Analysis of Differential Item Functioning on Selected Items Assessing Conceptual Knowledge of Descriptive Statistics for Spanish-Speaking ELL and non-ELL College Students

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ANALYSIS OF DIFFERENTIAL ITEM FUNCTIONING ON SELECTED ITEMS ASSESSING CONCEPTUAL KNOWLEDGE OF DESCRIPTIVE STATISTICS FOR SPANISH-SPEAKING ELL AND NON-ELL COLLEGE STUDENTS

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____________________________
Benjamin C. Flores, Ph.D.
Interim Dean of the Graduate School
to my

MOTHER and FATHER

with love
ANALYSIS OF DIFFERENTIAL ITEM FUNCTIONING ON SELECTED ITEMS ASSESSING CONCEPTUAL KNOWLEDGE OF DESCRIPTIVE STATISTICS FOR SPANISH-SPEAKING ELL AND NON-ELL COLLEGE STUDENTS

by

ANGELICA MONARREZ, B.S.

THESIS
Presented to the Faculty of the Graduate School of The University of Texas at El Paso in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

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Abstract

Recently, there has been growing interest in promoting conceptual understanding of statistical concepts in the classroom. The Assessment Resource Tools for Improving Statistical Thinking (ARTIST) project is a resource for maintaining and developing scales useful for measuring statistical conceptual knowledge. The focus of this study is to investigate whether items assessing conceptual knowledge of measures of center and variation from the (ARTIST) database show evidence of differential item functioning when administered to English Language Learners (ELLs). This is pertinent topic since the population of English Language Learners (ELL) in the United States has been growing rapidly in the past few years.

There is a large body of research about assessment of ELLs in mathematics. However, there is none that focuses just on statistics. Yet, statistics is an important application of mathematics and it requires an expanded vocabulary. In statistics we are not only dealing with numerical answers but also with written responses. For the purpose of this research, we studied assessments for ELL students in statistics focusing on the largest population of ELLs, native Spanish speakers. The items studied focus on measures of center and variability. This is an appropriate focus since all students encounter these concepts and these items are among those that utilize vocabulary that may be difficult for ELLs.

The survey was given to students taking an introductory statistics class at a large urban binational research university located in the Southwest and a large community college system in a large Southwestern urban environment both located by the Mexican border. There was some evidence of Differential Item Functioning (DIF) on some items taken from the ARTIST database on measures of center and variation. For some ability levels, ELLs had a lower probability of answering the item correctly and for other levels of ability that probability was higher for ELLs depending on the type of question. Overall the questions that showed DIF were about mean, median, interquartile range, spread, and average which
are common terms that students are expected to understand by the end of an introductory statistics course. Often, these terms are hard to understand even for non-ELLs, but may be even more difficult for ELLs. Students seemed to have issues when moving from the everyday register to the academic register of the word. In addition, ELLs may have a different everyday register of a word than non-ELLs which led them to answer differently.
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Chapter 1

Introduction

1.1 Importance of Assessment of Conceptual Knowledge

There has been growing interest in promoting conceptual understanding of statistical concepts in the classroom. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report, states that “(t)he desired result of all introductory statistics courses is to produce statistically educated students, which means that students should develop statistical literacy and the ability to think statistically” (Aliaga et al., 2010, p. 11). However, assessing statistical conceptual knowledge is very difficult, particularly for diverse student populations. Gal and Garfield wrote on the challenge of assessing statistical knowledge that “educators are further challenged by the need to make sure that students understand the real-world problems that motivate statistical work and investigations, and by the need to help students become familiar with the many nuances, considerations and decisions involved in generating, describing, analyzing, and interpreting data and in reporting findings” (Gal & Garfield 1997, p. 5). Thus, efforts need to be made to establish that current statistical assessments are measuring student conceptual knowledge in valid and reliable ways.
1.2 Tests or Scales

A test is an instrument used to measure conceptual knowledge. De Ayala (2009) defines measurement as “the process by which an attempt is made to understand the nature of a variable (cf. Bridgman 1928)” (p. 1). For the purpose of this study, we are considering variables that cannot be observed directly. These are latent variables or constructs. For example, if we want to measure statistical conceptual knowledge, we cannot directly observe the depth of knowledge. Rather, test items serve as an imperfect measure of knowledge.

There are some important points that need to be addressed before utilizing a test measuring conceptual knowledge. First, we have to look at the reliability of the test. Internal consistency, a component of scale reliability is used to assess the consistency of results across items within a test. If it is consistent across time, then it is said to have high reliability, otherwise it has low reliability. Second, we look at the validity of the test, or whether the test actually measures what it is supposed to measure. We want to accurately explain the latent variable by the measure. A test with good measurement properties will have high reliability as well as high validity. The third issue is the invariance of the test. Invariance is the property of independence between the measuring instrument and the subjects. Finally, we need a baseline for measuring the responses, for example, on a test we have nominal data because answers are right/wrong.

1.2.1 Example

Let us now consider an example in depth. Throughout this research we are going to be working with the Assessment Resource Tools for Improving Statistical Thinking (ARTIST) project (https://app.gen.umn.edu/artist/tests/index.html). This project was funded by the NSF to create an assessment instrument that would cover the wide array of students taking an introductory statistics course. Garfield and Gal wrote: “there is an increasing need to develop reliable, valid, practical, and accessible assessment items and instruments” (Garfield and Gal, 1999, p. 4). From the ARTIST project, an overall Comprehensive As-
essment of Outcomes in Statistics (CAOS) was created (delMas et al, 2006). The purpose was to find different items measuring concepts that students are expected to understand at the end of an introductory statistics course.

1.2.2 Validation

The CAOS project was a three year research project conducted by an experienced team of experts in education, statistics education and measurement. The ARTIST project had an advisory board created to help with the necessary content validity for the CAOS test as well as selection of test items. According to the advisory group feedback they created four versions of the CAOS test: CAOS, CAOS 2, CAOS 3, and CAOS 4. Each one is an improved version of the previous one. “An online prototype of CAOS was developed during summer 2004, and the advisors engaged in another round of validation and feedback in early August, 2004. The feedback was then used to produce the first version of CAOS, which consisted of 34 multiple-choice items” (delMas et al, 2006, p. 6). On the second round of evaluation the second version CAOS 2 was given as a pretest and post-test to students. After the results they made changes and created the CAOS 3.

According to delMas (2006) “the third version of CAOS was given to a group of 30 statistics instructors who were faculty graders of the Advanced Placement Statistics exam in June, 2005, for another round of validity ratings” (p. 7). With this feedback they created the CAOS 4 version with 40 multiple choice questions. There was a final analysis with a group of 18 members of the advisory and editorial boards of the Consortium for the Advancement of Undergraduate Statistics Education (CAUSE). They are well known statistics teachers at the college level as well as renowned experts in the statistics education community. The CAOS 4 version was given to this group of experts who unanimously agreed that the CAOS 4 measures important basic learning outcomes, and 94% agreement that it measures important learning outcomes (delMas et al, 2006). The validation population included females (57.3%) and males (40.5%). The ethnicity was White (74.3%), Black (5.1%), Asian (8.5%), and Chicano (3.6%) (Percentages do not add to 100% due to
missing data) (delMas et al, 2006).

1.2.3 Reliability Analysis

Out of 1028 students that took the CAOS test as a pretest and post-test 817 met the criteria used to select students for the reliability analysis of internal consistency. Students were required to answer all 40 questions on the test either in class using a paper test or take an online version lasting no more than 60 minutes. The internal consistency estimate was $\alpha = .77$ (Cronbach’s alpha). This implies that the CAOS test items have satisfactory internal consistency for the population of students taking an introductory statistics course (delMas, 2006).

1.3 Research Question

When tests are given to new populations, some issues arise regarding validity and reliability. When the scale is created for a certain population it might function differently when given to another population. People with a distinct age, education level, race, or language background might be in disadvantage when the scale was only tested for one kind of population. Our research question is to examine whether items from the ARTIST database on methods of center and variation function differently when administered to English Language Learners (ELLs).
Chapter 2

ELLs in the Mathematical Sciences

The population of English Language Learners (ELL) in the United States has been growing rapidly in the past few years. According to Goldenberg (2008), the population of ELLs in K-12 public schools grew from 1 out of 20 in 1990 to 1 out of 9 only fifteen years later. With this fast growth, he argues that 1 out of 4 K-12 students in the United States will be an English Language learner in 20 years. Even though not all of this population would attend college, there is a still a high population of ELLs in college. Just in Texas, which has the second highest population of ELLs (832,000 ELL students compared to California with 1.1 million ELL students): 46% of Asian ELLs, 26% of Black ELLs and 15% of Hispanic ELLs enroll in a 4-year public college (Flores, Batalova, and Fix, 2012, p. 17).

Academic language is widely used in college courses. If an ELL is more familiar with the everyday usage of English its very likely that he/she would struggle understanding the higher academic language used in college.

According to Cummins (1992), there are two proficiencies acquired when someone learns a new language: Basic Interpersonal Communicative Skills (BICS) and Cognitive Academic Language Proficiency (CALP). BICS are required for everyday communication such as reading, writing and listening, whereas CALP skills are necessary for an academic context. The later the person tries to learn a new language the harder would be to acquire the necessary CALP skills to succeed in school since the academic registers are already built in their own language. Lesser and Winsor (2009) state that “the challenge ELLs face is that the academic meaning of a term may be the same as the everyday meaning, different from everyday meaning, or not have an everyday counterpart at all” (p. 8). For the purpose of this research we would make the argument for a need of assessment for ELL students in
statistics focusing on the largest population of ELLs, Spanish native speakers. Figure 2.1 is a diagram illustrating the interaction between English and Spanish academic and everyday languages. Notice that an academic context is needed more in math and statistics but is also influenced by the everyday language at some point.

![Figure 2.1: Interaction between English and Spanish academic and everyday languages](image)

According to August (2005), “Skilled readers can tolerate a small proportion of unknown words in a text without disruption of comprehension and can even infer the meanings of those words from sufficiently rich contexts” (p. 50). On the other hand, for ELLs the proportion of unknown words is longer, thus making comprehension more difficult. In addition, context can complicate understanding rather than aid. Diane August also argues that ELLs are more likely to be diagnosed as learning disabled than non-ELLs. However, their poor performance might be due to their lack of English vocabulary rather than a learning disability.

When a person is trying to learn a new word in a language, the first thing that the person would look for is the definition of that word in his/her native language. Then we usually try to find a word in the first language that sounds similar to the new word. English and Spanish are very similar, there are many words that almost sound the same and mean
the same thing in English and Spanish. These words are called cognates. Words such as different/diferente or division/división might be easy to identify to a Spanish speaker learning English. However, there are false cognates as well that might be confusing for some such as embarrassed/embarazada sound the same but they have completely different meanings.

Many mathematics and statistics teachers may think that their job is to teach mathematics not language. However, mathematics is a new language that students need to master, a task possibly difficult for non-ELLs. Students usually need some time to adapt to mathematics language. Yet, at the end of a course students are expected to read, understand, and discuss mathematical ideas (Thompson, 2000). Thompson says that “teachers forget that the words and phrases that are familiar to us are foreign to our students” (p. 568).

In fact, there are many reasons why a non-ELL or ELL might get confused when learning mathematics vocabulary. One of the problems identified by Thompson (2000) is that “some words are shared by mathematics and everyday English, but they have distinct meanings”, for example in algebra: radical, origin, function or in statistics: mode, event, combination (p. 569). Therefore, if an ELL has just the BICS skills he/she would struggle to find the distinction between the mathematical meaning and the everyday meaning. Garrison (1999) states that when dealing with a linguistically diverse classroom, teachers must first consider the language needed as well as the language proficiency of the students in order to provide instruction.

Another issue pointed out by Thompson (2000) was, “a single English word may translate into Spanish or another language in two different ways” such as round (redondear), as in round off, or round (redondo), as in circular (p. 569). Hence, we have another reason to make the argument that understanding English is an important factor for ELLs in a mathematics classroom. Garrison (1999) states the importance of adding language in the mathematics classroom:

“As English-language learners make the transition from primary language into
English instruction, the English equivalents for the mathematical terms they learned in their primary language might not be covered in the upcoming lessons, creating gaps in their English vocabulary that are irregular and unpredictable. Therefore, mathematics teachers should review or preview all essential vocabulary at the beginning of a lesson or unit, especially when English-language learners are in the class. New mathematical vocabulary in the second language, however, is most effectively introduced after students have established the concepts the vocabulary words represent so that they learn the new ‘label’ for the known concept” (p. 49).

Previously, we have mentioned that poor achievement by ELLs might be confused with a learning disability. The following example, illustrating this point, is adapted from Garrison’s article (Garrison & Mora, 2008 pp. 43-44). The example is work done by a student that was a recent immigrant. When she was asked to explain the area of a triangle, her answer in English was very limited as we can see in Figure a. However, when she was asked to answer in Spanish her answer was considerably better (Figure b). This example illustrates that ELLs often have the right idea; they just don’t know how to express that idea in English.

![Figure 2.2: Answer in Spanish shows that the student knew the concept](image)
(a) Response in English
(b) Response in Spanish

Figure 2.2: Answer in Spanish shows that the student knew the concept
There is some research done on the language factor importance on assessing students in mathematics. Abedi & Lord (2001) found that ELLs were at disadvantaged when solving word problems and that modifying the linguistic structure of the problem resulted on a better performance for ELLs. There is another study that investigated math achievement differences between ELLs and non-ELLs students on a literacy-based performance assessment (LBPA) (Brown, 2005). Young et al (2011) also found some differences between ELLs, former ELLs and non-ELLs on a content based assessment in math. Mahoney (2008) states the need for large sample of students at different levels of English proficiency to create a good assessment for ELLs in mathematics. On the statistics area, there are some discussions about assessing students in statistics courses (Onwuegbuzie & Leech, 2003). There is also the CAOS assessment test mentioned above (Delmas et al, 2006). Yet, these assessments on statistics do not focus on ELLs.

There is a large body of research about assessment of ELLs in mathematics as mentioned in the previous paragraphs. However, there is none that focuses on statistics. Nevertheless, statistics is an important branch of mathematics and it requires an expanded vocabulary. In statistics we are not only dealing with numerical answers but also with written responses. Jennifer Kaplan et al (2009, 2011, 2012) have studied issues with vocabulary in statistics. However, it is important to point out that this and related studies did not include English Language Learners.

As we have seen, language is an important factor in mathematics and statistics achievement. There is a need to see statistics as a different subject on its own, especially for ELLs. According to Lesser and Winsor, the distinctiveness of statistics is relevant because one or more of the ways in which statistics is different from mathematics could plausibly affect how ELL issues play out in a concrete way (2009). We can summarize Lesser’s findings as follows (Lesser & Winsor, 2009 pp. 18-20):

- Students move among registers: confusion between the everyday usages of the word with academic usages of the word (in statistics or even mathematics).
• Context plays an important role when explaining statistics: the way the teacher is presenting the material is not familiar for the Spanish speaking audience.

• There may be an overlap between the previous two: confusion between mathematics and statistics registers.

For the purpose of this research we would make the argument for a need of assessment for ELL students in statistics focusing on the largest population of ELLs; that is native Spanish speakers. We build upon Lesser and Winsor’s research with Spanish speakers in an introductory statistics class concentrating on measures of center and variation.

Appendix A contains the survey items selected for this study. These items focus on measures of center and variability. This is an appropriate focus since all students encounter these concepts in all courses early in the semester. In addition, these items utilize vocabulary that may be difficult for ELLs. The Communication, Language and Statistics Survey (CLASS) identified that ELLs experience some difficulty differentiating between the words mean, median, and mode (Lesser, Wagler, Esquinca, & Valenzuela).
Chapter 3

IRT and DIF

3.1 Item Response Theory

Item Response models show the association between the correct response of an item and the ability to answer it measured by an instrument. The instrument in most cases is a survey or a test and the items are the questions in the instrument. IRT models can be either dichotomous (two categories) such as correct/incorrect questions or polytomous (more than two categories) such as poor, fair, good, excellent.

In order to predict a set of ability parameters for each person $\Theta$, there are a set of parameters from an IRT model called difficulty, discrimination, and guessing. First, we consider the parameter item difficulty which is the location parameter. The location parameter, under certain conditions is the likelihood of correct response $\beta$ in reference to the ability at which 50% of the examinees will answer correctly. In more general conditions it is the inflection point. Next, the discrimination parameter $\alpha$ is the difference between examinees and their latent trait. Large discrimination parameters can easily differentiate between examinees with high and low ability. Finally a lower asymptote or guessing parameter $\chi$ is also included in the IRT model. This parameter computes the probability that a person with the lowest ability answers the item correctly. The IRT model can include a lower or left asymptote and an upper or right asymptote. These are the parameters that an IRT model may include even though these may not always be freely estimated.

IRT models are often utilized for education scales that assess student knowledge, for example, the CAOS test. Validation studies for educational scales estimate the difficulty and discrimination parameters for each item as we can see in Table 3.1.
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<td>20</td>
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<td>.1889</td>
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<table>
<thead>
<tr>
<th>ITEM</th>
<th>DIFFICULTY $\beta$</th>
<th>DISCRIMINATION $\alpha$</th>
</tr>
</thead>
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<tr>
<td>21</td>
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<td>.3068</td>
</tr>
<tr>
<td>40</td>
<td>.5395</td>
<td>.3634</td>
</tr>
</tbody>
</table>

Table 3.1: Difficulty and Discrimination values

For a student with an average ability, we can see that on the CAOS test item 7 has a $\beta = .1469$ the probability that this randomly selected student will correctly respond to
this item is only .1469, whereas on item 11 the probability of answering the item correctly will be .8905. In addition, item 1 has a low discrimination value with $\alpha = .0684$ thus item 1 does not differentiate well among respondents. On the other hand, item 6 has a higher discrimination value with $\alpha = .4915$, so this item differentiates well among respondents.

### 3.2 Dichotomous models

Dichotomous models yield the probability of a score of 1 for a correct response or 0 for incorrect responses. The difference relies on the parameters that are being studied. In this section, different models and their respective parameters are discussed.

#### 3.2.1 Rasch Model:

The dichotomous Rasch Model (RM) assumes that there is a real-valued latent trait $\Theta_j$ for each examinee $j$ and a real-valued difficulty parameter $\beta_i$, for each item $i$, 

$$P(Y_{ij} = 1) = \frac{e^{(\Theta_j - \beta_i)}}{1 + e^{(\Theta_j - \beta_i)}} \quad (3.1)$$

for all examinees $j = 1, ..., N$ and all items $i = 1, ..., I$, and where $N =$ number of students and $I =$ number of items.

With this definition of RM, we have that the probability of responding correctly increases strictly with an increase in the parameter $\Theta_j$.

#### 3.2.2 One Parameter Logistic Model (1PL):

For the 1PL, the discrimination parameter $\alpha$ is the same among all examinees. The only parameter changing for each item is the difficulty parameter $\beta$ and $\Theta$, the latent trait varies for each examinee. The 1PL model is:

$$P(Y_{ij} = 1) = \frac{e^{\alpha(\Theta_j - \beta_i)}}{1 + e^{\alpha(\Theta_j - \beta_i)}} \quad (3.2)$$
for all examinees \( j = 1, \ldots, N \) and all items \( i = 1, \ldots, I \), where \( N \) = number of students and \( I \) = number of items. The difference between the 1PL and the RM model is that the RM model holds \( \alpha = 1 \) whereas in the 1PL model \( \alpha \) is freely estimated.

### 3.2.3 Two Parameter Logistic Model (2PL):

By allowing the discrimination parameter and difficulty parameter to vary the 2PL model is formulated. The 2PL model is given by,

\[
P(Y_{ij} = 1) = \frac{e^{\alpha_i(\Theta_j - \beta_i)}}{1 + e^{\alpha_i(\Theta_j - \beta_i)}}
\]

for all examinees \( j = 1, \ldots, N \) and all items \( i = 1, \ldots, I \) and where \( N \) = number of students and \( I \) = number of items.

On this model \( \Theta_j \) is the latent trait, \( \alpha_i \) is the location (discrimination) parameter and \( \beta_i \) is the difficulty parameter. With the 2PL model, an item provides the maximum probability of a correct response at \( \beta_i \). In contrast to the 1PL, in the two parameter model the maximum amount of information or reliability can vary from item to item as \( \alpha_i \) varies across items (DeAyala 2009, p. 119).

### 3.2.4 Three Parameter Logistic Model (3PL):

The 3PL model adds a lower asymptote parameter, called the guessing parameter. To better explain this parameter, we can look at an examinee that is taking a test in a completely different language than the one he/she speaks. The examinee does not understand this language so there is a probability of answering correct just by guessing. In this case the guessing parameter would be \( \chi_i \) and most of the time this parameter is higher than chance. We also add the difficulty and discrimination parameters as in in the 2PL model. The equation is:

\[
P(Y_{ij} = 1) = \chi_i + (1 - \chi_i) \frac{e^{\alpha_i(\Theta_j - \beta_i)}}{1 + e^{\alpha_i(\Theta_j - \beta_i)}}
\]

for all examinees \( j = 1, \ldots, N \) and all items \( i = 1, \ldots, I \), and where \( N \) = number of students and \( I \) = number of items.
3.3 Item Characteristic Curve

The item characteristic curve (ICC) is the relationship between examinees' item performance and the underlying variable of interest. We use a logistic function and the X-axis is the latent variable or ability and the Y-axis is the probability of getting the item correct. It is very common that the X-axis goes from $-3$ to $+3$.

The following table explains the ICC properties related to the IRT models:

<table>
<thead>
<tr>
<th>ICC Property</th>
<th>Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position along the X-axis</td>
<td>Item difficulty</td>
</tr>
<tr>
<td>($\beta$ parameter)</td>
<td>Amount of aptitude to get an item right</td>
</tr>
<tr>
<td>Slope</td>
<td>Item discrimination</td>
</tr>
<tr>
<td>($\alpha$ parameter)</td>
<td>Flat ICC does not differentiate among test takers</td>
</tr>
<tr>
<td>Y-intercept</td>
<td>Guessing</td>
</tr>
<tr>
<td>($\chi$ parameter)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Interpretation of ICC Properties for Knowledge measures

3.4 Estimation Method: Joint Maximum Likelihood

For the 3PL model, we will be using joint maximum likelihood to estimate the parameters. This estimation procedure consists of finding the set of item and person parameters that would maximize the likelihood of the observed item responses. The likelihood equation is as follows:

$$L(\theta, a, b, c; y) = \prod_{j=1}^{I} \prod_{i=1}^{n} P_i(\theta_j; a_i, b_i, c_i)^{y_{ij}} [1 - P_i(\theta_j; a_i, b_i, c_i)]^{1-y_{ij}}$$

(3.5)

Where $y_{ij}$ is the response to item $i$ by person $j$. 

15
We maximized the logarithm of the likelihood mentioned above and this is the set of estimation equations:

\[
\frac{\partial}{\partial \theta_j} = \sum_{i=1}^{I} [c_i a_i D + a_i (1 - c_i) E - a_i F] = 0
\]

\[
\frac{\partial}{\partial a_i} = \sum_{j=1}^{N} [c_i (\theta_j - b_i) D + (1 - c_i)(\theta_j - b_i) E - (\theta_j - b_i) F] = 0
\]

\[
\frac{\partial}{\partial b_i} = \sum_{j=1}^{N} [-c_i a_i D - a_i (1 - c_i) E + a_i F] = 0 \tag{3.6}
\]

\[
\frac{\partial}{\partial c_i} = \sum_{j=1}^{N} [D - a_i (\theta_j - b_i) E] = 0
\]

Where:

\[
D = \frac{y_{ij} \exp\{a_i (\theta_j - b_i)\}}{1 + c_i \exp\{a_i (\theta_j - b_i)\}}
\]

\[
E = (1 - y_{ij}) \tag{3.7}
\]

\[
F = \frac{\exp\{a_i (\theta_j - b_i)\}}{1 + \exp\{a_i (\theta_j - b_i)\}}
\]

We can find the solutions of these equations by starting with an initial value of the ability parameters and solving for the item parameters, then holding the item parameters fixed, and solving for an improved estimate of the ability parameter values, and so on. This technique was done by Birnbaum and it’s called the joint maximum likelihood (JML) (van der Linden & Hambleton, 1997 p. 15).

### 3.5 Differential Item Functioning

Differential Item Functioning (DIF) or measurement bias refers to “differences in the way a test item functions across demographic groups that are matched on the attribute measured by the test or the test item” (Osterlind 2009, p.8). For a more formal definition, we can have any of the previous models where \( Y = 1 \) is the response to a particular question on a
test or survey. We also have the ability or latent trait denoted by $\Theta$. So we would express the conditional probability of $Y$ given $\Theta$ as $f(Y|\Theta)$. On the DIF case we want to compare the answers of the conditional probability within two groups which we would called “focal” and “reference” groups. Even though there is no difference between which group will be the reference or the focal, it is common to name the group for which we think the test or survey will favor as the reference group. Thus, that group in disadvantage will be the focal group. If there is no DIF and the measurement errors distribution are the same for both group we have the following:

$$f(Y|\Theta, G = R) = f(Y|\Theta, G = F) \quad (3.8)$$

Where $G$ is the grouping variable, $R$ is the reference group, and $F$ is the focal group. We can use the properties of the ICC to determine whether there is DIF within the items. We can plot the same item for the different groups. If the ICCs are almost identical then the item does not display DIF. If the area within the curves is large then the item does display DIF. If the lines does not cross then the DIF is called uniform and if they cross is non-uniform.

Use of ICC to determine DIF within the items by plotting the same item for the different groups.

![Figure 3.1: Source: Zumbo, 1999 p. 17-21](image)

(a) Uniform DIF  
(b) Non-uniform DIF  
(c) No DIF is exhibited
3.6 Methods for Detecting DIF

There are many methods to detect DIF. The Logistic Regression method is widely used for binary data (correct/incorrect) and it detects non-uniform DIF. The SIBTEST can also detect non-uniform DIF.

3.6.1 SIBTEST: Simultaneous Item Bias Test

This method is a non-parametric test to detect uniform and with some changes it can also detect non-uniform DIF. For testing the null hypothesis of no DIF they created a test statistic $B_{uni}$ (for unidirectional or uniform) as:

$$B_{uni} = \frac{\hat{\beta}_{uni}}{\hat{\sigma}(\hat{\beta}_{uni})}$$

(3.9)

The null hypothesis is rejected with level of significance $\alpha$ if $B_{uni} > Z_{\alpha}$ where $P[N(0, 1) > Z_{\alpha}] = \alpha$ (Finch & French 2007, pp. 568-569)

3.6.2 IRTLR: Item Response Theory Likelihood Ratio Test

This is another method made for DIF. First we can fit the data by using the required model, for instance the 3PL. After that, we constrain all the item parameters to be the same in both the reference and focal groups. This gives the log-likelihood statistic (Finch & French 2007, pp. 569-570):

$$LL_{equal} = \sum_{G=1}^{2} \sum_{j=1}^{N} \ln[\sum_{i=1}^{q} \prod_{i=1}^{I} (T_{iG}(u_{ijG})\phi_{G}(\theta)d\theta)]$$

(3.10)

Where $T_{iG}(u_{ijG}) = \text{parameters for the ICC group when is constrained to be equal for both groups as mentioned above and } \phi_{G}(\theta) \text{ is the distribution of the latent trait for group G.}$ After that, the IRT model is fitted again but now it holds all item parameters equal among groups except for those when DIF is being assessed. Then, we would have another likelihood value ($LL_{unequal}$). The test statistic is a chi-square with 3 degrees of freedom of
the difference between the two values computed before $G^2 = -2(LL_{equal} - LL_{unequal})$. If the test is significant then there is DIF, we can also test each parameter individually by a chi-square with 1 degree of freedom.

3.6.3 Mantel-Haenszel

The Mantel-Haenszel method is based on contingency tables by creating $I \times 2 \times 2$ contingency tables (Hidalgo & Lopez-Pina 2004, pp. 905-907). It compares the focal and reference groups against the total observed, where I is the number of items. With the MH method we can find uniform DIF on an item when the odds of answering an item correct at a level $i$ is different for the two groups. We can define the odds ratio ($\alpha$) as:

$$\alpha = \frac{p_{Ri}/(1 - p_{Ri})}{p_{Fi}/(1 - p_{Fi})} = \frac{N_{1Ri}N_{0Fi}}{N_{0Ri}N_{1Fi}},$$

(3.11)

where $p_{Ri}$ and $p_{Fi}$ are the probability of answering an item correct for the reference and focal group, with the item $i$. The test statistics for detecting DIF in an item is distributed under the null hypothesis as a $\chi^2$ with 1 degree of freedom and is expressed as

$$MH = \frac{\left(\sum_{i=1}^{I} A_i - \sum_{i=1}^{I} E(A_i) \right)^2}{\sum_{i=1}^{I} Var(A_i)}$$

(3.12)

where $E(A_i) = (N_{Ri}N_{1i})/N_{..i}$ and $Var(A_i) = (N_{Ri}N_{Fi}N_{0i}N_{0Ri})/(N_{..i})^2(N_{..i} - 1)$ The common odds ratio, which assumes an equal odds ratio across items is, noted as $\hat{\alpha}_{MH}$ can be estimated as shown in (3.13):

$$\hat{\alpha}_{MH} = \frac{\sum_{i=1}^{I} N_{1Ri}N_{0Fi}/N_{..i}}{\sum_{i=1}^{I} N_{1Fi}N_{0Ri}/N_{..i}}$$

(3.13)

To better interpret $\hat{\alpha}_{MH}$, we can have a log transformation as given in 3.14:

$$\Delta_{\hat{\alpha}(MH)} = -2.35 \ln(\hat{\alpha}_{MH}).$$

(3.14)

Here are some guidelines to evaluate the size of DIF on the items:

- $\Delta_{\hat{\alpha}(MH)} < |1| \Rightarrow$ no DIF.
• $|1| \leq \Delta_{\alpha(MH)} \leq |1.5|$ and MH is statistically significant ⇒ moderate DIF

• $\Delta_{\alpha(MH)} > |1.5|$ and MH is statistically significant ⇒ large DIF

### 3.6.4 Logistic Regression

As mentioned above, the MH method is used when uniform DIF is present. However, there are cases when we have non-uniform DIF and logistic regression works well with crossing DIF. We can slightly change the logistic regression model by specifying equations for the two groups:

$$P(y_{ij} = 1 | \theta_j G) = \frac{e^{(\beta_0 G + \beta_1 G \theta_j G)}}{1 + e^{(\beta_0 G + \beta_1 G \theta_j G)}}, \ i = 1, \ldots, I \ j = 1, \ldots, N \ G = 1, 2. \ (3.15)$$

- if $\beta_{01} = \beta_{02}$ and $\beta_{11} = \beta_{12}$ ⇒ no DIF.
- if $\beta_{11} = \beta_{12}$ but $\beta_{01} \neq \beta_{02}$ ⇒ uniform DIF.
- if $\beta_{11} \neq \beta_{12}$ ⇒ nonuniform DIF.

### 3.6.5 Raju method

This method was proved by Raju (1988). It finds the area between the two ICCs. If we consider the 3PL model:

$$F_1 = F_1(\theta) = \chi_1 + (1 - \chi_1)P_1 \quad (3.16)$$

$$F_2 = F_2(\theta) = \chi_2 + (1 - \chi_2)P_2 \quad (3.17)$$

where:

$$P_1 = P_2(\theta) = \frac{e^{\alpha_{11}(\Theta_j 1 - \beta_{11})}}{1 + e^{\alpha_{11}(\Theta_j 1 - \beta_{11})}} \quad (3.18)$$

$$P_2 = P_2(\theta) = \frac{e^{\alpha_{12}(\Theta_j 2 - \beta_{12})}}{1 + e^{\alpha_{12}(\Theta_j 2 - \beta_{12})}} \quad (3.19)$$
for all examinees \( j = 1, \ldots, N \) and all items \( i = 1, \ldots, I \).

Where \( N \) = number of students and \( I \) = number of items.

Then we define the signed area (SA) as the difference between the two curves and the unsigned area (UA) as the distance between the curves. Thus we have the following equations:

\[
SA = \int_{-\infty}^{\infty} (F_1 - F_2) d\theta \\
UA = \int_{-\infty}^{\infty} |F_1 - F_2| d\theta
\] (3.20) (3.21)

**Case 1** If \( \chi = \chi_1 = \chi_2 \), then:

\[
SA = (1 - \chi)(\beta_2 - \beta_1) \\
UA = (1 - \chi) \left| \frac{2(\alpha_2 - \alpha_1)}{\alpha_1 \alpha_2} \ln \left( 1 + e^{\frac{\alpha_1 \alpha_2 (\beta_2 - \beta_1)}{\alpha_2 - \alpha_1}} \right) - (\beta_2 - \beta_1) \right| 
\] (3.22) (3.23)

**Case 2** If \( \chi_1 \neq \chi_2 \)

\[
SA = -\infty \text{ or } +\infty \\
UA = +\infty
\] (3.24) (3.25)

**Significance test of the Signed Areas**

Assuming that the SA is normally distributed, the test statistic is as follows:

\[
Z = \frac{SA - 0}{\sigma(SA)}
\] (3.26)

we reject if \( Z \) is outside of \((-z, z)\).

**Significance test of the Unsigned Areas**

To get the significance test of the unsigned area we first let

\[
\left| \frac{2(\alpha_2 - \alpha_1)}{\alpha_1 \alpha_2} \ln \left( 1 + e^{\frac{\alpha_1 \alpha_2 (\beta_2 - \beta_1)}{\alpha_2 - \alpha_1}} \right) - (\beta_2 - \beta_1) \right| \text{ be equal to } |H|.
\]

The distribution of \(|H|\) is half-normal. The test statistic for an observed \( H \) is defined by:

\[
Z = \frac{H - 0}{\sigma(H)}
\] (3.27)
where we also reject if $H$ lies outside the limits of $(-z, z)$.

Table 3.2 summarizes some different methods for detecting DIF and whether they can be used to detect uniform DIF or both uniform and non-uniform DIF.

![Methods for detecting DIF diagram]

Figure 3.2: Methods for Detection DIF
Chapter 4

Analysis

4.1 Population

The survey was given to students taking an introductory statistics class at a large urban bination research university located in the Southwest and a large community college system in a large Southwestern urban environment both located by the Mexican border. Approximately 76% of the student population at this university as well as the city population is Hispanic and a percentage of the student population are Mexican nationals. This population is a good target for this type of research due to the extensive proportion of Spanish speakers. The survey was administered during the Fall 2011 and Spring 2012 semesters. There was an option to take the survey as a paper based survey or an online version. Both versions took no more than 10 minutes to administer. The survey was not mandatory and they had the option to withdraw. The survey was administered around 2/3 of the Fall semester and in the middle of the Spring to assure that the teacher had covered the material needed for the survey.

From Table 4.1, it is evident that the majority of students were in their senior year and a high percentage of them were juniors and sophomores. Figure 4.1 shows that the majority of the students surveyed were approximately between 17-22 years of age, which is the average age of the students during their freshman to senior years at college.
Students would rate themselves as non-ELLs or non-ELLs by selecting their native language on question 3. It is important to note that students that answered anything other than English or Spanish were dropped from the study. Out of the 321 completed surveys, 156 students classified themselves as ELLs and 165 as non-ELLs by selecting their native language. The second criterion for classifying the populations of ELL or non-ELL was question 5 which asked students to provide their proficiency with speaking the English
language on the Interagency Language Roundtable (ILR) and American Council on the Teaching of Foreign Languages (ACTFL) scales respectively from 0-10. If the student answered a 10 on this question which states: “Able to speak like an educated native speaker” he/she was considered a non-ELL otherwise he/she was considered an ELL. With the second criterion, 152 students were marked as ELLs and 169 as non-ELL. Table 4.2 is providing the counts of students that were rated as ELLs and non-ELLs in the study. 50 students were classified as non-ELL on the second criterion but ELL on the first. On the other hand, 46 students were classified as non-ELL on the first criterion but ELL on the second. The proportion of times these classifications agree is 69%, which is relatively low. This implies both measures of ELL status are incomplete.

<table>
<thead>
<tr>
<th>Question 3</th>
<th>Question 5</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>ELL2</td>
</tr>
<tr>
<td>Question 3</td>
<td>ELL1</td>
</tr>
<tr>
<td></td>
<td>nonELL1</td>
</tr>
</tbody>
</table>

Table 4.2: Students that were rated as ELLs and non-ELLs according to their answer on Question 3 and 5.

Table 4.3 shows the percentages by answer choice in the 10 items by selecting their native language. We can see that there is a large discrepancy between the percentage of ELLs and non-ELLs on questions 8, 9, 11, and 15. The other questions hold a close percentage for both populations. On Table 4.4 referring to their English proficiency we see that there is a discrepancy on questions 9 and 14. The other questions have a very close percentage.
<table>
<thead>
<tr>
<th>Q</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
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<td>53.2</td>
<td>8.3</td>
<td></td>
<td></td>
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<td>4.5</td>
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<td>1.3</td>
<td></td>
<td></td>
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<tr>
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<td>33.3</td>
<td>3.2</td>
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<td>2.5</td>
<td>4.5</td>
<td>39.4</td>
<td>5.5</td>
<td>41.8</td>
<td>1.8</td>
<td>2.4</td>
<td>1.2</td>
<td>3.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 4.3: *Note: Boldfaced type signifies the correct answer.
Students’ answers in percentages (%) by answer choice by native language.

<table>
<thead>
<tr>
<th>Q</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
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<tr>
<td>Q11</td>
<td>19.7</td>
<td>46.0</td>
<td>19.0</td>
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<td>82.2</td>
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<td></td>
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<tr>
<td>Q14</td>
<td>3.3</td>
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<td>1.3</td>
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<td>3.5</td>
<td>66.2</td>
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<td>18.9</td>
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<td>Q15</td>
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<td>7.2</td>
<td>31.5</td>
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<td>1.3</td>
<td>2.6</td>
<td>3.2</td>
<td>4.6</td>
<td>42.6</td>
<td>2.9</td>
<td>43.2</td>
<td>1.7</td>
<td>1.7</td>
<td>0.5</td>
<td>2.3</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Table 4.4: *Note: Boldfaced type signifies the correct answer.
Students’ answers in percentages (%) by answer choice by their English proficiency.
4.2 Results

DIF analysis was conducted to detect both uniform and non-uniform DIF using the following methods from the package DifR in R for each population criterion: Transformed Item Difficulties (T.I.D.), Mantel-Haenszel (M-H), Standardization (Stand.), Breslow-Day (BD), Logistic regression (Logistic), Lord’s chi-squared test (Lord), Raju’s Area (Raju), and Likelihood Ratio test (LRT). Table 4.5 provides an overall assessment of uniform and non-uniform DIF utilizing the first classification for ELL status (i.e., asking what the student’s native language was). Notice that BD, Lord, and Raju detect DIF more often than the other methods this is because methods that reject more often tend to be more liberal and methods that do not reject as often are more conservative, for instance Logistic.

<table>
<thead>
<tr>
<th>T.I.D.</th>
<th>M-H</th>
<th>Stand.</th>
<th>Logistic</th>
<th>BD</th>
<th>Lord</th>
<th>Raju</th>
<th>LRT</th>
<th>#DIF</th>
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<td>NoDIF</td>
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<td>NoDIF</td>
<td>NoDIF</td>
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<td>DIF</td>
<td>NoDIF</td>
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<td>0/8</td>
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<td>NoDIF</td>
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<tr>
<td>Q12</td>
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<td>NoDIF</td>
<td>NoDIF</td>
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<td>DIF</td>
<td>NoDIF</td>
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<td>NoDIF</td>
<td>NoDIF</td>
<td>NoDIF</td>
<td>0/8</td>
</tr>
</tbody>
</table>

Table 4.5: Overall uniform and non-uniform DIF analysis for first ELL classification (native language)

Similarly Table 4.6 provides an overall assessment of uniform and non-uniform DIF utilizing the second classification (i.e., asking the students to provide their English profi-
Table 4.6: Overall uniform and non-uniform DIF analysis for second ELL classification (English proficiency)

Using the first classification for ELL status items 7, 10, 12, and 14 show some evidence of DIF. These items will be examined in detail in the following section. Similarly, items 6, 7, 8, 10, 12, 14, and 15 show some evidence of DIF when using the second classification and they will be examined as well.

### 4.3 Individual Item Analysis First Classification (native language)

With the overall analysis provided above an individual analysis based on the items that were detected as showing DIF is presented in this section. In the analysis, the focal group was ELL and the reference group was non-ELL, since we believed that the group in disadvantage
is ELLs (i.e. ELLs were not expected to do as well as non-ELLs).

4.3.1 Item 7 (Income Average)

Question 7 was flagged for DIF by the Breslow-Day and Lord tests. The ICC for this item is presented in Figure 4.2. The curve for ELLs on Figure 4.2 is almost a constant line suggesting the item doesn’t discriminate well among respondents who are ELLs. The ICC for non-ELLs does discriminate well among respondents where students at the lowest ability had a higher probability of answering the item right and students at the highest ability had a lower probability of getting a correct answer.

![Figure 4.2: Non-uniform DIF: ICC for focal and reference groups on Question 7](image)

4.3.2 Item 10 (Center and spread of a histogram)

The ICC for this item is presented in Figure 4.3. The curve for non-ELLs on Figure 4.3 is almost a constant line suggesting the item doesn’t discriminate well among non-ELLs respondents. The ICC for ELLs does discriminate well among respondents where students...
at the lowest ability had a lower probability of answering the item right and students at
the highest ability had a higher probability of getting a correct answer.

Figure 4.3: Uniform DIF: ICC for focal and reference groups on Question 10

### 4.3.3 Item 12 (Summarizing variability of a plot)

On Figure 4.4 the ICCs are very close at the lowest ability levels and separate at the highest
ability levels. For those with high ability (i.e. possessing high levels of statistical conceptual
knowledge), ELLs have a higher probability of a correct response than non-ELLs. On
the other hand, for the lowest ability (i.e. possessing low levels of statistical conceptual
knowledge), non-ELLs have a slightly higher probability of answering right.
4.3.4 Item 14 (Average definition)

The ICC on Figure 4.5 shows some evidence of DIF. At the lower ability levels, the probability of getting the item correct is higher for ELLs, and then the probability is higher for non-ELLs for students with a latent ability higher than zero.
4.4 Individual Item Analysis Second Classification
(English proficiency)

4.4.1 Item 6 (Average family size)

Both ICCs appear somewhat linear where at the lowest ability the probability of getting a correct answer was higher for ELL and then at the higher levels it is higher for non-ELLs.
4.4.2 Item 7 (Income average)

The ICC on Figure 4.7 shows little evidence of DIF since the area between the lines is very small. The probability of answering correctly is higher for non-ELLs.
4.4.3 Item 8 (Finding median after adding 5 to the top scores)

Figure 4.8 displays the ICC for the second classification on item 8. The ICC for ELLs is more linear with a positive slope, whereas the ICC plot for non-ELL follows a logistic curve. Also, at the lowest levels of ability the probability of getting a correct answer was higher for ELLs and the opposite for the higher levels.
4.4.4 Item 10 (Center and spread of a histogram)

On Figure 4.9, the ICCs are very close at the lowest ability levels and separate at the highest ability levels. For all ability levels the probability of getting the answer correct was higher for non-ELLs.
4.4.5 Item 12 (Summarizing variability of a plot)

Figure 4.10 displays the ICCs for item 12 on the second classification. At the lowest ability levels the probability of answering correctly is higher to ELLs but those probabilities are low and close to each other compared to the highest levels when the probabilities are now higher for non-ELLs.
4.4.6 Item 14 (Average definition)

This is very similar to what happened on Figure 4.10. However, it is important to notice that on the ICC on Figure 4.11 the probabilities of a correct answer are higher, and the separation is higher for the lowest ability.
4.4.7 Item 15 (Spread definition)

Figure 4.12 again is similar to the previous ICC on Figure 4.11. On this case it is clearer that the ICC for ELL has a linear shape with a positive slope. In addition, the separation on the probabilities of answering correct between ELLs and non ELLs is higher for the subjects with high ability.
Table 4.7 summarizes the results according to the results on the ICCs and when ELLs were favored or not at all, and whether we considered the item to be functioning poorly for either population.

<table>
<thead>
<tr>
<th>Results</th>
<th>Item #</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELLs favored at lower abilities</td>
<td>14(1), 6(2), 8(2), 12(2), 14(2), 15(2)</td>
</tr>
<tr>
<td>ELLs favored at higher abilities</td>
<td>7(1), 10(1), 12(1)</td>
</tr>
<tr>
<td>ELLs not favored at all ability levels</td>
<td>7(2), 10(2)</td>
</tr>
<tr>
<td>Poor Functioning Items</td>
<td>7(1)ELLs, 10(1)nonELLs, 6(2)ELLs, 7(2)Both, 8(2)ELLs, 10(2)Both, 12(2)ELLs, 14(2)ELLs, 15(2)ELLs</td>
</tr>
</tbody>
</table>

Table 4.7: *Note: (1) refers to native language and (2) refers to English proficiency classifications.

Results.
4.5 Analysis of the last 3 items

The last three items on the survey were not taken from the ARTIST items. We created them to check whether there is a difference on the understanding of the context of the words range, average, and spread among ELLs and nonELLs. If we recall Tables 4.3 and 4.4 show the proportions of answers among ELLs and nonELLs for both classifications respectively. For item 13 we had a p-value of .1385 and .2605 for each classification respectively. For item 14 we had a p-value of .5042 and .3677. For item 15 the p-value was .5301 and .2354. As we can see the p-values are big on all items so at a significance level $\alpha = .05$ we do not reject the null hypothesis that the answer is independent of whether the respondent was an ELL or nonELL for both classifications.
Chapter 5

Discussion

5.1 Individual Item Discussion

In the following, individual items are analyzed for DIF utilizing the ICCs. Only items flagged for DIF are considered.

**Item 6 (Average family size)**

Item 6 was flagged as DIF on the second classification. They were given the total number of households in a town and the average number of children per household and they were asked to choose a true statement. If we recall from Table 4.4 for both ELLs and non-ELLs the majority chose the correct answer b, with a higher percentage for non-ELLs (43.8% compared to 38.8% for ELLs). The second modal answer for ELLs was c (The most common number of children in a household is 2.2) with 28.3% and for non-ELLs the second most common answer was a (Half the households in the town have more than 2 children). This is perhaps due to a different understanding of the meaning of average for ELLs and non-ELLs, where ELLs may be assuming the everyday meaning of average is common. Additionally, ELLs also seem to be decontextualizing the answer from the context. That is, it is reasonable to obtain a numerical mean of 2.2, but, in context, this does not imply that the are 2.2 children on average on a household.
Item 7 (Income average)

For both classifications, Item 7 showed evidence of DIF. This item had skewed, mean, median in the wording of the problem. For the first classification, the most common answer was c for ELLs and non-ELLs when the correct answer was b. The percentages for the correct answer are higher for non-ELLs on Table 4.3. It is likely that both groups chose answer c (Not enough information to tell which is which) most often because they didn’t understand the question and decided that the question couldn’t be answered with the information given. For the second classification Table 4.4 shows that the percentages are very close for both groups and the correct answer was chosen as many times by ELLs than by non-ELLs.

Item 8 (Finding median after adding 5 to the top scores)

There was evidence of DIF on item 8 for the second classification only. The question was about what would happen to the median if out of 100 students the 10 students with the highest percentages were given a 5 point bonus. The majority of the subjects picked incorrectly answer c which was that the median would be higher than the original. A slightly higher percentage of non-ELsS chose the correct answer b compared to ELLs. This is perhaps because both groups only have a computational understanding of the median so since they were adding 5 points to the top 10% they thought that the median was going to be higher. Perhaps ELLs yield the incorrect answer more often than non-ELLS because ELLs rely more on computational knowledge rather than contextual knowledge. If we recall from the ICC plot on Figure 4.8 the ICC has a positive slope for ELLs meaning that the better understanding they had of the concept of median the more likely they were going to answer the item right.
Item 10 (Center and spread of a histogram)

Both classifications showed evidence of DIF for item 10. For this question respondents had a histogram with some results and they were asked to answer which two measures were most appropriate to describe center and spread for this distribution. Table 4.3 shows the percentages for the first classification and it is evident that for both groups the most common answer was not the right answer. For ELLs the most common answer was d whereas for non-ELLs it was b. Perhaps ELLs did not recognize the acronym for interquartile range (IQR) that was in the correct answer c and they picked mean and standard deviation as their answer. For non-ELLs it might be that mean and median are very common in statistics so they choose that answer even though the question was asking for measures of center and variation. On the second classification, the same happened, however, the percentages are closer to each other that is why the ICCs are more similar on Figure 4.9.

Item 12 (Summarizing variability of a plot)

Both classifications flagged item 12 as DIF. On this case the correct answer was the third most common answer by both groups on the first classifications. The modal answer for ELLs was b and for non-ELLs was c. Both b and c would mean that the answer to this item is standard deviation but for different reasons. Possibly ELLs choose b because is a more general definition of standard deviation and reflects reliance on computational knowledge. Perhaps they didn’t choose correctly interquartile range because they probably didn’t contextualize the probelem by looking at the given histogram carefully to be able to provide the correct answer.

Item 14 (Average definition)

Item 14 was a question about defining average. Both groups chose the correct answer which was the statistical meaning of average. The second most common answer was d which was giving a more general definition of average. It is important to note that more ELLs
answered d than non-ELLs. This is perhaps because ELLs might have less understanding of the definition of average so the statistical meaning of average is harder for them. On the second classification it is more evident the difference with a 26% for ELL against 18.9% for non-ELLs answering d.

**Item 15 (Spread definition)**

This is a similar question to item 14 only with the definition of spread. Table 4.3 shows that a higher percentage of ELLs chose answer a instead of the correct answer c. One reason might be that most subjects were not familiar with the statistical meaning of spread, and even more ELLs were unfamiliar with that definition. For the second classification, both groups chose the incorrect answer a more than the correct answer c. Notice that the percentage is higher between a and c for ELLs than for non-ELLs.

### 5.2 Limitations

The following describe the limitations associated with this study:

- Sample was smaller than anticipated. The target was to have at least 400 subjects and we had 321 subjects. The reason might be due to time and resource constraints. For one institution, we were only able to send email invitations to contact students, which led to low response from students since students do not check their emails regularly or the email might have been flagged as spam.

- For the last three questions, we were not specific in asking that we wanted the statistical meaning of the words.

- Item 12 had an acronym instead of the whole name, which might have confused the subjects.
• Limited to Spanish speaking ELLs. Speakers of other languages were dropped from the study since there were only 4; 1 German, 1 Korean, and 2 Hindi.

5.3 Conclusions and Recommendations for Teaching

There was some evidence of DIF on some items taken from the ARTIST database on measures of center and variation. For some ability levels, ELLs had a lower probability of answering the item correctly and for other levels of ability that probability was higher for ELLs, depending on the type of question. Some items function poorly for one or both populations. DIF items include those with a high level of technical vocabulary and those where mistakes may easily be made if relying on computational knowledge rather than contextualized interpretations. Overall, the questions that showed DIF were about mean, median, interquartile range, spread, and average which are common terms that students are expected to understand by the end of an introductory statistics course. Often, these terms are hard to understand even for non-ELLs, but may be even more difficult for ELLs. Students seemed to have issues when moving from the everyday language to the academic language of the word. In addition, ELLs may have a different everyday register of a word than non-ELLs which led them to answer differently. Table 5.1 provides teaching recommendation supported by the findings of this study.
<table>
<thead>
<tr>
<th>Recommendation</th>
<th>Items</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use vocabulary activities (Lesser &amp; Winsor, 2009)</td>
<td>6, 8, 10, 14, 15</td>
<td>There is evidence of confusion between academic terms (mean and median) and also between the everyday and academic use of words.</td>
</tr>
<tr>
<td>Emphasize context of problem when teaching (Lesser &amp; Winsor, 2009)</td>
<td>12</td>
<td>The explicit reference to the graphic confused both populations and perhaps ELLs more so.</td>
</tr>
<tr>
<td>Introduce new ideas conceptually first so that ELLs do not focus on procedural knowledge.</td>
<td>8, 12</td>
<td>ELLs had a good working knowledge of formulas without knowing how to properly apply them.</td>
</tr>
<tr>
<td>Make acronyms explicit</td>
<td>10</td>
<td>Many students may have been unable to identify what the IQR was.</td>
</tr>
<tr>
<td>Emphasize difference between everyday and academic meaning of words</td>
<td>6, 14, 15</td>
<td>Students seemed to be confusing everyday meaning of average and spread with the academic meaning.</td>
</tr>
<tr>
<td>Emphasize meaning and use of statistical graphics</td>
<td>10, 12</td>
<td>ELLs may have less familiarity with using graphics due to academic background.</td>
</tr>
</tbody>
</table>

Table 5.1: Recommendations for teaching
References


Appendix A

Survey

Circle the most appropriate answer for each question. Ask the proctor if you have any questions.

1. What year in school are you?  a) freshman  b) sophomore  c) junior  d) senior  
e) graduate student

2. What is your age? ______

3. What is your first language spoken?  a) English  b) Spanish  c)  
other:___________

4. Estimate what percentage of time in the past month you spoke Spanish. Give a  
number between 0 and 100%_____

5. Using the (0-10) scale below, give the number that best describes your  
proficiency with the English language__________

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Able to speak like an educated native speaker</td>
</tr>
<tr>
<td>9</td>
<td>Able to speak with a great deal of fluency, grammatical accuracy, precision of vocabulary and idiomaticity</td>
</tr>
<tr>
<td>8</td>
<td>Able to speak the language with sufficient structural accuracy and vocabulary to participate effectively in most formal or informal conversations</td>
</tr>
<tr>
<td>7</td>
<td>Able to satisfy most work requirements and show some ability to communicate on concrete topics</td>
</tr>
<tr>
<td>6</td>
<td>Able to satisfy routine social demands and limited work requirements</td>
</tr>
<tr>
<td>5</td>
<td>Able to satisfy most survival needs and limited social demands</td>
</tr>
<tr>
<td>4</td>
<td>Able to satisfy most survival needs and some limited social demands</td>
</tr>
<tr>
<td>3</td>
<td>Able to satisfy most survival needs and minimum courtesy requirements</td>
</tr>
<tr>
<td>2</td>
<td>Able to satisfy immediate need with learned utterances</td>
</tr>
<tr>
<td>1</td>
<td>Able to operate in only a very limited capacity</td>
</tr>
<tr>
<td>0</td>
<td>Unable to function in English</td>
</tr>
</tbody>
</table>
6. The school committee of a small town wanted to determine the average number of children per household in their town. They divided the total number of children in the town by 50, the total number of households. Which of the following statements must be true if the average children per household are 2.2 children?
   a. Half the households in the town have more than 2 children.
   b. There are a total of 110 children in the town.
   c. The most common number of children in a household is 2.2.
   d. None of the above.

7. The distribution of the top 1% of individual incomes in the US is strongly skewed to the right. In 1997, the two measures of center for the top 1% of individual incomes were $330,000 and $675,000. Which number represents the mean income of the top 1% and which number represents the median income of the top 1%? Choose the best answer.
   a. $330,000 is the mean and $675,000 is the median.
   b. $330,000 is the median and $675,000 is the mean.
   c. Not enough information to tell which is which.

8. You give a test to 100 students and determine the median score. After grading the test, you realize that the 10 students with the highest scores did exceptionally well. You decide to award these 10 students a bonus of 5 more points. The median of the new score distribution will be __________ that of the original score distribution.
   a. lower than
   b. equal to
   c. higher than
   d. depending on skewness, higher or lower than

9. Find the range of the following list: 32, 36, 48, 49, 50, 53, 54, 56, 60, 62
   a. 62
   b. 5
   c. 10
   d. 30
10. A test to measure aggressive tendencies was given to a group of teenage boys who were members of a street gang. The test is scored from 10 to 60, with a high score indicating more aggression. The histogram represents the results for these 28 boys. Which two measures would be most appropriate to describe center and spread for this distribution?

![Histogram of aggressive tendencies test scores]

- a. Range and mean
- b. Mean and median
- c. Median and IQR
- d. Mean and standard deviation

**Items 11 and 12 refer to the following situation:**
This is a distribution of how much money was spent per week for a random sample of college students.

![Histogram of weekly spending]

11. The range for this distribution is $132.50. Indicate your agreement or disagreement with the following statement: The range is not a useful summary of the variability of this data set.

- a. Agree, it is too vague.
- b. Agree, it is too easily influenced by outliers.
- c. Agree, it does not use information on the center of the data.
- d. Disagree; a range of $132.50 is a good measure of variability because students are apt to spend any amount of money between $0 and $132.50.
12. What is the best measure to use to summarize the variability of this data set?
   a. Range, because it tells you the overall spread of the data.
   b. Standard deviation, because it is based on all the information in the data set.
   c. Standard deviation, because it is the most commonly used measure of variability.
   d. Interquartile range, because it is resistant to outliers.

13. Range is defined as:
   a. The set of all output values produced by a function.
   b. The difference between the highest and the lowest values in a set.
   c. The numbered place in the list of ordered values.
   d. The size of a data set.
   e. The distance a charged particle travels before stopping.
   f. The set of notes a musical instrument can play, or that are used in a piece of music.

14. Average is defined as:
   a. Ordinary, Normal, typical, mediocre, not extraordinary, common, neither outstanding nor poor, standard
   b. Mean
   c. Median or in the middle
   d. Overall summary on something, general value that represents most of the data, overall outcome
   e. Mode, most common number
   f. Majority
   g. A value we can use to compare one person’s performance to the group

15. Spread is defined as:
   a. To scatter distribute, disperse; to go apart or separate; to extend over a larger space
   b. To distribute in a thin layer, smear or cover evenly
   c. Range or difference between two numbers as in point spread
   d. Extend, open out as in spread his wings
   e. Butter, jam, dip
   f. Spreadsheet
   g. A large group
   h. Graph
Protocol Title: Analysis of Differential Item Functioning on measures of center and variation for ELL and non-ELL students.

Principal Investigator: Angelica Monarrez and Amy Wagler

UTEP: Mathematical Sciences Department

1. Introduction

You are being asked to participate in the research project described below. Before agreeing to take part in this research study, it is important that you read the consent form that describes the study. Please ask the study researcher or the study staff to explain any words or information that you do not clearly understand.

2. Why is this study being done?

You have been asked to take part in a research study about how communication and language affect learning statistics in an introductory statistic class. The main purpose of this study is to understand better what strengths and difficulties students have with language and learning measures of center and variation in an introductory statistics course. You are being asked to be in the study because you were registered in an introductory statistics class this past semester. If you decide to participate in this study, your involvement will last about approximately 5-8 minutes.

3. What is involved in the study?

If you agree to take part in this study, you will be asked to answer a short survey. The responses are anonymous and cannot be traced to the participant.

4. What are the risks and discomforts of the study?

There are no known risks associated with this research.
5. Are there benefits to taking part in this study?

There will be no direct benefits to you for taking part in this study. Participating in this study may help you reflect upon or notice some details about your own process on learning in an online statistics class. This research may help the researchers understand what difficulties students face in a statistics class so that more effective teaching strategies can be developed for future students.

6. What other options are there?

Participation in this study is optional. There will be no penalties involved if you choose not to take part in this study. We hope, of course, that you will choose to participate because your participation will make the study stronger.

7. Who is paying for this study?

This study does not receive funding from any association.

8. What are my costs?

There are no costs to you beyond the time you spend on the survey.

9. Will I be paid to participate in this study?

You will not be paid for taking part in this research study.

10. What if I want to withdraw, or am asked to withdraw from this study?

Taking part in this study is voluntary. You have the right to choose not to take part in this study. If you do not take part in the study, there will be no penalty. If you choose to take part, you have the right to stop at any time. If you do stop, we encourage you to talk to a member of the research group so that they know why you are leaving the study.
11. Who do I call if I have questions or problems?

You may ask any questions you have now. If you have questions later, you may contact Dr. Amy Wagler (915-747-6847; awagler2@utep.edu). If you have questions or concerns about your participation as a research subject, you may contact the UTEP Institutional Review Board (IRB) at (915-747-8841) or irb.orsp@utep.edu.

12. What about confidentiality?

1. Your part in this study is anonymous. You are not providing your name on the survey and any results will be reported (at meetings or in publications) in a manner where no individual can be identified.
2. Every effort will be made to keep your information confidential, unless disclosure is required by law. Organizations that may inspect and/or copy your research records for quality assurance and data analysis include, but are not necessarily limited to:
   - The sponsor or an agent for the sponsor
   - Department of Health and Human Services
   - UTEP Institutional Review Board

13. Authorization Statement

I have read each page of this paper about the study (or it was read to me). I know that being in this study is voluntary and I choose to be in this study. I know I can stop being in this study without penalty. I can get a copy of this consent form now and can get information on results of the study later if I wish.

Participant Name: ___________________________ Date: __________

Participant Signature: ________________________ Time: __________

Consent form explained/witnessed by: ___________________________ Signature

Printed name: _____________________________

Date: _______________ Time: _______________
TO: Angelica Monarrez  
FROM: University of Texas at El Paso IRB  
STUDY TITLE: [272152-1] Analysis of Differential Item Functioning on measures of center and variation for ELL and non-ELL students.  
IRB REFERENCE #: 272152-1  
SUBMISSION TYPE: New Project  
ACTION: APPROVED  
APPROVAL DATE: October 17, 2011  
EXPIRATION DATE: October 17, 2012  
REVIEW TYPE: Expedited Review

Thank you for your submission of New Project materials for this research study. University of Texas at El Paso IRB has APPROVED your submission. This approval is based on an appropriate risk/benefit ratio and a study design wherein the risks have been minimized. All research must be conducted in accordance with this approved submission.

This study has received Expedited Review based on the applicable federal regulation.

Please remember that informed consent is a process beginning with a description of the study and insurance of participant understanding followed by a signed consent form. Informed consent must continue throughout the study via a dialogue between the researcher and research participant. Federal regulations require each participant receive a copy of the signed consent document.

Please note that any revision to previously approved materials must be approved by this office prior to initiation. Please use the appropriate revision forms for this procedure.

All SERIOUS and UNEXPECTED adverse events must be reported to this office. Please use the appropriate adverse event forms for this procedure. All FDA and sponsor reporting requirements should also be followed.

Please report all NON-COMPLIANCE issues or COMPLAINTS regarding this study to this office.

Please note that all research records must be retained for a minimum of three years after termination of the project.

Based on the risks, this project requires Continuing Review by this office on an annual basis. Please use the appropriate renewal forms for this procedure.

If you have any questions, please contact Athena Fester at (915) 747-8841 or afester@utep.edu. Please include your study title and reference number in all correspondence with this office.
Appendix B

R-Code

```r
setwd("C:\Users\amonarrez5\Documents\thesescoding")

library(difR)
dat=read.csv("survey_data.csv",header=T,sep="","")
head(dat)
dat=na.omit(dat)
library(xtable)

print(xtable((table(dat$Q1)/length(dat$Q1))*100), type="latex", file="output.tex")
print(xtable(table(dat$Q2)), type="latex", file="output.tex")
hist(dat$Q2)
table(dat$Q3)
summary(dat$Q4)
table(dat$Q5)

nonELL1=ifelse(dat$Q3==1,1,0)
nonELL2=ifelse(dat$Q5==10,1,0)

tab1=table(nonELL1,dat$Q13)
chisq.test(tab1)
fisher.test(tab1)
tab2=table(nonELL2,dat$Q13)
```

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chisq.test(tab2)
fisher.test(tab2)
tab3=table(nonELL1,dat$Q14)
chisq.test(tab3)
fisher.test(tab3)
tab4=table(nonELL2,dat$Q14)
chisq.test(tab4)
fisher.test(tab4)
tab5=table(nonELL1,dat$Q15)
chisq.test(tab5)
fisher.test(tab5)
tab6=table(nonELL2,dat$Q15)
chisq.test(tab6)
fisher.test(tab6)

print(xtable(tab5),type="latex",file="output.tex")
print(xtable(tab6),type="latex",file="output1.tex")

print(xtable(table(nonELL1)), type="latex", file="output.tex")
print(xtable(table(nonELL2)), type="latex", file="output.tex")
print(xtable(table(nonELL1,nonELL2)),type="latex",file="output.tex")

(table(dat$Q6,nonELL1)[,1]/156)*100
(table(dat$Q6,nonELL1)[,2]/165)*100
(table(dat$Q7,nonELL1)[,1]/156)*100
(table(dat$Q7,nonELL1)[,2]/165)*100
(table(dat$Q8,nonELL1)[,1]/156)*100
\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
1 & 0.96257500 \\
\hline
2 & 0.96257500 \\
\hline
\end{tabular}
\end{table}
(table(dat$Q11,nonELL2)[,2]/169)*100
(table(dat$Q12,nonELL2)[,1]/152)*100
(table(dat$Q12,nonELL2)[,2]/169)*100
(table(dat$Q13,nonELL2)[,1]/152)*100
(table(dat$Q13,nonELL2)[,2]/169)*100
(table(dat$Q14,nonELL2)[,1]/152)*100
(table(dat$Q14,nonELL2)[,2]/169)*100
(table(dat$Q15,nonELL2)[,1]/152)*100
(table(dat$Q15,nonELL2)[,2]/169)*100

dat[,6]=ifelse(dat$Q6==2,1,0)
dat[,7]=ifelse(dat$Q7==2,1,0)
dat[,8]=ifelse(dat$Q8==2,1,0)
dat[,9]=ifelse(dat$Q9==4,1,0)
dat[,10]=ifelse(dat$Q10==3,1,0)
dat[,11]=ifelse(dat$Q11==2,1,0)
dat[,12]=ifelse(dat$Q12==2,1,0)
dat[,13]=ifelse(dat$Q13==2,1,0)
dat[,14]=ifelse(dat$Q14==2,1,0)
dat[,15]=ifelse(dat$Q15==3,1,0)
items=dat[,6:15]

items1=data.frame(items,nonELL1)
dicho.uDIF=dichoDif(items1,group="nonELL1",focal.name=0,
method=c("TID","MH","Std","Logistic","BD","Lord","Raju","LRT"),
save.output=T,output=c("out","default"),alpha=.1,type="udif")
dicho.nuDIF = dichoDif(items1, group="nonELL1", focal.name=0, 
method=c("TID", "MH", "Std", "Logistic", "BD", "Lord", "Raju", "LRT"), 
save.output=T, output=c("out1", "default"), alpha=.1, type="nudif")
(dif1.Lord = difLord(items1, group="nonELL1", model="2PL", focal.name=0, 
save.output=T, output=c("out4", "default"), alpha=.1))

items2 = data.frame(items, nonELL2)
dif2.Lord = difLord(items2, group="nonELL2", model="2PL", focal.name=0, 
save.output=T, output=c("out4", "default"), alpha=.1)
dicho.uDIF2 = dichoDif(items2, group="nonELL2", focal.name=0, 
method=c("TID", "MH", "Std", "Logistic", "BD", "Lord", "Raju", "LRT"), 
save.output=T, output=c("out6", "default"), alpha=.1, type="udif")
dicho.nuDIF2 = dichoDif(items2, group="nonELL2", focal.name=0, 
method=c("TID", "MH", "Std", "Logistic", "BD", "Lord", "Raju", "LRT"), 
save.output=T, output=c("out7", "default"), alpha=.1, type="nudif")

print(xtable(dicho.uDIF2$DIF), type="latex", output="output2.tex")
print(xtable(dicho.nuDIF2$DIF), type="latex", output="output2.tex")
print(xtable(dicho.uDIF$DIF), type="latex", output="output2.tex")
print(xtable(dicho.nuDIF$DIF), type="latex", output="output2.tex")

plot(dif1.Lord, plot="itemCurve", item=2, save.plot=TRUE, 
save.options=c("plot24", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif1.Lord, plot="itemCurve", item=5, save.plot=TRUE, 
save.options=c("plot25", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif1.Lord, plot="itemCurve", item=7, save.plot=TRUE, 
save.options=c("plot26", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif1.Lord, plot="itemCurve", item=9, save.plot=TRUE, 
    save.options=c("plot27", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=1, save.plot=TRUE, 
    save.options=c("plot28", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=2, save.plot=TRUE, 
    save.options=c("plot29", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=3, save.plot=TRUE, 
    save.options=c("plot30", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=5, save.plot=TRUE, 
    save.options=c("plot31", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=7, save.plot=TRUE, 
    save.options=c("plot32", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=9, save.plot=TRUE, 
    save.options=c("plot33", "default", "jpeg"), group.names=c("nonELL", "ELL"))
plot(dif2.Lord, plot="itemCurve", item=10, save.plot=TRUE, 
    save.options=c("plot34", "default", "jpeg"), group.names=c("nonELL", "ELL"))
Angelica Monarrez was born on December 9, 1985, in Juarez, Chihuahua, Mexico. She is the last daughter of Rodolfo and Ana Monarrez, she graduated from Colegio de Bachilleres High School in 2004. She then entered the Autonomous University of Ciudad Juarez in hopes of pursuing a degree in Mathematics, she later transfer to The University of Texas at El Paso in 2006. In 2010 she succeeded her goal and received a bachelors in science. However, in the process of pursuing her degree she became interested in Statistics. It was then that she decided to focus all of her attention to the study of Statistics. Soon after graduation she was admitted in the Masters program in Statistics at The University of Texas at El Paso.

During her time as an undergraduate she worked as a tutor in mathematics as well as a peer leader for Pre-Calculus and Calculus courses. Later as a graduate student, she was a research assistant on the implementation of supplemental instruction for Pre-Calculus at El Paso Community College. She also worked as a teacher assistant on her last year as a master student. Her jobs have helped her realize the strong passion she has for teaching and research. After graduading Angelica plans to take her two passions and pursue her doctoral studies in math education.

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