Analysis of Vehicle Interactions on Interstate Highways: Discrete Choice and Linear System Approaches

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ANALYSIS OF VEHICLE INTERACTIONS ON INTERSTATE HIGHWAYS: DISCRETE CHOICE AND LINEAR SYSTEMS APPROACHES

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Dedication

To God for trusting me this endeavor and for giving me the strength to overcome many challenges.

To my beloved husband whose love and comprehension helped me reach my career goals.

To my parents and brothers who taught me the value of education and inspired me to pursue a professional career. Their motivation and support helped me to become a PhD.

I would like to dedicate this dissertation to my advisor Dr. Ruey Long Cheu. It was with his mentorship and guidance that I was able to reach my goal of becoming a PhD. With his support I have been able to find a path of being a successful researcher and I am very grateful for this.
ANALYSIS OF VEHICLE INTERACTIONS ON INTERSTATE HIGHWAYS: DISCRETE CHOICE AND CONTROL SYSTEMS APPROACHES

by

ALICIA ROMO

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Abstract

The research in this dissertation developed statistical and linear system models to predict driving behavior according to driver attributes, vehicle characteristics, car-following dynamics and/or driving conditions. The scope of this research was limited to the interaction of two vehicles traveling in the same direction on interstate highways.

The statistical models proposed in this research investigated the contributions of the different vehicle types on the manner and likelihood of collision. Discrete choice models were used to estimate the probability of the types of collision (rear-end, angle and sideswipe) as functions of driver attributes, striking and struck vehicle types, pre-crash driving actions and traffic and environmental conditions. The National Automotive Sampling System-General Estimates System (NASS-GES) crash data set from 2005 to 2009 was used to develop and validate the model. This research demonstrated that different types of vehicles have different coefficients in the utility functions of the discrete choice models.

Using linear system analysis, the states of a pair of vehicles over time were predicted based on vehicle-following dynamics and vehicle characteristics. The internal characteristics of the system composed of driver and type of vehicle were represented in a state-space equation. Assuming relative speed and space headway as the states of the system, the optimal safe distance between the vehicles was predicted using a gain feedback matrix and a state estimator. Using Kalman Filter, the best estimation of future states was obtained to control a “safe vehicle-following” based on the actions of the leader and the follower. Vehicle trajectories from the Next Generation SIMulation (NGSIM) database were used to calibrate and validate the state-space prediction model. This research identified the differences in the characteristic matrices of the different types of vehicle due to their dynamic properties.

This research recommended that different types of vehicle based on their dynamic properties should be differentiated to quantify safe and optimal interaction between cars and trucks.
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Chapter 1: Research Motivation

This chapter describes the problem background, states the objective of the research, and explains the significance of addressing the research questions to the profession.

1.1 Problem Background

In the United States, the annual number crashes in highways averaged 6.1 million from 2000 to 2009 according to Research and Innovative Technology Administration (RITA 2010). Many studies have been dedicated to address safety related problems on highways. However, far less attention has been paid to analyze factors that can lead to the different types of collisions on interstate highways using statistical analysis and linear system approach.

Highway driving is a complex task because drivers not only have to interact between vehicles of different types, but also need to make appropriate reactions to sudden changes in the traffic ahead. Constant negotiations are executed by drivers as to follow the vehicle ahead in the same lane, at a safe distance, while maintaining a comfortable velocity. In microscopic traffic flow theory, this behavior is termed “car-following”. When the risk of collision between two vehicles in the same lane is imminent, the driver of the following vehicle may apply brake or change lane to avoid a collision. An unsuccessful maneuver of the following vehicle may result in a rear-end, angle or sideswipe collision. Therefore, some collisions between two vehicles may be viewed as the outcome of an unsuccessful car-following interaction.

An increasing concern in highway safety analysis is the presence and increase in the volume of trucks in the traffic stream. Over the past years, registered large trucks in the United States have increased from approximately 8.5 million in 2005 to almost 11 million in 2009, as stated by Federal Motor Carrier Safety Administration (FMCSA 2011a). The size and dynamic characteristics of trucks are different from those of passenger cars. Passenger car drivers may misjudge the maneuver characteristics of trucks and take inappropriate actions, thus increasing the risk of collision.

Collisions that involve trucks usually result in more severe injuries or fatalities compared to collisions that involve only passenger cars. As reported in Islam and Hernandez (2012), about 80% of
truck-related fatalities occur in car-truck crashes in the United States. In 2009, 75% of fatal crashes involving large trucks had “collisions with vehicle in transport” as the most harmful event (FMCSA 2011b). It is not surprising that collisions that involve large commercial vehicles usually result in more severe injuries or fatalities compared to collisions that involve only passenger cars (Islam and Hernandez 2012). Based on the 2008 Status of The Nation’s Highways, Bridges and Transit Report (USDOT 2008), between 1997 and 2006, the number of fatalities for vehicle occupants by type of vehicle decreased among passenger cars and increased when large trucks were involved.

In addition, collisions involving large trucks usually result in significant financial cost. The estimated total cost to society of crashes involving large single-unit trucks was $14.7 billion, and for combination trucks $26.5 billion (FMCSA 2011b). It is necessary, then, to intensify efforts to study the close yet relatively risky interaction of different types of vehicles that share the road and identify inadequate or inappropriate actions that lead to these collisions.

In the analysis of crashes, crash type is an important descriptor as is number of crashes, crash rate, or crash severity. The study performed by Zaloshnja et al. (2004) looked into costs resulting from a particular type of collision at urban intersections. The purpose of the study was to evaluate traffic safety intervention effectiveness since the latter varies according to crash type. The comprehensive cost per crash including medical related costs, emergency services, property damage, lost productivity and monetized value of pain was estimated to examine crash effects by type. According to the results, the cost to society in 2001 dollars in no-intersection roads with a speed limit of 50 mph or higher is $25.6 billion per year for rear-end collisions and $25.1 billion per year for sideswipe collisions. Because the task of driving on highways is complex due to its interactivity and heterogeneity of traffic, and the propensity of high speed impact, it is necessary to extend transportation safety analysis to identify factors that lead to the different manners of collision on interstate highways.

Another important aspect of analyzing vehicle interaction is that it allows the identification of driving behavior that causes disruptions in traffic. Traffic delays are sometimes caused by an improper reaction of drivers and have a significant impact in the performance of the flow downstream.
Congested scenarios can aggravate overreactions of drivers and compromise urban mobility. As stated by Schrank et al. (2011) “stop-and-go roads carry half to two thirds of the vehicles as a smoothly flowing road.” As also highlighted in the 2011 Urban Mobility Report (Schrank et al. 2011), cars and trucks share the urban “congestion invoice” for costly delays, massive amount of time, fuel and money because of current transportation systems are operating inefficiently. Statistics reported that in 2010; 1.9 billion of gallons of fuel was wasted in addition to 4.8 billion hours of extra time. The cost of delay and fuel cost was $101 billion in which about 20% ($23 billion) compromised truck operations thus increasing prices for consumers. Another important fact reflects that trucks represent 26% of congestion cost in comparison to passenger cars, although trucks account only for 6% of miles traveled in urban areas.

1.2 Objective of Research

The objective of this research is to develop models that test the following hypotheses:

1. Car and Trucks have different driving behavior due to vehicle dynamic capabilities. Size and dimension of vehicles allow or constrain drivers to perform quick maneuvers and therefore influence driver decisions on the road. Type of vehicle driven affects driver’s comfort to follow closer or allow more space between vehicles.

2. Following vehicles have a different driving behavior when the vehicle which is leading is different. Same type of vehicles follow the leading vehicle closer (i.e. car following car, truck following truck) than if following a different type of vehicle (i.e. car following truck, truck following car).

3. Trucks are more cautious when following the vehicle in front. Trucks leave a bigger spacing gap when following in comparison to distance kept by cars.

4. Pre-crash actions and congested scenarios will expose critical vehicle-following fundamental in addressing safety related issues. Crucial decisions made by drivers in risky scenarios influence significantly the manner of collisions.
1.3 **Significance to the Profession**

As the volume of traffic in the United States interstate highway system increases, so will the amount of travel times, delay and crashes. This extends the need for transportation engineers to investigate vehicle interactions which significantly impacts safety and mobility.

Modeling driving behavior based on driver characteristics and vehicle type provides parameter configurations for traffic simulation modeling. This research can be used to forecast the impact that vehicle-following behavior have on capacity. Understanding the critical headways between different types of vehicles will lead to measures to optimize vehicular flow.

According to the Federal Highway Administration Safety Program (FHWA 2011), road safety is increased with the exposure of drivers to safety education, as well as, the manner in which vehicles are operated and engineered to minimize property damage and protect passengers during an impact. In addition, road safety depends on the design and maintenance of roads and on the safety available tools that exists to enforce it.

Recently the United Nations (UN) has proclaimed 2011 - 2020 as the “Decade of Action for Road Safety” to raise awareness about road safety and make an effort to eliminate fatal crashes (UN 2011). According to UN, each year there are approximately 1.3 million fatalities and up to 50 million injuries occurring on roads around the world.

This research contributes to the transportation profession by providing quantification and prediction of the safety performance on highways. The expected results are that the models will capture the behavior of the interaction between drivers and vehicles in a highway when different types of vehicles are following one another in the same direction. The prediction models can be applied to driving safety and safety measures that indicate the risk of impeding collision. The research has the potential to impact policy measures related to highway design, driving rules and safety in the future. An additional broader impact of this research is the improvement of automatic cruise control based on type of vehicle followed.
Chapter 2: Literature Review

This chapter describes the different approaches taken in the past for car-following, truck-following, crash analysis and longitudinal control. This chapter gives a review of the most relevant research work performed that are similar to the examinations in this dissertation. The review includes definitions, data collection and analysis performed by different researchers and compares the limitations of these models.

2.1 Models of Car-Following

According to Chandler et al. (1958) the study of car-following was initiated when the California Motor Vehicle Code established a minimum safe distance between vehicles: 15 feet (approximately the length of a car) for every 10 mph. Driven by an interest in this rule, Pipes (1951) performed studies to observe the safe distance between vehicles and modeled car-following behavior as an immediate response taken by the following driver according to the leader’s conduct. However, this fundamental model did not consider factors such as reaction time and sensitivity, which take place when a driver accelerates and decelerates as a response for the leader’s movement. Chandler et al. (1958) introduced lag time between perception and response when modeling car-following behavior. The behavior of the follower was described by the sensitivity term, and the safe space considered by the follower was strongly dependent on the leader’s speed. This means that the acceleration and deceleration behavior of the follower was dependent on the relative speed between the leader and the follower. Nevertheless, for the same driver, the sensitivities were the same for acceleration and deceleration. This assumption is not realistic when vehicles drive on highways at high speed. Edie (1961) proposed two different car-following models for different traffic states: free-flow and congested traffic. This study opened a new research for car-following models for heavy traffic conditions to give a better perception of driver’s behavior in peak hour scenarios.

Car-following models can analyze struggles faced by drivers when lane changing is restricted in congested traffic. According to the car-following historical review performed by Brackstone and Mcdonald (1999), researchers developed prototypes derived from the model tested by the General Motors (GM) team, commonly referred to as the GM model. The GM model has the form of:
\[
\dot{x}_f(t + \tau) = \lambda \frac{\left[\dot{x}_f(t)\right]^m}{\left[x_i(t) - x_f(t)\right]^n}\left[\ddot{x}_f(t) - \dot{x}_f(t)\right] \tag{2.1}
\]

where \( \dot{x}_f(t) \) = acceleration/deceleration (response) of the follower \( f \) at time \( t \);
\( \ddot{x}_f(t) \) = velocity of the follower \( f \) at time \( t \);
\( \dot{x}_i(t) \) = velocity of leader \( l \) at time \( t \);
\( x_i(t) \) = position of the leader \( l \) at time \( t \);
\( x_f(t) \) = position of the follower \( f \) at time \( t \);
\( \tau \) = reaction time;
\( \lambda \) = sensitivity constant as a function of spacing and speed of the follower; and
\( m, n \) = constants.

Most of the studies that used the GM model emphasized the calibration and validation of parameters. These studies proposed different optimal parameter values for different responses such as deceleration, acceleration and in different traffic conditions without a final decisive value.

Other car-following models were developed from different approaches in order to find a safe distance for vehicles to avoid collision. Such an approach was initiated by Kometani and Sasaki (1959) and promoted by Gipps (1981). However, since there is no specific function to account for a driver’s actions when following a car, assumptions must be made to simulate driver behavior. Psychological models (Evans and Rothery 1973; Michaels 1963; Reiter 1994) attempt to describe car-following by focusing on how decisions are made by the following drivers based on sensory evidence. They described how followers make decisions on following distance by visual judgment and relative speed perception. The disadvantage of this approach is that there is no satisfactory calibration for these models with field data and this makes their validity uncertain.

Moreover, car following behavior has been modeled using a system of functions that describe driver behavior from a set of driver’s attributes. This method, called fuzzy logic models, was introduced by Kikuchi and Chakroborty (1992). However, the selection of attributes in the choice set and the membership functions can be controversial.
Another form of models, the so-called linear models, was proposed by Helly (1959). The linear models incorporate an additional term, the following distance when the following vehicle is accelerating. This is the first attempt that describes the different driver behaviors when accelerating or decelerating.

In most of the studies reviewed above, researchers always compared their new models against the GM model. It appears that the GM model is still regarded as the benchmark in car-following studies.

2.2 Truck-Following Models

So far, all the car-following studies are based on a homogenous traffic. That is car following another car. In reality, microscopic traffic models must properly describe how driver’s behavior varies according to vehicle type used and vehicle type followed. There have only been a few studies that address cars interacting with trucks. Peeta et al. (2004) conducted a user’s survey to infer the discomfort factors of drivers when following cars. The factors of statistical significance were used to construct a fuzzy logic based model to capture the psychological discomfort level when following a truck. This research provides evidence from stated preference survey data that drivers behave differently when trucks are present around their vehicles. Peeta et al. (2004) identified gender, household size, weather, and congestion level as significant factors. However, not all the factors can be easily obtained and input into a microscopic simulation model. In addition, the data used was limited to pairs of car-truck following (a car following a truck) having at least two seconds time headway due to safety measures taken by the car drivers to enlarge their visibility. Moreover, this approach only encompasses car-truck behavior without taking into account different following combinations such as truck-car and truck-truck.

Siuhi and Kaseko (2010) used real data collected at Interstate-80 Freeway in Emeryville, California to calibrate the parameters of the GM car-following model. They proved that $\lambda$, $m$ and $l$ have different values when a car is following a truck (car-truck), a truck is following a car (truck-car) and a car is following a car (car-car). In particular, car drivers tend to follow trucks with a longer distance, and are more sensitive to the relative speed. However, they did not study the behavior when trucks are following a truck.
Although, Peeta et al. (2004) and Siuhi and Kaseko (2010) have contributed to spreading the scope of car-following study to heterogeneous traffic, the combinations of vehicle types were not exhaustive. For instance, truck-car and truck-truck combinations were not studied to the same level of detail. Even for car-car and car-truck pairs, there may be other forms of model that can better describe the driver’s behavior.

2.3 Crash Analysis Models

Statistical analysis of crashes has been widely studied in the past 20 years. A review on commonly used tools in transportation data analysis that focus on highway’s crash frequency and crash severity are found in Savolainen et al. (2011). In this review, it is mentioned that statistical analysis of motor-vehicle crash data has been used to relate highway collision outcomes to a variety of factors. Most crash analysis primarily focused on aftermath described by injury severity or property damage. Less attention has been paid to diverse failed interactions before the crash and that are explicitly found in the variety of manner of collision. This includes actions taken as a response to the impending danger, path of vehicle prior to crash, state of the driver, etc. which can give an insight of driving behavior that lead to a collision. However, the incorporation of vehicle type and car-following concepts in the analysis of collisions has been sparse.

2.3.1 Crash Analysis Models using Car-following

The analysis of vehicle collisions, including car-following attributes in the model, has been performed by Xin et al. (2008) and Zhang, et al. (2008). However, studies so far have assumed that failures in car-following only result in rear-end collisions, ignoring the possibility of angle and sideswipe collisions.

For instance, Xin et al. (2008) estimated the parameters of the model (maximum comfortable deceleration and acceleration rate and its oscillation, as well as, visual stimulus, following gap time, etc.) to obtain an accurate reaction time using non-crash data. The results were compared with crash data and it was found that the model could replicate the crashes based on the reaction time. However, the analysis of different types of crashes, from a perspective of microscopic traffic where interaction of vehicles
happen in same and adjacent lanes, may still need to be considered in the model to account for all possible manner of collision.

Zhang, et al. (2008) used Poisson and negative binominal distributions to relate frequencies of crashes in the same direction on rural roads as a function of mostly roadway features and the time of vehicle spent following another vehicle. This leaves behind variables that could be of great importance in determining a specific manner of collision. Moreover, detail variables describing car-following dynamics were not considered in their modeling approach.

2.3.2 Crash Analysis Models for Manner of Collision

Investigating crash type is crucial when identifying potential safety improvements in a roadway. Crash type analysis is implemented in the Highway Safety Improvement Program (HSIP) Manual (Herbel et al. 2010) to quantify the actual or expected safety of a roadway. The HSIP Manual uses crash type to identify high-risk facilities for potential safety improvement.

The manner of collision or crash types as an outcome of a crash that is related to driver, vehicle and roadway factors has not been studied extensively, especially in highways. Typically, disaggregate models in crash analysis focus on injury severity levels as the outcome. Although, studies have developed different statistical or econometric models to predict the manner of collision at intersections (see for examples, Abdel-Aty and Nawathe (2006), Ye et al. (2009), Kim et al. (2007), Keller et al. (2006)) far less attention has been paid to analyze factors that lead to the different types of collisions on interstate highways.

2.3.3 Crash Analysis Models using Multinomial Logit Models

The MultiNomial Logit (MNL) model has been used to predict the role of vehicle types in crashes based on factors that contribute to rear-end collisions (Abdel-aty and Abdelwahab 2003; Yan et al. 2009). In the work performed by Abdel-aty and Abdelwahab (2003), rear-end crashes involving light truck vehicles and passenger vehicles were analyzed using three models: MNL, Heteroscedastic Extreme Value (HEV), and BiVariate Probit (BVP). These three different models were compared to account for possible limitations when restricting the study to one specific model specification: (1) the BVP was used to account for the two possible (binary) outcomes (striker and struck), (2) the MNL was
used for predicting four types of vehicle combinations involved in rear-end collisions (e.g., car-car, car-light truck, light truck-light truck, and light truck-car), and (3) the HEV was used to account for an independence of irrelevant alternative (IIA) issues present in the MNL model. Although the approach for studying rear-end collisions among different vehicle types was innovative it could have been expanded to include other vehicles types (e.g., large trucks). Different manners of collisions could have been considered.

2.3.4 Crash Analysis Models using Mixed Logit Models

Recent literature in highway crash analysis using MiXed Logit (MXL) can be found in Milton, et al. (2008), Chen and Chen (2011), Bhat (2003), Kim et al. (2010), Eluru et al. (2008), Gkritza and Mannering (2008), Morgan and Mannering (2011). Although these studies are mostly dedicated to addressing accident or injury severity, they have consistently shown that MXL is the best modeling approach to justify unobserved effects and heterogeneity among observations. For example, in the case of Milton et al. (2008) and its analysis of highway severities, road characteristics, environment and driving behavior are the major contributing factors. These factors are expected to vary across roadway segments and therefore are treated as random across the samples. In addition, inclusion of unobserved characteristics of roads and the environment are addressed by using MXL. Similarly, the proposed MXL model considers the variability of pre-crash actions, driving behavior and communication between vehicle driven and vehicle interacting with to perform the analysis.

2.3.5 Crash Analysis Models for Large Truck

To address severe and costly crashes on highways, several safety studies have targeted collisions involving large trucks to capture possible factors influencing these collisions (Duncan et al. 1998; Council et al. 2003). However, factors describing pre-crash actions in highways between trucks and passenger cars were not included in these investigations.

Although sparse, crashes involving trucks have been previously studied using a MNL modeling framework as found in Yan et al. (2009). In this study, the role attributed to the vehicle was analyzed only in rear-end crashes as car-truck and truck-car. With regards to explanatory variables, they did not consider critical events experienced by the drivers before colliding with another vehicle (e.g.,
distraction, speed, and braking behavior). Although this approach for studying rear-end collisions among different vehicle types is new in the field, it has some limitations. This study did not consider critical events experienced by drivers before colliding (e.g., distraction, speed, and braking behavior) among the explanatory variables. A more comprehensive study should consider other possible manners of collision instead of exclusively analyzing rear-end crashes.

2.4 Models Using Accident Data

The National Automotive Sampling System-General Estimates System (NASS-GES) database has been used as a reliable source to draw national safety statistics obtained by the National Highway Traffic Safety Administration (NHTSA 2010a). The NASS-GES database documents annual records of all crashes involving all types of vehicles among different road systems in the United States. The NASS-GES crash data is collected from 400 police agencies across the U.S. Police crash reports are processed by the National Center for Statistics and Analysis, a branch of the Policy and NHTSA. This data set is a probability-based representative sample for all motor crashes that resulted in fatalities, injuries and property damage. The NASS-GES database is a sample of the 6 million crashes occurring nationwide every year, containing information for about 50,000 collisions. The NASS-GES database is managed via the Statistical Analysis System (SAS) software (SAS 2004) in which further data manipulation is possible. Data sets may be queried or created based on user specified conditions with respect to subsets of data: accident, vehicle, maneuver, distraction, etc.

Since 1988, it has served as an instructive tool for motor vehicle manufacturers, insurance companies, government agencies and researchers to enhance road safety. This is why; there are about 250 transportation studies that focused only on crash analysis on highways that used NASS-GES data. All previous mentioned research in the crash analysis models used NASS-GES as their source to develop the models. Therefore, this research has adopted this database as the only source for building statistical models.

2.5 Longitudinal Control Models

Although Automated Intelligent Cruise Control (AICC) and driverless vehicles has gained interest over the past years, this approach takes its first steps in the late 1960’s. One of the pioneer
studies in this matter can be found in Bender and Fenton (1969). It is one of the first in this nature to relate car-following theory with control systems. This study initiated further research for replicating driving based on the movements of the vehicle in front and most importantly, it intended to incorporate traffic densities in the study. Headway and relative speeds are considered in the estimation of the follower’s acceleration which is according to car-following theory. This research examined a steady-state car-following to model a control system for an automatic vehicle. Applying some limitations in the model (i.e. acceleration/deceleration must not exceed 0.1 g, response capabilities must be according to vehicle capabilities, etc.) the behavior of car-following behavior was observed in an automated highway. Braking, acceleration and steering of the test vehicle were achieved using electro-hydraulic control systems that include actuators to regulate throttle, brake-pedal and front-wheel. A phantom car was used as an input of velocity and headway to the system to observe the behavior of the controlled vehicle (follower). Longitudinal dynamics were represented by finding a relation between output and input in terms of throttle valve position, vehicle time constant and steady-state gain from throttle valve that led to a representation of car-following. The car-following model proposed assumed the acceleration of the follower vehicle to be described by a relation between the relative speed, the speed change of the lead car and a headway error term. Steady-state headway was derived when a change of speed of the leader occurs. The transfer function obtained describes the input of the system in terms of the leader’s velocity and the output as the follower’s velocity. The control configurations were based on a comparison of input signal and the position of the throttle valve. With this, a hydraulic actuator connected to the throttle valve was able to regulate the system output to reach the required behavior. Car-following behavior was studied using a test car on I-270 Freeway near Columbus, Ohio where two average speeds (60 mph and 40 mph) were used to collect data. Using different constant parameters, asymptotic stability from six different case studies were analyzed based on the estimation of the follower’s (controlled vehicle) acceleration. The results indicated that the system was unstable when the constant parameter was equal to one and time constant was 2 or 2.5. When constant parameter was increased and the time constant was not in the range previously mentioned, then the system was asymptotically stable indicating that there was a satisfactory relation between the vehicle response and the time headway. It
was found that when headway was omitted in the calculation of the acceleration of the follower, depending only on the relative speed, the system was also stable.

Another relevant study made by Sklar et al. (1979) gave a different perspective for modeling driver behavior under different velocity scenarios. This research compared different parameters used in vehicle follower controls, such as, constant time headway, constant separation and constant k-factor (arbitrary safety factor greater than or equal to one). The study proposed a safe approach for short and long headway where collisions are successfully avoided. This policy was different from others that suggest a constant time and/or space separation. A longitudinal control system that used velocity-spacing relationship was implemented instead. This control used a target-tracking method to calculate velocity commands in velocity-following control loops. It includes steady-state operating policies and dynamic separation-velocity constraints for a safe driving. Simulations showed that the safe approach controller was unconditionally stable and string stable overall. The results indicated that the follower mimics leader’s behavior when it changed its speed, performing similar but delayed maneuvers. Five vehicle were used in the simulations where following and overtaking were present. When constant separation policy was used, the simulation results showed that response disturbances were amplified along the stream making the system unstable. The constant k-factor policy resulted in collisions as compared with the safe approach which successfully negotiated all maneuvers.

The progress in the study of AHS has been mentioned in Fenton and Mayhan (1991) in their investigation conducted at the Ohio State University (OSU). This paper summarizes how different approaches done by OSU grew in complexity in order to achieve a more accurate model. It also gives a general knowledge of the theory and validation tests performed including a brief description of the results obtained, as well as, advantages and disadvantages. Some methods used in longitudinal control as described in this paper are: headway safety policy study to achieve asymptotic stability and avoid crash accidents, point-following study where an imaginary series of slots divided equally where a vehicle tracks a point (slot center) and uses it as a reference to adapt its longitudinal state. Car following models, although not considered a major part in the study, were still investigated from 1964-1971. Ohio State University did not create a detailed longitudinal controller design for car-following; instead the
controller was designed to meet specific conditions. For instance, small deviations of desired spacing and relative speed were considered as a requirement in the model. Also, the controller operates even with unexpected disturbance inputs. Acceleration capabilities of vehicles, as well as, the achievement of asymptotic stability in the platoon, were considered. Finally, a comfortable ride, efficient control of vehicle and adequate response to imminent danger were part of the requirements in the controller design. The first proposed control law describes the acceleration of the follower to be proportional to the relative speed of the leader with respect the follower. According to the results obtained in this study, the control law had a good performance in terms of small deviations, high speeds, and asymptotic stability. This was obtained when the time constant was kept as a value of one second or above. One of the difficulties faced in this study was the variation of desired spacing between vehicles for the follower vehicle which was limited when additional control laws were implemented. In addition, multimode control systems were explored as an attempt to include more operational situations instead of only focusing on steady-state car-following. In this way, multiple driving scenarios (e.g. overtaking, sudden braking, etc.) are identified from the resulted plotted graph of relative velocities. Six different control modes were employed accordingly to the six regions of driving behavior identified. Results from field testing proved the success of car-following models according to the desirable standards. In order to generate the input data for the simulation, such as the relative velocity, it was necessary the use of a “phantom” lead car and two vehicles attached by a mechanical take-up reel; a technology available at the time.

Previous works on controller designs used the leader’s characteristics in the controller to achieved platoon stability. Using the leader’s information as a feedback, that is, if the led followers know the leader’s velocity and acceleration, the platoon performance is stable. Other studies emphasized the possibility to adapt the follower’s behavior without being subject to the vehicle in front but rather, using a system that includes engine, transmission, drive train, and a brake system. An example of this can be found in Chien et al. (1994) where a powertrain model describes the longitudinal control. An imaginary preceding vehicle is used to set the initial conditions for the controller and model the transient response of the follower. This study used a safety distance headway policy for a following controller where the velocity of the follower is approximately equal to the leader commonly referred as tight
vehicle following. For the case of passing slower vehicles or merging, a desired k-factor headway policy is used. This factor represents the minimum and safe distance from one vehicle to another according to traffic capacity and vehicle capabilities. The design of the controllers allowed the activation/deactivation of actions (acceleration/deceleration or braking) according to the maneuver required. The longitudinal dynamic system is based on two inputs (throttle angle and brake torque) whereas the single output is the vehicle speed. The throttle controller performed the acceleration/deceleration and the brake controller is in charge of the deceleration operation. A switching logic alternates (activate/deactivate) between these controllers to perform the action needed. The model includes position and velocity, longitudinal direction, throttle angle, brake torque, gear ratio from the engine, vehicle mass, tire force, drag force, external disturbances, etc. Nonlinear validated models were used to prove the stability of the system when a vehicle was following another and when new leaders were introduced. In the first scenario asymptotic platoon stability was reached when four vehicles traveled in the same lane. In the second case, the initial leader changed to a different lane and a new leader was introduced. It was found that a smooth transient response was obtained since the follower’s dynamic behavior was reset to track the new vehicle in front. The third case focused in the follower rejection of disturbance which was successfully accomplished. The last simulation analyzed the entrainment controller performance when overtaking resulting in smooth acceleration magnitudes.

Another emphasis in longitudinal controls was the integration of these control laws to Intelligent Vehicle Highway System (IVHS). That means that the study of interactions between different platoons needs to explore the minimum yet safe spacing among groups of vehicles. In this case, an interrupted vehicular flow within the same group of vehicles is achieved due to the adaptation and actions of the group of vehicles in front of the platoon. In the approach supported by Godbole and Lygeros (1994), leader-following behavior was used to simplify the problem and avoid the use of communication systems integrated in the highway structure. An optimal velocity should be tracked and maintained by the platoon leaders. These leaders should engage certain maneuvers according to the traffic situation such as, joining other platoons, change lanes or split. A linear closed loop system was considered to make the input and output a linear relation. A feedback was then used to design the controllers of the
linear system. A full state feedback involving position, speed and acceleration was obtained using an
observer. Additional manipulations (i.e. pole placement) were done to avoid sensitive responds when
spacing was slightly changed. There was no improvement in the model to satisfy bounds for acceleration
and deceleration as considered as 2 m/s² and -5 m/s² respectively for normal operation, as well as for the
bounds for jerk considered to be ± 5 m/s³. To overcome this problem, different control laws for different
state space regions were developed. Region 1 placed the poles to achieve an error between actual and
safe inter-platoon spacing to be less than zero. Region 2 tracked the velocity of the leader when another
vehicle moving faster gets in front. Region 3 described the situation where another vehicle changes lanes
in the front with a slower velocity. Region 4, tracked the optimal velocity when there was no platoon in
the 60 meters in front of the leader and when there is a vehicle from the preceding platoon traveling
slower. A Proportional Integral Derivative (PID) controller is used to take care of disturbances. A
smooth transition between the changes of controllers according to the region was achieved using a
global unified controller. Finally, the leading platoon vehicle was able to perform maneuvers such as,
merging to another platoon, split the platoon so that a follower becomes a new leader, and changing
lanes where deceleration of either one car or the whole platoon was needed.

Driving performance and estimation of driver responses still represents a challenge in vehicle-
following. Based on the previous control models, driving behavior is controlled by the vehicle itself. An
alternative to this approach when modeling vehicle-following is the inclusion of human behavior in the
loop.

2.5.1 Kalman Filter in Car-Following

The concept of Kalman Filter (KF) was originally proposed by Rudolph Emil Kalman in the
1960’s (Kalman 1960). It has been widely used by electrical and mechanical engineers. In
transportation, KF has been mostly used for modeling position, speed and acceleration in trajectory data.
An example of KF filter as a smothering technique for filtering noise in car-following data is found in
Punzo et al. (2005), Ma and Andréasson (2007), and Ma and Jansson (2007). In these studies, KF has
been used as an estimation tool of the data used in car-following.
The work performed by Punzo et al. (2005) used KF to estimate a noise free time series of speeds and acceleration of following vehicles to estimate their inter-vehicle spacing. The state variables for the reference model (plant) were the measured velocities for leader and follower and their inter-vehicle spacing. The measured velocities and spacing were constrained to be equal to the previous time step. The state variables of the actual model (observer) were represented by the velocity of each vehicle and spacing between them, with the addition of error terms in each equation due to noise in the measurement and inaccuracies of the model. The state variables of the actual model were calculated based on the information from receiver antennas on two vehicles. With the KF algorithm, the velocities and accelerations were estimated from the positions so that there was no negative spacing between vehicles (errors). It was found that KF gives the closes estimates to the actual profile for speed and inter-vehicle spacing.

In Ma and Andréasson (2007), KF algorithm was used to cancel the noise in the measured physical states (acceleration, speed, and position) of the follower (observed vehicle) and instrumented leading vehicle (reference vehicle). The state variables of the leader and follower were represented by their: position, velocity and acceleration. For the reference vehicle (leader), its position and velocity were derived from physics law of motion, whereas the acceleration was calculated as a time series with an autoregressive model. For the follower, the position was calculated by subtracting the relative distance between the leader and the follower. Velocity is calculated from the relative speed with the vehicle in front. The input matrix in the system is defined by the measurement noise in the acceleration. The output is the estimated position, velocity and acceleration of the follower.

Another relevant study that considered KF for car-following modeling was performed by Ma and Jansson (2007). Here, KF was used to measure the general optimization function of behavioral response when driving. This study analyzed if acceleration outputs (when taken as the performance measure for car-following model) can give good results in trajectory and speed analysis. In this study, trajectory data from GPS and laser data was collected using an advanced instrumented vehicle. Kalman filter was used for the mathematical formulation of parameters used in car-following. The state equation for the plant and the observer are position, velocity, and acceleration. Position and velocity were taken from laws of
motion whereas acceleration was a function of time, follower’s relative velocity, follower’s relative position, velocity of leader and car-following dynamics represented by a constant. The observer’s state variables were defined as the plant states plus an additional term to account for measurement noise. The KF estimated parameters such as position, velocity, and acceleration. The GM model was used as the validation model in the estimation. It was found that car-following behavior from the collected dataset can be replicated in the simulation. When a closed-loop simulation was performed for 5000 data points of the same driver, the results indicated a close match in the velocity but with a significant difference in acceleration and inter-vehicle headway. The overall findings suggested that estimated parameters using KF did not converge due to an inadequate model configuration and or incorrect use of parameters. Also, the global optimum was not found since this computation will require long time to be reached.

The studies previously reviewed focus on how to estimate state variables for car-following by utilizing KF. Previous studies identified how the state variables are defined and what is considered as the input to the system. Car-following dynamics has been based on acceleration, velocity and position. Measurement noise has served as an external input to the system. This allows a better understanding on how following behavior is modeled in order to develop a new model where driver performance is integrated. The inclusion of vehicle characteristics and driving performance has been overlooked. Despite the progress of using an algorithm that can estimate linear systems, the potential of KF as a tool to examine the risk of collision has yet to be explored.

An important research question is whether there exists different behavior (aggressive vs. defensive driving) when different vehicles, i.e. cars and heavy vehicles, are maneuvered or followed. After reviewing the above related studies, a better model can be developed.

2.5.2 Heavy Vehicle-Following

The positive impact of AHS is greater when considering the automation of commercial vehicle. This is due to the large number of miles traveled and greater fuel consumption that each heavy vehicle has in comparison to passenger vehicles. In addition, the importance of studying heavy vehicles interactions is critical since heavy vehicles are more likely to travel in platoons because they can share origin/destination points and have a more established route. In order to overcome upstream disturbance
propagations that can be more predominant in large inter-vehicle spacing as observed in the presence of heavy duty vehicles, nonlinear spacing policies need to be analyzed as done by Eyre et al. (1998). The control objective introduced a positive design constant that regulated the response of the controller according to the relative velocity and separation error. This coefficient was a nonlinear function of the separation error. In addition, the controller did not need a switching logic to change from acceleration/deceleration to brake or vice versa. It used the command output instead to activate the accelerator or brakes (i.e. positive outcome activates fuel command and negative output is for the application of brakes). In addition, a Proportional Integral (PI) controller was proposed to account for the disturbances when regulating relative velocity and separation error. To compensate the lower actuation-to-weight ratio of commercial vehicles, a new term (signed quadratic) is incorporated to obtain a Proportional Integral Quadratic (PIQ) controller that respond to more aggressive large errors. In order to take into account the different vehicle characteristics information that different heavy trucks may have, adaptive gains were introduced. These gains were able to overcome different performance for different operating conditions. Variable time headway (i.e., the headway changes with relative speed) was taken from the concept that headway is reduced if the leader is traveling faster (positive relative speed) and vice versa. Note that neither negative nor large headways are desirable in this approach. The other headway approach proposed in this study, had a variable separation error gain. This was implemented to avoid aggressive accelerations when the follower was far behind the leader. Thus, in order to apply this headway policy, it is necessary a “platoon conscience” where the first vehicles sacrifice their performance in order to avoid amplification of decelerations that may result in collisions. When the separation error gain was tested, the results showed a smooth deceleration transition for all heavy vehicles in the platoon when the gains remained above a positive lower bound. There was no collision but it was noticeable that the first five vehicles compromised their behavior (i.e. have larger errors because their controllers did not react as aggressively as the others) in order to have a better overall performance. The controllers were also used in a merging maneuver of two platoons. The results indicate that the variable separation error gain is consistent. On the other hand, variable time headway policy alone did not provide successful results.
2.5.3 Mass-Spring-Damper Representation for Car-following

The design of AICC has been modified to approach driving conditions of the leader and follower using front and back information. This can be observed in the work done by Zhang et al. (1999) where, in order to apply this control, it is necessary for the vehicle to know the state of the back and front vehicles. Although not deeply described, a mass-spring system was taken into consideration to formulate the design of the controller. A safe distance policy that included the velocity of leader and follower vehicles, as well as, constants depending on braking capabilities was developed. A constant headway was derived for every follower assuming that its velocity in addition to the relative distance and relative velocity of the first and successive pair of vehicles were available for measurement. The last vehicle in the platoon used a virtual follower and the information of the preceding vehicle. For leader of the platoon a cruise control was assumed. Regarding longitudinal dynamics, this study considered a system that has two inputs represented as throttle angle and torque command, and one output described as vehicle speed. The controller also considered a switching logic that adjusted the vehicle behavior; acceleration/deceleration or deceleration, using a throttle controller or brake controllers accordingly. For the vehicle following throttle controller, spacing deviation between vehicles, length of follower vehicle, time headway and a chained inverse function of the preceding and following vehicles were included. In order to achieve stability in the platoon, the spacing and velocity errors remain bounded.

Over the years, research on longitudinal control of vehicles has focused on the safe interaction of a pair or a platoon of vehicles, and to achieve string stability in the platoon. Here, string stability refers to the phenomena that the amplitude of vehicle response decreases as it propagates upstream in the platoon. Although different methods have been employed to model and analyze vehicle-following behavior, the Mass-Spring-Damper (MSD) representation has only recently been studied.

In Eyre et al. (1998), this MSD modeling approach was proposed to represent the communication links between vehicles in a platoon. The springs and dampers in the system were treated as the electrical connections where vehicles communicate their speed and position to the adjacent vehicles, so that the relative speeds and distance could be calculated. In this way, the speed and distance information of the leader was used for the follower’s controller in the latter’s unidirectional control. By the same token, using the same vehicle information, and also, including the vehicle in front and back, a bidirectional
controller was designed. The paper investigated the string stability of the system using different vehicle-following policies, such as constant spacing policy, speed dependent spacing policy, and constant and variable time headway policies. Although this paper gave a detail mathematical formulation of vehicle interactions, no simulation was performed to study the behavior of the system with regards to the risk of collision. This paper also did not suggest any value for the spring constant and damping coefficient.

An example of vehicle-following simulation using the MSD representation can be found in Sapiee and Sudin (2009). This work studied the adaptation of the follower (in a pair of vehicles) based on the leader’s speed as an input. The system included internal vehicle dynamics with a time delay switch. In this work, an adaptive controller gain design was implemented to adjust the values of the spring constant and damping coefficient. A Model Reference Adaptive Control (MRAC) was used to measure these values based on the change in speed and position of the leader. This approach overruled the fixed spacing policy and considered instead a fixed headway policy. Since the value of the spring constant was related to the desired following headway, different values were necessary for the different following headway. The authors did not suggest how this can be implemented in a following vehicle’s controller. Ideally, a fixed spring constant and a fixed damping coefficient should be used in a controller, or have their values self-calibrated to the actual driving behavior.

2.6 Modeling Using Vehicle Trajectory Data

The development and validation of car-following models depend on data collected in actual driving environment. Early research in car-following models has faced challenges in collecting field data. For instance, developing the GM model, Chandler et al. (1958) started by wiring a test car (following vehicle) to a reel shaft mounted on the lead car. This first method found insignificant relative speed between vehicles when modeling car following behavior. Also, the driver reaction time had a high variation among drivers as well as the scalar constant as stated by Brackstone and Mcdonald (1999). Ozaki (1993) collected field data using a video camera positioned on the 32th floor of a building and calibrated constants according to acceleration and deceleration responds. However, the panoramic view was narrow and each pair of vehicles has only up to 10 seconds of recorded view. The surrounding traffic condition (such as downstream queuing) cannot be identified accurately from the video recording.
Brackstone and Mcdonald (1999). The introduction of technologies such as Global Positioning System (GPS) enabled Ranjitkar et al. (2003) and Punzo and Simonelli (2005) to record and obtain vehicle trajectory in car-following situations. Although, this resulted in a more accurate data set of the drivers’ performance, the data was collected in Japan and Italy respectively, and are not available publicly. Another procedure for data collection performed by Treiterer and Myers (1974) includes the use of a camera from a helicopter. The disadvantage of this method, however, is that a picture must be processed using a complex and time consuming system. Therefore, trajectory data must be obtained using a detailed, unbiased and accurate resource to provide an accurate traffic flow analysis.

The Federal Highway Administration (FHWA) funded the Next Generation SIMulation (NGSIM) project (USDOT 2010), which collected comprehensive sets of vehicle trajectory data. The NGSIM data sets were collected on the Interstate 80 Freeway at Emeryville, California, U.S. 101 Freeway in Los Angeles, California, Lankershim Boulevard in Los Angeles, California and Peachtree Street in Atlanta, Georgia. The data was collected by video cameras mounted on tall buildings. The images were processed and detailed vehicle trajectory information such as vehicle type, position, speed, acceleration, deceleration, headway, lane change, etc., at sub-second intervals were extracted. The NGSIM data set is readily available for researchers to perform traffic flow theory related studies. Examples of car-following research that have used NGSIM data are Leclercq et al. (2007), Ma and Ahn (2008), Hamdar et al. (2009), and Siuhi and Kaseko (2010). Using NGSIM vehicle trajectory data; car-following, lane changing and gap acceptance can be analyzed.

2.7 Summary of Literature Review

The development of car-following models from 1960’s to 2010’s has been reviewed in this chapter. Most of the car-following equations describe the following vehicle’s acceleration (or deceleration) as a function of several attributes such as the follower’s speed, the relative speed and distance with respect to the lead vehicle. The acceleration of the following vehicle is often regarded as the control or response variable that the following driver adjusts to keep up with the lead vehicle or to avoid a collision. So far, the calibration of parameters has been exhaustively studied by several researches, without applying it for crash analysis. In recent years, researchers have begun to study the
different driving behavior, sensitivity or aggressiveness of the different drivers according to vehicle types. However, the association of car-following with the manner of collisions that could take place in highways has largely been ignored. Hence, the microscopic motion of vehicle-pairs in car-following may be used as surrogates of the aggressiveness of driving behavior that is related to the type of collision. In other words, car-following attributes immediately before a crash may influence the manner of collision. The failure of car-following that results in crashes has been studied in limited cases, only in numerical simulation. However, stability studies so far have assumed that failures in car-following only result in rear-end collisions, ignoring the possibility of angle and sideswipe collisions.

Statistical models that address collisions have been restricted to estimate the frequency of collisions or injury severity, without including factors in the interaction of vehicles; mostly represented in car-following that can result in different types of collision.

Also, linear system model with state-space representation has not been related to study transportation safety issues. It has mostly been applied previously in transportation data collection to generate more accurate trajectory data, such as reducing the noise in measured speed, position, etc. The safe interaction among vehicles can be modeled by pre-crash car-following attributes and vehicle types represented in state-space equations to identify driving behavior that may result in a crash.

This study is the first to construct a model to construct statistical models to predict the outcome of a collision (which may be rear-end, angle and sideswipe) by including pre-crash car-following attributes and vehicle types in addition to driver characteristics and other factors normally considered in crash analysis. The results will contribute to a better understanding of the car-following dynamics and vehicle type on the manner of collision.

Even though the studies previously described suggest a different approach to estimate behavior measurement for car-following, this research proposes the use of linear systems to identify different behavior when different vehicle types interact. It is known that related car-following work measure performance based on acceleration. However, from following driver’s perspective, headway and/or relative speed may be more suitable for measuring external inputs. In addition, the inclusion of vehicle characteristics and/or other factors (aggressive vs. defensive driving) have been overlooked.
Chapter 3: Model Framework

This research has been executed in seven tasks. The tasks progress from reviewing the general aspects of car-following to the development of a crash prediction model and driving behavior model. A general review of current car-following, crash analysis and linear system models was used as the basis of identifying gaps in the existing approaches. Information gathered was developed into an analysis framework which is presented in this chapter. Crash prediction models using MXL and driving behavior model using linear system estimation are the two approaches in the research framework.

3.1 Research Methodology

Current research on traffic interactions, including the interactions between car and trucks and major safety problems associated with failures, i.e., crashes, was first reviewed. Impact on road safety was stated to relate the significance of addressing the failure in traffic interactions. After identifying the problem, crash prediction models that include car-following attributes was presented as an alternative tool that can be used by transportation agencies in designing, planning, operating, and maintaining highways. This modeling approach for car-following failure involved illustrating its potential application and identifying stake holders likely to be interested in the solution.

The history, definitions and concepts involved in car-following were next reviewed. Car-following studies were narrowed into models that describe vehicle trajectories and their relation to vehicle collisions. A detailed review of publications on car-following models and a comparison between them led to an understanding of the current state of car-following research. They were used to formulate a new model to predict the manners of collisions.

Mixed logit models were constructed to predict the probabilities of the manners of collisions that two vehicles can have when traveling in the same direction on interstate highways. Data describing the characteristics of the vehicle body type, car-following actions and driver behavior information, taken from the NASS-GES crash data was the source of developing and validating the model.

At this stage, it was important to underline the key variables for modeling car-following. These variables were classified as safety or state prediction variables. The proposed MXL and linear system models must be able to quantify the accident risk associated with trucks as opposed to cars, and relate
the risk with pre-crash situations according to vehicle’s dynamic characteristics and driver attributes. The MXL model was considered appropriate for predicting the probability of how two vehicles will collide among the different possible manners of collisions based on vehicle-driver and pre-crash car-following attributes. On the other hand, state-space representation is considered an appropriate tool in linear systems for predicting the behavior of a following vehicle that mimics the actions of the vehicle in front and adjusts its behavior in response to the risk of collision to achieve a desired steady state.

In order to develop MXL models, data collected on interstate highways involving the collision of two vehicles traveling in the same direction was analyzed. Data used in this part of the study is taken from NASS-GES. This database includes crashes where property damage, injuries and fatalities were reported by police officers. It provides information about all types of crashes involving all types of vehicles, as well as, environment conditions, vehicle-related factors, actions taken by driver, driver distractions, non-motorists actions, safety equipment, and etc. The MXL models were used to relate information on the above factors that contribute to rear-end, angle and sideswipe collisions. Results from the MXL models were used to quantify the relative contribution of the factors.

In the second modeling approach, linear systems was used to construct state-space representations that consider motion characteristics such as relative velocity, distance headway, acceleration, and etc., to evaluate the control variables that the driver adjusts to keep up with the leading car to avoid a collision. Vehicle trajectory data was obtained from the NGSIM project and was used for state-space analysis. This modeling approach used a linear system to capture the dynamics of car-following and KF to predict the behavior of the system. Kalman filter is an iterative algorithm that can be applied to predict the future state of a complex dynamically linear system when the system is subjected to an external excitation process. With an estimate of the future states, the behavior of the system can be controlled to achieve the desired objective. Kalman filter was used to predict the state of a pair of vehicles based on the characteristics of the vehicle (truck or car), the driver (aggressive or defensive), and the car-following attributes. This KF tool was applied to every pair of leader-follower to predict the collision risk of the vehicle pair in the immediate future. Trajectory data obtained from NGSIM data set was used to develop KF models. The NGSIM data set offers detailed vehicle trajectory
information, such as, vehicle type, position, velocity, acceleration, deceleration, headway, etc., at sub-second intervals. Measured data from this source, such as headway and relative velocity in the past few seconds, was used as inputs in order to predict the car-following state and collision risk in the next few time frames.

The expected results of MXL and linear system models can be used to predict the outcome of vehicle interactions in a highway when different types of vehicles are following one another. Based on the MXL and linear system modeling results, this research is able to determine if dynamical following behavior is different when a car is following a car, a car is following a truck, a truck is following a car, and a truck is following a truck. From the MXL model, parameters values that are unique to the vehicle type driven and vehicle type followed and/or striking and struck vehicles are identified. The KF in the linear system model is able to establish a crash prediction if the driving behavior of the follower remains unchanged and then apply corrective acceleration or deceleration.

Implementation of car-following in road safety depends on many aspects. By formulating a series of input variables and generating model outputs through software simulations, many “what if” scenarios were analyzed. Recommendations were formulated based on the models developed in this dissertation, as well as, advantages and disadvantages to address driving safety and safety measures. The recommendations will focus on identifying driving factors in impeding collisions, the associated risk and potential outcome when collisions may not be successfully avoided. Suggestions for further research involve the development of driving assist tools where drivers can be warned about danger in roadways. The research has the potential to impact policy measures related to driving rules and safety in the future.

3.2 Research Tasks

This dissertation research is represented in Figure 3.1 with seven tasks (developed as separate chapters) that are interconnected. The first task of the dissertation is the explanation of the purpose of the research (Chapter 1). The second task is the literature review of car-following models, crash analysis models and linear system models, the NASS-GES accident data and NGSIM vehicle trajectory data, respectively (Chapter 2). Task 3 formulates the model framework where the research methodology is planned (Chapter 3). Task 4 and Task 5 describe the development of the two models; MXL model for
pre-crash factor analysis (Chapter 4), and the KF estimation of linear system car-following model (Chapter 5). The results obtained from the models are discussed in Task 6 (Chapter 6). Finally, recommendations and conclusions are made in Task 7 (Chapter 7).
Figure 3.1: Research methodology flow chart.
3.2.1 Task 1: Research Motivation

This research contributes to better understanding of safety in car-following by modeling the behavior and interaction between two vehicles using the MXL and linear system approaches. It attempts to identify pre-crash conditions that depend on vehicle characteristics, driving behavior and road conditions that lead to collisions involving two vehicles traveling in the same direction in interstate highways. This research proposes models that can be used for highway safety analysis. These models make collision prediction that may be further developed into highway design policies or collision avoidance systems. The models may be used as analytical tools to assist in policy development to improve safety and potentially increase capacity in highways.

3.2.2 Task 2: Literature Review

The literature review focused on the following aspects of this research: current state of car-following models, crash analysis and linear system models. It also reviewed models that used accident data and models that used vehicle trajectory data. Car-following models were studied from their origins to the latest models in order to find research gaps that could be addressed and applied to traffic safety problems. Key variables in car-following were used as inputs in the proposed MXL and linear system models.

Crash data was studied to identify the most common types of collisions on interstate highways when traffic is traveling in same direction. Available accident data used in past research that includes crashes resulted in property damage, injuries and fatalities were identified. Factors that contribute to crashes, such as the types of crashes involving different types of vehicles, environment conditions, dynamic characteristics, actions taken by driver, driver distractions, non-motorists actions, safety equipment, and etc., were summarized.

A review of linear system models in car-following was performed. Kalman filter’s state-space estimator was included in the literature review as a tool to model the dynamics of car-following. Models that used vehicle trajectory data were analyzed in order to revise possible inputs into the KF iterative algorithm.
3.2.3 Task 3: Formulating the Analysis Framework

This task created a framework for the approaches taken by this research. It identified the tasks performed to achieve the desired results, milestones and deliverables. The research methodology considered how available data can be processed and the modeling techniques applied to conduct risk analysis in car-following.

3.2.4 Task 4: Development of Mixed Logit Model for Crash Prediction

This task involved the application of MXL model to relate crash type as a function of factors, such as driver, vehicle and road conditions. The data used in this study was taken from the NASS-GES accident database. Relevant data was organized into two sets so that the models could be calibrated and then validated, respectively. The original NASS-GES database was sorted to extract only crashes of two vehicles traveling in same direction that occurred on interstate highways under no adverse weather. The dependent variables of interest for the models found in NASS-GES database were vehicle body type, car-following behavior, visual circumstances, vehicle factors, driver distractions, and driver information. The independent variable was the manner of collision that resulted when traffic going in same direction interacted.

3.2.5 Task 5: Development of Linear System Model

This task constructed a state-space representation of the linear system model to evaluate if a pair of leader and follower has a safe interaction; it indicates the risk that the pair has to collide. Using KF, motion of a pair of vehicles was simulated to determine if the follower’s driving behavior can lead to a safe maneuver without changing lanes. Data from the NGSIM project was the basis of calibrating the characteristic matrices in the state-space equations. The matrices in state-space representation were obtained from an iteratively comparison of variables associated with driver-vehicle over a few seconds of information. Using calibrated model, the interactions of a pair of vehicles was predicted and studied. The expected output was an optimized control of the variables that based on vehicle characteristics and driving behavior reflect a safe interaction associated when following a vehicle.
3.2.6 Task 6: Discussion of Results

This task discussed the results obtained from the models developed in Tasks 4 and 5. In the MXL models, the relationship between each type of collision with the striking and struck body type and car-following attributes are discussed. In the car-following state-space estimation, prediction of the state a following vehicle in the next few time frames demonstrated safety maneuvers based on vehicle dynamics. In both approaches, the interactions between vehicles with the same and different body type were explicitly modeled. It was of special interest to identify safety factors found in car-following models and quantify their contributions to driving behavior in risky situations.

3.2.7 Task 7: Conclusions and Recommendations

This task presented the concluding remarks of the study. The research achievements, as well as the limitations encountered in this study were highlighted. Advantages and disadvantages of the approaches used were listed to guide future studies. Implications on policy measures related to driving rules and safety were recommended. Potential contributions to transportation safety and traffic operations were established. Gaps are identified so that they can be addressed in future research.

3.3 Chapter Summary

This section describes the research methodology, including the goal of each task. The research included a literature review of car-following theory and models. An analysis framework was formulated using MXL models, as well as a linear system design for car-following model was proposed. The potential data sources are identified, as well as, the suitable inputs for developing the MXL model and a state-space representation of car-following to perform an analysis of driving behavior.
Chapter 4: Development of Crash Prediction Models

In this chapter, discrete choice models were developed to predict the likelihood of the resulting manners of collision if there is a crash that involves two vehicles traveling in the same direction in a multilane interstate highway under no adverse weather conditions. The first goal of this chapter was to develop four MXL models for a specific role (striking or struck vehicle) played by a car or a truck involved in a two-vehicle collision. In each model, the response variable is the probability of a crash having a discrete manner of collision (rear-end, angle or sideswipe). The independent variables or attributes considered are mainly driving behavior prior to crash, especially vehicle-following and lane changing attributes. The second goal was to use the MXL models to examine the roles of the vehicles in a crash (striking and struck) and observe the actions performed by each driver prior to the collision. The third goal was to use the developed MXL models to deduce the differences in driving behavior of trucks and passenger cars when paired in a crash with same and different types of vehicle. Identifying similarities and differences in driver behavior of the same or different types of vehicle can contribute to a better understanding of events that caused the crash. The fourth goal of this chapter was to use the models to investigate the contribution of the different vehicle types on the manner and likelihood of collision. Finally, the fifth research goal was to compare the developed MXL models when predicting different collisions.

4.1 Empirical Setting

In this research, the NASS-GES database was selected because it captures situational, environmental, vehicle and human factors present during the collision. It also provides records of collisions involving all vehicles that traveled on U.S. highway systems. As mentioned before, this study considered two types of vehicle: passenger cars and trucks. Trucks include tractor-trailers, single-unit trucks, or cargo vans having Gross Vehicle Weight Rating (GVWR) greater than 10,000 pounds as defined by the Insurance Institute for Highway Safety (IIHS 2009).

All the crash records from 2005 to 2008 were combined to form a single dataset using Statistical Analysis System (SAS 2004). The combined dataset was preprocessed to extract cases that meet the desired characteristics: collisions involving only two vehicles in the same roadway in the same direction
on interstate highways under no adverse weather conditions. The constraint in traffic direction resulted
in three common manners of collision: rear-end, angle and sideswipe. To avoid double counting of the
same crash, information on the striking and struck vehicles involved in a collision were paired by the
same Case Number. Further data processing consisted of creating four separate datasets describing: (1)
Car striking Car (C-C); (2) Car striking Truck (C-T); (3) Truck striking Car (T-C); and (4) Truck
striking Truck (T-T), respectively. By having different datasets, it is possible to analyze differences on
the three manners of collisions. In this way, it is possible to examine pre-crash scenarios describing
vehicle-following and lane changing behavior, as well as, attributes of driver, road, etc. that are
exclusive for each vehicle pair. Additional information concerning the total crashes in each dataset and
total crashes corresponding to each manner is provided in Table 4.1.

<table>
<thead>
<tr>
<th>Crash Combination</th>
<th>Rear-End</th>
<th>Angle</th>
<th>Sideswipe</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-C</td>
<td>2804</td>
<td>225</td>
<td>734</td>
<td>3763</td>
</tr>
<tr>
<td>C-T</td>
<td>1049</td>
<td>337</td>
<td>1033</td>
<td>2419</td>
</tr>
<tr>
<td>T-C</td>
<td>780</td>
<td>236</td>
<td>782</td>
<td>1798</td>
</tr>
<tr>
<td>T-T</td>
<td>370</td>
<td>13</td>
<td>60</td>
<td>443</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>5003</strong></td>
<td><strong>811</strong></td>
<td><strong>2609</strong></td>
<td><strong>8423</strong></td>
</tr>
</tbody>
</table>

4.1.1 Defining Manner of Collision

In this research, the probability of each manner of collision is modeled as a function of the
driver-vehicle-road interaction that took place before a crash. The dependent variable is the probability
of the manner of collision that resulted from a failed driving maneuver between the two vehicles.

The manner of collision consists of three possible crash outcomes: rear-end, angle and sideswipe.
A detailed graphical representation and comprehensive description of manner of collision can be found
in the Fatality Analysis Reporting System (FARS) Manual (NHTSA 2010b). The NASS-GES and FARS
used similar sets of data but different coding and software. Figure 4.1 portrays (a) rear-end, (b) angle
and (c) sideswipe collisions used in this study which are depicted in the FARS Manual as Front-to-Rear (01), Front-to-Side, Same Direction (03) and Sideswipe-Same Direction (07) respectively.

![Image of possible manners of collision](image)

(a) Rear-end   (b) Angle   (c) Sideswipe

Figure 4.1: Possible manners of collision defined in FARS manual (NHTSA 2010b).

Following the exact definition by NHTSA (2010b), rear-end and same direction traffic collisions are literally outlined as:

a. **Front-to-Rear (includes Rear-End)**. “A rear-end collision is one in which the front end of one vehicle collides with the back of another vehicle, while the two vehicles are traveling in the same direction. Use Front-to-Rear (includes Rear-Ends) for all “rear-end” crashes and all crashes in which the front of one vehicle comes in contact with the rear of another in the First Harmful Event, regardless of the original direction of travel.”

b. **Front-to-Side, Same Direction** “is used for angle crashes where the front of one vehicle makes contact with any point along the side of another in the First Harmful Event and the orientation of the vehicles at impact is in the same direction. This does not include right angles or broadside crashes (See Front-to-Side, Right Angle).”

c. **Sideswipe, Same Direction** “occurs if the following conditions apply to both vehicles:

1. The initial engagement does not overlap the corner of either vehicle by more than four inches, so that there is no significant involvement of the front or rear surface areas.”
2. There is no pocketing of the impact in the suspension areas. The impact then swipes along the surface of the vehicle parallel to the direction of travel.

3. There is low retardation of the force along the surface of the vehicle.”

4.1.2 Independent Variables

The independent variables, as found in the NASS-GES database, include mainly vehicle activities prior to impact. In the NASS-GES data set, pre-crash variables describe what the vehicle was doing just prior to crash, what made the vehicle's situation critical, corrective action made, if any, to this critical situation, and location of the vehicle just prior to crash.

During the analysis, variables such as: road geometry characteristics (e.g. relation to interchange, un/divided roadway, and etc), traffic conditions at peak and off-peak hours (e.g. congestion) and driver characteristics (e.g. young, old, female, male) were also considered. However, these factors did not make statistical significant contributions to the model and therefore, were excluded.

Additional variables describe any distraction reported, road surface condition at the time of the crash and the geographical location where the crash occurred. Driver distraction is considered as a state of the driver, and thus, describes any inattention by the driver. Condition of surface can be dry, wet, snow or slush, ice, sand, dirt or oil. Geographical location is classified by the regions in the U.S.; each consists of several states.

A detailed description of variables used in this study can be found in Table 4.2. Table 4.3 lists the binary variables that are significant and common in the four estimated MXL models for C-C, C-T, T-C, T-T, respectively. The mean of every variable and its standard deviation in parenthesis are presented in Table 4.3. The descriptive statistics shows how these characteristics vary among the different datasets.
Table 4.2: Variable Dictionary.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAYLGT</td>
<td>General light conditions at the time of the crash (1 if daylight, 0 otherwise)</td>
</tr>
<tr>
<td>DRY</td>
<td>Condition of road surface at the time of the crash (1 if dry, 0 otherwise)</td>
</tr>
<tr>
<td>FAGVSI</td>
<td>Striker driving at higher speed than Struck vehicle (1 if yes, 0 otherwise)</td>
</tr>
<tr>
<td>FBRK</td>
<td>Action taken by striker in response to the impending danger (1 if braking, 0 otherwise)</td>
</tr>
<tr>
<td>FCHNGL</td>
<td>Vehicle pre-crash situation for striker (1 if changing lanes to the right or left, 0 otherwise)</td>
</tr>
<tr>
<td>FDISTR</td>
<td>Distraction reported by striker (1 if yes, 0 otherwise)</td>
</tr>
<tr>
<td>FENCRC</td>
<td>Critical event initiated by striker (1 if striker encroaching into struck vehicle's lane, 0 otherwise)</td>
</tr>
<tr>
<td>FGOING</td>
<td>Activity of striker prior to realization of critical event or prior to impact (1 if going straight, 0 otherwise)</td>
</tr>
<tr>
<td>FMANVR</td>
<td>Striker maneuvered to avoid something in the road (1 if maneuvered, 0 otherwise)</td>
</tr>
<tr>
<td>FNOTRL</td>
<td>Striking vehicle pulling no trailer unit (1 if no trailing unit, 0 otherwise)</td>
</tr>
<tr>
<td>FOVRDG</td>
<td>Critical event initiated by striker (1 if striker traveling over the line lane or off edge in the road, 0 otherwise)</td>
</tr>
<tr>
<td>FRDLEL</td>
<td>Path for striker prior to its first involvement in the crash (1 if vehicle stayed on roadway but left travel lane, 0 otherwise)</td>
</tr>
<tr>
<td>FSPEDR</td>
<td>Speed as a contributing factor to the cause of the crash according to striking vehicle (1 if yes, 0 otherwise)</td>
</tr>
<tr>
<td>FSTAYL</td>
<td>Path for striker prior to its first involvement in the crash (1 if vehicle stayed in travel lane, 0 otherwise)</td>
</tr>
<tr>
<td>FSTGHT</td>
<td>Pre-crash situation for striking vehicle (1 if travelling straight ahead on left or right lane, 0 otherwise)</td>
</tr>
<tr>
<td>INTERC</td>
<td>Relation to junction (1 if interchange area, 0 otherwise)</td>
</tr>
<tr>
<td>LDECST</td>
<td>Activity for struck vehicle prior to realization of critical event or prior to impact (1 if decelerating or stopped in traffic lane, 0 otherwise)</td>
</tr>
<tr>
<td>LGOING</td>
<td>Activity for struck vehicle prior to realization of critical event or prior to impact (1 if going straight, 0 otherwise)</td>
</tr>
<tr>
<td>LNOAVM</td>
<td>Action taken by the struck vehicle in response to the impending danger (1 if no avoidance maneuvered, 0 otherwise)</td>
</tr>
<tr>
<td>LNOTRL</td>
<td>Struck vehicle pulling no trailer units (1 if no trailing unit, 0 otherwise)</td>
</tr>
<tr>
<td>LOTHSP</td>
<td>Critical event initiated by struck vehicle (1 if traveling in same direction with lower speed, 0 otherwise)</td>
</tr>
<tr>
<td>LSTGHT</td>
<td>Pre-crash situation for struck vehicle (1 if straight ahead on left or right lane, 0 otherwise)</td>
</tr>
<tr>
<td>ONEAN</td>
<td>Alternative specific angle constant</td>
</tr>
<tr>
<td>ONERE</td>
<td>Alternative specific rear end constant</td>
</tr>
<tr>
<td>SOUTH</td>
<td>Region (1 if south, 0 otherwise)</td>
</tr>
<tr>
<td>SUMMER</td>
<td>Season (1 if summer, 0 otherwise)</td>
</tr>
<tr>
<td>WEST</td>
<td>Region (1 if west, 0 otherwise)</td>
</tr>
<tr>
<td>WINTER</td>
<td>Season (1 if winter, 0 otherwise)</td>
</tr>
</tbody>
</table>
4.2 Model Development

Discrete choice analysis was used to identify the effect vehicle, driver and road factors have on each possible manner of collision. It is possible to identify from the MXL model a particular combination of observed variables that describes the probability of an outcome, in this case, the manner of collision. A MXL model relaxes the assumptions related to Independence from Irrelevant Alternatives (IIA), Independent and Identically Distributed (IID) errors present in a MNL model and allows observing and unobserved heterogeneity (Greene 2007).

A better estimated result can be obtained if the coefficients of some of the attributes are treated as random because some unobserved attributes could be accounting for rear-end, angle and sideswipe collisions conditionally. Unobserved attributes are also assumed to exist due to the subjectivity of the data obtained from police crash reports. These variables can be hidden in the variability of perception.

### Table 4.3: Descriptive Statistics for Independent Variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Mean</th>
<th>Std Dev.</th>
<th>Mean</th>
<th>Std Dev.</th>
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<th>Std Dev.</th>
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<th>Std Dev.</th>
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<td>0.404</td>
<td>0.422</td>
<td>0.404</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SOUTH</td>
<td>0.505</td>
<td>0.500</td>
<td>0.230</td>
<td>0.444</td>
<td>0.230</td>
<td>0.444</td>
<td>0.230</td>
<td>0.444</td>
<td>0.230</td>
<td>0.444</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SUMMER</td>
<td>0.425</td>
<td>0.430</td>
<td>0.425</td>
<td>0.430</td>
<td>0.425</td>
<td>0.430</td>
<td>0.425</td>
<td>0.430</td>
<td>0.425</td>
<td>0.430</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WEST</td>
<td>0.472</td>
<td>0.499</td>
<td>0.472</td>
<td>0.499</td>
<td>0.472</td>
<td>0.499</td>
<td>0.472</td>
<td>0.499</td>
<td>0.472</td>
<td>0.499</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Descriptive Statistics for Independent Variables.
when measuring driving actions and their consequences by the police officer or the parties involved in an accident investigation.

A MXL model should be an alternative approach to improve the performance of the model. For instance, some attributes with random coefficients could be: movement prior to crash (traveling straight, stopped, changing lanes, etc.), as well as region and road surface conditions. We define \( I \) as the set of outcomes, \( I=\{\text{rear-end collision, angle collision, sideswipe collision}\} \). The MXL probability of an observation or collision \( n \) that results in manner of collision \( i \ (i \in I) \) may be expressed as (Washington et al. 2011):

\[
P_in = \frac{\exp(V_in)}{\sum_{\forall j \in I} \exp(V_j)} f(\beta | \varphi) d\beta
\]

Eq. (4.1)

where \( P_{in} \) = probability of a two vehicle crash \( n \) resulting in manner of collision \( i \) given that a two-vehicle collision has occurred;

\( V_{in} \) = deterministic component of the utility value of manner of collision \( i \) associated with collision \( n \);

\( f(\beta | \varphi) \) = density function that determines the weighted average of the probability;

\( \varphi \) = parameter vector describing mean and standard deviation.

The deterministic component of the utility value of manner of collision \( i \) associated with collision \( n \) is a linear function:

\[
V_{in} = \beta_i X_{in}
\]

Eq. (4.2)

where \( \beta_i \) = row vector of parameters associated with the attributes of manner of collision \( i \);

\( X_{in} \) = column vector of observed attribute values of manner of collision \( i \) for collision \( n \).

Marginal effects were calculated in order to see how the probability of a manner of collision is influenced when changing a binary variable associated with attribute \( k \) from 0 to 1. The marginal effect formula used is described as follows Washington et al. (2011) (the subscript \( n \) is dropped for simplicity):
\[
\frac{\partial P_i}{\partial x_{ki}} = [1 - P_i] P_i \beta_k
\]
Eq. (4.3)

In addition, to test the practical use of random coefficients (i.e. MXL model) versus fixed coefficients (i.e. MNL model), a likelihood ratio test was performed (Jones and Hensher 2007):

\[
\chi^2 = -2 \left[ LL_{MNL}(\beta^{MNL}) - LL_{MXL}(\beta^{MXL}) \right]
\]
Eq. (4.4)

where: \( LL_{MNL}(\beta^{MNL}) \) = log-likelihood at convergence of the MNL model; 
\( LL_{MXL}(\beta^{MXL}) \) = log-likelihood at convergence of the MXL model.

4.3 Results and Discussion

The econometric software NLOGIT 4.0 (Greene 2007) was used to develop the MXL models. With this software, it was possible to separate the different categories of responses (manners of collision) and create new explanatory variables to better capture the effect the variable of interest has in the outcome.

The results summarized in Tables 4.4 and 4.5 give the estimated results for the MXL models corresponding to C-C, C-T, T-C and T-T collisions, respectively. In this section the positive and negative influence of the independent variables used in each model as dictated by their positive or negative coefficient is discussed.

The alternative specific constant accounts for the systematic bias of all unobserved attributes that contribute to rear end, angle and sideswipe utility values. A negative alternative specific constant means that attributes not accounted for or found to have insignificant influence collectively reduces the probability of this outcome and therefore are less likely to contribute to the relevant manner of collision. For this study, the base case scenario is sideswipe collision since no alternative specific constant was specified in the utility function. The opposite is true for a positive alternative specific constant. The alternative specific constants across the different models are further discussed at the end of this chapter.
The developed MXL model considers all unobserved attributes assumed to exist due to the subjectivity of the data obtained from police crash reports. These variables can be hidden in the variability of perception when measuring driving actions and their consequences by the police officer and/or the parties involved in an accident investigation. Random parameters were considered to vary across observations according to a normal distribution since it resulted in a good statistical fit. Other distributions, such as, normal, lognormal, triangular and uniform were tested but were not found to be statistically significant. The random parameters are obtained from repeated simulated draws. In this study, 1000 random draws were employed using Standard Halton Sequence (SHS) intelligent draws as recommended by Bhat (2001).

The statistical significance of random parameters is also shown in Table 4.4 for each model. Further discussion as of unobserved factors taking part in the modeling can be found at the end of the discussion of the model when estimated random parameters were considered.
Table 4.4: Mixed Logit Model Results; Coefficients, t-Statistics and Marginal Effects.

<table>
<thead>
<tr>
<th>Variable</th>
<th>C-C</th>
<th>C-T</th>
<th>T-C</th>
<th>T-T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>t-Stats</td>
<td>M.E</td>
<td>Coef.</td>
</tr>
<tr>
<td>Rear-End</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ONEERE</td>
<td>-2.128</td>
<td>-7.583</td>
<td></td>
<td>0.879</td>
</tr>
<tr>
<td>DAYLGT</td>
<td>-0.602</td>
<td>-4.011</td>
<td></td>
<td>1.060</td>
</tr>
<tr>
<td>DRY</td>
<td>0.558</td>
<td>3.144</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>FAVGSI</td>
<td>1.523</td>
<td>4.629</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>FBRK</td>
<td>0.546</td>
<td>2.966</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>FOING</td>
<td>-0.675</td>
<td>-3.737</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>FMANVR</td>
<td>1.097 (1.837)</td>
<td>4.279 (6.393)</td>
<td>0.003</td>
<td>0.674</td>
</tr>
<tr>
<td>FNOTRL</td>
<td>1.566</td>
<td>6.958</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>FSPEDR</td>
<td>1.907 (1.649)</td>
<td>5.504 (5.112)</td>
<td>0.002</td>
<td>2.717</td>
</tr>
<tr>
<td>FSSTAYL</td>
<td>0.600</td>
<td>2.864</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>FREXHA</td>
<td>1.648</td>
<td>2.452</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>LOTHSP</td>
<td>4.232</td>
<td>16.227</td>
<td>0.084</td>
<td></td>
</tr>
<tr>
<td>FSTAYL</td>
<td>0.469</td>
<td>2.661</td>
<td>0.011</td>
<td></td>
</tr>
</tbody>
</table>

| Angle          |      |      |     |      |      |     |      |      |     |      |
| ONEAN          | 0.522 | 1.974 | -0.125 | 2.122 | 9.312 | 0.030 | 0.779 | 3.895 | -4.374 | -8.414 |
| DRY            | -0.975 | -3.787 | -0.142 | 1.537 | 6.133 | 0.030 | -0.705 | -3.800 | -0.013 |      |      |     |
| FCHNLG         |      |      |     |      |      |     |      |      |     |      |      |     |
| FDISTR         |      |      |     |      |      |     |      |      |     |      |      |     |
| FOING          | 2.029 | 6.853 | 0.084 |     |      |      |     |      |     |     |
| FMANVR         |      |      |     |      |      |     |      |      |     |      |      |     |
| FSSTAYL        |      |      |     |      |      |     |      |      |     |      |      |     |
| FNOTRL         |      |      |     |      |      |     |      |      |     |      |      |     |
| FSPEDR         |      |      |     |      |      |     |      |      |     |      |      |     |
| LOTHSP         |      |      |     |      |      |     |      |      |     |      |      |     |
| FSTAYL         |      |      |     |      |      |     |      |      |     |      |      |     |
| LOTHSP         |      |      |     |      |      |     |      |      |     |      |      |     |
| WINTER         |      |      |     |      |      |     |      |      |     |      |      |     |

| Sidestrip      |      |      |     |      |      |     |      |      |     |      |      |     |
| FDISTR         | -0.469 | -2.361 | -0.011 | 3.878 | 18.986 | 0.219 | -4.282 (2.745) | -2.612 (2.034) | -0.034 |      |      |     |
| FCHNLG         |      |      |     |      |      |     |      |      |     |      |      |     |
| FMANVR         | 2.276 | 7.949 | 0.051 | 1.645 (2.521) | 5.647 (6.371) | 0.012 |      |      |     |      |      |     |
| FFOVRDG        | 2.116 | 12.528 | 0.199 |      |      |      |     |      |     |     |      |      |     |
| FRDLEL         | -1.37 | -4.838 | -0.030 | -0.634 | -2.481 | -0.010 | -0.299 | -1.331 | -0.003 |      |      |     |
| INTERC         |      |      |     |      |      |     |      |      |     |      |      |     |
| FLEQST         |      |      |     |      |      |     |      |      |     |      |      |     |
| FSPEDR         |      |      |     |      |      |     |      |      |     |      |      |     |
| FSPEDR         |      |      |     |      |      |     |      |      |     |      |      |     |
| LOTHSP         |      |      |     |      |      |     |      |      |     |      |      |     |
| LOTHSP         |      |      |     |      |      |     |      |      |     |      |      |     |
| SUMMER         | -0.387 | -2.737 | -0.029 | 5.498 | 9.736 | 0.300 | 0.633 | 2.957 | 0.020 |      |      |     |

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Table 4.5: Mixed Logit Model Results; Model Statistics.

<table>
<thead>
<tr>
<th>Model Statistics</th>
<th>C-C</th>
<th>C-T</th>
<th>T-C</th>
<th>T-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Parameters</td>
<td>16</td>
<td>18</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Log-likelihood at convergence</td>
<td>-1118.004</td>
<td>-1246.184</td>
<td>-1048.044</td>
<td>-151.023</td>
</tr>
<tr>
<td>Log-likelihood at zero</td>
<td>-4134.078</td>
<td>-2657.543</td>
<td>-1975.305</td>
<td>-486.685</td>
</tr>
<tr>
<td>Chi squared</td>
<td>6032.149</td>
<td>2822.718</td>
<td>1854.523</td>
<td>671.323</td>
</tr>
<tr>
<td>Number of observations</td>
<td>3763</td>
<td>2419</td>
<td>1798</td>
<td>443</td>
</tr>
<tr>
<td>Fixed Parameter Log-likelihood at convergence</td>
<td>-1134.079</td>
<td>-1261.569</td>
<td>-1051.057</td>
<td>-151.9251</td>
</tr>
<tr>
<td>Likelihood Ratio (Chi squared)</td>
<td>32.15</td>
<td>30.77</td>
<td>6.026</td>
<td>1.804</td>
</tr>
<tr>
<td>Significance Level</td>
<td>99.99%</td>
<td>99.98%</td>
<td>95.09%</td>
<td>99.82%</td>
</tr>
</tbody>
</table>

A log-likelihood ratio test between MNL and MXL models was performed for each model. As can be seen in Table 4.5, the significance level obtained indicates that MXL is more appropriate. According to the results, the null hypothesis that MXL model is not statistically superior to the MNL model, is rejected. This means that MXL model provides a better approach to model the manner of collision.

4.3.1 C-C (Model 1): Rear-End Collisions

For this crash type, the results indicate that the striking vehicle is traveling straight and the vehicle in front is traveling at a lower speed. Although the striker’s response is to brake and to stay in the same travel lane, the crash was not avoided. All these pre-crash variables influence the occurrence of a rear-end collision and therefore they have a positive sign. According to the marginal effects, if the struck car maintains a different speed, in this case, a lower speed than the striker, the probability of rear-end collision increases by 8.4%. If the striker prior to crash stays in the same lane, the probability of rear-ending the car in front increases by almost 4%. A breaking performed by the striker to avoid collision also increases the chances of being involved in a rear-end collision but at a smaller magnitude of 0.7%.

The random parameters estimated for the C-C model are related to the actions of the striking vehicle and struck vehicle. Interestingly, these variables describe the actions that both parties contribute to a rear-end collision and may be blamed for. The first random parameter describes possible speeding
for the striker, and the second one describes the struck vehicle decelerating or stopping in the travel lane. It could be that unobserved variability exists in these factors because speeding and stopping in lane may be interpreted as accepting responsibility for the crash. Therefore, these reported actions are not fixed across observations. The first random parameter has a mean of 1.097 and a standard deviation of (1.837). This indicates that for about 72% of car crashes, speeding increases the likelihood of a rear-end collision. The second random parameter has a mean of 1.907 and a standard deviation of (1.649). This suggests that in 88% of car crashes, a slower lead vehicle increases the likelihood of a rear-end collision.

4.3.2 C-C (Model 1): Angle Collisions

The only contributing factor in this model for an angle collision between two cars is an avoiding maneuver before the crash. This maneuver was a reaction to avoid an object on the road. The probability of an angle collision between cars increases by 8.4% if the striking car maneuvered to avoid something on the road. However, angle collision is less likely among cars that crashed on a dry surface. This road condition decreases the probability of this outcome by 12.5%. This conversely implies that slippery road conditions may also increase the risk of an angled impact if a car steered to avoid impacting an object on its travel lane.

4.3.3 C-C (Model 1): Sideswipe Collisions

According to the results in Table 4.4, sideswipe collisions among cars are influenced by the maneuvers performed before crash, as well as any distraction present. If the striker maneuvered to avoid impacting an object and drove over the lane marker, it is more likely to be a sideswipe collision. If a car is driving over the lanes or if maneuvered to avoid something in the road, the probability to hit the vehicle on the side increases by almost 20% and 5.1%, accordingly. If the crash occurred in the south region (Alabama, Arkansas, Delaware, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, West Virginia, and Washington D.C.), the crash is less likely to be sideswipe. Sideswipe collisions decreases by 1.1% if the striker reported a distraction. If a crash between cars occurred in the south region, the probability of having a sideswipe is decreased by 3%.
4.3.4 C-T (Model 2): Rear-End Collisions

When analyzing a crash where the front of a car collided with the rear of a truck, the following variables were observed as significant in the MXL model. The expected traveling straight, speeding factors and sudden decelerations or stoppings are present as observed with C-C collisions. A negative coefficient is found for daylight, meaning that C-T crashes during daylight are less likely to be rear-end collisions. This could lead to the assumption that the occurrence of a car colliding with a truck is due to a wrong perception of spacing between vehicles when there is no daylight. Daylight decreases the probability of a car rear-end a truck by 3.6%. Another negative coefficient decreasing the likelihood of rear-end collision is if the car maneuvered to avoid something on the road. The probability of rear-end collisions between a car and a truck decreases by 1.3% if the car maneuvered to avoid something on the road.

Additional variables increasing the outcome of a car-truck crash to be rear-end are: dry road conditions at the time of crash, and geographical location in the west region in the U.S. (Montana, Idaho, Washington, Oregon, California, Nevada, New Mexico, Arizona, Utah, Colorado, Wyoming, Alaska and Hawaii). Some possible reasons for this variables being significant can be attributed to driving behaviors (e.g. aggressive following) in this region under ordinary weather conditions. As also described in (Knipling 2013), occurrence of car-truck crash is higher (80.8%) when pavement is dry and under no adverse weather conditions. According to the marginal effects, the most influential variables are the road is dry, when the truck decelerates or stopped in the lane and when the car keeps its direction. These variables increase the probability of rear-end collisions by 10.2%, 4.4% and 9% respectively.

4.3.5 C-T (Model 2): Angle Collisions

In the case of a car traveling alongside a heavy vehicle, the model indicate that an intended maneuver was attempted by the car but failed. This can be inferred from the pre-crash situation influencing the likelihood of an angle collision due to lane changing. If a car changes lane, the probability of hitting the truck traveling to the side increases by 3%. It seems that cars misjudge the gap from the adjacent lane and the speed and position of the truck traveling on the side.
A negative effect is obtained from the variable describing a distraction by the striker. The probability of an angle crashes decreases by 1.3% if the car driver did not experience a distraction. This reinforces the idea that performing a risky maneuver, rather than distraction, influences the likelihood of this type of crash.

An additional variable with a negative influence in this crash outcome is a truck with no trailing unit. The collision between a car and a truck at an angle deceases by 1.5% if the truck has no trailing unit. It could be that vehicle dimensions play a role in angle collisions. The driving behavior to drive a truck with no unit may differ than when having a cargo and this may be misleading for a car attempting to change lanes.

4.3.6 C-T (Model 2): Sideswipe Collisions

Having a different combination of vehicles but keeping the same striker vehicle type as car gives an interesting perspective on sideswipe collisions. A sideswipe crash in a car and a truck collision is more likely to occur if a truck is traveling straight ahead on the left or right lane of the car, or if a crash occurred in winter. According to the marginal effects, the probability of a car crashing into a truck in the manner of sideswipe increases by 30% if the truck is going straight. If a crash occurs during winter, the probability of a sideswipe collision increases by 2%. It is less likely to occur if the striker left the lane prior to crash. Sideswipe collisions slightly decreases by 3% if car lefts the travel lane.

The likelihood of sideswipe collision is increased if the car encroach the next lane where the truck was traveling. However, this variable behaves differently across observations since some unobserved factors not captured in the dataset may also be influencing this variable. For instance, for a crash in winter, the car could have involuntarily skidded to the other lane. The parameter has a mean value of 1.645 and standard deviation of 2.621. This indicates that 73% of car-truck collisions are more likely to be sideswipe if the striking vehicle encroach the next lane.

4.3.7 T-C (Model 3): Rear-End Collisions

When a crash involves a truck striking a car, it is more likely to be rear-end if truck was speeding and the car in front took no action to avoid the collision. Again, speed is considered a contributing factor to the cause of the rear-end collision. For this combination of vehicles, it is interesting to see that the
struck car did not respond to the impeding danger. The marginal effect indicates that rear-end collisions between a truck and a car, where the truck is blamed for, increases by 3.4%. And 1.6% when there is a speed related collision and the car does not make an evasive maneuver when realizing the imminent impact. A crash is less likely to be rear-end if the car was going straight before the truck ran into a car. The probability of a car going straight decreases by 1.2% for this type of collision. Although this might seem unexpected, a possible explanation could be that the car suddenly merged into the truck’s lane and going straight was not the path prior to the impact.

4.3.8 T-C (Model 3): Angle Collisions

If the truck is traveling straight ahead in the adjacent lane of the vehicle before crashing, it will influence the likelihood of an angle collision. The chances for a truck to collide at an angle with a car increases by 2.7% if the truck travels straight ahead on the left or right lane. However, if a car decelerates or stops in a traffic lane, an angle crash outcome is less likely to occur. Car decelerating or stopping decreases the outcome probability by 0.5%.

Both random estimates have a negative influence in the outcome of angle collisions when a crash has occurred, meaning that it is less likely to be an angle crash. The first random estimate describes the truck to be going straight prior to colliding with the car. This random parameter has a mean of -1.451 and a standard deviation of 1.849. This suggests that in 78% of crashes where a truck hit a car, the outcome is less likely to be an angle collision if the truck was going straight. Other unobserved behavior such as merging before crash may cause random effects in the model. The second random parameter relates to the season in which the crash occurred. In this study, summer season is defined as the months of June, July and August. Its variability across observations is assumed to exist because other months may be reported as summer. The statistical values of -1.696 for mean and 2.2226 for standard deviation indicates that 78% of these crashes involving a truck and a car are less likely to be angle collisions.

4.3.9 T-C (Model 3): Sideswipe Collisions

One of the actions prior to impact that can result in a sideswipe collision between a truck and a car is changing lane. The pre-crash variables with negative coefficients that affect having a crash that is
not a sideswipe are truck leaving travel lane and car decelerating or stopping in travel lane. Further, a crash on an interchange contributes to collision types other than sideswipe.

The variable with the highest marginal effect among the attributes in this utility function is truck changing lanes. This event increases the probability of this outcome by 21.9%. Other variables have less influence on the outcome. Truck stayed on roadway but left travel lane decreases the probability of a sideswipe collision by 1%. If the car decelerated or stopped, this diminishes the outcome probability by 0.6%. Interestingly, if the crash occurs at an interchange, sideswipe collisions decreases by 0.3%.

4.3.10 T-T (Model 4): Rear-End Collisions

According to the model, the pre-crash actions performed by the striker that affects the outcome of a rear-end collision between two trucks is braking and staying in the travel lane. If truck stays in the same lane, the probability of crashing into the truck in front increases by 10.6%. If the truck breaks as a response to the impending danger, the probability of striking the leading truck increases by 1.1%. Also, if the struck truck was not pulling any trailing unit, it also affects the likelihood of this particular manner of collision. When the truck in front does not carry a trailing unit, the probability of a rear-end collision among trucks increases by 0.9%. Interestingly, if the striker also has no cargo unit, it is less likely for the outcome to be a rear-end collision. If the truck striking another truck that has no trailing unit, the chances of collision decreases by almost 1%. This could mean a truck misjudges the trajectory of the truck in front because of its smaller dimension. It could motivate the trailing truck to behave more aggressively and tailgate the truck in front. Another reason for this type of impact could be attributed to the differences in braking capabilities of larger and smaller trucks.

4.3.11 T-T (Model 4): Angle Collisions

The likelihood of an angle collision between trucks is influenced mainly by the maneuver of the striker to avoid an object on the road. This is supported by the negative alternative specific constant which includes additional variables not accounted for. If the striker performs a maneuver to avoid something on the road, the chance of hitting the adjacent truck at an angle increase by 10.5%.
4.3.12 T-T (Model 4): Sideswipe Collisions

This manner of collision was challenging, as the only significant variables in the model have a negative influence that decreases the likelihood of the collision. However, this variable may be useful since it can describe how a sideswipe collision does not occur. The only fixed parameter that describes the struck vehicle to decelerate or stop in the lane. If the struck truck decelerates or stops, the chances of this outcome decrease by almost 4%. A possible explanation of why any deceleration of two vehicles traveling side by side will not result in sideswipe can be that the decelerated vehicle would be left behind and this action may finally result in a different manner of collision.

The other parameter which is treated as random describes a negative effect in the outcome because of its negative sign. This variable describes the striker as going straight before collision. For an impact along the side of both vehicles, it is expected that an activity that indicates a movement, such as steering, is more suitable. With regards to unobserved factors affecting this variable, strikers may use this definition to be excused for not paying attention to the road. Assuming this, this action may be reported differently in the observations and therefore can be considered as random. The coefficient of this random parameter has a mean of \(-4.282\) and a standard deviation of 2.745. The likelihood of getting involved in a sideswipe collision is 94% less likely when the striker was going straight.

4.4 Comparing Different Driving Behavior

This section compares the different possible manners of collision that are more or less likely to occur according to the developed MXL models. The results for each vehicle crash combination point out the different manner of collisions when a vehicle strikes the same and different type of vehicle. The manner of collision will reflect unsafe driving behavior and vehicle dynamics when interacting with same or different vehicles. This analysis is based on the sign of the alternative specific constants for rear-end and angle collisions for each model as shown in Table 4.4.

For instance, if a car hit a truck, according to the positive coefficients of 0.879 and 2.122 for rear-end and angle respectively, and all attributes being equal, it is less likely to be a sideswipe collision. In the same way, in a crash where a truck ran into a car, the positive values of 0.477 and 0.779 for rear-end and angle constants, indicate that it is more likely to be rear-end or angle collision rather than
sideswipe. However, in a collision that involves two trucks the negative values of -2.546 and -4.374 in rear-end and angle collisions tell that it is more likely to be as a sideswipe. For crashes involving two cars, the coefficients of -2.128 and 0.522 for rear end and angle accordingly, demonstrates that the manner of collision is more likely to be angle.

Knowing the previous likelihood in the manner of collision, it can be interpreted that cars can better interact with same vehicles in front and large vehicles on the side traveling parallel. However, cars are more likely to fail to follow a truck in a safe manner and change lanes when a truck or a car is in the adjacent lane. This is supported by the findings in Knipling (2013) that points to a higher frequency of aggressive driving in car drivers than truck drivers and errors attributed to car drivers. Accordingly, a failed interaction among trucks describes a possible failure for trucks to interact with cars traveling in front or positioned in the adjacent lane ahead or behind the truck’s position. For longitudinal interaction errors, it could be in part because of “trucks’ relative inability to evade an errant car” as mentioned in Knipling (2013). For trucks failing to interact with cars in the adjacent lane, a possible hypothesis can be referred to the lack of visibility when trucks move from left to right as suggested in Knipling (2013). Also, it indicates unsafe driving behavior when the truck matches the position of another truck traveling in the adjacent lane. This can be partly due to trucks’ lack of stability and the aerodynamics involved when two trucks are traveling side by side.

4.5 Policy Implications

One of the aims of developing these MXL models is to aid in the decisions taken by public officials to reduce the likelihood of rear-end, angle and sideswipe collisions. The following measures are suggested as an attempt to contribute to safety improvements on highways. For a more holistic perspective on the effectiveness of operational strategies to reduce car-truck interactions referred to Peeta et al. (2004).

This research suggests that roadway improvements should considered sight distance, acceleration/deceleration capabilities and vehicular dimensions of cars and trucks to reduce rear-end, angle, and sideswipe collisions.
Restricting traffic to left or right lanes, for example prohibit trucks from using the left lane, can allow for safe car-following behavior based on vehicle capabilities. This could be complemented by designated different speed limits to trucks and cars. In addition, a separation policy when following a different type of vehicle can also help eliminating rear-end collisions.

Standards on vehicle design may be needed to be modified in order to alert drivers of vehicles changing lanes. New configurations of turning signals (for example, lane change indicators at side mirrors) and/or electronic on-board devices (such as blind spot vehicle detection system) may warn drivers on time and prevent crashing into the car in the next lane. This may result in the reduction of angle and sideswipe collisions.

The improvement of state-highway education programs and law enforcement are crucial in succeeding reducing possible rear-end, angle and sideswipe collisions on highways. Providing education to drivers on the different vehicles on the road and their capabilities can be the first step towards promoting safety awareness and reducing aggressive driving behavior.

4.6 Conclusions

This chapter has explored the use of MXL model to quantify the contributions of, among others, vehicle characteristics, vehicle-following and pre-crash attributes on the probabilities of three manners of collision in interstate highways.

The activity of the striker prior to collision is of special interest since vehicle-following attributes are incorporated in the model to try to explain unsafe interactions between two vehicles prior to the rear-end crash. An angle collision between same or different vehicle types can be commonly attributed to conscious yet aggressive lane changing or unpredicted maneuver that changed vehicle’s path. In particular, this manner of collision, between cars and trucks, regardless of their role, is expected to occur due to the difference in size and dynamic characteristics between two distinct types of vehicles whether one changes lanes on purpose or not. In interstate highways, trucks are driven mostly in the right lane(s), thus there exists an inherent lane separation between cars and trucks. This behavior is expected to contribute to angle collisions in crashes between a car and a truck. It is expected that a collision in a sideswipe manner occurs when same vehicles are traveling next to each other in adjacent lanes. It is
suspected that the occurrence of this collision among cars and trucks will be attributed to the difference in longitudinal dimensions. Identifying and understanding driver behavior leads to responsible countermeasures for road and insurance policy and exposes risks implications for truck companies.

This chapter has proven the hypothesis that car and truck drivers have different driving behavior (hypothesis 2 in Section 1.2). The results demonstrated that cars failed to follow trucks and trucks struggled following cars. Thus, driving behavior is different when vehicles follow different types of vehicles as stipulated in the research hypothesis 2. This research supported that pre-crash actions influenced the likelihood of collisions confirming the hypothesis that unsafe interactions can expose vehicle-following behavior (hypothesis 4).
Chapter 5: Development of Driving Behavior Model for Vehicle-Following

This chapter describes the model development used to simulate driving behavior and the process followed to validate the proposed model. The linear system state-space model developed in this chapter mimics human driving behavior according to relative velocity and distance between the vehicles travelling in the same lane. The linear system is intended to optimize driving performance when following different types of vehicles. Using equations that relate the motion related variables in a vehicle-following scenario, the state of the follower can be estimated. In the development of the linear system model, two experiments were conducted. The first experiment estimated the system parameters. The second experiment predicted car-following behavior using a state estimator. The state estimation consists of comparing the position and velocity of the follower with the state of the leader. Using a KF algorithm, a target following distance, which is implicit for a particular driver, can be identified so that the follower can adapt to this behavior at consecutive time steps.

5.1 Linear System for Driving Behavior

In this chapter, the safe interaction between two vehicles is analyzed using the concept of control systems. In this way, it is possible to identify the system of driving behavior and represent it using mathematical equations. Once the dynamic behavior of a system is represented analytically, a state estimator can then be used to optimize the control deviation (error) of a feedback system and estimate parameter values required for computing the desired following distance.

Control systems are used to direct, regulate and command a system to perform a desired task. If a system needs to be controlled, a sensor to measure the actual behavior is necessary so that it can be later compared with the desired behavior. The illustration of the three basic components of a control system, defined as input as a function of time $u(t)$, a physical system formed by different components, and the output as a function of time $y(t)$, are shown in Figure 5.1. Using control systems, it is possible to identify the characteristics of the system and their influence in the performance of the system. Also, by applying the control system approach, it is possible to model the physical system and track the time history or its dynamic behavior over time.
In the case of a vehicle control system, when analyzing its safe interaction with a vehicle ahead, the inputs in the process are the position, velocity and acceleration of this vehicle. After giving these inputs to the system, a controller, in this case the driver, will direct, regulate and command these inputs to avoid a collision, giving this vehicle its new position, velocity and acceleration in the next time step as outputs. Since these outputs must be adjusted constantly with the reference inputs when driving, the system is also known as a closed-loop system, feedback system or compensatory system, as described in detail in Figure 5.2.

From Figure 5.2, it can be seen that distance, velocity and acceleration of a vehicle are regulated by the driver (or the so-called controller). However, velocity and acceleration can be derived from knowing the positions of two vehicles in the road over time. Therefore, regulating the physical gap
between vehicles can be referred as the target variable when modeling vehicle-following behavior. Gap is defined as the distance between the front bumper of the following vehicle and the rear bumper of the leading vehicle. It can be said that the steady state in the traffic stream can be achieved when the gap is kept constant. By doing this, vehicles do not interfere with each other as long as the relative velocity (leader’s velocity minus follower’s velocity) maintains a positive value. Otherwise, when a vehicle does not maintain a constant distance with the vehicle in front, the possibility of a collision exists. In human-driver vehicle-following models, the state of a vehicle may be defined as its gap and relative velocity from its leader similar to the GM Model in Eq. (2.1). For the follower, the inputs in the vehicle-following process are based on the position, velocity and acceleration of the preceding vehicle. After giving these inputs to a system, the follower’s driver will direct, regulate and command the vehicle. This implies giving a new value for the state variables denoted as gap and relative velocity. There is a constant need for the follower to adjust its state in response to the change in state of the leader. The driver must continuously judge the distance, velocity and position of his/her vehicle by visually measuring these factors and comparing them with the leading vehicle. In order for a vehicle to avoid becoming a striker in a rear-end collision, the driver must decide whether to accelerate, brake and/or steer when comparing these factors between his/her vehicle and his/her desired behavior. Because of this, it can be said that the system is a closed-loop system, feedback system or compensatory system.

5.5.1 Control System - Definition and Concepts

State-space representations model complex systems consisting of multiple variable inputs and outputs using equations. State-space representation allows the modeling of the system in time domain. The state variables are defined as the variables that change when an input into the system is made. For example, at time $t_0$, the state of a system is the information of the initial conditions of the system. After that, as time increases, the state of the system together with the input will determine the output.

The states of a linear system, which change with time $t$, represent the nature of the modeled system and are denoted by a vector $\mathbf{x}(t)$. This vector contains the numerical representation regarding the current state of the system. At any time, the system receives inputs $\mathbf{u}(t)$ and produces outputs $\mathbf{y}(t)$. Note that, $\mathbf{x}(t)$, $\mathbf{u}(t)$ and $\mathbf{y}(t)$ are column vectors which may have different dimensions. Although $\mathbf{x}(t)$
may not be measured directly, it is possible to measure the output $y(t)$. Thus, knowing $y(t)$, the state variables $x(t)$ of the system can be derived. The estimation process is defined by the following two linear equations:

State equation for a dynamic system:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$  \hspace{1cm} \text{Eq. (5.1)}

Output equation for a dynamic system:

$$y(t) = C(t)x(t) + D(t)u(t)$$  \hspace{1cm} \text{Eq. (5.2)}

where $x(t) =$ state-space representation;

$x(t) =$ vector of the state variables;

$A(t) =$ state matrix with the eigenvalues or the internal characteristics of the system;

$B(t) =$ input vector with coefficients of input variables;

$C(t) =$ output matrix containing the coefficients of state variables;

$D(t) =$ feedback matrix;

$u(t) =$ input vector of variables; and

$y(t) =$ vector of output measures.

The state-space representation in Eq.(5.1) and Eq.(5.2) can be formulated for a Multiple Inputs Multiple Outputs (MIMO) system as (the subscript $t$ is dropped for simplicity of the expression):

$$\begin{align*}
\dot{x}_1(t) &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_{11}u_1 + \cdots + b_{1r}u_r, \\
\dot{x}_2(t) &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_{21}u_1 + \cdots + b_{2r}u_r, \\
\vdots & \quad \vdots \\
\dot{x}_n(t) &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_{n1}u_1 + \cdots + b_{nr}u_r.
\end{align*}$$  \hspace{1cm} \text{Eq. (5.3)}

They can be represented in matrix form as:
\[
\frac{d}{dt} \begin{bmatrix}
  x_1 \\
  \vdots \\
  x_n
\end{bmatrix} = \begin{bmatrix}
  a_{11} & \cdots & a_{1n} \\
  \vdots & \ddots & \vdots \\
  a_{n1} & \cdots & a_{nn}
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  \vdots \\
  x_n
\end{bmatrix} + \begin{bmatrix}
  b_{11} & \cdots & b_{1r} \\
  \vdots & \ddots & \vdots \\
  b_{n1} & \cdots & b_{nr}
\end{bmatrix} \begin{bmatrix}
  U_1 \\
  \vdots \\
  U_r
\end{bmatrix}
\]

Eq. (5.4)

In the same way, given the output equations as:

\[
\begin{align*}
\dot{y}_1(t) &= c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n + d_{11}U_1 + \cdots + d_{1r}U_r \\
\dot{y}_n(t) &= c_{m1}x_1 + c_{m2}x_2 + \cdots + c_{mn}x_n + d_{m1}U_1 + \cdots + d_{mr}U_r
\end{align*}
\]

Eq. (5.5)

The output equation can be represented in matrix form as:

\[
\frac{d}{dt} \begin{bmatrix}
  y_1 \\
  \vdots \\
  y_n
\end{bmatrix} = \begin{bmatrix}
  c_{11} & \cdots & c_{1n} \\
  \vdots & \ddots & \vdots \\
  c_{m1} & \cdots & c_{mn}
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  \vdots \\
  x_n
\end{bmatrix} + \begin{bmatrix}
  d_{11} & \cdots & d_{1r} \\
  \vdots & \ddots & \vdots \\
  d_{m1} & \cdots & d_{mr}
\end{bmatrix} \begin{bmatrix}
  U_1 \\
  \vdots \\
  U_r
\end{bmatrix}
\]

Eq. (5.6)

In order to describe a system, it is necessary to represent the system with mathematical formulations, mostly differential equations. Since the system is described using algebraic equations, a mathematical tool that gives the ability to obtain linear approximations of a physical system is necessary. Thus, the application of Laplace transforms in a dynamic system will allow solving the mathematical operations to trace the time behavior or dynamic behavior. The Laplace transform is a mathematical transformation which allows a function of time \( f(t) \) to be represented by a new variable \( S \), the Laplace operator. In this manner, the function of time \( f(t) \) can be mapped as a function of the Laplace operator \( F(S) \) to simplify a mathematical problem and obtain the solution easier. To obtain the Laplace transform of \( f(t) \), the application of the following formula is needed:

\[
F(S) = \int_0^\infty f(t)e^{-st} \, dt
\]

Eq. (5.7)
Laplace transform is an optimal mathematical tool that can be used to manipulate such complex estimation. By using the Laplace transform, it is possible to represent the cause and effect on the behavior of the system by the transfer function, as shown in Eq. (5.8):

\[
\text{Transfer Function} = \frac{Y(s)}{U(s)} = \frac{\text{Laplace } \{y(t)\}}{\text{Laplace } \{u(t)\}} = G(s)
\]

Eq. (5.8)

This formula describes the relationship between the output and input as the ratio of the Laplace transform of single output over the Laplace transform of single input. A transfer function is often formulated to achieve an optimal measure between output and input.

5.5.2 Kalman Filter - Definition and Concepts

Kalman Filter (KF) is an algorithm designed to optimize the prediction of the state of a dynamic linear system. A KF reprocesses data using the same inputs as the state-space equations at each time step, estimates the system parameters and enables the linear system to make the best prediction of future states. The repetitive process of the interaction between the linear system and KF reaches stability and optimality when the error between the estimated value of the reference model and the actual model is minimized. In order to do so, the system uses an estimation matrix at each time step to adjust the dynamic change in the model parameters.

In a KF, a set of equations is used iteratively to enable the linear system to produce the best estimation of state at a future instance with the least possible error. This tool (KF) is applied into a multivariate linear system until a specified condition is met. It uses measured data and observed outputs in the past as inputs, feedback the error in order to predict the state of the system in the future. A KF can be applied to assist a linear system to predict the future state of a complex system when the system is subjected to an external excitation process. With an accurate estimate of the future states, the behavior of the linear system can be controlled to achieve the desired objective.

When applying KF to a linear system, a plant (the reference model) and an observer (actual model) are compared to measure the error among them (see Figure 5.3). Then, a KF estimator iteratively
adjusts the system variables using an estimator matrix until the error is minimized. In the vehicle-following model, the KF estimator is important in defining the characteristics of the follower’s behavior.

![Diagram](image)

Figure 5.3: Estimation of deviation between reference model (real) and actual model (observer).

It can be said that, when a KF is integrated with a linear system of equations, the system adds an estimator matrix \( L(t) \) in its state-space representation, leading to the following formula (with the cap “^” representing estimated value):

\[
\dot{x}(t) = A(t) \hat{x}(t) + B(t)u(t) + L(t)(\hat{y}(t) - \hat{y}(t))
\]

Eq. (5.9)

with an output equation represented as:

\[
\hat{y}(t) = C(t) \hat{x}(t) + D(t)u(t)
\]

Eq. (5.5)

When subtracting Eq. (5.9) from Eq. (5.1), and Eq. (5.10) from Eq. (5.2) new equations are obtained for the estimation of state variables and output:
\[
\begin{align*}
(\dot{x}(t) - \dot{\hat{x}}(t)) &= A(t)(x(t) - \hat{x}(t)) + L(t)(\dot{y}(t) - \hat{\dot{y}}(t)) \quad \text{Eq. (5.11)} \\
(\dot{y}(t) - \dot{\hat{y}}(t)) &= C(t)(x(t) - \hat{x}(t)) \quad \text{Eq. (5.12)}
\end{align*}
\]

Rearranging the above equations by substituting Eq. (5.12) in Eq. (5.11), a new formula is obtained to find the displacement error of the known matrices \(A(t)\) and \(C(t)\), and the new \(L(t)\) matrix:

\[
(\dot{x}(t) - \dot{\hat{x}}(t)) = (A(t) - L(t)C(t))(x(t) - \hat{x}(t)) \quad \text{Eq. (5.6)}
\]

In this way, the characteristic matrix \((A(t) - L(t)C(t))\), commonly known as eigenvalues, can be used in the prediction of the behavior of the model.

5.5.3 Feedback Gain Matrix - Definition and Concepts

In the design of a controller, a control law is often introduced in order for the system to achieve the desired output. The control law without a reference input is represented by:

\[
u(t) = -k(t)x(t) \quad \text{Eq. (5.14)}
\]

where \(u(t)\) = Input of the system;

\(k(t)\) = Gain feedback matrix; and

\(x(t)\) = State variable vector.

When Eq. (5.14) is substituted into Eq. (5.1) and (5.2) we obtain the modified behavior for the state variables and output of the system described by:

\[
x(t) = (A(t) - B(t)k(t))x(t) \quad \text{Eq. (5.15)}
\]

\[
y(t) = (C(t) - D(t)k(t))x(t) \quad \text{Eq. (5.16)}
\]
These formulas enable the controller to accomplish the behavior required, since the state feedback gain will shift the performance of the system.

5.5.4 Design of a Controller and Full-State Observer- Definition and Concepts

In modern controls, the design of a controller and an observer are based on two principles:

1) Controllability - Can the system be controlled?
2) Observability - Can the system be observed (measured)?

To determine if a system is controllable, a solution must exist from state \( x(0) \) to \( x(t) \). If the determinant of a system described by the matrices \( A \) and \( B \) is not zero, then it can be said that the system is controllable. The formulation for controllability at one time step is described as follows:

\[
\text{det} \begin{vmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{vmatrix} \neq 0
\]

Eq. (5.17)

Observability in a system is determined by finding that the determinant of \( Q \) is not equal to zero as described in Eq. (5.18):

\[
Q = \text{det} \begin{vmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{vmatrix} \neq 0
\]

Eq. (5.18)

The optimal performance of the system (vehicle-following behavior) is obtained by integrating a gain feedback \( K \) and a state estimator \( L \). The gain feedback describes the optimal action to be performed by the follower in order to catch up with the leader. This can be represented by an amplifier that intensifies the states of the follower (spacing, relative velocity) and goes back into the system to obtain a best possible reaction (acceleration/deceleration). The state estimator (KF) will obtain the optimal information needed to predict follower behavior. In this way, the perception of the observer (follower) is continuously adjusted to account for unknown parameters. Gaining a new perspective allows to
approximate as close as possible the “true” behavior of the system, which in this case is the vehicle-following behavior.

**Controller**

To design a state feedback control, it is necessary to measure the sensitivity of elements (state variables $x(t)$) and introduce a gain feedback $K$ into the input. In a complex system, measurement may be inaccurate, expensive or not accessible. Then, instead of measuring all elements, an estimation based on the observed state $\hat{x}(t)$ is performed. A gain observer matrix $K$ will be estimated by introducing the control law into the state-space representation:

Based on the control law found in Eq. (5.14), it is necessary to determine the characteristic polynomial given by:

$$\det |sI - A + Bk| = 0$$

Eq. (5.19)

where $s = \text{scalar eigenvalue}$; and

$I = \text{identity matrix}$.

To make the estimations, it is necessary to select the desired characteristic equation with the desired poles. The characteristic equation or eigenvalues contains the specifications for a more stable and optimal performance. The desired eigenvalues are represented by a conjugate pair $\mu_{1,2} = -a \pm jb$ that is located anywhere on the left region of a Laplace plane (s-plane) as shown in Figure 5.4 in the shaded region.
The desired characteristic equation will take the form of:

\[(s - \mu_1)(s - \mu_2) \cdots (s - \mu_n) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1s + a_0 = 0\]

Eq. (5.7)

Then by establishing an equivalency between the characteristic polynomial Eq. (5.19) and the desired characteristic equation Eq. (5.20), it is possible to solve for the K matrix to be used in the design of the controller.

**Observer**

The state estimator matrix compares the output of the observer (follower) with the plant (leader), in this case the position, and uses state estimators so the observer can meet the desired responses. The transient response for the observer can be described by the system response shown in Figure 5.5.
Figure 5.5: Transient response.

where: $\Delta h =$ Overshoot;
$T_R =$ Rise time;
$T_P =$ Peak time; and
$T_S =$ Settling time.

The desired observer’s behavior can be measured using the natural frequency $w_n$ and damping ratio $\varphi$ described in a second order linear Ordinary Differential Equation (ODE) with constant coefficients. The standard form for ODE for a free response is expressed as:

$$\ddot{x} + 2\varphi w_n \dot{x} + w_n^2 x = 0$$  \hspace{1cm} \text{Eq. (5.8)}$$

This equation provides the desired characteristic equation describing the pole placement. To find the values in the $L$ matrix, an equivalency with the state estimator equation must exist. The characteristic polynomial equation to be used in finding the solution is represented as:
\[
\det \left| sI - A + LC \right| = 0 \quad \text{Eq. (5.21)}
\]

### 5.5.5 Fuzzy Similarity Formulation

This research used fuzzy similarity measure to obtain an accurate parameter of state estimators in the L matrix. Fuzzy similarity looks for a grade of compatibility between sets (in this vehicle-following case the vehicle types of the leader and follower). To do this, the minimum and maximum of the variables of interest is computed. On order to find the relation that encloses similarities between sets, the following expression is computed:

\[
S(X, Y) = \frac{\sum \min (x_i, y_i)}{\sum \max (x_i, y_i)} \quad \text{Eq. (5.9)}
\]

For example, if \( X = [0 \ 1 \ 1 \ 1] \) and \( Y = [1 \ 1 \ 1 \ 0]^T \), the similarity between these two vectors is:

\[
\frac{0 + 1 + 1 + 0}{1 + 1 + 1 + 1} = \frac{2}{4} = 0.5
\]

Similarity analysis between cars and truck will be based on: mass, moment of inertia, center of gravity, velocity and acceleration. It is expected that same combinations such as car following car and truck following truck give a similarity value of 1. This indicates that same vehicles share same vehicular capacities. On the other hand, a small value between cars and trucks is expected due to the significant disproportion of mass and moment of inertia.

### 5.2 Vehicle-Following According to Vehicle Dynamics

The state transition matrix, known as \( A \), is proposed as the key factor for influencing car-following behavior. The matrix \( A \) is calculated based on vehicle’s mass, velocity, acceleration, moment of inertia, and etc., giving specific values for each element in the matrix. These values are the different
dynamic characteristics that can be observed and measured. During simulation, the eigenvalues are constantly changing. Knowing the parameters for each element in the matrix allows quantifying driving behavior. It is expected that each vehicle will have a different $A$ matrix depending on the vehicle used: heavy truck or passenger car.

5.2.1 Data

The study used the NGSIM data from the northbound direction of Interstate I-80 Freeway in Emeryville, California. Traffic information regarding the trajectory of vehicles traveling along a segment of 1650 ft was collected on April 3, 2005 by Cambridge Systematics (2005). Figure 5.6 shows an aerial view of the data collection area and illustrates how the freeway was segmented to collect data.

![Figure 5.6: US I-80 highway study area (from Cambridge Systematics (2005)).](image)

Data collection was performed in three different 15-minute periods: 4:00-4:15 p.m., 4:30-4:45 p.m. and 5:15-5:30 p.m. This study only used the first set of the data (from 4:00-4:15 p.m.) when the
traffic congestion was building up and driving behavior started to change. As can be seen in Figure 5.7, the vehicular flow is almost 8,500 vehicles per hour.

![Figure 5.7: Vehicle flow (from Cambridge Systematics (2005)).](image)

The speed is almost three times slower compared to non-congested condition. Figure 5.8 illustrates the time mean speed and space mean speed of the traffic flow. On average, vehicles speed ranges from 25 to 20 mph. The high density traffic state is of special interest to this research because drivers are forced to follow the car in front for longer periods of times since lane-changing maneuvers are limited. This study aims to capture the actions of drivers based on their perception of slow traffic.

![Figure 5.8: Time mean speed and space mean speed (from Cambridge Systematics (2005)).](image)
As the vehicles move along the road segment, cameras tracked their movements as observed in Figure 5.9. Special software identified each vehicle and extracts position coordinates from video. Additional information on velocity, acceleration, space headway, time headway are derived from the video images. The trajectory data for vehicles collected is paired to provide information about the interaction between leader and follower.

![Video processing](from Cambridge Systematics (2005)).

From the selected data set, different types of vehicles were recorded and monitored to extract car-following information every 0.1 seconds. But only trucks and automobiles were selected for this study. Table 5.1 describes the different type of vehicles found in the traffic stream when the data collection study was conducted.
Table 5.1: Percentage of Vehicles According to Type (from Cambridge Systematics (2005)).

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>Vehicles</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorcycle</td>
<td>14</td>
<td>0.7%</td>
</tr>
<tr>
<td>Automobile</td>
<td>1,942</td>
<td>94.6%</td>
</tr>
<tr>
<td>Truck and Buses</td>
<td>96</td>
<td>4.7%</td>
</tr>
<tr>
<td>Sum</td>
<td>2,052</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

5.2.2 Variable Definition

This section describes the horizontal dynamics of vehicle, where the forces exerted by the tires, as well as, mass, moment of inertia, center of gravity and velocity of the vehicle are considered. By changing a distinctive factor in the equation, such as, mass or moment of inertia, differences in the dynamics between a small or large vehicle; four scenarios of vehicle-following can be analyzed. In this way, the horizontal movement of different types of the vehicles, especially, heavy trucks and passenger cars can be interchanged to represent different leader-follower pairs. Figure 5.10 shows the acting forces in horizontal dynamics that will be used to formulate the equations used.

![Figure 5.10: Vehicle design components.](image)

As indicated by Jazar (2008), the acceleration of a car is proportional to the friction under its tires. This can be represented by the following equations:
\[ F_{uv} = \mu F_{zv} \quad \text{Eq. (5.23)} \]

\[ F_{uh} = \mu F_{zh} \quad \text{Eq. (5.24)} \]

where \( F_{uv} \) = Front wheel circumferential force (N); a value of zero is considered if the vehicle is a rear wheel drive car;

\( F_{uh} \) = Rear wheel circumferential force (N); a value of zero is considered if the vehicle is a front wheel drive car;

\( \mu \) = Pavement friction coefficient considered as 0.1;

\( F_{zv} \) = Vertical force under front wheel (N); and

\( F_{zh} \) = Vertical force under rear wheel (N).

The vertical forces under the front and rear wheels can be calculated using the following equations (Jazar 2008):

\[ F_{zv} = \frac{1}{2} mg \frac{b}{l} - \frac{1}{2} mg \frac{h_a}{l} \frac{a_{cc}}{g} \quad \text{Eq. (5.25)} \]

\[ F_{zh} = \frac{1}{2} mg \frac{a}{l} - \frac{1}{2} mg \frac{h_a}{l} \frac{a_{cc}}{g} \quad \text{Eq. (5.26)} \]

where: \( m \) = Mass (kg);

\( g \) = Gravity, 9.81 m/s\(^2\);

\( a \) = Distance from center of gravity to front wheel (m);

\( b \) = Distance from center of gravity to rear wheel (m);

\( l \) = Distance from front wheel to rear wheel (m);

\( h_a \) = Distance from road surface to center of gravity (m); and

\( a_{cc} \) = Acceleration of vehicle (m/s\(^2\)).

The important input and output variables of the horizontal dynamics are represented in Figure 5.11. The definition of these variables can be found in the state-space equation definition later.
According to Figure 5.11, the longitudinal motion of a vehicle is governed by a state-space representation as:

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} \lambda
\]

Eq. (5.27)

where:
\[\dot{z}_1 = \text{Angular velocity (rad/s);}\]
\[\dot{z}_2 = \text{Angular acceleration (rad/s^2);}\]
\[z_1 = \text{Heading angle (rad);}\]
\[z_2 = \text{Rotational speed (rad/s);}\]
\[\lambda = \text{Input (rad);}\]

\[
a_{11} = -\frac{F_{uv} + F_{uv}}{mV}
\]

Eq. (5.28)

\[
a_{12} = -\left(\frac{F_{uv} b + F_{uv} a}{mV^2} - 1\right)
\]

Eq. (5.29)

\[
a_{21} = -\frac{F_{uv} b + F_{uv} a}{\Theta}
\]

Eq. (5.10)

\[
a_{22} = -\frac{F_{uv} b^2 + F_{uv} a^2}{\Theta V}
\]

Eq. (5.11)

For front-wheel drive (cars)
\[ b_1 = \frac{F_{uv}}{mV} \]  
Eq. (5.122.a)

\[ b_2 = \frac{F_{uv} a}{\Theta} \]  
Eq. (5.13.b)

For rear-wheel drive (trucks)

\[ b_1 = \frac{F_{uh}}{mV} \]  
Eq. (5.33.a)

\[ b_2 = \frac{F_{uh} b}{\Theta} \]  
Eq. (5.33.b)

\[ \Theta = \text{Pitch moment of inertia (kg.m}^2\); \]

\[ V = \text{Velocity (m/s).} \]

The \( \textbf{A} \) matrix in Eq. (5.27) relates how the current heading angle and rotational speed affects the angular velocity and angular acceleration, respectively. This system matrix portrays the internal characteristics of the system consisting of vehicle dynamics and driver actions. The \( \textbf{B} \) vector determines how the input affects the change of the states. This control vector is indirectly related to driving habit and/or sensitivity of driver because of its close relation to the input. To estimate each element in the \( \textbf{A} \) matrix and \( \textbf{B} \) vector, it is necessary to know vehicle specifications and driver’s response. As observed in Eq. (5.28) to (5.33), if vehicle’s mass, moment of inertia and center of gravity are known, only the velocity maintained by the driver will be needed to estimate \( \textbf{A} \) and \( \textbf{B} \). It is in the scope of this research to consider that the state-space representation of a vehicle is genuine and meets the description of horizontal vehicle motion.

5.2.3 Empirical Setting

The first step in analyzing the data was to group vehicles identified as followers according to their vehicle classification. It was observed from the NGSIM data that the duration of vehicle-following behavior within the data collection area varied from just a couple of seconds up to one or two minutes. Although it was desirable to target vehicles who engaged in vehicle-following for long time periods, the
sample size was considerately reduced. A minimum of 30 seconds was set as the minimum duration for vehicle-following intending to find more pairs of vehicles. A total of 32 cars following a leader (car or truck) were selected for the analysis, as well as, 25 trucks following either a car or another truck. Data points were averaged into 0.5 second for practicality. Table 5.2 gives more details about the pairs selected for each combination.

The NGSIM data classified vehicles into three categories: motorcycle, car and truck. The data was filtered to only select pairs classified as car or trucks. Further selection of the truck classification was performed since 18-wheelers are the only interest in this study. Trucks specified in the NGSIM data to be more than 60 ft in length were only considered in the analysis.

Table 5.2: Number of Vehicles and Following Duration.

<table>
<thead>
<tr>
<th>Pair Combination</th>
<th>Pairs</th>
<th>Total Vehicles</th>
<th>Duration (sec)</th>
<th>Data Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car Following</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car following Car (C-C)</td>
<td>15</td>
<td>30</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Car following Truck (C-T)</td>
<td>17</td>
<td>34</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Truck Following</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truck following Truck (T-T)</td>
<td>5</td>
<td>10</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Truck following Car (T-T)</td>
<td>20</td>
<td>40</td>
<td>35</td>
<td>70</td>
</tr>
<tr>
<td>Total</td>
<td>57</td>
<td>114</td>
<td>155</td>
<td>310</td>
</tr>
</tbody>
</table>

For the purpose of this study, the information related to position was mainly used. Although velocity was also available in the data set, this variable was estimated from the coordinates. In this way, any possible errors in the data set due to data processing were disregarded (Punzo et al. 2012). Additional information taken from the NGSIM database used in this research was time headway and gap. An average for the derived velocity, given time headway and estimated gap was taken for every 0.5 seconds from the values in the NGSIM data set. The average velocity, time headway and gap values of all following vehicles according to pair combination are illustrated in Figures 5.12, 5.13, and 5.14 accordingly.

As can be observed in Figure 5.12, all pairs of vehicles, except truck-truck combination, follow a similar pattern with respect to velocity. The following vehicles were decelerating during the first 15
seconds and accelerating afterwards. Trucks that follow other trucks do not have a prominent deceleration. Although velocity decreases, the acceleration/deceleration rate is more frequent in this combination than in others.

![Figure 5.12: Average velocity of following vehicles](image)

The time headway is measured by taking the time that passes between two vehicles to reach the same spot. For years, recommended safety distance in driver’s state manual has been based on headway rules. One example of this is the “two-second rule” that gives the minimum safe distance to avoid collisions under normal conditions. Form the Figure 5.13 below, it can be concluded that most combination of vehicles allowed a time headway greater than the common average, which is two seconds in congested scenarios. Only car-car combination followed this safety rule (close to two seconds) most of the time. Cars that followed trucks and trucks following trucks have the highest headways. On average truck combinations allowed a seven-second headway. Cars following trucks struggled the most when traffic decelerated. Although all vehicles have higher average values during this time, there is an abrupt disruption in the pattern for car following truck combination. Headway may
have been increased due to sight limitations or for changing lanes. Bigger gaps may be preferred by cars to see traffic ahead or in order to merge to be in the fastest lane.

![Figure 5.13: Average time headway of following vehicles](image)

Figure 5.14 below shows the different following gap for all car and truck combinations. As expected, trucks allowed more gap when following another truck. Truck drivers may anticipate cars cutting off and may feel more comfortable falling way behind when following another truck. Bigger following distance is also observed when a car is leading a truck. Greater gaps are also given between a car and a truck, but a more aggressive behavior is noted. For instance, when traffic slows down, the gap gets shorter in the first ten seconds. This indicates that the truck rapidly gets closer to the car as the car decelerates. On the contrary, when traffic speeds up, trucks increase their gap between them and the car in front. It could be that trucks have slower accelerations and take more time to speed up. Truck driver may not want to jeopardize their acceleration by a sudden stop of the car in front.
Because vehicle dynamics information was not given in the NGSIM data set, it was assumed that all cars exhibit same vehicular capacities and all trucks are the same. For simplicity, cars were considered to be 1998 Honda Civic. The vehicle dynamic specifications for cars were taken from Heydinger et al. (1999). Inertial and mass properties of a dual rear-axle white freight empty truck were considered for all trucks (Winkler 1983). The following table describes the general values considered for all cars and all trucks.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Car</th>
<th>Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>1,143</td>
<td>36,364</td>
</tr>
<tr>
<td>Longitudinal Center of Gravity (m)</td>
<td>1.04</td>
<td>2.24</td>
</tr>
<tr>
<td>Vertical Center of Gravity (m)</td>
<td>0.51</td>
<td>0.92</td>
</tr>
<tr>
<td>Wheel Base (m)</td>
<td>2.62</td>
<td>4.44</td>
</tr>
<tr>
<td>Pitch Moment of Inertia (kg.m²)</td>
<td>1,617</td>
<td>43,367</td>
</tr>
</tbody>
</table>
It was assumed that all cars are front-wheel drive whereas trucks are rear-wheel drive. The formulas to calculate the $A$ and $B$ matrix were adjusted to reflect these assumptions. This means that for a front wheel-drive the circumferential forces in the rear wheel were assumed to be zero. Likewise, rear-wheel drive vehicles will have zero force at the front wheels.

5.2.4 Results and Discussion

The goal of this first analysis is to capture different driving behavior due to different vehicle dynamics and vehicle-following roles (leader versus follower). The objective of this experiment is to understand how the parameters specific to the vehicle pair combination and driving role varies over time. It is of special interest to compare the $A$ matrix and $B$ vector settings across each pair combination based on the vehicle dynamics. In this manner, it is possible to measure the internal characteristics of the system embedded in the $A$ matrix in the state-space representation. At the same time, the $B$ vector can also be estimated. Mass, velocity, moment of inertia and vertical forces are some of the common variables that both matrices used. The relation between the elements $a_{11}$, $a_{12}$ and $b_i$ in the $A$ matrix and $B$ vector are inversely proportional to mass. This means that a small mass, such as of a car, will result in a higher absolute number. Otherwise, a heavy mass (e.g truck) will give a smaller absolute value of these elements. In a similar way, the relation between the elements $a_{11}$, $a_{12}$ $a_{22}$, and $b_i$ in the $A$ matrix and $B$ vector are inversely proportional to velocity. When the vehicle decelerates or applies the brakes, the velocity is reduced. This will result in a higher absolute value for the elements. One the contrary, increased velocity will produce a lower absolute value of these elements. The same can be said about the elements $a_{21}$ and $b_2$ with respect to the moment of inertia. A higher absolute value is obtained when the moment of inertia is small (e.g. in a car). The opposite is obtained with higher values of moment of inertia. A higher moment of inertia is considered for truck instead. The $a_{21}$ and $b_2$ elements will have smaller absolute values. The following section describes the results obtained for each pair combination.

Car following Car

An average value of 15 pairs of vehicles was used for estimating the $A$ matrix (i.e. $a_{11}$, $a_{12}$, $a_{21}$, and $a_{22}$). In the same manner, the $B$ vector ($b_1$ and $b_2$) were computed at each time step. The variability of
the internal characteristics (A matrix) and input vector (B) of each follower and leader can be observed in the figures below.

As can be observed from Figure 5.15, the common trend (average value of 15 followers) in all the different elements is that the follower behaves almost independently from the actions of the leader in the first 20 seconds of car following mode. This period of time is characterized by having low velocities as shown in Figure 5.12. The results indicate that the leader’s actions are not replicated by the follower when the followers are decelerating (i.e., the follower is closing in to the leader). After this period, however, the follower generates a delayed yet similar output as expected. Car-following another car reflects a strong impact of the driving behavior based on the role actions rather than vehicular dynamics.

One of the singularities of the results obtained is that there is a disruption in the pattern from the 10th to the 20th second. These generated outliers suggest that most of the cars selected encountered slower traffic and were forced to stop and resume control of the vehicle. One possible explanation can be that the shock wave propagated from upstream reached the incoming traffic at this point in time. Because acceleration is used in calculating A matrix elements and is a common denominator in the formula, a low value will cause a higher outcome. The same observations can be applied for the input vector B as can be observed from the Figure 5.16 below.
Figure 5.15: **A** matrix for car following car.

Figure 5.16: **B** vector for car following car.
**Car following Truck**

When comparing the vehicular responses between cars and trucks it is expected to find different trends. What it is distinctive from the results is that the shape of the curves of the follower is somewhat similar in all elements of the A matrix, regardless of the type of vehicle driven. However, it is impossible to replicate exact same values for cars following trucks. The significance of this is that the cars that follow a truck tend to replicate same acceleration/deceleration trend. In this car-truck combination the truck exerts a different influence in the driving behavior of a car. When comparing Figure 5.17 with Figure 5.15, it can be observed that the leader’s A matrix for trucks are different from cars because of higher inertia values. In this case, the response of the car reflects an amplified response to the acceleration/deceleration experienced by the truck. The average vehicle-following behavior of the 16 pairs of vehicles simulated for 30 seconds can be seen from Figures 5.17 and 5.18.
Figure 5.17: **A** matrix for car following truck.

Figure 5.18: **B** vector for car following truck.
**Truck following Car**

The average behavior of 19 pairs of trucks following a car is shown in Figures 5.19 and 5.20. As can be seen, similar behavior is obtained from the follower (truck) when responding to the changes of the leader (car) in front. As previously observed, the phenomena of replicating the behavior of the vehicle in front is more common among different types of vehicles. This again confirms that when trucks follow a car, the only difference in the results is the magnitude of the values obtained for each vehicle. This variability in the magnitude of the elements in the matrices is to be expected due to the differences in mass and inertial properties between vehicles. This finding suggests that following different type of vehicles constrained vehicle-following behavior to be more predictable.
Figure 5.19: **A** matrix for truck following car.

Figure 5.20: **B** vector for truck following car.
For the five pairs of trucks considered, vehicle-following seems more consistent as can be observed in Figure 5.21 and 5.22. Same vehicle dynamics seem to encourage a more dependent “follow-the-leader” behavior in trucks, especially when traffic is slowing down. The distinction between leader and follower is not as clear as when a truck follow a car as seen in Figure 5.19. Still, it is interesting to observe that abrupt changes in the leader’s behavior do not seem to match the reaction of the follower. One explanation could be that the reaction time for reacting is not fast enough. Changes in driving behavior may come before the follower can response to the last action of the leader. Action/reaction may overlap because of the capabilities of a truck. Breaking force, large mass, limited maneuver, could be very demanding in a congested highway for trucks following another truck. Despite of this, trucks were able to adapt more successfully to changes in traffic in comparison to when a car was leading them. Having limited visibility when following another truck may encourage drivers to “trust” the truck in front and replicate actions.
Figure 5.21: A matrix for truck following truck.

Figure 5.22: B vector for truck following truck.
Car-Following

The intention of comparing vehicle-following among same followers but different leaders is to identify differences in driving behavior exclusive to followers and vehicle type. In addition, comparing car-following with different types of leaders lead to the different ranges of values in the elements of matrix A and vector B that are exclusive to each follower combination.

The following figures illustrate how cars follow another car versus following a truck. Some of the findings when comparing car-following indicate the follower responds in a similar way when there is a truck or a car in front. The actions taken by the car when following a truck produce a shifted curve different in scale mostly due to mass differences. The values of the matrix A and input vector B are shifted but same behavior is replicated. The curves obtained from the follower regardless of the type of vehicle leading, is constantly oscillating at all times meaning cars are constantly adjusting in congested traffic. Also, cars seem to react faster to changes in velocity a truck performs. This is demonstrated by observing a similar curve for both scenarios. Also, from Figures 5.23 and 5.24, it can be observed that some drivers overacted to changes in speed and altered the average curve. Their characteristics can be observed in the outliers located away from the rest of the path. These drivers were more sensitive, even more aggressive when following than the average car follower. They may have even experienced an emergency brake because of a distraction.

One possible conclusion could be that, in general, cars experienced same car-following behavior when driving behind trucks or cars. Neither size nor dynamics of the leading vehicle encouraged drivers to change their driving behavior.
Figure 5.23: A matrix for car-following behavior.

Figure 5.24: B vector for car-following behavior.
Table 5.5 and 5.6 shows the range of car-following values for A matrix and B vector, accordingly. These parameters can be used to compare driving behavior when a car follows same type of vehicle and when follow a truck. Among all elements in the A matrix, $a_{11}$ has a noticeable difference when following different vehicles. This variability indicates the effect that different size and mass proportions has on the driving behavior of cars. Also, forces exerted on tires also influence this variable. Forces act in the front wheels in cars and trucks have a rear driven force. On the other hand, values in the car-following B vector are similar.

Table 5.4: Car-following Values for A matrix Elements.

<table>
<thead>
<tr>
<th>Matrix Element</th>
<th>Car-Following</th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11}$</td>
<td>Car</td>
<td>-0.66</td>
<td>-0.09</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.10</td>
<td>0.35</td>
<td>0.57</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>Car</td>
<td>0.95</td>
<td>1.06</td>
<td>1.73</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>0.95</td>
<td>1.00</td>
<td>1.67</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>Car</td>
<td>0.22</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>0.20</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>Car</td>
<td>-0.53</td>
<td>-0.08</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.49</td>
<td>-0.06</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

Table 5.5: Car-following Values for B Input Elements.

<table>
<thead>
<tr>
<th>Vector Element</th>
<th>Car-Following</th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>Car</td>
<td>-0.66</td>
<td>-0.09</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.64</td>
<td>-0.08</td>
<td>-0.04</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Car</td>
<td>0.22</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>0.20</td>
<td>0.21</td>
<td>0.21</td>
</tr>
</tbody>
</table>
**Truck-Following**

The results shown in Figure 5.25 and 5.26 suggest that followers driving a truck are more confident to embrace changes in speed when another truck is leading them. Among followers who drive a truck, vehicle-following behavior has a curve with fewer fluctuations when the leading vehicle is also a truck. This describes a constant and gradual adaption to the vehicle in front. This “accordion” behavior tells the follower is replicating the leader’s movement in a natural manner. This follow-the-leader behavior is commonly experienced when the gap between vehicles is small.

On the other hand, trucks that follow a car appear to be more hesitant to engage in a closer vehicle-following engagement. Possible explanations could be hidden in the mass and dynamic related variables. Vehicle capabilities of trucks may be influencing how they adapt to traffic. Trucks may need more time and distance to accelerate and decelerate and therefore, keep a larger distance when following a truck so that there is a slower transition from deceleration to acceleration. This effect can be seen as a u-shape curve in Figures 5.26 and 5.27 where the truck slowly decelerates until there is enough gap, or incentive, to accelerate again. Truck drivers may be more experienced when following a truck as demonstrated by small changes in the matrix elements. The curve when following a truck maintains a constant trend or oscillates within a small range. This suggests there was not much change of speed or following was not close. Trucks following trucks choose a gap such that the truck can drive more efficiently (less stop-and-go) when driving.
Figure 5.25: A matrix for truck-following behavior.

Figure 5.26: B vector for truck-following behavior.
Table 5.25 and 5.26 describe the minimum, maximum and average values for each element in the truck-following A matrix and B vector, respectively. One observation that can be made is that the values in the A matrix are similar when examining the average and the maximum value. But when the minimum value between cars and trucks are compared, it is observed that the difference is significant. Variability in the lower limit, for example 0.37 (car) and 0.98 (truck), indicates the diversity in comfort for truck drivers when following a car or truck.

Table 5.6: Truck-following Values for A matrix Elements.

<table>
<thead>
<tr>
<th>Matrix Element</th>
<th>Car-Following</th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11}$</td>
<td>Car</td>
<td>-0.12</td>
<td>-0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>Car</td>
<td>0.37</td>
<td>0.94</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>Car</td>
<td>0.28</td>
<td>0.46</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>0.41</td>
<td>0.46</td>
<td>0.65</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>Car</td>
<td>-0.49</td>
<td>-0.23</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.13</td>
<td>-0.16</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Table 5.7: Truck-following Values for B Input Elements.

<table>
<thead>
<tr>
<th>Vector Element</th>
<th>Car-Following</th>
<th>Minimum</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>Car</td>
<td>-0.12</td>
<td>-0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Car</td>
<td>-0.67</td>
<td>-0.46</td>
<td>-0.28</td>
</tr>
<tr>
<td></td>
<td>Truck</td>
<td>-0.53</td>
<td>-0.46</td>
<td>-0.41</td>
</tr>
</tbody>
</table>
5.3 Proposed Vehicle-Following Control Model

This section utilizes the estimated $A$ matrices related to vehicle dynamics obtained in section 5.2. Giving a control input to a system consisting of a follower (observer) and a leader (plant), the safe interaction between different types of vehicles is optimized using KF. The goal of this experiment is to construct a linear system for different pairs of vehicles that is able to adjust the gap and relative velocity of the follower to minimize the gap between vehicles.

5.3.1 Approach for Representing Driving Task

According to the many tasks involved in driving a vehicle, to successfully adapt to the changes of the vehicle in front, three different levels of decisions are proposed. In this manner, it is possible to describe how the decisions are made by the driver, in addition to, how the vehicle dynamics are involved. The visualization for the driving tasks in strategic, tactical and operational levels is used. The framework for this approach is illustrated in the following Figure 5.27.

![Figure 5.27: Driving task at a strategic, operational and tactical level.](image)

As represented in Figure 5.27, the *strategic level* can be described as the “strategic” output that is desired. For instance, in this case, the ultimate objective is to attain a safe distance between the pair of vehicles. The *tactical level* is the human actions or decisions to avoid collision with the vehicle in front. It is assumed that this aim is based on keeping a safe gap. These actions controlled by the follower take place when estimating the change in velocity and distance of the vehicle in front with respect the vehicle.
being driven. The *operational level* considers vehicle dynamics of four different scenarios where the vehicle driven follows the same type and when following a different type. For this research, vehicle dynamics will take into account the different size and mass proportions of heavy trucks and passenger cars.

This research builds upon this platform to measure and analyze driving behavior and to propose a new methodology to estimate vehicle-following behavior.

Figure 5.28: Architecture of vehicle-following.

Figure 5.28 represents the modeling approach used in the section to model driving behavior. In this approach, the state variables are the relative velocity of the follower with respect to the leader, and the gap. As represented above, the states of the follower and the leader are compared with the required behavior. The required behavior is described by a constant distance between the two vehicles or by maintaining a constant velocity or acceleration. Thus, the required behavior can be measured by the difference in velocity and gap among the pair of vehicles. An important assumption that needs to be
mentioned is that the leader portrays the prototype dynamics whereas the follower behavior is treated as the observer. In this way, if the required behavior is the same as the leader, then it is say that the vehicle also follows the desired behavior. On the other hand, the actual behavior is expected to describe how the follower behaves differently from the required behavior when trying to adapt to the leader’s trajectory. The actual behavior is described by the random changes in the velocity and acceleration of the follower that results in an oscillatory gap. When the desired behavior is compared with the actual behavior knowing the similarities of vehicle capabilities, and there is no deviation, it can be said that a safe vehicle-following behavior has been achieved. On the contrary, if the output results in a deviation, then a motoric correction must be made by the follower. This adjustment can be predicted using a state estimator that serves as a feedback to the system.

5.3.2 Model Formulation

The state variables considered were heading angle and heading velocity in the state-space equation. This state-space formulation in Eq. (5.27) accounts for lateral dynamics that occur when negotiating a curve, such as, steering. Knowing this, the output equation must be transformed to be consistent with this criterion and reflect the interaction of vehicles, especially vehicle-following behavior. Because of this, a new specification needs to be made in the equation to capture exclusively the motion representation of the car when traveling in a straight line. To analyze the risk of rear-end collisions happening only in straight line, the units in the state-space equation must also be consistent to those used in the output. It is required that radians are changed to meters in the state variables to reflect motion going straight. This will result in the translation of the state variables to reflect distance and velocity in meters and meters per second, respectively rather than angular position and motion. By multiplying the radius of the tires, the angular velocity will be transformed into linear velocity.

Following the new specification, the output proposes a following behavior that is safe and optimal. This safe spacing rule can be then expressed as the output equation in the form of:
\[ y = \left[ C_1 \quad C_2 \right] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \left[ D_1 \quad D_2 \right] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]

Eq. (5.34)

where:  
\( y \) = Safe following distance (m);  
\( C_1 \) = Safety factor;  
\( C_2 \) = Time headway (s);  
\( z_1 \) = gap (m);  
\( z_2 \) = Relative velocity \((V_i - V_f)\) (m/s);  
\( D_1 \) = Comfort zone factor;  
\( D_2 \) = 0 (zero);  
\( u_1, u_2 \) = Sine input (m).

By the variables used in this equation, especially by the terms safety factor and comfort zone, a new approach is suggested for estimating the optimal and safe distance. For instance, safety factor will accommodate any under or over estimation or any misjudgment in measuring distance. The comfort zone is a way to describe the willingness of the driver to follow closely. This term is expected to play a critical role when analyzing a car following a truck. This is because, although a car is keeping a minimum distance and still avoiding a collision, the driver might feel more comfortable by allowing an extra gap. Usually, this action is to evade debris coming from the tires of the truck and damaging the windshield or front bumper of the car.

In this analysis, the overall aim is for the follower to avoid collision. This can be visualized as maintaining a constant yet safe distance between the vehicle driven and the vehicle in front. An important assumption to make is that the leader behaves correctly at all times when driving. In this way, leader’s behavior serves as the reference or, in technical context, as the plant from which the follower (observer) controls its behavior. In this light, it is in the follower’s best interest to command, direct and regulate its behavior to approach the leader in a controlled and safe manner. Therefore, to achieve this required behavior, that is, maintain a constant gap, the relative velocity and difference in position of both vehicles must be considered. By measuring the relative velocity and position of the follower with respect to the leader, the target distance can be evaluated. This is true as long as the distance is
considered as a function of relative velocity and relative acceleration. Then, the output of both the leader (plant) and the follower (observer) is compared to test if the required distance has been met. If not, the error between the state of the leader (plant) and the estimated states of follower (observer) can be used with the state estimators $L_1$ and $L_2$ in Eq. (5.13) to adjust the follower’s (observer) behavior. At the same time, the follower’s (observer) state variables will be used in a state feedback that will be fed back as an input through gain $K$.

Nevertheless, a simple comparison between different estimated distance outputs from leader and follower are not enough. One of the innovative approaches taken in this research to simulate driving behavior is the inclusion of fuzzy logic. This research suggests that driving regulations in a simulation controller should include vehicle capabilities. When driving, it is crucial to regulate the vehicle position to avoid a collision. Maintaining a constant gap between vehicles, results in a smooth transition of acceleration/deceleration rates. This is especially desirable when highways are congested. Constant breaking and start up accelerations creates oscillations in traffic that propagates downstream and disrupts the flow.

Setting an efficient vehicle-following as a goal, the creation of a computational intelligence is implemented in the state estimator. This modification is based on similarity analysis used in fuzzy logic. In this case, the vehicular similarities shared by cars and truck are considered to make a better judgment of the actions to be taken in the next state. Using properties such as mass, velocity, acceleration, moment of inertia, position of center of gravity, it is possible to observe how vehicles differ. Knowing this, the state estimator can have a new and better point of reference to predict the future states. The way similarity analysis is implemented into the framework is shown in Figure 5.29. The state estimator adopts the values of the fuzzy similarity as the parameters of the $L$ matrix.
For illustration purposes only, a detailed representation of the output model for the plant (leader) can be observed in Figure 5.30. The representation of the output for the observer (follower) follows the same structure and has been omitted. In this figure, the proposed formulation of the plant is broken down into its single matrix and vector elements. Also, the state and output equations are combined in a single model to simulate the position of the observer. The state equations are located on the left side of the close loop whereas the output equation is located on the right hand side.
5.3.4 **Empirical Setting**

Having estimated the internal characteristics of the system and the input vectors \((A\) and \(B)\) in section 5.2, the next step in the simulation was to incorporate the state-space representation and the output equation. It is important to remember that the states of a system may not be easy to measure from the current system. Obtaining the states from a system may be expensive, or inaccurate. For this reason, establishing an output equation will allow a derivation of the states to be input into the equation. The output in this study is described in terms of a safe and controlled following distance. By selecting the values of the \(C\) and \(D\) vectors, the computation of this output can be achieved.
Several pairs of vehicles were considered in the simulation, but only those ranges who gave a reasonable output were chosen. Time headway and gap distance observed in the real traffic and recorded in the NGSIM data were taken as a starting point in the selection process. For this reason, adopted ranges for simulation purposes have similar values to ranges recorded in the data. The values for vectors C and D from the output equation are shown in Table 5.8. The two values on each element represent the range (minimum and maximum) value given to each parameter. A range of values for every vector element were generated randomly and input into the simulation. Only the first element in the vector D, i.e., $D_1$, was considered for an easier computation. The second term $D_2$ was set to zero.

<table>
<thead>
<tr>
<th></th>
<th>Safety Factor $C_1$</th>
<th>Time Headway (seconds) $C_2$</th>
<th>Comfort Zone Factor $D_1$</th>
<th>Comfort Zone Factor $D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car following Car</td>
<td>Leader (1,1)</td>
<td>(2,8)</td>
<td>(9,12)</td>
<td>(0,0)</td>
</tr>
<tr>
<td></td>
<td>Follower (1,4)</td>
<td>(2,8)</td>
<td>(9,12)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Car following Truck</td>
<td>Leader (1,1)</td>
<td>(2,8)</td>
<td>(9,12)</td>
<td>(0,0)</td>
</tr>
<tr>
<td></td>
<td>Follower (1,4)</td>
<td>(2,8)</td>
<td>(9,12)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Truck following Truck</td>
<td>Leader (1,1)</td>
<td>(2,10)</td>
<td>(18,47)</td>
<td>(0,0)</td>
</tr>
<tr>
<td></td>
<td>Follower (0,1)</td>
<td>(2,10)</td>
<td>(18,47)</td>
<td>(0,0)</td>
</tr>
<tr>
<td>Truck following Car</td>
<td>Leader (1,1)</td>
<td>(1,4)</td>
<td>(2,20)</td>
<td>(0,0)</td>
</tr>
<tr>
<td></td>
<td>Follower (1,4)</td>
<td>(1,4)</td>
<td>(2,20)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Based on the reasoning of considering driver and vehicle as a single system, a computational intelligence component was included to approximate driving behavior. In order to do so, the specifications of each vehicle are identified and diversified into four different combinations to capture their possible range of similarity. The similarities between a pair of vehicles are captured using fuzzy relations in terms of mass, moment of inertia, velocity, acceleration and center of gravity.
Following this reasoning, the similarities between each pair of vehicles are presented in Table 5.9. The parameters in the state estimator (L matrix) will be replaced by this compatibility values to impose a prediction of driving behavior based on vehicle capabilities. The similarity values between same vehicles are 1 whereas, different among vehicles are 0.05.

<table>
<thead>
<tr>
<th>Vehicle Pair Combination</th>
<th>Similarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car following Car</td>
<td>1.00</td>
</tr>
<tr>
<td>Car following Truck</td>
<td>0.05</td>
</tr>
<tr>
<td>Truck following Truck</td>
<td>1.00</td>
</tr>
<tr>
<td>Truck following Car</td>
<td>0.05</td>
</tr>
</tbody>
</table>

5.3.5 Results and Discussion

This section describes the behavior of the controlled system in terms of input, state variables and output. The simulation consists of a 30-second time period. A detailed description of each model is given to understand how the system was simulated based on the controller and state estimators. For instance, this can be observed in the control input law which modified the original input to the system to achieve the desired output. Also, the change of the state variables will describe the internal changes in the system needed to make the system behave according to the conditions. The expected result in the output equation should reflect an optimum performance of the vehicle-following system. This means that the difference of the position of the leader with respect the follower should have a small and stable value. A relatively small gap in vehicle-following represents a smooth adaption to the changes of the leader. The advantage of having a minor difference in gap is that the simulation can be applied additionally to a pre-determined gap. This represents an opportunity for automated systems to adopt a standard gap (e.g. one car length) and control a car to accelerate/decelerate without disrupting the passengers comfort when driving.

From previous results when the state-space representation was computed, it was observed that vehicles slowed down in the first 15 seconds. In general, all pairs of vehicles in the NGSIM data
traveled at the slowest velocity around the 10th second. This study is interested to observe how the proposed controller regulates the action by the drivers and vehicles after this critical point.

In this research, as long as the follower keeps its distance oscillating within a small range, the controller has regulated the driving behavior to its optimum performance. This means that the follower accelerates or decelerates according to the leader’s stimulus and subsequently the deviation in the position of both cars is reduced to its optimal. Optimal following distance can be translated to efficient traffic mobility with no disruption and no rear-end collisions. The purpose of this vehicle-following simulation is to improve simulation tools to help achieving this.

**Input**

The input to this system was considered in a form of a sinusoidal wave. The sine wave’s amplitude is 1 m with a period of 30 seconds. This external force \( u(t) \) applied to the system represents the gap oscillation in meters. The sinusoidal function is believed to represent the traffic motion in stop-and-go traffic.

Figure 5.31 illustrates the input given to all combinations of vehicles along with the control law used by the controller to regulate the behavior of the system and achieve a desirable output. It can be seen that the control law for trucks following trucks is very similar to the original input but it has shifted up to compensate any behavior not captured in the original input. In comparison, the control curves for a car following another car or a truck following a car, the shapes maintain a closer and similar form as the original input. When a car is following a truck the control law appears to lose the sine wave shape and modifies the input in order to achieve an optimal distance for this vehicle combination.
State Variables

A state of a system describes the current status of the internal elements of a dynamic system. In this case, the dynamic system is the combination of driver-vehicle whom makes decisions at every time step on how close or apart it should follow the vehicle in front. State variables are a subset of system variables that can be measured over time when an input is given. State variables account for the values of the internal components in a dynamic system and change over time independently to the output of the system. Since states variables change over time in a dynamic system, this dynamic response can be seen as a path traced in the state-space equation.

The state variables used in the analysis are gap between vehicles and relative velocity as stated in Eq. (5.34). These variables are commonly found in car-following models. Because these variables can indicate safe or risky interactions between vehicles, gap gap and relative velocity have a direct impact on safety and traffic operational performance. Thus, gap and relative velocity are the values from inside the system that will represent driver-vehicle system at any given time. These state variables will serve to inform the current state of a dynamic system.
The figures below show the gap and relative velocity used in the proposed model. Figure 5.32 describes the combination of car following car (car-car). Figure 5.33 shows the dynamic change of state variables when the system is composed of a car following a truck. Figure 5.34 describes the changes in gap and relative velocity for truck following trucks. The last figure (Figure 5.35) indicates the state variables that account for trucks following cars. State variables indicate the adjusted behavior performed by the controller and state estimator when modeling vehicle-following. Small values are obtained because this model intends to adjust velocity at a comfortable rate for the driver and still following at a safe distance.

As can be observed, followers driving a car have a curve with a similar shape; almost replicating the sine input curve. This curve indicates that cars were regulated to increase their spacing and relative velocity in the first 15 seconds. Then, the system modulated the state variables to get closer to the leading vehicle after by decreasing the gap and subsequently the relative velocity. On the other hand, trucks have an S-shape curve when following other vehicles. This means that trucks maintained a constant, yet greater, spacing and relative velocity after the first 15 seconds. This is only the case when trucks follow other trucks. When trucks follow cars, there is only an increasing gap and relative velocity to keep up with the traffic.
Figure 5.32: Changes in state variables for car following car.

Figure 5.33: Changes in state variables for car following truck.
The relationship between gap and relative velocity is shown in Figure 5.36. Car-following models are based on the assumption that vehicles perform continuous adjustments in terms of distance and relative velocity. Adjustments on driving behavior are influenced by the driver’s perception of how
close or apart the leading vehicle is. Driving behavior that is graphically represented by gap and relative velocity results in “spirals”. Negative relative velocity values indicate the vehicle is closing in and reducing the gap with the leading vehicle. Positive velocity values denote vehicles getting apart and increasing the following distance. This “natural” behavior is seen in cars following another car or truck, but, it is not reflected on trucks. As previously mentioned, trucks are constantly increasing their relative velocity and therefore, increasing their distance gap. The behavior observed in trucks is influenced by vehicle dynamics and capabilities influencing driving behavior in a short period of time.

![Figure 5.3: Relative velocity versus gap.](image)

In the proposed method, the states of the system (gap and relative velocity) are calibrated through the controller in order to find the suitable values that maintain a reasonable distance between the two vehicles. For illustration purposes, the actual distance of followers in the different vehicle combinations are included in the following graphs. Actual behavior has been factored by 10 in order to fit in the same graph without minimizing the details of the output. Although curves representing real gap
may seem to have a smoother path, their changes are 10 times bigger than the scaled curve shown in the figures (Figures 5.37 to 5.40).

**Output Equation**

In this section, the interaction of cars is analyzed in terms of the closing or opening of the gap between the follower and the leader. This gap illustrates how the actions of the follower were adjusted through a controller to keep up with the vehicle in front. Table 5.10 provides the ranges (minimum and maximum) of the change in gap between vehicles.

<table>
<thead>
<tr>
<th>Vehicle-Following</th>
<th>Minimum Change in Distance (m)</th>
<th>Maximum Change in Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car following Car</td>
<td>-0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Car following Truck</td>
<td>-0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Truck following Truck</td>
<td>-0.01</td>
<td>0.9</td>
</tr>
<tr>
<td>Truck following Car</td>
<td>-0.05</td>
<td>1</td>
</tr>
</tbody>
</table>

The change in following distance (gap) for cars following cars is represented in Figure 5.37. The range in the oscillation varies from 0.2 meter to -0.5 meters. Negative values mean that the follower is closing in to the leader from the initial following distance, and there is the risk of collision if no action is taken. Another interesting behavior can be found in the first 10 seconds of the simulation. Cars using this adaptive controller do not change the gap nor feel any incentive to accelerate or decelerate. This is only observed when the initial gap is comfortable for the follower and there is almost without movement during the first 10 seconds. After this period, it can be seen that the follower feels stimulated to make changes in its behavior. The oscillation keeps a steady variation with acceleration and deceleration translated into more and less gap accordingly. For the last period of the simulation, cars increased their gap with the car in front. This could be due to experiencing a different traffic setting (i.e. more
congestion or less congestion) where cars speed up or slow down and increase the gap. Overall, the controlled output has a similar pattern than the actual distance kept by cars in real traffic.

Figure 5.37: Optimal following distance for car following a car.

As observed from Figure 5.38, cars seemed to struggle more when following a different type of vehicle, in this specific case, a truck. The change in gap when a car follows a truck is mostly negative, meaning that the car tends to follow more closely (from the initial gap) if there is a truck leading. The change in gap ranges between -0.5 m (when getting closer) and 0.3 m (allowing more distance). This gap represents the optimum output according to the controller and state estimator. For the first 10 seconds, the vehicle is controlled to follow closely. It can be said that the simulation accurately predicted a reduction in gap which can be verified by the real case. The controlled behavior of the car averaged out the sensitive changes of drivers who suddenly increased their distance and after a short period of time had to follow closely again as observed in the first two seconds of the actual behavior. After this period of time, gap was adjusted to follow the truck in front and at the same time to compensate traffic changes by increasing the distance for the rest of the simulation.
Vehicle-following between trucks is characterized by larger gaps as can be concluded from the results shown in Figure 5.39. The gap between trucks is maintained positive and increasing most of the time, indicating that trucks are limited by their vehicle dynamics and thus allow for more space. The controller takes this into consideration and reaches a 0.9 m increase in gap which is much greater than the gaps observed in cars. The range of values varies between -0.01 m to 0.9 m. The -0.01 m gap represents the vehicle maintaining a constant gap. Afterwards the gap continuously increases but still adjustments are made at every time step. It is unexpected to see a mismatch between the controlled gap and the actual. Actual behavior describes more aggressive truck drivers who follow other trucks more closely. However, the controlled distance suggested in this analysis sustains a more conservative approach. That is to allow more space when following another truck. One common characteristic is that both actual and simulated gap has few oscillations. Most of the times, gap varies slowly and takes more time to make drastic changes.
For trucks following a car, the change in gap ranges between -0.05 m and 1 m as can be seen from Figure 5.40. The pattern of the gap in general has the same trajectory as the distance kept by trucks in real traffic. Both describe a truck following more closely during the first 10 seconds and increasing distance afterwards. This graph in particular has more oscillations present in comparison to trucks lead by trucks. The simulation results proposed that trucks have to adjust their gaps more often when following cars.

Figure 5.39: Optimal following distance for truck following a truck.
Figure 5.40: Optimal following distance for truck following a car.

5.4 Conclusions

This chapter investigated the application of modern controls in transportation. A closed-loop vehicle control system was used to represent driving behavior subject to vehicle dynamics of the car driven and followed. Vehicle-following for pairs of cars and trucks was considered to be a linear system that includes a leader (plant) and a follower (observer). As part of the analysis, internal parameters in the system were identified to describe driving dynamics of cars and trucks when following same and different vehicles. The use of a state estimator (KF) captured and modeled the future states of a pair of vehicles. Predicting future vehicle-following states such as, relative velocity and gap, allowed to address rear-end clearance. The optimal gap was estimated according to a change in following distance. In this way it was possible to model and study how following vehicles: (1) perceive a congested traffic environment; (2) process information emitted by the leading vehicle; and (3) identify the action (of the follower) to adjust its position.

This chapter has proven hypothesis 1 that cars and trucks are driven and followed differently and this leads to different optimal following gaps. The results from this chapter indicated that cars approach vehicles more closely than trucks. On the other hand, trucks tend to allow more space when following (hypothesis 3). This research has also demonstrated that vehicle dynamics influence the way vehicles are operated as hypothesized. Cars continuously change their gap to avoid collision when following cars or
trucks. Trucks adjust their following gap more often when following a car. This research proved that proper selection of following gap according to vehicle type of vehicle is critical for the system to replicate efficient vehicle-following behavior and reduce the risk of collision.
Chapter 6: Discussion of Developed Models

6.1 Relation between Approaches

The approaches taken to study car-following were based on statistical models and linear systems. Both approaches bring into light the importance of studying driving behavior to improve safety and operational performance on highways. On one side, statistical modeling identified driving movements, driver and road characteristics commonly present in collisions. Knowing the factors that caused crashes allowed one to predict the consequences of “what-if” interactions. On the other hand, simulating vehicle-following at a microscopic level by means of state-space representation helped one to understand the constant position adjustments of vehicles required to avoid collision.

Despite differences in modeling techniques, both approaches adopted car-following behavior to perform the analysis. Whereas statistical modeling identified driving actions before colliding with the leading vehicle, linear systems focused on the minimum safe distance to follow. In statistical models common responsive actions of vehicles before colliding were described as speeding, braking and not perceiving slower traffic. Similarly, linear system simulations followed gap, velocity and acceleration to model car-following behavior.

Another similarity shared by both approaches is that pairs of vehicle were grouped into four different combinations: car-car, car-truck, truck-truck, and truck-car. Both simulations emphasized on the actions taken by the follower, or striker, to describe vehicle-following interactions. Additionally, dynamical properties of vehicles were taken into consideration in both approaches. Statistical analysis looked into the pre-crash actions of drivers. Explanations on why a maneuver was performed are directly related to size and capabilities of the vehicles. Limited visibility, slower breaking capabilities of truck drivers were found to be possible factors influencing truck related crashes. On the other hand, vehicle specifications of cars and trucks were taken into consideration when designing a controller. Mass, length, center of gravity, moment of inertia, accounted for specifying vehicle capabilities for each combination analyzed using control systems.

Common findings related to driving behavior were also obtained, although two different approaches were considered. Findings from statistical and control systems indicated that interactions
between cars and trucks are more problematic. Different combinations of vehicles in a statistical modeling resulted in rear-end collision crashes. Mostly, because vehicles of different types did not realized the other vehicle in front was stopping or decelerating. Vehicle following models based on linear systems allows the coding of vehicle dynamic properties in the state-space equations.

6.2 Research Contributions

This research incorporates statistical modeling and modern control systems to analyze vehicle-following behavior. This represents an improvement of current car-following simulation tools by using advanced theories from other disciplines. Transportation analysis through statistical modeling and modern linear systems provides an improvement of vehicle-following by considering human actions before a crash, preferred and optimal driver’s gap choice and vehicle characteristics. This can be achieved by understanding human factors in risky situations and driving capabilities of vehicles using statistical modeling and modern controls, respectively.

The combination of both approaches ensures a comprehensive analysis of driving behavior from the safety and mobility aspects. This is the first contribution from investigating vehicle interactions. Safety is the main the priority of transportation agencies. The safety contributions of this research target crashes on highways. These collisions resulted because of failed driving maneuvers and failing to respond with other types of vehicles. Separating the roles of drivers in crashes (striking or struck) contributes to the understanding of follower’s behavior. The scope of work not only identified the factors that contribute to same-direction manners of collisions but also performs an analysis of responsive actions consciously or unconsciously made before collisions. This investigation is one of the few that emphasize the importance of investigating truck and car crashes. Fatalities, severe injuries and costly accidents are some of the unfortunate yet common outcomes from truck-car crashes. This research has drafted some safety policy implications of truck and car crashes that look into accommodating different vehicles. Modifying lane widths, enforcing driving rules, accessibility of technology devices on vehicles, and etc. are some indications arising from this research.

At the same time, when drivers fail to change velocity smoothly while following too close, it generates a disruption in the flow that is aggravated in congested traffic. Several car-following models
have been proposed for the past sixty years. Nevertheless, increased volume of trucks and cars in addition to the heterogeneity of vehicle types present in highways requires an alternate car-following approach. This research has designed a controller that can regulate velocity transitions in an effective manner. That is, that the controller will adjust the gap when following without crashing the vehicle in front. By doing this, drivers drive safe on highways.

The research performed in this dissertation recognized the different driving behavior and vehicle dynamics of cars and trucks that must be included in simulation tools. In addition, understanding driving behavior specific to car and truck drivers helps modeling complex vehicle controllers used for adaptive cruise control. Adaptive cruise controls are based on in-vehicle sensors that can detect vehicles in the surroundings using technologies such as cameras, radar, LIDAR, etc. Acceleration/deceleration adjustments are made based only on an estimation of how far away the vehicle in front is without recognizing if the other vehicle is a car or a truck. This research provides evidence on the variability of driving behavior when different vehicles are paired.

Driving behavior based on what type of vehicle is leading and following has not been integrated in automated cruise controls. Such limitation ignores driver’s comfort when following a truck or a car. This research proposes that driver comfort is related to vehicle capabilities. Accelerations and deceleration responses are slower for trucks in comparison to cars. Findings from this analysis suggested that car and truck driver are aware of the differences on vehicle dynamics and adjust their following distance based on this. This research built a controller that regulates the gap between two vehicles. The minimum change in gap can be implemented in current vehicles that allow the user to choose a short, medium and long gap that is comfortable to the driver when activate the automated cruise control. The controller designed in this analysis, recognizes what type of vehicle is followed, allows the vehicle to consider the gap set by the user and make the appropriate modifications. This is possible because of a computational intelligence that compares how similar the capabilities of the vehicle in front are with the vehicle driven.

Consideration of restricted traffic movements where variability of driving behavior is unforeseen and risk of collision is imminent is another contribution made by this research. Automated cruise
controls are usually activated when vehicles are traveling at a high velocity and can change lanes if they do not want to follow the leader. The proposed controller takes into consideration follow-the-leader behavior in congested traffic where drivers are forced continue in the same lane. By doing this, it was possible to calibrate and validate vehicle-following parameters that reflect real congested traffic conditions. The developed parameters are a significant contribution to micro-simulations. Estimating parameters exclusive to driving behavior according to types of vehicles paired improves the accuracy of simulation.
Chapter 7: Conclusions and Recommendations

7.1 Research Summary

Statistical models helped to understand the complexity of human actions when driving. Reported crashes between cars and trucks served as a platform to investigate what caused such collisions from the perspective of driving interactions. Statistical modeling provided insightful information about close interactions of cars and trucks. By grouping pair of vehicles according to their type, that is car or truck, it was possible to identify specific actions taken before crashing into another vehicle. The analysis performed in this research investigated possible types of collisions when traffic travels in the same directions. Because of this, rear-end, angle and sideswipe collisions, were studied based on vehicle-following and lane changing actions. Roles of the vehicles in the crash denoted as striking and struck vehicle were also incorporated in the study. By doing this, the study of failed interactions allowed to traced back critical driving movements and classified them as failed responses (for striking vehicles) and unanticipated actions (initiated by struck vehicles), when applicable.

A general conclusion suggests that driver characteristics, environmental circumstances, road features, and vehicle attributes affect the outcome of a crash. This means that distractions, human perception, geometric road design, type of vehicle driven, etc., can jeopardize safe interactions. It was interesting to select only crashes that occurred under normal driving conditions (e.g. no extreme weather, at daylight, dry road surface, etc.). This has allowed the study of the relationship between human actions and constrains imposed by vehicle dimensions and capabilities. Human errors, such as, not realizing the vehicle in front stopped or decelerated, or getting into the next lane without noticing, were some of the crucial findings in the analysis. Separating the equation into the type of vehicle driven permitted to realize vehicle capabilities and limitations of cars and trucks. Drivers in cars were unable to follow trucks in front because of misperceptions of distance. But when following a car, cars were less likely to rear-end the car in front. The same finding was obtained when the roles are reversed. Trucks also failed to adjust to the actions made by the car in front. This research demonstrated that when cars and trucks travel in different lanes, trucks were unable to detect cars in blind spots and they ended up running into them. Trucks also struggled driving next to other trucks. Size may also be influencing.
collisions because of vehicular stability and aerodynamics of trucks. In conclusion, all the findings from the statistical analysis suggest that driving actions of trucks and cars that led to collisions are influenced by their vehicle capabilities. This indicates car and trucks behave differently in terms of the comfortable gap and left for other vehicles and comfortable travel velocity. Following too close and speeding, for example, were common indicators of failed car-following interactions.

Linear system is a crucial part of this research because corrective measures can be modeled to enable follower to avoid a collision. Understanding driving behavior in heavy traffic represents an ideal scenario to focus on car-following. This is why traffic at evening rush hours was selected for the study in order to investigate driving behavior under a congested regime. Acceleration and deceleration rates were present as expected in stop and go scenarios. Driving in congested scenarios makes drivers consciously follow the lead vehicle. Because of the risk of crashing into the vehicle in front, in addition to, frequent stops in the traffic, modulating car-following behavior represents an opportunity to address safety and operational issues by linear systems model.

Differentiation between cars following the same type of vehicles or trucks, as well as, trucks following other trucks or cars was also made under this approach. Because of this, it was possible to regulate driving behavior based on type of vehicle driven and followed. Driving behavior differences in cars and trucks were also present in terms of how close or apart to follow to achieve a safe and efficient. After understanding how cars and trucks interact in real life, it was possible to identify the following distance that most vehicles are comfortable with based on who is leading them. Real traffic data indicate truck drivers allow greater gaps when following trucks than cars. Cars were more comfortable keeping shorter gaps than trucks; in particular, cars follow other cars more closely than when following trucks. Based on this fact, a modern control system was designed to account for driver preferences, as well as, vehicle dynamics related to mass, size and inertial properties. The results indicated the corrective actions made by the system according to distance kept between vehicles to adjust to sudden changes in traffic flow. The output distance from the simulated scenario followed same distance preferences. However, it ensured a more efficient following distance by making changes more wisely. This implies that drivers
with this type of controller built into their vehicles will approach other vehicles in a safer and more efficient manner.

7.2 Research Limitations

Statistical analysis performed in this study can be improved if data is collected to target specific driving behavior. For, instance, data used for statistical analysis was taken from police crash reports. Testimonies given by parties involved in a crash may be inaccurate because of the fear to be blamed. Judgments made by police officers to classify manners of collision may also not be precise. Errors in the data may come from using different sources to compile national statistics. Regions may follow their own guidelines when making a crash report. Another limitation was that social, economic and demographic characteristics of the driver were unknown. In order to convey accurate information, it may be necessary to limit the study to a single region, or state. By doing this, information may describe driver and road characteristics better and results may be more specific.

Data used for vehicle-following simulations also present limitations. The NGSIM data was collected using cameras mounted at the top of buildings. Because of this, only the top view and a small road segment were considered. Knowing what the driver is looking ahead, as well as, selecting larger road segments may provide additional information to analyze traffic scenarios before, during, and after congestion. In addition, driver attributes cannot be identified from the video recordings. Another shortcoming is that video processing introduced errors in the data.

One possible solution is to use instrumented vehicles that records driving behavior in congested scenarios to capture driving behavior and actions performed when driving. Naturalistic driving behavior could be another source to gather information about drivers that were involved in a collision or that successfully evaded a crash. Under this study, cameras allow to see what happen before crashing or almost running into another car. At the same time, following behavior can be captured in terms of gap and velocity.

The limitation when developing a vehicle-following control using available data was the presence of stopped traffic. Low velocity values that are close to zero, could not be incorporated in the simulation because unstable values were obtained in A matrix and B vector elements. Therefore, the
number of vehicles considered in the simulation was reduced and considered only those who did not completely brake.

Although this research analyzed human perception using statistical modeling, the incorporation of human reasoning was not incorporated in the control. A block diagram that describes human behavior is a crucial element in the analysis to include human in the loop.

7.3 Further Work

This research has the potential to be expanded to examine connected vehicle technologies. To do this, vehicle-to-vehicle communication will require to model actions performed by leading vehicles and transmit the vehicle type to follower. Both leader and follower are included in the closed-loop system in this research but only followers were targeted. Further research can extend the scope of this study to look into regulating the actions of the leader who at the same time needs to keep up with the vehicle ahead. Vehicles in the middle of a platoon may need to adjust driving behavior with the vehicle ahead and at the same time prevent follower to tailgate.

This research can also be tailored to study driving behavior specific to trucks. One possible research can focus on truck convoys that travel longer distances. Looking into automated trucks that form a platoon could be a subject of interest for private truck companies.
References


Vita

Alicia Romo was born in El Paso, TX and has lived in the region all of her life. Alicia has a Bachelor of Science in Civil Engineering from The University of Texas at El Paso (UTEP) and is a registered Engineer In Training. Alicia Romo will graduate from The University of Texas at El Paso with a Civil Engineering PhD. Alicia has been a civil engineering teaching assistant and a graduate researcher at UTEP Border Intermodal Gateway Transportation Lab from 2009 to 2013. For the past two summers, Alicia has worked for the Federal Highway Administration as a research engineer intern at the Turner-Fairbank Highway Research Center (TFHRC) in Mclean, Virginia. She has recently been offered a post-doc position at the Safety Operations Department at the TFHRC. Her research interests are: car-following modeling, control systems, and human factors. Alicia’s personal goal is to encourage the participation of more women in civil engineering research.

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