On Different Techniques For The Calculation Of Bouguer Gravity Anomalies For Joint Inversion And Model Fusion Of Geophysical Data In The Rio Grande Rift

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ON DIFFERENT TECHNIQUES FOR THE CALCULATION OF BOUGUER GRAVITY ANOMALIES FOR JOINT INVERSION AND MODEL FUSION OF GEOPHYSICAL DATA IN THE RIO GRANDE RIFT

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Para mi
familia
con amor

Sin ustedes
nada de esto
seria posible
ON DIFFERENT TECHNIQUES FOR THE CALCULATION OF
BOUGUER GRAVITY ANOMALIES FOR JOINT INVERSION AND MODEL
FUSION OF GEOPHYSICAL DATA IN THE RIO GRANDE RIFT

by

AZUCENA ZAMORA

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Abstract

Density variations in the Earth result from different material properties, which reflect the tectonic processes attributed to a region. Density variations can be identified through measurable material properties, such as seismic velocities, gravity field, magnetic field, etc. Gravity anomaly inversions are particularly sensitive to density variations but suffer from significant non-uniqueness. However, using inverse models with gravity Bouguer anomalies and other geophysical data, we can determine three dimensional structural and geological properties of the given area. We explore different techniques for the calculation of Bouguer gravity anomalies for their use in joint inversion of multiple geophysical data sets and a model fusion scheme to integrate complementary geophysical models. Various 2- and 3- dimensional gravity profile forward modeling programs have been developed as variations of existing algorithms in the last decades. The purpose of this study is to determine the most effective gravity forward modeling method that can be used to combine the information provided by complementary datasets, such as gravity and seismic information, to improve the accuracy and resolution of Earth models obtained for the underlying structure of the Rio Grande Rift. In an effort to determine the most appropriate method to use in a joint inversion algorithm and a model fusion approach currently in development, we test each approach by using a model of the Rio Grande Rift obtained from seismic surface wave dispersion and receiver functions. We find that there are different uncertainties associated with each methodology that affect the accuracy achieved by including gravity profile forward modeling. Moreover, there exists an important amount of assumptions about the regions under study that must be taken into account in order to obtain an accurate model of the gravitational acceleration caused by changes in the density of the material in the substructure of the Earth.
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Chapter 1

Introduction

Geophysical problems dealing with imaging of the Earth structure and determining its processes and evolution have seen a growing interest in the scientific community throughout the last decades. This endless need to accurately determine the physical and geological properties of the Earth has made the quest for novel methodology for the solution of applied inverse problems in the area a relevant topic in geophysics. Increasing needs for fossil fuels and water, environmental issues with pollutants, and earthquake risk evaluations make the 3-Dimensional modeling of the Earth’s structure a critical mission for scientists and governmental agencies (Sharma, 1997).

There are different types of geophysical datasets that can be used for modeling of the Earth’s structure: receiver functions, surface wave dispersion, gravity anomalies, and magnetotellurics (MT), among others. Each dataset has its own characteristics and focuses on particular aspects dealing with specific physical and geological properties of the Earth. These datasets are often classified as being seismic and non-seismic depending on the nature of the observations (coming from earthquakes and controlled explosions or coming from potential fields). However, they all have something in common: they were designed as tools to detect discontinuities and changes in subsurface’s investigations and their power lies on determining where underground regions differ sufficiently from their surroundings in terms of physical properties such as density, magnetic susceptibility, conductivity, or elasticity (Sharma, 1997). Models of the Earth’s structure in a given region can be calculated using these geophysical datasets obtained from observations generated at specific locations in the area of study and determining where the changes in structures occur according to the chosen physical property.

Important issues to consider in geophysical problems are non-uniqueness and noise in the observations. Non-uniqueness refers to the fact that there are usually an infinite number of
models of the Earth that fit the given observations while noise refers to errors arising from faulty instrument readings and/or numerical round-off. The existence of these two setbacks dealing with real world geophysical inverse problems indicate that given two different datasets modeling the same region using the same kind of information–i.e., teleseismic P-wave receiver functions and surface wave dispersion velocities characterizing the Earth as a layered structured parameterized using seismic shear velocities–these usually result in two different models of the Earth that highlight different aspects of the given structure. However, given the characteristics of each one of these surveys and their results, it may be beneficial to use a combination of geophysical techniques in order to improve the reliability and accuracy of the final model. The use of multiple datasets depends on their complementarity, the type of physical property to be highlighted, and the characteristics of the structure to be modeled. Moreover, additional information obtained from the combination of multiple datasets has been proven to be very helpful in ultra-deep water exploration and sub-salt and sub-thrust exploration where imaging using only seismic information encounters significant problems (Tartaras et al. 2011). By using various types of observations, the extent of the ambiguities and uncertainties related to each of the individual datasets may be reduced. With this in mind, there have been many efforts in recent years that show that the use of different complementary datasets obtained from a region can constitute an improvement for exploration techniques as research of increasingly complex geological environments are explored (Moorkamp et al. 2010; Julia et al. 2000; Heincke et al. 2006; Vermeesch et al. 2009).

Another important aspect to consider is how datasets are processed in order to obtain the optimal geophysical representation that meets the corresponding physical and geological properties. The level of integration of multiple geophysical datasets has evolved throughout the years, going from sequential cooperative inversion–in which one set of data is inverted independently and the result is used to constrain the subsequent independent inversion of the second set of data–to different types of joint inversion, or simultaneous fitting, of multiple datasets. In a joint inversion scheme, the multiple datasets may be sensitive to the same physical property, responsive to different physical properties but related by an established analytic relationship, or responsive to different physical properties without an analytic relationship available between
the properties (also called disparate datasets) which instead enforce structural or compositional similarities between the property models (Gallardo et al. 2004). Each one of these schemes has been shown to improve the Earth models obtained for the given regions when compared to the results obtained from their corresponding single dataset inversions.

Throughout the years, seismic information has been the principal component of a vast majority of research explorations for imaging the subsurface. However, it has been shown that non-seismic methods such as electromagnetic (EM) methods and gravity and magnetic fields and gradient measurements, have characteristics that provide additional information that can be used to further constrain the proposed Earth models obtained using purely seismic observations (Tartaras et al. 2011). The objective of this work is to build foundations necessary to analyze gravity anomaly information obtained from the Rio Grande Rift (RGR) region. This work analyzes three different forward model techniques in order to determine the most effective 2- or 3-dimensional gravity forward modeling method that can be used in combination with the joint inversion scheme for receiver functions and surface wave dispersion datasets as proposed by Sosa et al. (2013a, 2013b) and a model fusion scheme for seismic and gravity datasets as proposed by Ochoa et al. (2011). A description of each one of them, implementations through the use of synthetic and real data obtained from the Rio Grande Rift (RGR), and a discussion on the best alternative are included.

The methodology explained here is part of an effort aiming to implement a constrained optimization method to improve the results obtained from the joint inversion of two geophysical datasets, i.e., teleseismic P-wave receiver functions and gravity anomaly data, in order to determine a consistent 3-D Earth structure model that meets the properties of all the involved individual datasets obtained from the RGR at once.

1.1 Background

The level of integration of multiple geophysical datasets has evolved throughout the years, going from sequential cooperative inversion to different levels of joint inversion. Literature on each one of these schemes has shown that there are improvements on the Earth structure
models obtained for the referenced regions when compared to the results obtained from the corresponding single domain inversions.

Previous work on cooperative sequential and joint inversion schemes include:

- Lines et al. proposed a cooperative sequential inversion scheme using surface and borehole observations of seismic and gravity responses (1988).

- Li et al. proposed a joint inversion scheme of surface and three-component borehole magnetic data (2000).

- Heincke et al. uses magnetotelluric (MT), gravity, and seismic data for joint inversion (2006)

- Tartaras et al. constrained MT models using well log resistivities (2011)

- Moorkamp et al. proposed the joint inversion of receiver functions, surface wave dispersion, and magnetotelluric data (2010) and a 3-dimensional joint inversion for seismic, MT and scalar and tensorial gravity data (2011).

- Sosa et al. (2013a, 2013b) proposed a novel approach to jointly invert surface wave dispersion and receiver functions data by using interior point method optimization techniques where constraints (obtained from the geological information available and the physical properties of the region) are used in the formulation of the problem.

- Ochoa et al. (2011) proposed an approach to fuse the Earth models coming from different datasets such as gravity and seismic information. The importance of this model fusion technique is more evident when the resulting models have different accuracy and/or spatial resolution at different depths given that the reliability of each one of the techniques is increased or decreased at different levels (using weights).

The key in the use of multiple datasets for sequential or joint inversion is finding the types of data that will contribute the most to the final Earth structure model. These datasets should provide sufficient information to determine a fair approximation of the actual substructure when used individually, but should represent an even greater advantage when used together.
Moreover, Julia et al. proposed that for sequential or joint inversion of two independent datasets to provide a meaningful estimate of the Earth’s structure, the datasets used in the inversion should be consistent and complementary (2000). Consistency requires that “both signals sample the same portion of the propagating medium” (so that the information contained in the waveforms sample the same part of the Earth) while complementarity “refers to the desire that the joint data improve the constraints provided by each independent dataset” (Julia et al. 2000). Table 1.1 shows the different types of information that can be used for the imaging of the Earth and the physical property that is the most sensitive for each one.

<table>
<thead>
<tr>
<th>Method</th>
<th>Measured parameter</th>
<th>Operative physical property</th>
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<tbody>
<tr>
<td>Seismic</td>
<td>Travel times of reflected/refracted seismic waves</td>
<td>Density and elastic moduli, which determine the propagation velocity of seismic waves</td>
</tr>
<tr>
<td>Gravity</td>
<td>Spatial variations in the strength of the gravitational field of the Earth</td>
<td>Density</td>
</tr>
<tr>
<td>Magnetic</td>
<td>Spatial variations in the strength of the geomagnetic field</td>
<td>Magnetic susceptibility and remanesc</td>
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<tr>
<td>Electrical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistivity</td>
<td>Earth resistance</td>
<td>Electrical conductivity</td>
</tr>
<tr>
<td>Induced polarization</td>
<td>Polarization voltages or frequency-dependent ground resistance</td>
<td>Electrical capacitance</td>
</tr>
<tr>
<td>Self-potential</td>
<td>Electrical potentials</td>
<td>Electrical conductivity</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>Response to electromagnetic radiation</td>
<td>Electrical conductivity and inductance</td>
</tr>
<tr>
<td>Radar</td>
<td>Travel times of reflected radar pulses</td>
<td>Dielectric constant</td>
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</table>

It can be determined that seismic and gravity surveys are both sensitive to density changes in the substructure of the Earth. Given that all geophysical methods are limited by their inherent non-uniqueness and uncertainties, it may be beneficial to use these two datasets in combination to improve the results obtained from single domain inversions by limiting the range
of velocity and density models to those that fit the data equally well (Vermeesch et al. 2009). By the simultaneous use of gravity and seismic surveying, “ambiguity arising from the results of one survey method may often be removed by consideration of results from a second survey method” (Kearey et al. 2002) which would represent improvements on the velocity and density distributions with respect to the single inversion models.

Gravity and magnetic surveys, also called “potential fields” surveys, are used to give an indirect way to determine the Earth’s substructure by analyzing the the density and magnetization of rocks respectively (USGS, 1997). Some of the benefits of these explorations include their ability to locate faults, mineral and petroleum resources, and ground water, the relatively low cost (with respect to other seismic surveys), and the large areas of ground that can be quickly covered (USGS, 1997). Gravity information is obtained by measurements of the Earth’s gravitational acceleration (also called gravity or gravitational field) across the survey area. Changes in the gravitational field usually correspond to discontinuities or changes in the subsurface features. Given that “gravity anomalies decrease in amplitude and increase in wavelength with increasing depth”, this type of exploration technique has its greatest resolving power in the shallow substructures (Maceira et al. 2009). The information obtained from gravity anomalies is often used to determine the constraints of variations in the rock densities of the substructure (Maceira et al. 2009) and to obtain additional information of the area to fill in gaps in seismic coverage and track regional deep structures (Heincke et al. 2006; Vermeesch et al. 2009).

With respect to seismic information, the most commonly used techniques are receiver functions (RF), surface wave (SW) dispersion, and travel times in reflection and refraction tomography. Their characteristics are:

- Receiver functions: Commonly used to determine the Moho and other discontinuities in seismic velocities in the crust and upper mantle through the identification of P to S conversions in teleseismic data. Receiver functions are time-series that are sensitive
to the structure near the receiver (Julia et al. 2000) where “time is a proxy for depth and significant positive or negative amplitudes correspond to an increase or decrease in seismic velocity, respectively” (Moorkamp et al. 2010). The primary sensitivity of receiver function inversions is to velocity contrasts and relative travel time and they are used to constrain small-scale relative shear-velocity (Ammon et al. 1990).

- Surface wave dispersion: Given that surface wave dispersion is primarily sensitive to seismic shear wave velocities, variations in shear velocity are the usual parameters for this type of model in inversion studies. Surface waves are ideal to study the structure of the crust and upper mantle. The variations in shear wave speed can be determined with good vertical resolution by using periods shorter than around 40 seconds for strong sensitivity in the crustal structure and longer periods of waveform for an increase sensitivity within the upper mantle (Moorkamp et al. 2010). Having good path coverage makes it possible to obtain a reasonably good lateral resolution with few seismic stations in order to model the horizontal propagation of surface waves (Moorkamp et al. 2010).

- Reflection tomography: Reflection tomography looks at the propagation through the Earth of the waves originated at source points (controlled explosions). Through its use, reflections of the waves at boundaries separating two rock layers of different physical properties are captured at the Earth’s surface by geophones. Arrival times of the reflections are used to determine a subsurface velocity model to calculate synthetic traveltimes that best match the “picked” traveltimes through the use of raytracing. This type of survey is best used to resolve the shallow subsurface. The horizontal resolution of reflection tomography is greater than the vertical resolution (Etgen, 2004).

- Refraction tomography: Provides deeper models and estimates directly the velocity of the compressional P-wave (Re et al. 2010). Refraction based techniques make use of the body wave energy that is refracted in the near surface and observed in seismograms
as first arrivals. Refraction tomography is used to determine the velocity profiles of a region’s subsurface through the analysis of the fastest raypaths associated with first-break arrivals with which an estimate of the compressional wave velocity, $v_p$, can be calculated (Re et al. 2010).

Researchers need to determine the focus of their studies and, based on that, choose the seismic survey that best images the Earth’s substructure of interest (e.g., salt dome, aquifer, oil deposit, etc).

Gravity field and seismic information depend mainly on two different physical properties, rock densities and seismic wave velocities. These properties can be related through the use of empirically derived equations obtained from laboratory experiments and well log data; depending on the type of rock and its location’s depth, it may be convenient to use the Nafe-Drake (Nafe et al. 1963) for sedimentary rocks and a linear Birch’s law for denser rocks (in the basement) (Maceira et al. 2009).

1.2 Motivation

Recent work proposed by Sosa et al. (2013a, 2013b) supports the idea of using complimentary geophysical information of the Earth structure through the joint inversion of Earth models coming from different datasets.

Some of the advantages of the proposed joint inversion technique are:

- It represents a more accurate scheme for the use of complimentary geophysical information.

- It provides improvements of accuracy and/or spatial resolution in different areas of coverage of gravity, geologic, and seismic data.

- It allows an easier manipulation of different types of data with respect to other joint inversion schemes by placing constraints instead of weighting schemes.
• It increases the resolution of the 3-D model of the Earth obtained as a result, given the complementary and consistent nature of the inverted models.

The work proposed by Sosa et al. consists on the implementation of a joint inversion least-squares algorithm (LSQ) for the characterization of one-dimensional Earth’s substructure through the use of seismic shear wave velocities as a model parameter (2013a). The geophysical datasets used for the inversion are Receiver Functions (RF) and Surface Wave (SW) dispersion velocities (both sensitive to shear wave velocities). The novel methodology used in the joint inversion consists on posing the problem as a constrained minimization problem and using Primal-Dual Interior Point (PDIP) methods as the optimization scheme (Sosa et al. 2013a). Through the use of synthetic crustal velocity models and datasets obtained from the Rio Grande Rift (RGR), Sosa et al. were able to conduct numerical experimentations and conclude that PDIP method provides a “robust approximated model in terms of satisfying geophysical constraints, accuracy, and efficiency” (2013a). Given that both RF and SW dispersion datasets are obtained from seismic surveys, we would like to include gravity information in the implementation of the joint inversion using PDIP methods in order to further constrain the model for the RGR.

Another approach that we would like to implement for the complementary use of seismic and gravitational acceleration datasets is proposed by Ochoa et al. (2011). In this paper, the authors describe a novel technique to fuse the models obtained from different types of seismic and non-seismic information rather than working on the inversion of datasets as in joint inversion and cooperative inversion techniques. The fusion scheme starts with an estimate model originated from gravity measurements (with have a lower spatial resolution than seismic information) that covers a considerable depth and an estimate model originated from seismic measurements (with the higher spatial resolution) that covers depths above the moho surface. The accuracy and uncertainty of the information obtained from both seismic and non-seismic models of the given region is also taken into consideration in the model fusion technique which
can be important aspects of the data available. The result obtained from model fusion is an Earth model that takes into consideration the spatial resolution, accuracy and uncertainty of both original models and provides an improved model that takes combines their inherent characteristics for an improved Earth model.

In order to continue the work for joint inversion and model fusion, the appropriate forward gravity model has to be chosen to calculate the differences (through the calculation of root mean square RMS) obtained between observed and calculated gravity anomalies of the region. The purpose of this work is to show the different techniques available in the literature for the mathematical calculation of changes in gravity generated by anomalous masses. We explain the most common techniques proposed by Talwani (1959), Telford et al. (1990), and Plouff (1976) and discuss the most helpful scheme to use in the joint inversion and model fusion techniques through their implementation using synthetic data.

1.3 Potential Fields

There are different types of potential fields that can provide information about the formations found in the subsurface of the Earth including those dealing with gravitational, electric, elastic, electromagnetic, nuclear, and chemical potential energy. All these types of energy are associated with the position of bodies in a system and measure “the potential or possibility for work to be done” within the system (Young et al. 2004). For the purposes of this thesis, the focus will be solely on the gravitational potential field and the potential energy associated with the gravitational force that acts on a body and depends only on the body’s location in space. The factors that affect the gravitational potential energy of a body are its location with respect to a reference point (e.g., the Earth’s center of mass), its mass, its density, and the strength of the gravitational field surrounding it.
1.3.1 Gravity Data

Newton’s Law of gravitation is the basis for gravity prospecting methods dealing with changes in the lateral distribution of density in subsurface geology. The law states that there is a force of attraction between two particles of mass \( m_1 \) and \( m_2 \) which is directly proportional to the product of the masses and inversely proportional to the square of the distance between the centres of mass (Telford et al. 1990). This is represented in the relationship

\[
F_g = G \frac{m_1 m_2}{r^2}
\]

where \( F \) is the force of attraction between the masses, \( G \) is the universal gravitational constant \((6.6725985 \times 10^{-11} \text{ m}^3 \text{ kgs}^{-2})\), and \( m_1 \) and \( m_2 \) represent the masses in the system.

Assuming the Earth has a spherical form with mass \( M \) and radius \( R \), the force exerted by the Earth on a point mass, \( m \) resting on the Earth’s surface is

\[
F_g = G \frac{m M}{R^2}
\]

According to Newton’s second law of motion, the acceleration \( a \) of a body is parallel and directly proportional to the net force \( F \) acting on the body and inversely proportional to the mass \( m_1 \) of the body. This relationship is represented by:

\[
a = \frac{F}{m}
\]

Using Equation (1.2) in this context, it is possible to calculate the acceleration of a point mass \( m_2 \) due to the presence of mass \( m_1 \). In particular, if \( m_1 \) is considered to be the mass of the Earth \( M_E \), the acceleration of point mass \( m \) at the surface of the Earth can be found using

\[
g = G \frac{M_E}{R^2}
\]
Figure 1.1: Geometries for the gravitational attraction of (a) 2 point masses, (b) a point mass outside a sphere, and (c) a point mass on the surface of the sphere.

Diagrams of the geometric representations of different systems are shown in Figure 1.1.

This acceleration, called the gravitational acceleration, was first measured by Galileo in his famous experiment in Pisa (dropping objects from the tower to demonstrate that the mass of the objects didn’t affect their time of descent from the tower) and it is measured in cm/s$^2$ or gals. We can conclude then that the gravitation is “the force of attraction between two bodies”, such as the Earth and a body on the surface of the Earth; it’s strength “depends on the mass of the two bodies and the distance between them” (USGS, 1997).

Since the Earth’s shape is an oblate ellipsoid, the absolute value of the acceleration of gravity at the Earth’s surface is around 983.2 Gals near the poles and 978.0 Gals near the equator. A fraction of these changes are related to many known and measurable factors such
as location and elevation of the observation point, local topography, and tidal forces (USGS, 1997). Gravity surveys exploit the local variations in the gravity field related to the density distribution of rocks located near the surface (Telford et al. 1990); by doing this, high gravity values may help determine the location of rocks with higher density with respect to their surrounding area, while low gravity values are found above rocks with a lower density (USGS, 1997).

Scientists measure gravitational acceleration $g$ using gravimeters for absolute gravity or relative gravity. Gravity meters measure very small variations in this acceleration, hence, it is often preferable to use milliGals (1 milliGal = 1 mGal = 0.001 Gal) or gravity units (1 gu = 0.000001 m/s$^2$ = 0.1 mGal) for exploration purposes.

High resolution investigations can help determine the density distribution in the substructure of the Earth of a small area by using a small distance of only a few meters between measurement stations (Lowrie, 2007). For regional gravity surveys, where the identification of hidden structures of greater dimensions is the focus, the distance between stations may be several kilometers (Lowrie, 2007).

### 1.3.2 Corrections to Gravity Data

In several geophysical survey methods the local variation in a parameter, with respect to some normal background value, is the primary interest rather than the absolute fields (Kearey et al. 2002). In this case, corrections are made for the known factors affecting the variations and the anomalies that remain help geophysicists obtain information about the changes in density and mass that are of interest (Telford et al. 1990). These geophysical anomalies are normally “attributable to a localized subsurface zone of distinctive physical property and possible geological importance” (Kearey et al. 2002). Assuming a uniform density subsurface in the Earth, its gravitational field would be constant everywhere after the appropriate corrections have been applied. On the other hand, gravity anomalies would be any “local variation from the other-
wise constant gravitational field” resulting from any lateral density variation associated with a change of subsurface geology (Kearey et al. 2002). A brief description of these corrections is included next (additional details can be found in Appendix A).

**Instrument Drift and Tidal Effect Corrections** Instrument drift and tidal effect corrections are temporal based variations included in the observed acceleration measurements that are based solely on time changes. These changes in the observed acceleration would occur even if the gravimeter used for the survey was not moved from its original location at a base stations (Lowrie, 2007).

The *instrument drift* is the effect that a change in the gravimeter’s response over time has on the observed gravitational readings. Gravimeters are very precise and sensitive instruments; any minor readjustments in their internal mechanisms or external settings (e.g., temperature) would have an effect on the gravity readings over time. The correction for these changes can be approximated by using the best linear representation of the change in gravitational acceleration in the period of time between base observations.

The *tidal effect correction* relates to the Earth’s motions induced in its solid (the crust and the mantle) and liquid (the core and the oceans) materials and the changes in its gravitational potential by the tidal forces exerted by external bodies (e.g., the sun and the moon). The gravitational forces of the sun and the moon deform the Earth’s shape, and cause tides in the oceans, atmosphere, and body of the Earth (Lowrie, 1997). The effects of the tides on gravity measurements are well known and are often calculated and tabulated for any place and time before a survey is performed (Lowrie, 1997).

**Latitude Correction** The *latitude correction* is used to subtract the theoretical gravity acceleration expected at a given station based solely on its latitude position. The Normal Gravity Formula is used to calculate $g_n$ and assumes a uniform homogeneous elliptical Earth as its theoretical shape for calculation purposes (Lowrie, 1997). The Normal Gravity, $g_n$, is
subtracted from the absolute gravity on the reference ellipsoid using the Reference formula:

\[ g_n = g_e \left(1 + \beta_1 \sin(\lambda)^2 + \beta_2 \sin(2\lambda)^4\right) \] (1.5)

where \( g_e = 9.780327 \text{ m/s}^2 \), \( \beta_1 = 5.30244 \times 10^{-3} \), and \( \beta_2 = -5.8 \times 10^{-6} \).

Relative gravity with respect to a base station can be corrected by differentiating the reference formula, \( g_n \) with respect to \( \lambda \) such that a change in the distance from the base station would result in a change in gravitational attraction based on latitude given by the formula \( \Delta g_{\text{lat}} = 0.814 \sin(2\lambda) \) mGals per kilometer of displacement in the North-South direction. This correction would be subtracted from those stations closer to the pole than the base station.

**Terrain Correction**  Topographical structures, such as hills and valleys, can have an effect on the gravity acceleration measured at a point on the surface. In order to compensate for such effects, the terrain correction is applied to the area under study. The behavior of gravimeters near topographic features such as hills or valleys affects the measured gravity acceleration by decreasing the observed value of gravity. The effect of the topography on gravity acceleration is calculated and considered to always be positive. Hence, to compensate for the topography of an area, the terrain correction is calculated and added to the measured gravity (Lowrie, 1997). This correction works the same (positive terrain corrections \( +\Delta g_T \)) even if the topographic feature observed in the area is a valley (which represents a mass deficiency) instead of a hill (which pulls up on the mass in the gravity meter)(Lowrie, 1997). The magnitude of topographic corrections in mountainous regions can be as large as 10s of mGals hence the importance of these corrections (see Appendix A).

**Bouguer Plate Correction**  The Bouguer Plate Correction, \( \Delta g_{BP} \), applied after the terrain correction, compensates the observed gravitational acceleration for the difference in the layer of rock with thickness, \( \Delta h \) and density \( \Delta \rho \). Here \( \Delta h \) refers to the change of elevation of the gravity station where the measurement took place and the reference level (sea level of the
reference ellipsoid).

The formula for this correction is:

\[ \Delta g_{BP} = 2\pi G \Delta \rho \Delta h \]  

(1.6)

Assuming that density is given in kg/m\(^3\), \( \Delta g_{BP} = (0.0419 \times 10^{-3}) \rho \) mGals per meter. The use of a particular value of \( \Delta \rho \) depends on the region and can vary according to the bulk density of the area under study. This correction must be subtracted, unless the station is located below sea level (in which case a layer of rockl should be added to reach the reference level).

**Free-Air Correction** The *Free-Air Correction* is usually applied after the Bouguer Plate and relates to the change in height between the actual position of the station and its position on the reference ellipsoid. This is called the Free-Air correction because after the Bouguer correction has been applied, stations appear to be suspended in free air and not placed on the land. The correction takes care of the additional height of the station with respect to the actual survey and compensates for the decrease in gravity caused by the additional distance from the surface of the Earth to its center. This correction must be added for all stations above sea level. The formula for this correction is:

\[ \Delta g_{FA} = \frac{\partial}{\partial r} \left( -G \frac{m_E}{r^2} \right) = \frac{-2}{r_E} g_0 \]  

(1.7)

Using a value of \( g_0 = 9.8331 \text{ m/s}^2 \) (the mean sea-level gravity), \( m_E = 5.9736 \times 10^{24} \text{ kg} \) (the mass of the Earth), and \( r_E = 6.371 \times 10^6 \text{ m} \) (the Earth’s radius), the \( \Delta g_{FA} = 0.3086 \text{ mGals per meter of elevation} \).
1.3.3 Free-Air and Bouguer Anomalies

Assuming that the shape of the Earth is the reference ellipsoid and the distribution of density inside the Earth is homogeneous, if gravity was measured at the surface of the Earth, the observed gravity acceleration would be the same as the theoretical gravity acceleration (as given by the normal gravity formula in Equation (1.5)). The terrain, Bouguer, and free-air corrections presented previously are used to compensate for the actual situation of the gravity station given that it is usually not on the ellipsoid. Differences between the corrected measured gravity and the theoretical gravity are the so-called gravity anomalies. These anomalies are the result of density distributions in the subsurface that originate from the inhomogeneity of the Earth’s interior and are the basis to understand the internal structure of the planet.

There are two types of anomalies that are commonly used in the literature: Free-air and Bouguer anomalies. The difference between them is the type of corrections that are applied to the measured gravitational acceleration at the different stations.

The Bouguer gravity anomaly, $\Delta g_B$ is obtained by applying all the corrections described previously:

$$\Delta g_B = g_{obs} + (\Delta g_{FA} - \Delta g_{BP} + \Delta g_T + \Delta g_{tide}) - g_n \tag{1.8}$$

Here, $g_{obs}$ is the measured gravity value, $g_n$ is the theoretical (or normal) gravity value and the corrections used are free-air ($\Delta g_{FA}$), Bouguer plate ($\Delta g_{BP}$), terrain ($\Delta g_T$) and tidal ($\Delta g_{tide}$).

The Free-air gravity anomaly, $\Delta g_F$ is obtained by applying only the free-air, terrain, and tidal corrections to the observed gravity:

$$\Delta g_F = g_{obs} + (\Delta g_{FA} + \Delta g_T + \Delta g_{tide}) - g_n \tag{1.9}$$

There may be an important difference between Bouguer and Free-air anomalies across the
same structure as illustrated by Figure 1.4 which contains two simplified representations of a mountain range.

Neglecting terrain and tidal corrections and focusing on the differences between the Free-air and Bouguer anomalies, it can be determined that:

- For Figure 1.4a:

  1. Given that in computing Bouguer anomalies corrections for the landmass above the ellipsoid (the mountain range) are made, and the underground structure does not vary laterally, the corrected measurement will be equal to the theoretical gravity and the Bouguer anomaly will always be zero across the mountain range.

  2. There is no effect of the mountain top on the Bouguer anomalies.

  3. For the Free-air anomaly, the terrain \( \Delta T \) and free-air \( \Delta q_{FA} \) corrections are applied; hence the “part of the measured gravity due to the attraction of the landmass above the ellipsoid is not taken into account” (Lowrie, 1997) and although the theoretical and actual positions of the stations are now considered to be both on
the ellipsoid, there is a fictive uniform layer of rock between the gravity station and the reference ellipsoid whose effect is still included in the measured gravity (since the Bouguer plate correction is not applied).

4. Therefore, over the mountain range there is a positive Free-air anomaly caused by the mass of the mountain block (the measured or observed gravity is greater than the reference value since there is an excess in mass) while away from the mountain-block the free-air anomaly is zero.

• For Figure 1.4b:

1. In this case, since there is a block of less-dense crustal rock (the 'root-zone' of the mountain) projecting down into the denser mantle, there is a lateral change in the density distribution that will affect both types of gravity anomalies. Since the density of the second layer is less than expected (with respect to the underlying mantle), this causes a negative effect on the gravity anomaly (a deficit in mass).

2. This change in density across the mountain range will cause the attraction recorded on the gravimeters at stations on the profile to be less than the reference value; this constitutes a negative Bouguer anomaly along the profile.

3. Away from the mountain block, both the Bouguer and the Free-air anomalies are equal (but not zero), this is because the Bouguer anomaly is now also affected by the root-zone in the same way as the Free-air anomaly.

4. Over the mountain-top, the Free-air anomaly now has the effects of the top of the mountain and the mountain root while the Bouguer anomaly is only affected by the mountain root. Since both are affected by the same root-zone, there is a constant positive offset between the Free-air anomaly and the Bouguer anomaly (as seen in part (a) of the figure).

The preference on the use of either Free-air or Bouguer anomaly profiles for a given region
depends on the main objectives of the survey. In general, Free-air anomalies are used for geodetic applications while Bouguer anomalies are used for geophysical applications since it shows the effects of different subsurface rock density distributions on the gravity anomaly observations.

1.3.4 Interpretation of Gravity Anomalies

As previously stated, gravity anomalies result from the inhomogeneous density distribution in the Earth (Lowrie, 1997). In order to calculate the gravity anomalies originated by a subsurface body with density $\rho$, it is necessary to calculate the density contrast of the body with respect to the surrounding rocks, $\Delta \rho = \rho - \rho_0$ ($\rho_0$ is the density of the rocks surrounding the body). A body that has a positive density contrast has a density higher than the host rock while a body with negative density contrast has a density lower than that of the host.

In general:

- A high-density body would result in a positive gravity anomaly.
- A low-density body would result in a negative gravity anomaly.
- Gravity anomalies in a profile are indicators of a body or structure that is different (has a density contrast different to zero) with respect to the surrounding area.
- The sign of the anomaly and the density contrast are the same and indicates whether the density of the body is higher or lower than expected.

The contribution to gravitational acceleration of an anomalous body located in the subsurface of the Earth depends on its dimensions, density contrast, and depth with respect to sea-level. The wavelength of an anomaly is its horizontal extent and represents the depth of the anomalous mass. Hence, a large deep body usually causes a broad (or long-wavelength) low amplitude anomaly, while a small shallow body causes a narrow (or short-wavelength) high
amplitude anomaly. It is considered that long-wavelength anomalies due to deep density con-
trasts represent regional anomalies and short-wavelength anomalies, due to shallow anomalous
masses represent residual (or local) anomalies (Lowrie, 1997). Regional anomalies and residual
anomalies are usually used together and superimposed in Bouguer gravity maps. They serve
different purposes; regional anomalies are often used to understand “the large-scale structure
of the Earth’s crust under major geographic features, such as mountain ranges, oceanic ridges
and subduction zones”, while residual anomalies are generally used for commercial exploitation
(e.g., the location of petroleum or natural gas reservoirs) (Lowrie, 1997).

The modeling of subsurface anomalous bodies and their effect on anomalies in the observed
gravitational attraction at different station locations on the Earth’s surface will be discussed
in further sections.
In order to model the substructure of the Earth, an initial estimation of the physical properties (seismic wave propagation, density distribution, etc.) of the region is used. The conversion of the chosen physical property to a different parameter (e.g., density distribution, compressional wave velocity, etc.) can be computed by using empirical relationship and existing physical laws.

An important aspect of physical science is the ability to make inferences about physical parameters from data obtained from observations and measurements. Forward models provide the means to compute the data values given a model while inverse models aim to reconstruct the model from a set of measurements. The relationships between forward and inverse problems can be observed in Figure 2.1.

To better understand how the forward and inverse gravity formulations are obtained, it is important to state the general structure for forward and inverse problems.

![Figure 2.1: Definition of forward and inverse problems.](image)
2.1 Forward Modeling

Geophysicists frequently deal with problems in which they need to relate physical parameters that characterize a model, $m$, with collected observations that comprise a data set, $d$. Assuming that the fundamental physics exist and are well-understood, a function $G$, may be found to relate $m$ and $d$

$$G(m) = d$$

Here $d$ represents the collection of discrete observations, while $G$ can represent an ordinary differential equation (ODE), a partial differential equation (PDE), or a linear or nonlinear system of algebraic equations (Aster et al. 2005). $G$ is called an operator when $m$ and $d$ represent functions, while $G$ is a function when $m$ and $d$ are vectors.

The forward problem would be to find $d$ given $m$. In the case of gravitational attraction and gravity anomalies, this is equivalent to say that if we know the structure of the anomalous mass, its depth, and its density contrast, using the forward model for gravity anomalies, we can determine the contribution to the gravitational acceleration at a given point on the surface that is due to this body. Computing $G(m)$ is not always straightforward and might involve solving PDEs or ODEs, evaluating integrals, or applying algorithms that may not have explicit analytical formulations for $G(m)$ (Aster et al. 2005). For geophysical problems, the laws of physics provide the appropriate structure to compute data values given a model (Snieder et al. 1999).

2.2 Inverse Modeling

Although the definition and use of an inverse problem are not the main topics of this thesis, the use of gravity acceleration information through the implementation of inverse methodology will be analyzed as part of my future work. Therefore, the following definition of inverse problems
should be kept in mind for future references. In inverse problems, the goal is to determine the model parameters that best reconstruct the set of measurements. Ideally, “an exact theory exists that prescribes how the data should be transformed in order to reproduce the model” (Snieder et al. 1999). In reality an exact solution may not exist, but it may be sufficient to solve for model parameters that approximate the data in the best fit sense. In other words, it may be enough to find the best approximate solution that produces a minimum misfit or residual (Aster et al. 2005).

Given an observed data vector, \( d \in \mathbb{R}^n \), we want to find the unknown model, \( m \), such that \( G(m) \) approximates \( d \) as much as possible, i.e.,

\[
\min_m \| G(m) - d \|^2 = \min_m \sum_{i=1}^{n} (G_i(m) - d_i)^2
\]  
(2.1)

In this case, the best approximation in the least squares sense will be found (using the traditional 2-norm or Euclidean length). There are additional misfit measures that can be used for this purpose; one of these alternatives is the 1-norm

\[
\min_m \| G(m) - d \|_1 = \min_m \sum_{i=1}^{n} |G_i(m) - d_i|
\]  
(2.2)

In the case of gravitational attraction and gravity anomalies, the inverse problem would be to determine the geometrical shape of the anomalous body that is responsible for the observed gravitational attraction measurements obtained at different points on the surface of the Earth.

Finding mathematically acceptable answers to inverse problems is not simple. There may be infinitely many models that fit the data in an adequate way. It is essential to determine how good the solution is, how feasible it is, and if it meets additional constraints and is consistent (Aster et al. 2005). There are three important aspects that must be considered when solving inverse problems: solution existence, solution uniqueness, and instability of the solution process (Aster et al. 2005). Existence refers to the idea that there may not exist a model that fits the
given data set exactly given that the physics of the mathematical model is approximate and 
there may be noise in the data (Aster et al. 2005). Uniqueness refers to the likelihood of having 
more than one solution to the inverse problem that satisfy the data exactly. Instability relates 
to the behavior shown by the implied models when there are small changes in measurements. 
This behavior determines if a problem is ill-posed (for continuous systems) or ill-conditioned 
(for discrete systems), which occurs when “small features of the data ... drive large changes 
in inferred models” (Aster et al. 2005).
The contribution to gravitational acceleration of an anomalous body located in the subsurface of the Earth depends on its dimensions, density contrast, and depth with respect to sea-level. The modeling of subsurface anomalous bodies and their effect on the observed gravitational attraction at different locations on the Earth’s surface can be determined by using different forward modeling techniques to approximate the contribution of each one of the bodies involved in a region. The algorithm for the forward modeling of gravitational acceleration, and three different forward model techniques for the modeling of anomalous bodies are explained in the following sections.

### 3.1 Forward Model of Gravitational Acceleration

The forward problem formulation for the gravitational acceleration of anomalous bodies in the Earth’s subsurface can be stated as follows: given a density profile and depths of an initial underlying structure of the Earth, we determine the vertical component of the gravitational attraction \( \Delta g_z \) obtained at various points on the surface.

As stated by Sharma (1997), forward modeling (also iterative modeling) is a technique for the interpretation of geophysical data that involves the following steps:

1. An initial model of the density distribution is obtained from known geology of the region.

2. The gravity anomaly (\( \Delta g_{\text{calc}} \)) of one or two principal profiles is computed by using the appropriate and most reasonable formulation (e.g., using cylinders, sheets, slabs, etc.)
3. Compute gravity anomaly ($\Delta g_{\text{calc}}$) and determine the difference between observed and computed gravity anomalies at all available points: ($||\Delta g_{\text{obs}} - \Delta g_{\text{calc}}||$).

(a) If the difference between gravity anomalies computed in the previous step is higher than a threshold specified by the use (e.g., $\pm 10$ mGals), improve the model by adjusting the appropriate model parameters (density, depths, or distances).

(b) Otherwise, terminate the process.

4. Repeat step 3 until the difference ($||\Delta g_{\text{obs}} - \Delta g_{\text{calc}}||$) becomes smaller than the given threshold.

This modeling scheme contains elements of both forward and inverse modeling for gravity anomalies. A purely forward modeling scheme would consist of Steps 1 and 2, while the inverse modeling scheme would consist of implementing all of the steps in the algorithm a finite number of times until the difference between observed and calculated gravity anomalies is below the threshold specified by the user (e.g., ($||\Delta g_{\text{obs}} - \Delta g_{\text{calc}}|| < 10$ mGals).

Since the main factor in this process is the type of forward model that we use to determine the gravity anomaly at different points on the surface of the Earth given an initial substructure (densities and depths of various layers of different materials), it is important to choose the most favorable formulation to model the anomalous masses or layers in our crustal scale models. The most simple one consists of the use of simple geometrical figures to model the anomalous bodies found in the subsurface of the Earth. An additional technique uses rectangular prisms (Plouff, 1976) to model any type of buried geological body (by varying the dimensions of the prisms as needed). Polygonal prisms, as proposed by Talwani et al. (1959), goes further in this calculation by using 2.5 dimensions in the modelling of the subsurface’s density distribution. The details of each one of these techniques are discussed further in the following sections.
### Table 3.1: Formulas used for the gravitational anomalies caused by simple bodies

(Sharma, 1997)

<table>
<thead>
<tr>
<th>Simple Body</th>
<th>Representation</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point mass</td>
<td>Salt Domes</td>
<td>[ g_z = 2\pi G R^2 \Delta \rho \frac{z}{(x^2 + z^2)} ]</td>
</tr>
<tr>
<td>Sphere</td>
<td>Salt Domes</td>
<td>[ g_z = \frac{4}{3} \pi G R^3 \Delta \rho \frac{z}{\sqrt{(x^2 + z^2)^{3/2}}} ]</td>
</tr>
<tr>
<td>Infinite Horizontal Cylinder</td>
<td>Buried channels, anticlines, etc.</td>
<td>[ g_z = 2\pi G R^2 \Delta \rho \frac{z}{\sqrt{x^2 + z^2}} ]</td>
</tr>
<tr>
<td>Semi-infinite Horizontal Sheet</td>
<td>Narrow Horizontal Bed</td>
<td>[ g_z = 2\pi G \Delta \rho h \left(\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{z}\right)\right) ]</td>
</tr>
<tr>
<td>Horizontal Thin Sheet</td>
<td>Faulted Sills</td>
<td>[ g_z = 2\pi G \Delta \rho h \left(\tan^{-1}\left(\frac{L - x}{z}\right) + \tan^{-1}\left(\frac{x}{z}\right)\right) ]</td>
</tr>
<tr>
<td>Vertical Cylinders</td>
<td>Volcanic plugs and salt domes</td>
<td>[ g_z = 2\pi G \Delta \rho \left(L + \sqrt{R^2 + z^2} - \sqrt{(z + L)^2 + R^2}\right) ]</td>
</tr>
<tr>
<td>Vertical Thin Rod</td>
<td>Volcanic plugs and salt domes</td>
<td>[ g_z = \pi G R^2 \Delta \rho \left(\frac{1}{\sqrt{x^2 + z^2}} - \frac{1}{\sqrt{x^2 + (z + L)^2}}\right) ]</td>
</tr>
<tr>
<td>Infinite Slabs</td>
<td>Sedimentary basins, plutons, ice caps, etc</td>
<td>[ g_z = 2\pi G \Delta \rho h ]</td>
</tr>
</tbody>
</table>

### 3.2 Simple Bodies

Telford et al. (1990) states the formulation needed to model the gravity effects of simple shapes such as spheres, cylinders, thin rods, and sheets, among others. More interesting bodies such as slabs, faults, and dipping beds are also included in the book.

Here is a compilation that states the formulas for simple bodies:

Due to the simplistic nature of this technique, the results obtained from its use are only useful for areas in which the structure of the Earth is well-known and assumptions can be made to model the buried anomalous bodies with simple bodies. Therefore, this technique is included in this thesis only for documentation purposes and will not be further analyzed in the numerical experimentation section of this work.

The use of simple geometrical bodies is very helpful and can provide a clear example on
the functionality and use of gravitational acceleration datasets for structural anomalies. This method provides a naive approximation to model geological structures of interest and highlights the characteristics found on some ideal (and not common) bodies in the subsurface.

Figure 3.1 shows an example on the use of simple bodies to determine the Bouguer gravity anomalies associated with the given region of the Earth.

![Figure 3.1: Non-uniqueness in the calculation of Bouguer gravity anomalies using simple bodies.](image)

Figure 3.1 (a) shows the Bouguer gravity anomaly associated with a shallow anomalous body of positive density contrast with respect to the surrounding material, (b) shows that two bodies of different shape, depth, and possibly density contrast can have the same gravity signature portraying the non-uniqueness behavior of gravity anomalies, and (c) shows three bodies with different shape, depth, and possibly density contrast that have the same gravity signature again showing how non-uniqueness can greatly affect the modeling of anomalies in the shallow crust. Here the non-uniqueness refers to having more than one crustal density structures that have the same gravity anomaly signature (observed gravity). Although this problem does not affect the forward modeling calculation (given that each anomalous body will generate a unique gravity signature), it becomes a significant problem in the inverse problem calculation in which we want to determine the shape of the anomalous body buried in the Earth that originates the given gravity signature or observations on the surface.
3.3 Rectangular Prisms

An additional technique that allows the portrayal of complex anomalous bodies was shown by Plouff who proposed that any realistically shaped geologic body or topographic feature "can be synthesized to any standard of accuracy or esthetics by combining the effects of a sufficiently large number of small prisms (1976).

This technique has been used previously by Maceira et al. (2009) to calculate the gravitational acceleration caused by changes in the density of the substructure of the central Asian Basin. Figures 3.2 and 3.3 illustrate the scheme used by Plouff (1976) using rectangular prisms to calculate their corresponding gravity anomaly and an example on their use to model a definite body (as proposed by Maceira et al. (2009)) as a combination of many rectangular prisms with constant depth and varying density.

![Rectangular Prisms Scheme](image)

Figure 3.2: Rectangular prisms scheme proposed by Plouff (Maceira et al. 2009).

The equation associated with this scheme is the following:

\[
g_z = G \rho \sum_{i,j,k=1}^2 s \left[ z_k \tan^{-1} \left( \frac{x_i y_j}{z_k R_{ijk}} \right) - x_i \ln (R_{ijk} + y_j) - y_j \ln (R_{ijk} + x_i) \right] \tag{3.1}
\]

where \( R_{ijk} = \sqrt{x_i^2 + y_j^2 + z_k^2} \), \( s = s_i s_j s_k \), \( s_1 = -1 \), and \( s_2 = +1 \).
Figure 3.3: Rectangular prisms used to model layers of different density material (modified from Maceira et al. 2009).

In this technique, the total contribution of the rectangular cylinder to gravity is determined by the contributions of each one of the vertices of the cylinder. There are 8 calculations involved in each cylinder which makes this a very computationally demanding technique for the determination of gravity acceleration at different points on the surface (each gravity station). Moreover, there is a trade-off between accuracy and cost given that a better approximation can be obtained by making the rectangular prisms sufficiently small to better portray the geological structures of the area, but this would represent an increase in the number of operations performed by the algorithm.

An example on the use of this technique with a rectangular prism with three different density values ($\Delta \rho_1 = 600$ kg/m$^3$, $\Delta \rho_2 = 800$ kg/m$^3$, and $\Delta \rho_3 = 1000$ kg/m$^3$) is shown in Figure 3.4. The dimensions of the rectangular prism are: $x = [-50, 50]$ (in and out of the screen), $y = [-2, 2]$ (left to right in the same direction as the gravity stations), and $z = [-2, -6]$ (up and down). From this example, we can see that the gravity signature of the prism with the lowest density contrast ($\Delta \rho_1 = 600$ kg/m$^3$) has a gravity signature with a lower amplitude with respect to that of the other two bodies. Also, given that the wavelength of an anomaly is related to the depth to the anomalous mass (in this case the prisms are located in
the same position and have the same depth), it can be seen that the wavelengths of the three prisms are about the same.

Figure 3.4: Rectangular prisms code used with a prism of three different density contrast values.

3.4 Polygonal Prisms

The most commonly used scheme to model substructures for the calculation of gravity anomalies in a 2-dimensional setting was proposed by Talwani et al. (1959) and consists in the use of polygonal prisms to model complex anomalous bodies found in a given region. Figure 3.5 shows the theoretical geometry proposed for the polygonal prisms scheme.

The equation associated with this technique is the following:

\[
Z = A \left[ (\Theta_1 - \Theta_2) + \left( \frac{z_2 - z_1}{x_2 - x_1} \right) \ln \left( \frac{r_2}{r_1} \right) \right]
\]  
(3.2)

where \( A = \frac{(x_2 - x_1)(x_1 z_2 - x_2 z_1)}{(x_2 - x_1)^2 + (z_2 - z_1)^2} \), and \( r_i^2 = x_i^2 + z_i^2 \).

The Talwani code has been widely used in the last few decades and it allows the user
to construct 2-dimensional (infinite strike extent) or 2.5 dimensional bodies. Following the theory published in Talwani et al. (1959) and Cady (1980), the 2.5 dimensionality means that "the polygons that comprise the model do not implicitly extend to infinity in and out of the computer screen. Instead, one can look at a gravity map of the region of the profile being modeled and determine the strike length of the main anomalous bodies in and out of the screen and enter these values into the program" (Blakely, 1996). When the strike extent is considered to be a large number (e.g., 1000 km), the program is considered to represent a 2-dimensional model.

An example on the use of this approach can be found in Robbins (1971). In this report, authors used the Talwani technique in two different 2-dimensional profiles to determine the gravity anomalies of the areas of study and used this information to determine the geological activity in the vicinity of San Jose, California.

Figure 3.6 shows an example on the use of this technique by using a rectangular horizontal prism with three different density contrasts. The structure in Figure 3.6 consists in a rectangular horizontal prism that extends in and out of the screen (has a strike of) 1000 km, extends in the y direction 4 km and has a thickness of 4 km. The density contrasts in this case are \( \Delta \rho_1 = -600 \text{ kg/m}^3 \), \( \Delta \rho_2 = -800 \text{ kg/m}^3 \), and \( \Delta \rho_3 = -1000 \text{ kg/m}^3 \).

From this example (see Figure 3.6), we can see that the gravity signatures of these prisms are
negative given that the density contrasts (the differences between the density of the prisms and the surrounding area) are all negative. In this case the highest density contrast ($\Delta\rho_1 = -600 \text{ kg/m}^3$) has a gravity signature with a lower amplitude with respect to that of the other two bodies. Also, since the three prisms have the same depth to the center of mass, the wavelengths of the three prisms are about the same.

Now that we have discussed the functionality of each one of the techniques, a list of the advantages and disadvantages associated to each one of them and their implementation is included in following chapters.
Chapter 4

Gravity Dataset

The gravity dataset used in the present work was part of a compilation and processing effort made by Raed Aldouri for the Pan American Center for Earth and Environmental Studies (PACES: http://paces.geo.utep.edu) which is currently hosted at the Cyber-ShARE Center of Excellence at UTEP. Part of this dataset was used in Averill (2007) to show the regional anomalies in gravitational acceleration due to changes in the density distribution of the Rio Grande Rift (RGR) region. Figure 4.1 shows the Bouguer gravity anomaly map for the southern portion of the Rio Grande Rift with the locations from the stations as used by Averill (2007).

Using information from previous studies of the area (including Averill (2007), Adams et al. (1996), Keller et al. (1999), and Berglund et al. (2012), among others) and the dataset from PACES, Shearer et al. (2000), and Chang et al. (1999) it was possible to improve the model of the Rio Grande Rift. A further constrained Bouguer gravity anomaly map covering a wider area was obtained using the compiled dataset and incorporating receiver function results, gravity, and magnetic information and interpretations from seismic reflection, refraction, and velocity models to create an enhanced subsurface crustal scale model (Thompson et al. 2013). The Bouguer gravity anomaly map for the RGR and the locations of the stations used are illustrated in Figure 4.2.

The Bouguer gravity anomaly map with topography details for the RGR is shown in Figure 4.3. Terrain corrections for the region were calculated as shown in Webring (1982) and a density of 2670 kg/m³ was used for the Bouguer gravity correction. There were 45,945 Bouguer gravity observation points used to create the Bouguer gravity anomaly map. The average error for the data in these Bouguer gravity anomaly maps ranges from 0.05 to 2 mGals (Thompson
Figure 4.1: Bouguer gravity anomaly map for SRGR. Inset map contains locations of gravity readings (black dots) (Averill, 2007).

A preliminary Bouguer gravity anomaly map for a cross section on latitude 32° in the West-East direction is shown in Figure 4.4. Most recent cross sections for three different profile lines AA′ (at latitude 32°), BB′ (at latitude 34°), and CC′ (a diagonal line as shown in Figure 4.3) were also obtained for the RGR. These cross-sections are illustrated in Figures 4.5-4.7.

These 2-dimensional cross-sections and the Bouguer gravity anomaly map from the RGR shown in Figure 4.3 constitute the basis for the dataset that will be used in the joint inversion and model fusion approaches combined with seismic information obtained from the area.

The gravity lows are associated with a thick crust under the Colorado Plateau and the Delaware Basin while the gravity highs are associated with a thin crust located near El Paso and the central Rio Grande Basin (Thompson et al. 2013). The purpose of these crustal scale
Figure 4.2: Bouguer gravity anomaly map for RGR with location of all 45,945 gravity stations

models is to illustrate the differences between the different segments of the rift (north, central and south) and compare the results with previous studies along the area.
Figure 4.3: Modified Bouguer gravity anomaly map for RGR (Thompson et al. 2013)

Figure 4.4: Preliminary gravity anomaly cross-section from West-East of the RGR
Figure 4.5: Gravity anomaly cross-section at latitude 32° (Thompson et al. 2013)

Figure 4.6: Gravity anomaly cross-section at latitude 34° (Thompson et al. 2013)
Figure 4.7: Gravity anomaly at Northwest-Southeast cross-section (Thompson et al. 2013)
Numerical Experimentation

In this chapter we present the analysis of two different approaches for the forward gravity model in order to determine the best alternative as part of a joint inversion scheme based on interior point methods and a model fusion technique for complementary seismic (RF and SW dispersion) and gravity data sets obtained from the RGR.

5.1 Synthetic Problems

In order to compare the behavior of each one of the techniques explained previously, the following synthetic examples were used to test them. The results obtained are reported in the next subsections.

5.1.1 Rectangular Prisms

The first synthetic problem used consisted on a rectangular prism with width = 100 km (x coordinate; coming in and out of the screen), heigth = 4 km (z coordinate; going up and down), and length = 4 km (y coordinate; moving to the left and right ) buried at a depth of 2 km. There are 80 station points located on the surface of the region (z = 0), where the gravity acceleration caused by the anomalous body and the rectangle has a positive density contrast of $\Delta \rho = 1000$ kg/m$^3$ with respect to the surrounding material (prism is denser than the rest of the material).

Figure 5.1 shows that the results obtained from the rectangular and polygonal prisms code coincide in all points. This is true because the rectangular prisms algorithm considers the 3-D
Figure 5.1: Gravity anomaly results using rectangular (Plouff) and polygonal (Talwani) modeling in the given prism.

structure of the prism while the polygonal prism uses a 2-dimensional representation but the
strike in and out is also used (in this case it is $x = 1000$ Km).

The second example uses two rectangular prisms located on the same region: Prism 1) heigth = $[-1, -6]$, length = $[0, 5]$, and $\Delta \rho_1 = 1250$ kg/m$^3$ and Prism 2) heigth = $[-2, -6]$, length = $[18, 22]$, and $\Delta \rho_2 = 1000$ kg/m$^3$. Figures 4.2 and 4.3 show the results obtained from the polygonal and rectangular algorithms respectively.

Figure 5.2: Gravity anomaly results using polygonal prisms algorithm for two bodies


In this case, since prisms can be exactly modelled using polygonal prisms too, the results are the same. However, this additional example using two anomalous bodies is useful to determine that although both approaches have different formulations, they are equivalent and provide the same results for some appropriate simple geometrical forms.

### 5.1.2 Horizontal Cylinder

Another synthetic problem consists on using a sphere as the geometrical form for the anomalous body and calculating the changes in gravitational acceleration that are obtained from the different techniques. For this synthetic problem, a horizontal cylinder (with length going in the x-direction in and out of the screen) was used having the following dimensions: $R = 2$ km ($y$ coordinate), density contrast of $\rho = 600$ Kg/m$^3$ (prism is denser than the surrounding material), and buried at a depth of 4 km (distance from the surface to the center of the cylinder). The same number of stations were used (80 stations) where the gravity acceleration was measured.

Figure 5.4 shows the 2-dimensional cross section used for the polygonal prisms algorithm and the results obtained for the corresponding Bouguer gravity anomaly, while Figure 5.5 shows the alternative model obtained using rectangular prisms and the corresponding gravity
Figure 5.4: 2-D modeling of the cross-section of a horizontal cylinder using polygonal prisms

Figure 5.5: 3-D modeling of a horizontal cylinder using rectangular prisms algorithm

anomaly calculated. The polygonal prisms algorithm was implemented using 12 vertices for the polygon. The rectangular prisms implementation consisted on 10 rectangular prisms located on the same region all with a $\Delta \rho_1 = 600 \text{ kg/m}^3$. Figures 5.4 and 5.5 show the results obtained from the polygonal and rectangular algorithms respectively.

While the rectangular prisms implementation for the horizontal cylinder calculates the corresponding gravitational anomaly for each one of the rectangular prisms (each contribution is included in the top part of Figure 5.5) and adds the total contribution at the end, the polygonal prisms algorithm calculates the contribution of each vertex (a total of 12 in this
case) to the gravity anomaly and the total gravitational attraction due to the anomalous body at each one of the stations located on the surface (blue dots with \( z = 0 \) in the bottom part of Figure 5.5).

## 5.2 Rio Grande Rift Dataset

After running the two codes for polygonal and rectangular prisms using these synthetic examples, a preliminary profile of a cross-section of the Rio Grande Rift (see Figure 4.4) obtained by Thompson et al. (2013) was used to test the behavior of the polygonal and rectangular prisms techniques.

Figure 5.6 shows the gravity anomaly obtained with the implementation of the polygonal prisms algorithm using the preliminary profile of the RGR. The top part of the figure contains the gravity anomaly observations obtained for the profile (observed gravity) and the calculated gravity anomalies using the polygonal prisms algorithm (calculated gravity). The bottom part of the figure shows the front face of the polygonal prisms used to represent the layers in the corresponding cross-section profile. For this particular example, I used a tie-point (a point in the \( y \)-direction in the profile where the observed gravity anomaly is assumed to be exact) of 40 km and 100 km to see the difference in the root mean square (RMS) of the implementations. The results are reported in Figures 5.6 and 5.7.

The corresponding RMS for the 40 km tie-point was 40.4856 while the RMS for the 100 km tie-point was 32.0151. A discussion on the results obtained from the forward modeling of the RGR using the polygonal prisms algorithm is included in the following chapter.

With respect to the rectangular prisms forward modeling, an illustration of the composition of the layers of the 2-dimensional cross-section of the RGR using rectangular prisms is included in Figure 5.8. In this implementation there were around 1770 prisms used. The cross section covered an area of 560 kilometers in the \( y \)-direction (left-to-right) plus end bodies of 50 km on either side of the profile. In the \( x \)-direction, prisms were constructed every 10 or 15 kilometers
Figure 5.6: Gravity anomaly results obtained using the polygonal prisms for the density profile of the RGR and a tie-point of 40 km.

Figure 5.7: Gravity anomaly results obtained using the polygonal prisms for the density profile of the RGR and a tie-point of 100 km.
to cover a length of 100 km in the $x$-direction (from $-50$ to 50 kilometers). With respect to the $z$-direction, the prisms were created such that they had a depth equal to the thickness of each one of the five layers they corresponded to.

![3-D modeling of a cross section of the RGR using rectangular prisms](image1)

Figure 5.8: 3-D modeling of a cross section of the RGR using rectangular prisms

![Gravity anomaly results using the rectangular prisms algorithm for the initial density profile of the RGR.](image2)

Figure 5.9: Gravity anomaly results using the rectangular prisms algorithm for the initial density profile of the RGR.

In order to determine the gravitational anomalies of the area, a density of $\Delta \rho = 2670 \text{ kg/m}^3$ was subtracted from all the densities in the cross-section (since using the actual densities of the area would provide the whole gravitational acceleration measured at each one of the
stations instead of the gravity anomaly observations). Figure 5.9 shows the gravity anomaly observations obtained from the cross-section profile (the observed gravity) and the calculated gravity anomaly using the rectangular prisms algorithm. The RMS of this particular model was calculated to be equal to 36.4215.
Chapter 6

Discussion

In this chapter, I discuss the advantages and disadvantages that I found in the implementation of the three techniques introduced in Chapter 3 – simple bodies, rectangular prisms, and polygonal prisms – and further discuss some of the results reported in the numerical experimentation reported in chapter 5.

6.1 Advantages and Disadvantages of Techniques

The advantages and disadvantages inherent to each one of the forward modeling techniques for gravity anomalies are reported here. These characteristics are related to the formulation of the techniques, the computational challenges I found, and the accuracy and sensitivity that each technique may have to changes in the dataset.

6.1.1 Simple Bodies

The advantages of using simple geometrical forms to model anomalous bodies include:

1. It has a simple formulation: the gravity anomaly of a simple body is calculated by plugging in the corresponding values in a formula; no need for further calculations.

2. Results are produced immediately: the gravity anomaly of a simple model is obtained with one calculation.

3. It produces good preliminary results: this approach can be used to obtain preliminary results of the gravity anomaly caused by a simple body representation that may be a
rough representation for a more complex area where the edges of the body are not as
smooth or well-defined.

4. One of these models may actually be a good representation of the actual anomalous body:
there are specific cases where spheres, cylinders, and/or slabs may be the optimal model
for the anomalous mass being analyzed. For example, when salt domes, buried channels,
anticlines, tunnels, basins, and plutons are being modeled the appropriate simple body
representation can be used and the results would be very accurate.

5. A combination of different representations of simple bodies can help model complex
structures: for those areas in which there are many anomalous masses or a unique simple
body representation may not be enough to model the gravity anomalies of the region, a
combination of different shapes of bodies can be used to obtain a better representation
and approximation to the observed gravity anomalies.

The disadvantages associated with modeling anomalous bodies using simple representations
include:

1. The innacuracy related to the simplistic approach may influence the results greatly: The
use of simple bodies may be a very inaccurate representation of the actual anomaly (due
to the heterogeneous nature of the shallow crust) and hence the results obtained from
the forward modeling may be very different to the observed values.

2. The use of inappropriate parameters may influence the result: this disadvantage is in-
herent to all three modeling techniques given that the use of inappropriate or erroneous
depths and/or density values for the anomalous masses found in the subsurface of the
Earth will affect the calculations of gravity anomalies since this type of information is
very sensitive to small changes in the data (more so in the shallow crust).
6.1.2 Rectangular Prisms

The advantages associated to this technique are:

1. The algorithm is easy to program and implement: the article published by Plouff (1976) contains a Fortran algorithm that can be easily replicated and improved on using Fortran or Matlab interfaces for modern computers.

2. This approach can be used for the modeling of any type of irregular anomalous mass: The method provides a simple algorithm to model geologically complex structures that may not be clearly defined by using finitely many prisms.

3. It has been proven to be very accurate and efficient: The use of this approach has been used in Seber et al. (2001) and Maceira et al. (2010) and has been proven to provide very accurate results and improvements to the 3-dimensional modeling of the Earth’s substructure when combined with complementary seismic information.

4. The algorithm can be adapted easily: this code can be modified to run automatically by reading the coordinates of the rectangular prisms and their densities from a .txt file.

5. This program calculates the total gravitational acceleration observed at a point on the surface instead of the gravitational anomalies of the region: depending on the information needed, reductions and corrections can be made to determine the changes in gravitational acceleration caused by those changes in density that are due to anomalous bodies in the region (Maceira et al. 2010) or the total gravitational acceleration can be used.

The disadvantages associated with modeling of anomalous bodies in gravity surveying using rectangular prism representations include:

1. This is a computationally expensive algorithm: This approach needs the calculation of the contribution of each one of the vertices of the rectangular prism to its final gravity
anomaly. There are 8 vertices in a prism and the algorithm is implemented for each one of the prisms. Compared to the simple body representations’ algorithm and even the polygonal prisms algorithm (discussed next) this approach involves a greater number of operations per anomalous body.

2. A large number of prisms may be needed to better approximate the anomalous bodies found in the subsurface: it is possible to approximate any kind of structure by changing the parameters of the rectangular prisms (thickness, width, and length); however, for very detailed structures, a large number of prisms may be needed to improve the level of accuracy of the approximation which constitutes an increase in the number of operations associated with the implementation of this algorithm.

3. There is a trade-off between the accuracy and computational cost: A considerable trade-off is associated with using very small thicknesses for changes in the $x$ and $y$ directions given that very small values for $\Delta x$ or $\Delta y$ can provide more accuracy to the calculations but will increase significantly the number of calculations needed to obtain the total gravity anomaly caused by the structure. Hence, a large value for $\Delta x$ and $\Delta y$ would provide less accuracy in less time while small values for $\Delta x$ and $\Delta y$ provide more accuracy but require a larger number of operations and computational time.

6.1.3 Polygonal Prisms

The polygonal prisms scheme has the following advantages:

1. This approach can be used for the modeling of any type of irregular anomalous mass: given that a finite number of vertices can be used to model the anomalous masses found in the substructure of the Earth, this approach makes it easy to model geometrically complex and geologically detailed structures by using polygons of $m$ number of vertices.
2. Layers are considered polygons: using this approach, there is no need to divide layers into smaller bodies to obtain a better approximation of the gravity anomaly which makes the manipulation of the dataset easier.

3. The algorithm can be adapted to run in Fortran or Matlab: through the use of for-loops and commands to read information from .txt documents the algorithm can be adapted to run automatically using any of the two interfaces.

4. This algorithm is the basis for many inverse modeling software packages: many software packages for geophysical computations such as magnetics and gravity anomalies use Talwani et al. (1959) and Cady (1980) as the basis for their algorithms. In this type of inverse calculation, the initial model is modified a finite number of times by changing the depths and/or densities of the layers of the cross-section profile in order to determine the optimal model that minimizes the residual between observed \( g_{\text{obs}} \) and calculated gravity \( g_{\text{calc}} \) at different points on the surface.

The disadvantages associated with this scheme include:

1. This is a computationally expensive algorithm: This approach needs the calculation of the contribution of each one of the vertices of the polygonal prism to its final gravity anomaly. Compared to the simple body representations’ algorithm and even the rectangular prisms algorithm (depending on the structure) this approach involves a greater number of operations per anomalous body.

2. This technique has a rigorous format in which information has to be calculated: in order for the final gravity anomaly to be calculated correctly, the vertices of the polygonal prism have to be entered in clockwise order. Hence there are format restrictions that must be followed when information is entered into the algorithm or read from a .txt file.

3. Additional information from the base station(s) used during the survey may be needed to obtain a better approximation: for this algorithm there may be a need for a tie-point
where the gravity anomaly observation is assumed to be correct in order to obtain the appropriate results.

4. End-bodies (those bodies that are surrounding the area under study) are needed in the algorithm: this is a requirement in the program in order for gravity anomalies to be calculated correctly. However, these values can be defined by default and won’t affect the results greatly.

5. There are assumptions needed for the algorithm to work correctly: given that Talwani’s code works with density contrasts instead of actual densities, there are assumptions that need to be made about the densities of the region in order for the code to determine the Bouguer gravitational anomaly profile (e.g., all the densities of the cross-section are compared to 2670 kg/m$^3$, the density of granite, or another density that represents an average density for the region under study). The assumption of this type of information can affect greatly the results obtained for the gravity anomalies of a region, hence, it is important to determine first-hand the types of rock present in the region of study, the possible geological activity of the area, and any other geological or geophysical information that can provide greater insight of the area prior to the modeling.

6.1.4 Comments on Implementation Results

Through the implementations of the rectangular and polygonal prisms for different substructure models, it was observed that for those crustal models that had a homogeneously layered composition both, the rectangular and polygonal prisms schemes, were very easy to implement and provided good approximations to the observed gravity anomalies. For those areas where the crustal models had a heterogeneous composition and more irregular shaped anomalous masses where present on the first 10 km in depth, the polygonal prisms algorithm may have an advantage over the rectangular prisms algorithm given that a polygon can be easily used
to model the irregularities in the body while doing the same with rectangular prisms can be challenging and the results obtained may not be as accurate.

Most of the advantages and disadvantages of the techniques presented in this work are inherent to the calculations involved within the techniques themselves regardless of the crustal model that is used. However, when presented with very detailed and not well-defined information about the boundaries of the anomalous masses, it seems easier and more logical to use polygonal prisms as the forward modeling of choice given that it gives more space for manipulation with respect to rectangular prisms.

As shown in Section 5.2, where two different tie-points are used for the example using the dataset obtained from the RGR very different results can be obtained from the forward model of choice if the cross-sectional crustal model used for the area is not appropriate, the density values for the different types of rock are not correct, or the assumptions of the rocks located in the region are not accurate. This is true for both the rectangular and polygonal prisms approaches. Hence, it is important to consider that the examples used in this work assume a lot of information (the extent of the end bodies, the extent in the $x$-direction of the anomalous masses, the density used to obtain the density contrast $\Delta \rho$ for each calculation, etc). For a different survey in a different region of the Earth these assumptions may change and the appropriate adjustments must be made prior to the analysis of the results obtained.

### 6.2 Recommendation on Technique

Keeping in mind the results obtained from the implementation and numerical experimentation of the rectangular and polygonal prisms forward model algorithms for the calculation of gravity anomalies, the final step is to look at the format, type, and characteristics of the datasets available for use in the joint inversion and model fusion schemes on which we plan to use gravity datasets.

Sosa et al. use Receiver functions (RF) and Surface wave (SW) dispersion in their joint
inversion scheme using interior point methods (2013a). Although Ochoa et al. (2011) doesn’t include any reference to the types of seismic data that can be used for the implementation of his proposed model fusion approach it can be assumed that crustal models obtained from RF and SW dispersion can also be used with this method.

The seismic stations located in the RGR and used by Sosa et al. (2013a, 2013b) are shown in Figure 6.1.

The joint inversion scheme proposed by Sosa et al. (2013a, 2013b) applies a “constrained optimization approach for joint inversion of surface wave and receiver functions using seismic S-wave velocities as a model parameter”. The inversions produces 1-dimensional S-wave velocity profiles at different points on the surface based on the locations of the seismic stations shown in Figure 6.1 and these profiles are interpolated using a Bayesian kriging scheme to develop a 3-D velocity model of the RGR. Hence, the available dataset for the gravity anomalies of the region should be related to its corresponding S-wave velocity conversion (using the Nafe-Drake (Nafe et al. 1963) and Birch’s Law empirical relationships as appropriate (Birch, 1961)).

The format of the information is the most important detail in the implementation of gravity
anomalies in joint inversion. The use of polygonal prisms involves using cross sections of crustal structures of a region that are reported in 2-dimensions (or 2.5 depending on the extent of the end-bodies used in the algorithm). These 2-dimensional profiles reflect the substructure of the corresponding line profile but doesn’t necessarily reflect the structure of the region as a whole. Using polygonal prisms we are assuming that the structure reflects an area that covers the region more thoroughly and this can be a very important factor in those areas where sudden changes in the composition of the crust are common. In this sense, the use of rectangular prisms to model the gravity anomalies along 2-D cross-section profiles of the RGR may be a better option given that this algorithm allows the user to enter information from a 3-dimensional perspective and calculates the contribution of each rectangular prism to the total calculated gravity anomaly individually. This characteristic of polygonal prisms makes it easier to take information from the 3-D gravity anomaly map available from the RGR, take the 2-D characteristics of the same point from the appropriate 2-D crustal model and use this with the available information obtained from seismic surveys.

Although there is not a definite answer as to why rectangular prisms is more appropriate for the use of gravity acceleration information from a region for its use in joint inversion and model fusion schemes, in this particular case there are more favorable conditions to recommend this technique based on the idea that rectangular prisms can be more easily adapted to the joint inversion and model fusion schemes that are of interest for the constrained optimization of seismic and non-seismic datasets for the 3-D modeling of the substructure of the RGR.

6.3 Significance of the Result

The advantages that the use of complementarity datasets would provide includes the ability to better constrain irregular bodies in the shallow crust through the use of gravity information that provides better resolution in this region. Hence, due to its particular characteristics, gravity complements seismic information such as surface wave dispersion and receiver functions
to model the shallow subsurface. The use of information obtained from the gravitational acceleration measured at various points of the surface of Earth can help to better constraint the first 50 km of the crustal distribution of a region. The non-uniqueness and high complexity associated to this type of dataset makes it difficult to trust gravitational data by itself; however, when it is used in conjunction with seismic or magnetic information, it can be used to improve the resolution of the models in the shallow crust (Maceira et al. 2009). Maceira et al. shows an example on the use of rectangular prisms for the calculation of gravity anomalies in the Asian basin region which was shown to provide additional information for a more detailed model of the Earth’s structure using surface wave velocity and gravity observations (2009).

As shown in previous sections, gravity has a high sensitivity to changes in density and depth of anomalies which can cause two very different crustal density models to have the same gravitational signature. In order to minimize the likelihood of coming up with a model that does not reflect the geological structure of the region under study, there are additional geological techniques that can be implemented in the field prior to gravity surveys to get a glimpse of the possible crustal model that better resembles the geological processes of the region throughout the years. Geological information from the region can be compiled using boreholes, sediment cores, gamma logs, single-point resistance logs, spontaneous-potential logs, and seismic surveys in order to determine the principal types of rock present in the region and the possible structural processes that may have an influence on the density and rock distribution of the area.

The use of all available geophysical and geological information from a region can help scientists to understand the geological evolution and tectonic setting of the area under study by improving the imaging of its substructure. Through the use of Bouguer gravity anomaly datasets the work performed on the RGR region can help to better constrain the irregular anomalous masses that are found in the shallow crust and would provide more information about the evolution of the rift throughout the last centuries.
6.4 Future Work

Given that Bouguer gravity anomaly maps can help further constraint the models obtained from joint inversion and model fusion that rely solely on seismic information, the next step in this research effort is to integrate the rectangular prisms gravity forward model as part of the joint inversion and model fusion schemes proposed by Sosa et al. (2013a, 2013b), and Ochoa et al. (2011).

The information obtained from the joint inversion of different types of seismic datasets consists on 1-dimensional velocity profiles that provide information from the structure of the Earth right beneath the corresponding seismic station. Therefore, there are some details that need to be taken into consideration in order to be able to integrate the 3-dimensionality of gravity information and the 1-dimensional characteristics obtained from the joint inversion of surface wave dispersion and receiver functions. Sosa et al. (2013b) include in their results a 3-dimensional model of RGR region that may help us to further determine if the use of gravity information in joint inversion provides an improved model of the Earth’s structure with respect to the results obtained from the use of seismic information only.

With respect to the model fusion scheme proposed by Ochoa et al. (2011), the resultant fused model obtained from the model fusion of different seismic models from the RGR region would still consist on 1-dimensional velocity models describing the geological structure of the Earth right beneath the corresponding seismic stations. Therefore, a similar approach to the one for joint inversion should be generated.

In order to determine if the rectangular prisms gravity forward model is actually the best strategy to use in this particular setting, additional tests will be done by implementing the polygonal prisms algorithm for forward model of gravity anomalies with the same joint inversion and model fusion algorithms.

An additional effort will be taken to improve the running time of the rectangular and polygonal prisms algorithms through the implementation of parallel programming strategies.
using MPI and OpenMP. The inherent structure of both algorithms (polygonal and rectangular prisms) consist of nested loops that could be parallelized very easily through the use of parallel for-loops and be implemented in a cluster of computers in order to improve the algorithms’ average running time. This could improve the accuracy of the results obtained from the rectangular prisms algorithm without having to sacrifice running-time.
Chapter 7

Conclusions

Density variations in the Earth result from different material properties, which reflect the tectonic processes attributed to a region. Density variations can be identified through measurable material properties, such as seismic velocities, gravity field, magnetic field, etc. Gravity anomaly inversions are particularly sensitive to density variations but suffer from significant non-uniqueness. However, using inverse models with gravity Bouguer anomalies and other geophysical data, we can determine three-dimensional structural and geological properties of the given area. Through this work, we explored the use of three different techniques – simple bodies, rectangular prisms, and polygonal prisms – for the calculation of Bouguer gravity anomalies. The formulas and calculations involved in each one of the techniques were explained. The purpose of this study was to determine the most effective gravity forward modeling method that can be used to combine the information provided by complementary datasets, such as gravity and seismic information, to improve the accuracy and resolution of Earth models obtained for the underlying structure of the Rio Grande Rift. This was done by running numerical experimentations and analyzing the behavior of the rectangular and polygonal prisms schemes with different synthetic examples and information obtained from the Rio Grande Rift region in previous studies. We found that there are different uncertainties and important assumptions associated with each methodology that affect the accuracy achieved by each one of the gravity profile forward modeling techniques. Moreover, there may exist a bigger margin of error associated to the 2-D methods due to the simplification of calculations that do not take into account the 3-D characteristics of the Earth’s structure.

By analyzing the advantages and disadvantages associated to the implementation of each
one of the techniques mentioned before, it was found that due to the type of data obtained from seismic surveys, its format (1-dimensional velocity profiles at different stations) and its characteristics the rectangular prisms forward modeling scheme is the best alternative to use in the joint inversion and model fusion schemes discussed in previous sections. Although there is not a definite answer as to why rectangular prisms is more appropriate for the use of gravity acceleration information from a region over the polygonal prisms algorithm (given the many assumptions that both of the techniques apply), in the particular case of the datasets available for the RGR the rectangular prisms approach can be more easily adapted to the joint inversion and model fusion algorithms that are of interest for the 3-D imaging of its substructure.

The importance of this work resides on the benefits that the use of complementary datasets obtained from the Earth can provide to the imaging of the shallow crust. Through the use of Bouguer gravity anomaly datasets and seismic information obtained from the work performed previously on the RGR region, scientists will be able to better constrain the irregular anomalous masses that are found in the shallow crust and to obtain additional information about the evolution of the rift throughout the last centuries. It is expected that through the use of complementary datasets such as gravity and seismic information, questions about the deformation of the southern extent of the RGR and its influence on the evolution of adjacent areas within the North American Plate can be further analyzed and clarified.
References


Appendix A

Gravity Corrections

Instrument Drift and Tidal Effect Corrections  A drift is defined by the Encyclopedic Dictionary of Exploration Geophysics as “A gradual and unintentional change in the reference value with respect to which measurements are made” (Sheriff, 1991). The instrument drift is the effect that changes in the gravimeter’s response over time have in the observed gravitational readings. Although gravimeters are very stable and precise instruments, their high sensitivity makes them prone to be affected by changes in temperature, changes in the structure of the internal springs, and any minor readjustments in their internal mechanisms.

A commonly used example to illustrate this gravity variation at a base station over time was published in 1940 by Wolf. Using a gravity data set collected at a single station over a two day period, it was shown that the instrument drift can be represented by a least squares line of best fit to the data. A graph illustrating the data obtained from this publication is shown in Figure A.1.

Using the same information from Figure A.1, it can be seen that there is a more noticeable oscillatory behavior in the observed gravitational acceleration. These changes in gravitational acceleration are due to the tidal attraction of the sun and the moon – often called the tidal effect. Together, the effects of the sun and moon cause an acceleration at the surface of the Earth of about 0.3 mGals. In order to eliminate tidal effects from gravity observations, it is necessary to know the time at which each measurement is taken (Lowrie, 1997).

Latitude Correction  The normal gravity formula is used to determine the theoretical gravity at a given location based on its latitude. This formula is based on the assumption that the
interior of the Earth is uniform and uses the international reference ellipsoid as the theoretical shape of the Earth. If the observed gravitational acceleration is recorded as an absolute value of gravity, the latitude correction is performed by subtracting the predicted value $g_n$ (obtained with the normal gravity formula) from the observed value $g_{obs}$. Whenever the gravity survey is made with gravimeters, the quantity measured is the gravity difference between the observed gravity at a station and the value at the base station. In this case, given that the gravity acceleration increases towards the poles, “the correction for stations closer to the pole than the base station must be added to the measured gravity” (Lowrie, 1997).

The formula used for the latitude dependent changes based on Geodetic Reference Formula of 1967 is:

$$g_n = 978.03185 (1 + 0.005278895 \sin^2(\phi) - 0.000023462 \sin^4(\phi)) \quad (A.1)$$

where $\phi$ is the latitude of the station.

Some additional reference formulas include:

- First internationally accepted International Gravity Formula (IGF) in 1930:

$$g_n = 9.78049 (1 + 0.0052884 \sin^2(\phi) - 0.0000059 \sin^2(2\phi))$$
• The Geodetic Reference System in 1967 provided the 1967 IGF:

\[ g_n = 9.78031846(1 + 0.0053024 \sin^2(\phi) - 0.0000058 \sin^2(2\phi)) \]

• The Geodetic Reference System of 1980, leading to World Geodetic System 1984 (WGS84):

\[ g_n = 9.7803267714 \left( \frac{1 + 0.00193185138639 \sin^2(\phi)}{\sqrt{1 - 0.00669437999013 \sin^2(\phi)}} \right) \]

Here the Normal Gravity \( g_n \) is the gravitational acceleration expected to be generated by a homogeneous rotating ellipsoidal Earth with no geological complication and no surface features at the given latitude. This constitutes the reference value for gravity measurements. However, in practice it is commonly impossible to measure gravity on the reference ellipsoid at the places where the reference value is known. The positions of the measurement stations may be situated above or below the ellipsoid and the station may be surrounded by hills or valleys that affect gravitimeter’s readings. Because of this, before this reference gravity value can be used and compared to the measured gravity, there are additional corrections that must be made to the theoretical gravity (Lowrie, 1997).

**Terrain Correction** An additional correction for stations located in areas of non-uniform rugged terrain has to be applied in order to correct “for the departure of the terrain from a plane surface” (Sharma, 1997). This correction is the *terrain correction* and can be illustrated using Figure A.2. Using this figure as reference, it can be seen that a hill in position 1 located above station S will give an upward component of attraction, which will reduce the gravity attraction caused by the rest of the Earth (Sharma, 1997). Likewise, any valley located near station S (e.g., position 2) will correspond to a negative mass in the Bouguer slab which also tends to reduce the gravity effect at station S (Sharma, 1997). The terrain correction, \( g_{TC} \) originated by both topographical irregularities is always positive and should be added to the
measured gravity difference.

![Figure A.2: The terrain correction takes into consideration the effects caused by topographic rises (1 and 3) and depressions (2 and 4) (Sharma, 1997).](image)

The terrain correction is usually obtained following the procedure shown in Hammer (1939). Here, the corrections are obtained by “dividing the region around a station into segments bounded by concentric rings at suitable angular intervals \( \phi \). The difference in mean elevation \( \Delta h \) between each segment and the gravity station is determined from a topographic map, without regard to its sign” (Sharma, 1997). Figure A.3 illustrates the terrain correction and the transparent overlay used to determine the contribution of each element.

The terrain correction caused by the attraction of the material in each segment is given by:

\[
g_{TC} = G \rho \phi \left[ (r_2 - r_1) + \sqrt{r_1^2 + (\Delta h)^2} - \sqrt{r_2^2 + (\Delta h)^2} \right]
\]

The correction for terrain effects within 50 meters of the location of a gravity station can be very significant and must be taken into consideration. For those features that are located hundreds of meters or more away, the effects will influence each survey station to an equal extent and need not be considered even if they are possibly quite large in magnitude (Sharma, 1997).

**Bouguer Plate Correction** Once the terrain correction has been performed and the topography of the region has been “levelled” (valleys have been filled out and hills have been cut), there is a uniform layer of rock, with density \( \rho \), that is located between the gravity station and
the reference ellipsoid. This layer is called the Bouguer plate and it is the basis for the Bouguer plate correction. Figure A.4 (a-b) illustrate the terrain and Bouguer plate corrections.

The contribution of this uniform layer of rock to the total gravitational acceleration at a station can be calculated by using the Bouguer plate correction, $\Delta g_{BP}$, which “compensates for the effect of a layer of rock whose thickness corresponds to the elevation difference between the measurement and reference levels. This is modelled by a solid disk of density $\rho$ and infinite radius centered at the gravity station $P$” (Lowrie, 1997).

The Bouguer plate correction, $\Delta g_{BP}$, is given by:

$$\Delta g_{BP} = 2\pi G \rho h$$

This expression can be reduced to $0.0419 \times 10^{-3} \rho \text{ mGals/m}$ where the density, $\rho$, is in kg/m$^3$. The appropriate value of $\rho$ depends on the type of region that is being modeled (e.g., for
crustal rocks $\rho = 2670$ kg/m$^3$ marine gravity surveys use $(\rho - 1030)$ kg/m$^3$, while large deep lake surveys use $(\rho - 1000)$ kg/m$^3$ (Lowrie, 1997).

**Free-Air Correction** After the Bouguer Plate correction has been applied to the theoretical gravity value, there is a difference in height between the station and the position of the station on the reference ellipsoid, which gives the impression that “the measurement station is [now] floating in air above the ellipsoid” (Lowrie, 1997). In other words, this effect is due to a change of elevation in the stations, as if the stations were now suspended in free-air and not situated on the land. Hence, the free-air correction accounts for changes in the gravitational acceleration due solely to an increase in the distance from the center of the Earth and is given by:

$$\Delta g_{FA} = \frac{\partial g}{\partial r} = -\frac{2g}{r}$$

The expression can be reduced to $\Delta g_{FA} = 0.3086$ mGal/m after using the Earth’s radius (6371 Km) and average value of gravity (981,000 mGals).
**Combined Elevation Correction**  Free-Air and Bouguer Plate corrections can often be combined into a single elevation correction:

\[ \Delta g_{COMB} = (0.3086 - (0.0419 \times 10^{-3} \rho)) \text{ mGals/m} \]

An example for the combined elevation correction of stations located above typical crustal rocks with density 2670 kg/m³ would result in 0.197 mGals/m. This quantity should be added to the measured gravity whenever the gravity station is situated above the reference ellipsoid and subtracted when it is below (Lowrie, 1997).
Azucena Zamora was born in Juarez, Chihuahua, Mexico as the second daughter of Alejandra Reyes and Isaias Zamora. Her interest in Mathematics started in middle school when she was first introduced to the art of algebra by her teacher. After moving to the United States in 2000, she attended the El Paso Community College and later transferred to the University of Texas at El Paso. Majoring in applied mathematics, she obtained her bachelor in science degree in 2007. While studying for her bachelor’s degree, she worked as a Supplemental instructor (SI) leader for Calculus I students under the direction of Dr. Emil Schwab. Upon completion of her bachelor’s degree, she started her studies in the Mathematics Department in the Master of Arts degree in Teaching Mathematics. During this stage of her academic career, Ms. Zamora worked with Dr. Hamide Dogan-Dunlap as part of an NSF grant focused on inquiry learning in matrix algebra courses for undergraduates. This work was the basis for her thesis “Use of Cognitive Constructs in Linear Algebra”. Once she finished her master’s degree, Ms. Zamora enrolled in the doctoral program in Computational Science at UTEP in which she worked as a research associate for the Cyber-ShARE center of excellence with supervision of Dr. Aaron Velasco. Recently, Ms. Zamora was awarded a fellowship from the NSF Graduate STEM Fellows in K-12 Education program in which she works with early college high school students. Ms. Zamora is currently enrolled in her third year of the Computational Science PhD program.

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