\pm} = a'_{\pm}\gamma^{\mu}k_{\mu}^{\pm} \tag{2.59}$$

It is not necessary to have all the coefficients of energy projectors, $\Lambda^{\pm}$ to be known in (k+p)-space at this stage. These can be derived easily. We will do this later.

2.5 Traces

$$\begin{aligned} \mathrm{Tr}S(k)S(k+p) &= \mathrm{Tr}G_k^{+}G_{k+p}^{+} + \mathrm{Tr}(G_0^{+}\Delta^{-}G^{-})_k(G_0^{-}\Delta^{+}G^{+})_{k+p} \\ &+ \mathrm{Tr}(G_0^{-}\Delta^{+}G^{+})_k(G_0^{+}\Delta^{-}G^{-})_{k+p} + \mathrm{Tr}G_k^{-}G_{k+p}^{-} \end{aligned} \tag{2.60}$$

Here we calculate each of the four terms separately.

1. $\mathrm{TrG}_k^+ G_{k+p}^+$

First, we find the expression for $G_k^+ G_{k+p}^+$ which is

$$\begin{aligned} G_k^+ G_{k+p}^+ &= \left(\alpha_+ \gamma_0 \Lambda^+ + \beta_+ \gamma_0 \Lambda^+\right)_k \left(\alpha'_+ \gamma_0 \Lambda^+ + \beta'_+ \gamma_0 \Lambda^+\right)_{k+p} \\ &= (\alpha_+ \alpha'_+) \Lambda_k^- \Lambda_{k+p}^+ + (\alpha_+ \beta'_+) \Lambda_k^- \Lambda_{k+p}^- \\ &+ (\beta_+ \alpha'_+) \Lambda_k^+ \Lambda_{k+p}^+ + (\beta_+ \beta'_+) \Lambda_k^+ \Lambda_{k+p}^- \end{aligned} \tag{2.61}$$

where

$$\begin{aligned} \Lambda_k^- \Lambda_{k+p}^+ &= \frac{1}{4} + \frac{\gamma_0 \gamma_j \hat{q}_j}{4} - \frac{\gamma_0 \gamma_i \hat{k}_i}{2} - \frac{\gamma_0 \gamma_i \hat{k}_i \gamma_0 \gamma_j \hat{q}_j}{4} \\ \Lambda_k^- \Lambda_{k+p}^- &= \frac{1}{4} - \frac{\gamma_0 \gamma_j \hat{q}_j}{4} - \frac{\gamma_0 \gamma_i \hat{k}_i}{2} + \frac{\gamma_0 \gamma_i \hat{k}_i \gamma_0 \gamma_j \hat{q}_j}{4} \\ \Lambda_k^+ \Lambda_{k+p}^+ &= \frac{1}{4} + \frac{\gamma_0 \gamma_j \hat{q}_j}{4} + \frac{\gamma_0 \gamma_i \hat{k}_i}{2} + \frac{\gamma_0 \gamma_i \hat{k}_i \gamma_0 \gamma_j \hat{q}_j}{4} \\ \Lambda_k^+ \Lambda_{k+p}^- &= \frac{1}{4} - \frac{\gamma_0 \gamma_j \hat{q}_j}{4} + \frac{\gamma_0 \gamma_i \hat{k}_i}{2} - \frac{\gamma_0 \gamma_i \hat{k}_i \gamma_0 \gamma_j \hat{q}_j}{4} \end{aligned} \tag{2.62}$$

Here $\hat{q} = \frac{\vec{k}+\vec{p}}{|\vec{k}+\vec{p}|}$. Using $\mathrm{Tr}(\gamma^\mu \gamma^\nu) = 4\ g^{\mu\nu}$, we get

$$Tr\left(G_k^+ G_{k+p}^+\right) = \left(\alpha_+ \alpha'_+ + \beta_+ \beta'_+\right)\left(1 - \hat{k}.\hat{q}\right) + \left(\alpha_+ \beta'_+ + \beta_+ \alpha'_+\right)\left(1 + \hat{k}.\hat{q}\right) \tag{2.63}$$

2. $\mathrm{TrG}_k^- G_{k+p}^-$

$$\begin{aligned} G_k^- G_{k+p}^- &= \left(\alpha_- \gamma_0 \Lambda^+ + \beta_- \gamma_0 \Lambda^+\right)_k \left(\alpha'_- \gamma_0 \Lambda^+ + \beta'_- \gamma_0 \Lambda^+\right)_{k+p} \\ &= (\alpha_- \alpha'_-) \Lambda_k^- \Lambda_{k+p}^+ + (\alpha_- \beta'_-) \Lambda_k^- \gamma_0 \Lambda_{k+p}^- \\ &+ (\beta_- \alpha'_-) \Lambda_k^+ \Lambda_{k+p}^+ + (\beta_- \beta'_-) \Lambda_k^+ \Lambda_{k+p}^- \end{aligned} \tag{2.64}$$

Similarly,

$$Tr\left(G_k^- G_{k+p}^-\right) = \left(\alpha_- \alpha'_- + \beta_- \beta'_-\right)\left(1 - \hat{k}.\hat{q}\right) + \left(\alpha_- \beta'_- + \beta_- \alpha'_-\right)\left(1 + \hat{k}.\hat{q}\right) \tag{2.65}$$

Adding (2.63) and (2.65), we get

$$\begin{aligned} Tr\left(G_k^+ G_{k+p}^+\right) &+ Tr\left(G_k^- G_{k+p}^-\right) = \left(\alpha_+\alpha'_+ + \beta_+\beta'_+ + \alpha_-\alpha'_- + \beta_-\beta'_-\right)\left(1 - \hat{k}.\hat{q}\right) \\ &+ \left(\alpha_+\beta'_+ + \beta_+\alpha'_+ + \alpha_-\beta'_- + \beta_-\alpha'_-\right)\left(1 + \hat{k}.\hat{q}\right) \end{aligned} \tag{2.66}$$

where

$$\begin{aligned} &\left(\alpha_+\alpha'_+ + \beta_+\beta'_+ + \alpha_-\alpha'_- + \beta_-\beta'_-\right) \\ = &\frac{\left(k_0^- - |\vec{k}|\right)\left(q_0^- - |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| + \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| + \mu\right)^2\right]} \\ + &\frac{\left(k_0^- + |\vec{k}|\right)\left(q_0^- + |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| - \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| - \mu\right)^2\right]} \\ + &\frac{\left(k_0^+ - |\vec{k}|\right)\left(q_0^+ - |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| - \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| - \mu\right)^2\right]} \\ + &\frac{\left(k_0^+ + |\vec{k}|\right)\left(q_0^+ + |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| + \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| + \mu\right)^2\right]} \end{aligned} \tag{2.67}$$

and

$$\begin{aligned} &\left(\alpha_+\beta'_+ + \beta_+\alpha'_+ + \alpha_-\beta'_- + \beta_-\alpha'_-\right) \\ = &\frac{\left(k_0^- - |\vec{k}|\right)\left(q_0^- + |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| + \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| - \mu\right)^2\right]} \\ + &\frac{\left(k_0^- + |\vec{k}|\right)\left(q_0^- - |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| - \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| + \mu\right)^2\right]} \\ + &\frac{\left(k_0^+ - |\vec{k}|\right)\left(q_0^+ + |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| - \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| + \mu\right)^2\right]} \\ + &\frac{\left(k_0^+ + |\vec{k}|\right)\left(q_0^+ - |\vec{q}|\right)}{\left[k_0^2 - \Delta^2 - \left(|\vec{k}| + \mu\right)^2\right]\left[q_0^2 - \Delta^2 - \left(|\vec{q}| - \mu\right)^2\right]} \end{aligned} \tag{2.68}$$

3. Tr $(G_0^+\Delta^- G^-)_k(G_0^-\Delta^+ G^+)_{k+p}$

$$\begin{aligned}
(G_0^+\Delta^- G^-)_k(G_0^-\Delta^+ G^+)_{k+p} &= (a_+\gamma^\mu k_\mu^+)(-\Delta\gamma_5)(\alpha_-\gamma_0\Lambda_k^+ + \beta_-\gamma_0\Lambda_k^+)(a'_-\gamma^\mu q_\mu^-)(\Delta\gamma_5) \\
&\quad \times(\alpha'_+\gamma_0\Lambda_{k+p}^+ + \beta'_+\gamma_0\Lambda_{k+p}^-) \\
&= -\Delta^2\Big\{a_+\alpha_- a'_-\alpha'_+\left(\gamma^\mu k_\mu^+\gamma_5\gamma_0\Lambda_k^+\gamma^\nu q_\nu^-\gamma_5\gamma_0\Lambda_{k+p}^+\right) \\
&+ a_+\alpha_- a'_-\beta'_+\left(\gamma^\mu k_\mu^+\gamma_5\gamma_0\Lambda_k^+\gamma^\mu q_\mu^-\gamma_5\gamma_0\Lambda_{k+p}^-\right) \\
&+ a_+\beta_- a'_-\alpha'_+\left(\gamma^\mu k_\mu^+\gamma_5\gamma_0\Lambda_k^-\gamma^\nu q_\nu^-\gamma_5\gamma_0\Lambda_{k+p}^-\right) \\
&+ a_+\beta_- a'_-\beta'_+\left(\gamma^\mu k_\mu^+\gamma_5\gamma_0\Lambda_k^-\gamma^\nu q_\nu^-\gamma_5\gamma_0\Lambda_{k+p}^-\right)\Big\}
\end{aligned} \tag{2.69}$$

In the above equation, there are four terms which need to be simplified and then we take the trace. So, we simplify each of the four terms separately and take the trace, which are

$$\begin{aligned}
Tr\left(\gamma^\mu k_\mu^+\gamma_5\gamma_0\Lambda_k^+\gamma^\nu q_\nu^-\gamma_5\gamma_0\Lambda_{k+p}^+\right) &= (k_0q_0+\vec{k}.\vec{q}) + (k_0\vec{q}+q_0\vec{k}).\hat{q} - \mu k_0 - \mu\vec{k}.\hat{q} \\
&+(k_0\vec{q}+q_0\vec{k}).\hat{k} + [(\hat{k}.\hat{q})k_0q_0 + (\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
&-(\vec{k}.\vec{q})(\hat{k}.\hat{q}) + (\vec{k}.\hat{q})(\hat{k}.\vec{q})] \\
&-\mu\vec{k}.\hat{k} - \mu(\hat{k}.\hat{q})k_0 + \mu q_0 + \mu\vec{q}.\hat{q} \\
&-\mu^2 + \mu\hat{k}.\vec{q} + \mu(\hat{k}.\hat{q})q_0 - \mu^2\hat{k}.\hat{q}
\end{aligned} \tag{2.70}$$

$$\begin{aligned}
Tr\left(\gamma^\mu k_\mu^+\gamma_5\gamma_0\Lambda_k^+\gamma^\nu q_\nu^-\gamma_5\gamma_0\Lambda_{k+p}^-\right) &= (k_0q_0+\vec{k}.\vec{q}) - (k_0\vec{q}+q_0\vec{k}).\hat{q} - \mu k_0 + \mu\vec{k}.\hat{q} \\
&+(k_0\vec{q}+q_0\vec{k}).\hat{k} - [(\hat{k}.\hat{q})k_0q_0 + (\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
&-(\vec{k}.\vec{q})(\hat{k}.\hat{q}) + (\vec{k}.\hat{q})(\hat{k}.\vec{q})] - \mu\vec{k}.\hat{k} \\
&+\mu(\hat{k}.\hat{q})k_0 + \mu q_0 - \mu\vec{q}.\hat{q} - \mu^2 + \mu\hat{k}.\vec{q} \\
&-\mu(\hat{k}.\hat{q})q_0 + \mu^2\hat{k}.\hat{q}
\end{aligned} \tag{2.71}$$

$$
\begin{aligned}
Tr\left(\gamma^{\mu}k^{+}_{\mu}\gamma_5\gamma_0\Lambda^{-}_{k}\gamma^{\nu}q^{-}_{\nu}\gamma_5\gamma_0\Lambda^{+}_{k+p}\right) \;=\;& (k_0q_0+\vec{k}.\vec{q})+(k_0\vec{q}+q_0\vec{k}).\hat{q}-\mu k_0-\mu\vec{k}.\hat{q} \\
& -(k_0\vec{q}+q_0\vec{k}).\hat{k}-[(\hat{k}.\hat{q})k_0q_0+(\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
& -(\vec{k}.\vec{q})(\hat{k}.\hat{q})+(\vec{k}.\hat{q})(\hat{k}.\vec{q})]+\mu\vec{k}.\hat{k} \\
& +\mu(\hat{k}.\hat{q})k_0+\mu q_0-\mu\vec{q}.\hat{q}-\mu^2 \\
& -\mu\hat{k}.\vec{q}-\mu(\hat{k}.\hat{q})q_0+\mu^2\hat{k}.\hat{q} \qquad (2.72)
\end{aligned}
$$

$$
\begin{aligned}
Tr\left(\gamma^{\mu}k^{+}_{\mu}\gamma_5\gamma_0\Lambda^{-}_{k}\gamma^{\nu}q^{-}_{\nu}\gamma_5\gamma_0\Lambda^{-}_{k+p}\right) \;=\;& (k_0q_0+\vec{k}.\vec{q})-(k_0\vec{q}+q_0\vec{k}).\hat{q}-\mu k_0+\mu\vec{k}.\hat{q} \\
& -(k_0\vec{q}+q_0\vec{k}).\hat{k}+[(\hat{k}.\hat{q})k_0q_0+(\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
& -(\vec{k}.\vec{q})(\hat{k}.\hat{q})+(\vec{k}.\hat{q})(\hat{k}.\vec{q})]+\mu\vec{k}.\hat{k} \\
& -\mu(\hat{k}.\hat{q})k_0+\mu q_0-\mu\vec{q}.\hat{q}-\mu^2 \\
& -\mu\hat{k}.\vec{q}+\mu(\hat{k}.\hat{q})q_0-\mu^2\hat{k}.\hat{q} \qquad (2.73)
\end{aligned}
$$

Putting all these back into (2.69), we find

$$
\begin{aligned}
Tr\Big[(G_0^+\Delta^-G^-)_k(G_0^-\Delta^+G^+)_{k+p}\Big] = & - \Delta^2\left(a_+\alpha_-a'_-\alpha'_-\right)[(k_0q_0+\vec{k}.\vec{q})+(k_0\vec{q}+q_0\vec{k}).\hat{q} \\
& - \mu k_0 - \mu\vec{k}.\hat{q} + (k_0\vec{q}+q_0\vec{k}).\hat{k} + [(\hat{k}.\hat{q})k_0q_0 + (\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
& - (\vec{k}.\vec{q})(\hat{k}.\hat{q}) + (\vec{k}.\hat{q})(\hat{k}.\vec{q})] - \mu\vec{k}.\hat{k} - \mu(\hat{k}.\hat{q})k_0 + \mu q_0 \\
& + \mu\vec{q}.\hat{q} - \mu^2 + \mu\hat{k}.\vec{q} + \mu(\hat{k}.\hat{q})q_0 - \mu^2\hat{k}.\hat{q}] \\
& - \Delta^2\left(a_+\alpha_-a'_-\beta'_-\right)[(k_0q_0+\vec{k}.\vec{q})+(k_0\vec{q}+q_0\vec{k}).\hat{q} \\
& - \mu k_0 - \mu\vec{k}.\hat{q} - (k_0\vec{q}+q_0\vec{k}).\hat{k} - [(\hat{k}.\hat{q})k_0q_0 + (\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
& - (\vec{k}.\vec{q})(\hat{k}.\hat{q}) + (\vec{k}.\hat{q})(\hat{k}.\vec{q})] + \mu\vec{k}.\hat{k} + \mu(\hat{k}.\hat{q})k_0 + \mu q_0 \\
& - \mu\vec{q}.\hat{q} - \mu^2 - \mu\hat{k}.\vec{q} - \mu(\hat{k}.\hat{q})q_0 + \mu^2\hat{k}.\hat{q}] \\
& - \Delta^2\left(a_+\beta_-a'_-\alpha'_+\right)[(k_0q_0+\vec{k}.\vec{q})+(k_0\vec{q}+q_0\vec{k}).\hat{q} \\
& - \mu k_0 - \mu\vec{k}.\hat{q} - (k_0\vec{q}+q_0\vec{k}).\hat{k} - [(\hat{k}.\hat{q})k_0q_0 + (\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
& - (\vec{k}.\vec{q})(\hat{k}.\hat{q}) + (\vec{k}.\hat{q})(\hat{k}.\vec{q})] + \mu\vec{k}.\hat{k} + \mu(\hat{k}.\hat{q})k_0 + \mu q_0 \\
& - \mu\vec{q}.\hat{q} - \mu^2 - \mu\hat{k}.\vec{q} - \mu(\hat{k}.\hat{q})q_0 + \mu^2\hat{k}.\hat{q}] \\
& - \Delta^2\left(a_+\beta_-a'_-\beta'_+\right)[(k_0q_0+\vec{k}.\vec{q})-(k_0\vec{q}+q_0\vec{k}).\hat{q} \\
& - \mu k_0 + \mu\vec{k}.\hat{q} - (k_0\vec{q}+q_0\vec{k}).\hat{k} + [(\hat{k}.\hat{q})k_0q_0 + (\vec{k}.\hat{k})(\vec{q}.\hat{q}) \\
& - (\vec{k}.\vec{q})(\hat{k}.\hat{q}) + (\vec{k}.\hat{q})(\hat{k}.\vec{q})] + \mu\vec{k}.\hat{k} - \mu(\hat{k}.\hat{q})k_0 + \mu q_0 \\
& - \mu\vec{q}.\hat{q} - \mu^2 - \mu\hat{k}.\vec{q} + \mu(\hat{k}.\hat{q})q_0 - \mu^2\hat{k}.\hat{q}] \qquad (2.74)
\end{aligned}
$$

This is a big expression. We can make it simple by rewriting it. First, let us find the expressions for $\left(a_+\alpha_-a'_-\alpha'_-\right)$, $\left(a_+\alpha_-a'_-\beta'_-\right)$, $\left(a_+\beta_-a'_-\alpha'_+\right)$ and $\left(a_+\beta_-a'_-\beta'_+\right)$

i)

$$
\begin{aligned}
\left(a_+\alpha_-a'_-\alpha'_+\right) = & \frac{1}{\left(k_0+\mu+|\vec{k}|\right)\left(q_0-\mu+|\vec{q}|\right)} \\
& \times\frac{1}{\left(k_0^2-\Delta^2-(|\vec{k}|-\mu)^2\right)\left(q_0^2-\Delta^2-(|\vec{q}|+\mu)^2\right)} \qquad (2.75)
\end{aligned}
$$

ii)

$$\left(a_+\alpha_-a'_-\beta'_+\right) = \frac{1}{\left(k_0+\mu+|\vec{k}|\right)\left(q_0-\mu-|\vec{q}|\right)} \times \frac{1}{\left(k_0^2-\Delta^2-(|\vec{k}|-\mu)^2\right)\left(q_0^2-\Delta^2-(|\vec{q}|-\mu)^2\right)} \tag{2.76}$$

iii)

$$\left(a_+\beta_-a'_-\alpha'_+\right) = \frac{1}{\left(k_0+\mu-|\vec{k}|\right)\left(q_0-\mu+|\vec{q}|\right)} \times \frac{1}{\left(k_0^2-\Delta^2-(|\vec{k}|+\mu)^2\right)\left(q_0^2-\Delta^2-(|\vec{q}|+\mu)^2\right)} \tag{2.77}$$

iv)

$$\left(a_+\beta_-a'_-\beta'_+\right) = \frac{1}{\left(k_0+\mu-|\vec{k}|\right)\left(q_0-\mu+|\vec{q}|\right)} \times \frac{1}{\left(k_0^2-\Delta^2-(|\vec{k}|+\mu)^2\right)\left(q_0^2-\Delta^2-(|\vec{q}|-\mu)^2\right)} \tag{2.78}$$

and the following simplifications

a)

$$\begin{aligned}
&(k_0q_0+\vec{k}.\vec{q})+(k_0\vec{q}+q_0\vec{k}).\hat{q}-\mu k_0-\mu\vec{k}.\hat{q}+(k_0\vec{q}+q_0\vec{k}).\hat{k}\\
&+[(\hat{k}.\hat{q})k_0q_0+(\vec{k}.\hat{k})(\vec{q}.\hat{q})-(\vec{k}.\vec{q})(\hat{k}.\hat{q})+(\vec{k}.\hat{q})(\hat{k}.\vec{q})]\\
&-\mu\vec{k}.\hat{k}-\mu(\hat{k}.\hat{q})k_0+\mu q_0+\mu\vec{q}.\hat{q}-\mu^2+\mu\hat{k}.\vec{q}+\mu(\hat{k}.\hat{q})q_0-\mu^2\hat{k}.\hat{q}\\
&=\left(1+\frac{\vec{k}.\vec{q}}{|\vec{k}||\vec{q}|}\right)\left(k_0+\mu+|\vec{k}|\right)(q_0-\mu+|\vec{q}|)
\end{aligned} \tag{2.79}$$

b)

$$\begin{aligned}
&(k_0q_0+\vec{k}.\vec{q})-(k_0\vec{q}+q_0\vec{k}).\hat{q}-\mu k_0+\mu\vec{k}.\hat{q}+(k_0\vec{q}+q_0\vec{k}).\hat{k}\\
&-[(\hat{k}.\hat{q})k_0q_0+(\vec{k}.\hat{k})(\vec{q}.\hat{q})-(\vec{k}.\vec{q})(\hat{k}.\hat{q})+(\vec{k}.\hat{q})(\hat{k}.\vec{q})]\\
&-\mu\vec{k}.\hat{k}+\mu(\hat{k}.\hat{q})k_0+\mu q_0-\mu\vec{q}.\hat{q}-\mu^2+\mu\hat{k}.\vec{q}-\mu(\hat{k}.\hat{q})q_0+\mu^2\hat{k}.\hat{q}\\
&=\left(1-\frac{\vec{k}.\vec{q}}{|\vec{k}||\vec{q}|}\right)\left(k_0+\mu+|\vec{k}|\right)(q_0-\mu-|\vec{q}|)
\end{aligned} \tag{2.80}$$

c)

$$
\begin{aligned}
&(k_0q_0+\vec{k}.\vec{q})+(k_0\vec{q}+q_0\vec{k}).\hat{q}-\mu k_0-\mu\vec{k}.\hat{q}-(k_0\vec{q}+q_0\vec{k}).\hat{k}\\
&-[(\hat{k}.\hat{q})k_0q_0+(\vec{k}.\hat{k})(\vec{q}.\hat{q})-(\vec{k}.\vec{q})(\hat{k}.\hat{q})+(\vec{k}.\hat{q})(\hat{k}.\vec{q})]\\
&+\mu\vec{k}.\hat{k}+\mu(\hat{k}.\hat{q})k_0+\mu q_0+\mu\vec{q}.\hat{q}-\mu^2-\mu\hat{k}.\vec{q}-\mu(\hat{k}.\hat{q})q_0+\mu^2\hat{k}.\hat{q}\\
&=\left(1-\frac{\vec{k}.\vec{q}}{|\vec{k}||\vec{q}|}\right)\left(k_0+\mu-|\vec{k}|\right)(q_0-\mu+|\vec{q}|)
\end{aligned}
\tag{2.81}
$$

d)

$$
\begin{aligned}
&(k_0q_0+\vec{k}.\vec{q})-(k_0\vec{q}+q_0\vec{k}).\hat{q}-\mu k_0+\mu\vec{k}.\hat{q}-(k_0\vec{q}+q_0\vec{k}).\hat{k}\\
&+[(\hat{k}.\hat{q})k_0q_0+(\vec{k}.\hat{k})(\vec{q}.\hat{q})-(\vec{k}.\vec{q})(\hat{k}.\hat{q})+(\vec{k}.\hat{q})(\hat{k}.\vec{q})]\\
&+\mu\vec{k}.\hat{k}-\mu(\hat{k}.\hat{q})k_0+\mu q_0-\mu\vec{q}.\hat{q}-\mu^2-\mu\hat{k}.\vec{q}+\mu(\hat{k}.\hat{q})q_0-\mu^2\hat{k}.\hat{q}\\
&=\left(1+\frac{\vec{k}.\vec{q}}{|\vec{k}||\vec{q}|}\right)\left(k_0+\mu-|\vec{k}|\right)(q_0-\mu-|\vec{q}|)
\end{aligned}
\tag{2.82}
$$

help us to rewrite (2.74) as

$$
\begin{aligned}
Tr\Big[(G_0^+\Delta^-G^-)_k(G_0^-\Delta^+G^+)_{k+p}\Big] &= \frac{-\Delta^2\left(1+\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|-\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|+\mu)\right]}\\
&+\frac{-\Delta^2\left(1-\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|-\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|-\mu)\right]}\\
&+\frac{-\Delta^2\left(1-\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|+\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|+\mu)\right]}\\
&+\frac{-\Delta^2\left(1+\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|+\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|-\mu)\right]}
\end{aligned}
\tag{2.83}
$$

Similarly,

$$
\begin{aligned}
Tr\Big[(G_0^-\Delta^+G^+)_k(G_0^+\Delta^-G^-)_{k+p}\Big] &= \frac{-\Delta^2\left(1+\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|+\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|-\mu)\right]}\\
&+\frac{-\Delta^2\left(1-\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|+\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|+\mu)\right]}\\
&+\frac{-\Delta^2\left(1-\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|-\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|-\mu)\right]}\\
&+\frac{-\Delta^2\left(1+\hat{k}.\hat{q}\right)}{\left[k_0^2-\Delta^2-\left(|\vec{k}|-\mu\right)\right]\left[q_0^2-\Delta^2-(|\vec{q}|+\mu)\right]}
\end{aligned}
\tag{2.84}
$$

Substituting all the four traces into (2.60) and simplifying all terms, it is found that

$$
\begin{aligned}
Tr\Big[S(k)S(k+p)\Big] &= \frac{2\Big(1-\hat{k}.\hat{q}\Big)\Big(k_0q_0-\Delta^2+|\vec{k}||\vec{q}|+\mu|\vec{k}|+\mu|\vec{q}|+\mu^2\Big)}{\Big[k_0^2-\Delta^2-\Big(|\vec{k}|+\mu\Big)^2\Big]\Big[q_0^2-\Delta^2-\Big(|\vec{q}|+\mu\Big)^2\Big]} \\
&+ \frac{2\Big(1-\hat{k}.\hat{q}\Big)\Big(k_0q_0-\Delta^2+|\vec{k}||\vec{q}|-\mu|\vec{k}|-\mu|\vec{q}|+\mu^2\Big)}{\Big[k_0^2-\Delta^2-\Big(|\vec{k}|-\mu\Big)^2\Big]\Big[q_0^2-\Delta^2-\Big(|\vec{q}|-\mu\Big)^2\Big]} \\
&+ \frac{2\Big(1+\hat{k}.\hat{q}\Big)\Big(k_0q_0-\Delta^2-|\vec{k}||\vec{q}|+\mu|\vec{k}|-\mu|\vec{q}|+\mu^2\Big)}{\Big[k_0^2-\Delta^2-\Big(|\vec{k}|+\mu\Big)^2\Big]\Big[q_0^2-\Delta^2-\Big(|\vec{q}|-\mu\Big)^2\Big]} \\
&+ \frac{2\Big(1+\hat{k}.\hat{q}\Big)\Big(k_0q_0-\Delta^2-|\vec{k}||\vec{q}|-\mu|\vec{k}|+\mu|\vec{q}|+\mu^2\Big)}{\Big[k_0^2-\Delta^2-\Big(|\vec{k}|-\mu\Big)^2\Big]\Big[q_0^2-\Delta^2-\Big(|\vec{q}|+\mu\Big)^2\Big]} \qquad (2.85)
\end{aligned}
$$

2.6 Matsubara Sum

Since Eq.(2.85) is big, we name each of the four terms as F_{1T}, F_{2T}, F_{3T} and F_{4T} respectively and do the Matsubara sum over k_4 separately. Before we introduce the notation $\vec{k}+\vec{p}=\vec{q}$ and go to the Euclidean space, we put $k_0 = ik_4$ and $p_0 = ip_4$.

The Matsubara sum for the first term is

$$\sum_{z_i} F_{1T} = -\frac{1}{\beta}\Big[Res\ \ F_{1T}(z_i).f(z_i)\Big] \qquad (2.86)$$

where the auxiliary function is

$$f(k_4) = \frac{i\beta}{1+e^{i\beta k_4}} \qquad (2.87)$$

Defining

$$
\begin{aligned}
A &= -2\Big(1-\hat{k}.\frac{\vec{k}+\vec{p}}{|\vec{k}+\vec{p}|}\Big) &(2.88)\\
B_1 &= \Delta^2-\Big(\mu+|\vec{k}|\Big)\Big(\mu+|\vec{k}+\vec{p}|\Big) &(2.89)\\
\epsilon_{11}^2 &= \Delta^2+\Big(|\vec{k}|+\mu\Big)^2 &(2.90)\\
\epsilon_{12}^2 &= \Delta^2+\Big(|\vec{k}+\vec{p}|+\mu\Big)^2 &(2.91)
\end{aligned}
$$

The first term of (2.85) becomes

$$F_{1T} = \frac{A\Big[k_4^2 + k_4 p_4 + B_1\Big]}{\Big(k_4^2 + \epsilon_{11}^2\Big)\Big[\,(k_4 + p_4)^2 + \epsilon_{12}^2\Big]} \tag{2.92}$$

which has poles at $-i\epsilon_{11}$, $i\epsilon_{11}$, $-p_4 - i\epsilon_{12}$ and $-p_4 + i\epsilon_{12}$. Evaluating residues and the auxiliary function at these poles, eq.(2.86) becomes

$$\begin{aligned}\sum_{z_i} F_{1T} = -i\Bigg[& \frac{A\Big[-\epsilon_{11}^2 - i\epsilon_{11}p_4 + B_1\Big]}{(-2i\epsilon_{11})\Big[p_4 - i(\epsilon_{11} - \epsilon_{12})\Big]\Big[p_4 - i(\epsilon_{11} + \epsilon_{12})\Big]}\left(\frac{1}{1 + e^{\beta\epsilon_{11}}}\right) \\ & + \frac{A\Big[-\epsilon_{11}^2 + i\epsilon_{11}p_4 + B_1\Big]}{(2i\epsilon_{11})\Big[p_4 + i(\epsilon_{11} - \epsilon_{12})\Big]\Big[p_4 + i(\epsilon_{11} + \epsilon_{12})\Big]}\left(1 - \frac{1}{1 + e^{\beta\epsilon_{11}}}\right) \\ & + \frac{A\Big[-\epsilon_{12}^2 + i\epsilon_{12}p_4 + B_1\Big]}{(-2i\epsilon_{12})\Big[-p_4 + i(\epsilon_{11} - \epsilon_{12})\Big]\Big[-p_4 - i(\epsilon_{11} + \epsilon_{12})\Big]}\left(\frac{1}{1 + e^{-i\beta p_4 + \beta\epsilon_{12}}}\right) \\ & + \frac{A\Big[-\epsilon_{12}^2 - i\epsilon_{12}p_4 + B_1\Big]}{(2i\epsilon_{12})\Big[-p_4 - i(\epsilon_{11} - \epsilon_{12})\Big]\Big[-p_4 + i(\epsilon_{11} + \epsilon_{12})\Big]}\left(\frac{1}{1 + e^{-i\beta p_4 - \beta\epsilon_{12}}}\right)\Bigg]\end{aligned} \tag{2.93}$$

Putting $p_4 = 0$, we get

$$\sum F_{1T} = \left[\frac{A\Big[-\epsilon_{11}^2 + B_1\Big]}{(2\epsilon_{11})\Big[\epsilon_{11}^2 - \epsilon_{12}^2\Big]}\left(1 - \frac{2}{1 + e^{\beta\epsilon_{11}}}\right) + \frac{A\Big[-\epsilon_{12}^2 + B_1\Big]}{(2\epsilon_{12})\Big[\epsilon_{11}^2 - \epsilon_{12}^2\Big]}\left(\frac{2}{1 + e^{\beta\epsilon_{12}}} - 1\right)\right] \tag{2.94}$$

Matsubara sum over k_4 for the rest of therms can be carried out in the similar way.

$$\sum_{z_i} F_{2T} = \left[\frac{A\Big[-\epsilon_{21}^2 + B_2\Big]}{(2\epsilon_{21})\Big[\epsilon_{21}^2 - \epsilon_{22}^2\Big]}\left(1 - \frac{2}{1 + e^{\beta\epsilon_{21}}}\right) + \frac{A\Big[-\epsilon_{22}^2 + B_2\Big]}{(2\epsilon_{22})\Big[\epsilon_{21}^2 - \epsilon_{22}^2\Big]}\left(\frac{2}{1 + e^{\beta\epsilon_{22}}} - 1\right)\right] \tag{2.95}$$

$$\sum_{z_i} F_{3T} = \left[\frac{A'\Big[-\epsilon_{31}^2 + B_3\Big]}{(2\epsilon_{31})\Big[\epsilon_{31}^2 - \epsilon_{32}^2\Big]}\left(1 - \frac{2}{1 + e^{\beta\epsilon_{31}}}\right) + \frac{A'\Big[-\epsilon_{32}^2 + B_3\Big]}{(2\epsilon_{32})\Big[\epsilon_{31}^2 - \epsilon_{32}^2\Big]}\left(\frac{2}{1 + e^{\beta\epsilon_{32}}} - 1\right)\right] \tag{2.96}$$

$$\sum_{z_i} F_{4T} = \left[\frac{A'\Big[-\epsilon_{41}^2 + B_4\Big]}{(2\epsilon_{41})\Big[\epsilon_{41}^2 - \epsilon_{42}^2\Big]}\left(1 - \frac{2}{1 + e^{\beta\epsilon_{41}}}\right) + \frac{A'\Big[-\epsilon_{42}^2 + B_4\Big]}{(2\epsilon_{42})\Big[\epsilon_{41}^2 - \epsilon_{42}^2\Big]}\left(\frac{2}{1 + e^{\beta\epsilon_{42}}} - 1\right)\right] \tag{2.97}$$

Adding (2.94), (2.95), (2.96), (2.97) and using the relations that $\epsilon_{11}^2 = \epsilon_{31}^2$, $\epsilon_{12}^2 = \epsilon_{42}^2$, $\epsilon_{21}^2 = \epsilon_{41}^2$ and $\epsilon_{22}^2 = \epsilon_{32}^2$, we get the final Matsubara sum, which is

$$
\begin{aligned}
\sum_{z_i} F(k_4) = \Bigg[& \left(\frac{A\Big[-\epsilon_{11}^2 + B_1\Big]}{(2\epsilon_{11})\Big[\epsilon_{11}^2 - \epsilon_{12}^2\Big]} + \frac{A'\Big[-\epsilon_{11}^2 + B_3\Big]}{(2\epsilon_{11})\Big[\epsilon_{11}^2 - \epsilon_{22}^2\Big]} \right) \left(1 - \frac{2}{1+e^{\beta\epsilon_{11}}} \right) \\
+ & \left(\frac{A\Big[-\epsilon_{21}^2 + B_2\Big]}{(2\epsilon_{21})\Big[\epsilon_{21}^2 - \epsilon_{22}^2\Big]} + \frac{A'\Big[-\epsilon_{21}^2 + B_4\Big]}{(2\epsilon_{21})\Big[\epsilon_{21}^2 - \epsilon_{12}^2\Big]} \right) \left(1 - \frac{2}{1+e^{\beta\epsilon_{21}}} \right) \\
+ & \left(\frac{A\Big[-\epsilon_{12}^2 + B_1\Big]}{(2\epsilon_{12})\Big[\epsilon_{11}^2 - \epsilon_{12}^2\Big]} + \frac{A'\Big[-\epsilon_{12}^2 + B_4\Big]}{(2\epsilon_{12})\Big[\epsilon_{21}^2 - \epsilon_{12}^2\Big]} \right) \left(\frac{2}{1+e^{\beta\epsilon_{12}}} - 1 \right) \\
+ & \left(\frac{A\Big[-\epsilon_{22}^2 + B_2\Big]}{(2\epsilon_{22})\Big[\epsilon_{21}^2 - \epsilon_{22}^2\Big]} + \frac{A'\Big[-\epsilon_{22}^2 + B_3\Big]}{(2\epsilon_{22})\Big[\epsilon_{11}^2 - \epsilon_{22}^2\Big]} \right) \left(\frac{2}{1+e^{\beta\epsilon_{22}}} - 1 \right) \Bigg] \qquad (2.98)
\end{aligned}
$$

After substituting the values of A's and ϵ's, it becomes

$$
\begin{aligned}
= \quad \Bigg[& -\frac{\mu + |\vec{k}|}{2\epsilon_{11}} \left(\frac{A}{|\vec{k}| - |\vec{q}|} + \frac{A'}{|\vec{k}| + |\vec{q}|} \right) \left(1 - \frac{2}{1+e^{\beta\epsilon_{11}}} \right) \\
& + \frac{\mu - |\vec{k}|}{2\epsilon_{21}} \left(\frac{A}{|\vec{k}| - |\vec{q}|} + \frac{A'}{|\vec{k}| + |\vec{q}|} \right) \left(1 - \frac{2}{1+e^{\beta\epsilon_{21}}} \right) \\
& - \frac{\mu + |\vec{q}|}{2\epsilon_{12}} \left(\frac{A}{|\vec{k}| - |\vec{q}|} - \frac{A'}{|\vec{k}| + |\vec{q}|} \right) \left(\frac{2}{1+e^{\beta\epsilon_{12}}} - 1 \right) \\
& + \frac{\mu - |\vec{q}|}{2\epsilon_{22}} \left(\frac{A}{|\vec{k}| - |\vec{q}|} - \frac{A'}{|\vec{k}| + |\vec{q}|} \right) \left(\frac{2}{1+e^{\beta\epsilon_{22}}} - 1 \right) \Bigg] \qquad (2.99)
\end{aligned}
$$

Here

$$
\frac{A}{|\vec{k}| - |\vec{q}|} + \frac{A'}{|\vec{k}| + |\vec{q}|} = -4 \left(\frac{|\vec{k}| - \hat{k}.\vec{q}}{|\vec{k}|^2 - |\vec{q}|^2} \right) \qquad (2.100)
$$

$$
\frac{A}{|\vec{k}| - |\vec{q}|} - \frac{A'}{|\vec{k}| + |\vec{q}|} = -4 \left(\frac{|\vec{q}| - \hat{q}.\vec{k}}{|\vec{k}|^2 - |\vec{q}|^2} \right) \qquad (2.101)
$$

Thus Eq.(2.99) becomes

$$\sum_{k_4=\frac{2n+1}{\beta}} F(k_4) = 2\left[\frac{\mu+|\vec{k}|}{\sqrt{\Delta^2+(\mu+|\vec{k}|)^2}}\left(\frac{|\vec{k}|-\hat{k}.\vec{q}}{|\vec{k}|^2-|\vec{q}|^2}\right)\left(1-\frac{2}{1+e^{\beta\epsilon_{11}}}\right)\right.$$
$$-\frac{\mu-|\vec{k}|}{\sqrt{\Delta^2+(\mu-|\vec{k}|)^2}}\left(\frac{|\vec{k}|-\hat{k}.\vec{q}}{|\vec{k}|^2-|\vec{q}|^2}\right)\left(1-\frac{2}{1+e^{\beta\epsilon_{21}}}\right)$$
$$-\frac{\mu+|\vec{q}|}{\sqrt{\Delta^2+(\mu+|\vec{q}|)^2}}\left(\frac{|\vec{q}|-\hat{q}.\vec{k}}{|\vec{k}|^2-|\vec{q}|^2}\right)\left(1-\frac{2}{1+e^{\beta\epsilon_{12}}}\right)$$
$$\left.+\frac{\mu-|\vec{q}|}{\sqrt{\Delta^2+(\mu-|\vec{q}|)^2}}\left(\frac{|\vec{q}|-\hat{q}.\vec{k}}{|\vec{k}|^2-|\vec{q}|^2}\right)\left(1-\frac{2}{1+e^{\beta\epsilon_{22}}}\right)\right] \quad (2.102)$$

2.7 Infrared Limit

In the infrared limit, we calculate Eq.(2.102) at $|\vec{p}| \to 0$. Then

$$\lim_{|\vec{p}|\to 0}\left(\frac{|\vec{k}|-\hat{k}.\vec{q}}{|\vec{k}|^2-|\vec{q}|^2}\right) = \frac{1}{2|\vec{k}|} \quad (2.103)$$

$$\lim_{|\vec{p}|\to 0}\left(\frac{|\vec{q}|-\hat{q}.\vec{k}}{|\vec{k}|^2-|\vec{q}|^2}\right) = -\frac{1}{2|\vec{k}|} \quad (2.104)$$

$$\lim_{|\vec{p}|\to 0}\epsilon_{11} = \lim_{|\vec{p}|\to 0}\epsilon_{12} = \sqrt{\Delta^2+(\mu+|\vec{k}|)^2}$$
$$\lim_{|\vec{p}|\to 0}\epsilon_{21} = \lim_{|\vec{p}|\to 0}\epsilon_{22} = \sqrt{\Delta^2+(\mu-|\vec{k}|)^2} \quad (2.105)$$

If we put the above limiting values into (2.102), it looks like

$$\lim_{|\vec{p}|\to 0}\left(\sum_{k_4=\frac{2n+1}{\beta}} F(k_4)\right) = \frac{2\left(\mu+|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu+|\vec{k}|)^2}} - \frac{2\left(\mu-|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu-|\vec{k}|)^2}}$$
$$+\frac{4\left(\mu-|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu-|\vec{k}|)^2}}\left(\frac{1}{1+e^{\beta\sqrt{\Delta^2+(\mu-|\vec{k}|)^2}}}\right)$$
$$-\frac{4\left(\mu+|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu+|\vec{k}|)^2}}\left(\frac{1}{1+e^{\beta\sqrt{\Delta^2+(\mu+|\vec{k}|)^2}}}\right) \quad (2.106)$$

Chapter 3

Results and Discussion

In this chapter we present our result for the scalar polarization operator in the superfluid medium. In the last chapter we have taken traces, performed Matsubara summation and found our resulting calculation in the infrared limit. Now we do the integration to get the polarization operator.

In the zero temperature limit, $\beta \to \infty$, last two terms of (2.106) don't survive and we are left with the following

$$\lim_{\beta\to\infty,|\vec{p}|\to 0}\left(\sum_{k_4=\frac{2n+1}{\beta}} F(k_4)\right) = \frac{2\left(\mu+|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu+|\vec{k}|)^2}} - \frac{2\left(\mu-|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu-|\vec{k}|)^2}} \tag{3.1}$$

Now we recall the polarization operator again

$$\Pi = g^2 \sum_{k_4} \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}\left[\Gamma S(k)\Gamma S(k+p)\right] \tag{3.2}$$

After taking the traces, Matsubara summation and the infrared limit, the above equation reduces to

$$\Pi = g^2 \int \frac{d^3k}{(2\pi)^3}\left(\frac{2\left(\mu+|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu+|\vec{k}|)^2}} - \frac{2\left(\mu-|\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2+(\mu-|\vec{k}|)^2}}\right) \tag{3.3}$$

Taking this integral into spherical coordinates, we get

$$
\begin{aligned}
\Pi &= g^2 \int_0^{2\pi} d\phi \int_0^{\pi} sin\theta d\theta \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3} \left(\frac{2\left(\mu + |\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2 + (\mu + |\vec{k}|)^2}} - \frac{2\left(\mu - |\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2 + (\mu - |\vec{k}|)^2}} \right) \\
&= g^2 \int_0^{\Lambda} \frac{k^2 dk}{\pi^2} \left(\frac{\left(\mu + |\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2 + (\mu + |\vec{k}|)^2}} - \frac{\left(\mu - |\vec{k}|\right)}{|\vec{k}|\sqrt{\Delta^2 + (\mu - |\vec{k}|)^2}} \right) \qquad (3.4)
\end{aligned}
$$

Here Λ is the cut off.

Changing the variables of the integrand as

$$\mu + k = k'$$

$$dk = dk'$$

and

$$\mu - k = k''$$

$$-dk = dk''$$

it is found that

$$\Pi = \frac{g^2}{\pi^2} \int_{\mu}^{\mu+\Lambda} dk' \frac{(k')^2 - \mu k'}{\sqrt{\Delta^2 + (k')^2}} - \frac{1}{\pi^2} \int_{\mu}^{\mu-\Lambda} dk'' \frac{(k'')^2 - \mu k''}{\sqrt{\Delta^2 + (k'')^2}} \qquad (3.5)$$

Rearranging the limits of the integral, this might look like

$$\Pi = \frac{g^2}{\pi^2} \int_{\mu-\Lambda}^{\mu+\Lambda} dk \frac{k^2}{\sqrt{\Delta^2 + k^2}} - \frac{\mu}{\pi^2} \int_{\mu-\Lambda}^{\mu+\Lambda} dk \frac{k}{\sqrt{\Delta^2 + k^2}} \qquad (3.6)$$

which gives the following result after the integration is carried out:

$$
\begin{aligned}
\Pi = \frac{g^2}{\pi^2} \Bigg[& \frac{\mu}{2} \left(\sqrt{(\mu - \Lambda)^2 + \Delta^2} - \sqrt{(\mu + \Lambda)^2 + \Delta^2} \right) \\
& + \frac{\Lambda}{2} \left(\sqrt{(\mu - \Lambda)^2 + \Delta^2} + \sqrt{(\mu + \Lambda)^2 + \Delta^2} \right) \\
& - \frac{\Delta^2}{2} ln \left| \frac{\frac{\mu}{\Lambda} + 1 + \sqrt{(\frac{\mu}{\Lambda} + 1)^2 + \frac{\Delta^2}{\Lambda^2}}}{\frac{\mu}{\Lambda} - 1 + \sqrt{(\frac{\mu}{\Lambda} - 1)^2 + \frac{\Delta^2}{\Lambda^2}}} \right| \Bigg] \qquad (3.7)
\end{aligned}
$$

With the help of Taylor's expansion, some of the terms can be approximated as

$$\sqrt{(\mu-\Lambda)^2+\Delta^2} \simeq \Lambda\left(1+\frac{\mu^2+\Delta^2}{2\Lambda^2}-\frac{\mu}{\Lambda}\right)$$
$$\sqrt{(\mu+\Lambda)^2+\Delta^2} \simeq \Lambda\left(1+\frac{\mu^2+\Delta^2}{2\Lambda^2}+\frac{\mu}{\Lambda}\right) \tag{3.8}$$

and

$$\begin{aligned} ln\left|\frac{\frac{\mu}{\Lambda}+1+\sqrt{(\frac{\mu}{\Lambda}+1)^2+\frac{\Delta^2}{\Lambda^2}}}{\frac{\mu}{\Lambda}-1+\sqrt{(\frac{\mu}{\Lambda}-1)^2+\frac{\Delta^2}{\Lambda^2}}}\right| &= ln\left|\frac{\frac{\mu}{\Lambda}+1+1+\frac{\mu^2+\Delta^2}{2\Lambda^2}+\frac{\mu}{\Lambda}}{\frac{\mu}{\Lambda}-1+1+\frac{\mu^2+\Delta^2}{2\Lambda^2}-\frac{\mu}{\Lambda}}\right| \\ &= ln\left|\frac{4\left(1+\frac{\mu^2+\Delta^2}{4\Lambda^2}+\frac{\mu}{\Lambda}\right)}{\frac{\mu^2+\Delta^2}{\Lambda^2}}\right| \\ &\simeq \frac{\mu^2+\Delta^2}{4\Lambda^2}+\frac{\mu}{\Lambda}-\left(ln\frac{\mu^2+\Delta^2}{\Lambda^2}\right) \end{aligned} \tag{3.9}$$

Putting these approximated values back into (3.7), the polarization operator becomes

$$\Pi \simeq \frac{g^2}{\pi^2}\left[-\frac{\mu^2}{2}+\frac{\Delta^2}{2}-\frac{\Delta^2\left(\mu^2+\Delta^2\right)}{8\Lambda^2}-\frac{\Delta^2\mu}{2\Lambda}+\frac{\Delta^2}{2}ln\left(\frac{\mu^2+\Delta^2}{\Lambda^2}\right)+\cdots\right] \tag{3.10}$$

where $\cdots$ are terms independent of μ or Δ.

The above result indicates that the superfluid medium modifies the scalar mass making it dependent on the density and the superfluid gap. The effective mass decreases with the chemical potential, so the scalar field becomes lighter and hence more relevant for the transport properties of the system.

Notice that the effective scalar mass can be approximated as $m_s^2 \simeq \tilde{M}_s^2 - \mu^2$, where $\tilde{M}_s$ is the renormalized mass of the scalar field. In the case the chemical potential is larger than the renormalized mass, the scalar mass becomes imaginary, that is tachyonic indicating an unphysical situation and an unstable ground state. To find the correct ground state in this situation, we would need to go back and study the problem considering a condensate of the scalar field in addition to fermion-fermion condensate. This could be an interesting continuation of the present work.

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Curriculum Vitae

Sajib Kumar Barman is the oldest son of Subal Chandra Barman and Shafali Rani Barman, born and raised in Bangladesh on December 7, 1985. After completing his high school education, he was admitted to the University of Dhaka, a top ranked public university in Bangladesh. He earned his Bachelor of Science in Physics in 2009. He also completed his Master's Degree in Condensed Matter Physics from the same university in 2010. He was employed as a high school physics teacher in a reputed English medium school at Dhaka, Bangladesh where he had served for one and half years, while undergoing his Master's Degree. But the thirst of higher knowledge in physics led him to come to the USA where he enrolled at The University of Texas at El Paso. In the Fall of 2011, he began as a graduate student in the physics department and began working for another Master of Science degree. Here, he has worked in superfluidity with Dr. Vivian Incera; now, he is going to pursue his PhD at The University of Texas at Arlington.

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