2009-01-01

Third Grade Students' Challenges and Strategies to Solving Mathematical Word Problems

Elizabeth Bernadette
University of Texas at El Paso, ebernadette2@gmail.com

THIRD GRADE STUDENTS' CHALLENGES AND STRATEGIES TO SOLVING MATHEMATICAL WORD PROBLEMS

ELIZABETH BERNADETTE

Department of Teacher Education

APPROVED:

Mourat Tchoshanov, Ph.D., Chair

Olga Kosheleva, Ph.D.

Lawrence Lesser, Ph.D.

Patricia D. Witherspoon, Ph.D.
Dean of the Graduate School

DEDICATION

I would like to dedicate this paper to my father for instilling an “all or nothing” outlook on life. He refused to let me quit anything as a child and gave me the strength to complete this mission, which I have set out to do. I would also like to dedicate this to my mother, who nurtures and supports me one hundred percent in anything I decide to do. I am so lucky to have parents that I know I can always count on to help me get my act together. Thank you to my sisters and brother for always picking up the phone to listen to me vent when things get tough. Without the support system that has been built around me, none of this would have been possible and I am humbled to have grown up in such a wonderful family.

I would like to acknowledge three people who served as professors, mentors, and advisors to me. Dr. Tchoshanov, Dr. Kosheleva, and Dr. Lesser, you have inspired me to not only further my education but to be the best teacher that I am capable of to the future leaders of the world. I hope that many others will find your guidance the way I did.

Most of all I would like to thank my students who are always eager to learn. You are our future, and I am honored to be able to work in a profession that allows me to be a part of your destiny.

THIRD GRADE STUDENTS' CHALLENGES AND STRATEGIES TO SOLVING MATHEMATICAL WORD PROBLEMS

by

ELIZABETH BERNADETTE

THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF ARTS

COLLEGE OF EDUCATION

THE UNIVERSITY OF TEXAS AT EL PASO

ABSTRACT

This project explores the difficulties and challenges that third grade students face solving mathematical word problems. Three students were asked to be participants and share their knowledge on this topic as well as their work. After extensive interviews it was concluded that the challenges of mathematical word problems include but are not limited to the level of reading comprehension, conceptual understanding of mathematical concepts, and the belief that math is a compilation of computations and unexplainable procedures. The participants provided insight as to what strategies are helpful to students. These strategies include group discussion on problem solving strategies, self-assessment, incorporating games into math lessons, and learning that math makes sense. Graphic representations were found to assist the students in all three cases. When using graphic representation, each student was able to show better understanding of the word problem. This project helps to understand what challenges the students face and how teachers can improve instruction in a way that benefits each learner.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF APPENDICES

Chapter I

INTRODUCTION

A. Statement of the problem

Problem solving is a skill that is required every day in any situation. Many children are prepared from a young age to handle these situations. Schooling prepares students to handle problem situations in both real experiences and on paper. Not only is it something that is taught, but it is also an ongoing struggle in elementary third-grade mathematics. Given a problem in a real-life situation, a student should be able to rationalize and find an appropriate solution. If a student shares something from his/her lunch with multiple friends, that student is able to determine how to divide it equally. That is called problem solution. Why then is it so difficult for students to handle mathematical word problems? Students tend to see mathematical problem solving as purely a mathematical skill. It is frustrating and confusing for students to be told to read, understand, and carry out a mathematical solution. In a child's mind, and other's as well, reading is done in reading class and math is done in math class. Teaching students to read for comprehension in math class is more difficult than it seems. Children may understand how to add, subtract, divide, and multiply, but if they cannot comprehend what a word problem is asking of them, the ability to compute numbers becomes worthless. Finding a way to bridge math and reading is a big challenge for most students and teachers alike. Mathematical word problems are a challenge for students as well as a requirement, which occurs both in school and in every day situations, meeting this challenge will become a life-long skill for the students.

B. Focus of study

Throughout the study, the difficulties of problem solving will be discussed, as well as the strategies that are found to be helpful. Three students were chosen to be a part of the study. They were identified to be students who showed the most improvement. Also, the work that the students displayed was tagged as being valuable and worthy of discussing at length and opened for discussion.

C. Research questions

1. What challenges do third-grade students face in solving mathematical word problems?
2. What strategies are found to be helpful to a third-grade student when solving mathematical word problems?
3. To what extent does graphic representation provide assistance to the three case studies presented and why were they successful?

D. Significance

Problem solving is an ever-growing skill that will continue to develop from toddlerhood well into adulthood. The problem solving in this study is displayed in the form of mathematical word problems. In mathematics, many difficulties arise for students in third-grade. Some of the challenges that students face are reading comprehension, choosing an appropriate mathematical operation, conceptually understanding the mathematical operation, and using only necessary information in a problem. Looking at student work will help to identify not only the difficulties that other students may have, but also helpful strategies that other students can benefit from. Strategies that may be useful to students include approaching a word problem as a story, group discussion on problem solving strategies, hands-on mathematical manipulatives, schema-based instruction, self-assessment, and using graphic representation. Graphic representations have proved to be useful in previous research. Teachers and other professionals can help students use these strategies and they will benefit from analyzing student work, including the work presented here.

E. Limitations

1. Low level ability group.

From the beginning of the academic year, the third grade teachers at this particular school had been preparing for a challenging year in mathematics. It was confirmed that this particular group of students had been carefully observed in this area since kindergarten. More often than not, the group was found to be a good mixture of students' abilities; however, occasionally an entire grade level can be classified as either high or low. Although these limitations were indeed a factor, the question of why the students fail to see mathematical word problems in a manner in which they become solvable is still raised.

2. Previous data.

It is helpful to analyze any previous information or data on a participant. With third grade being the first year students take a standardized Math test there was no previous test data to compare. The only information that was collected from second grade was the end-of-year average. This is helpful; however, with test scores, more information could have been analyzed.

F. Variables

The variety in strategies that were taught to each class is a variable in this study because all three participants did not receive identical instruction. The value of the strategies could alter the success that the students obtained in solving mathematical word problems. Another variable is the students' level of engagement in small group conversation. If the students exchanged valuable information with each other, the success rate could have been altered as well. In addition, the students' reading level was a variable as it was found by Tchoshanov (2006) that reading level drastically affects students' problem solving abilities. The lower the reading level, the most likely the student will have a poor performance on his/her work.

The student's ability to solve word problems also affected their problem solving ability. If a student was familiar with these particular types of problems before the study, it is likely that they would answer the problems with more ease than those participants who had never had experience with them. The second dependent variable is the students' use of relevant graphic representation. Because graphic representation is believed to help a student answer a word problem, whether or not they were used will alter the students' success.

Finally, every child learns differently and thrives in different classroom settings. One child may need 100% silence to operate to the best of his/her abilities. Another child may be able to function just the same in a very chaotic setting. It is hard to determine what the most appropriate classroom setting is for all of the students and something that simple could drastically alter a student's performance. The second variable deals with the time of day the students were given the tests. For the students in my class, there was a specific time in which problem solving would take place. The students were aware of what it was, when it was going to happen, and what was expected of them. In giving the second class of participants the pre and post-tests, the setting was not quite as controlled. I administered the tests usually at the end of the day when the students were restless and looking forward to going home. It was difficult to encourage the students to concentrate on the math presented to them. In addition, the class didn't respond to me as they might to their own teacher. It was more difficult for me to manage. The teacher may not have managed her class in the same manner as I did mine, which may have caused a lack of concentration on their part.

G. Definition of terms

Abstract- something that is not concrete in appearance; rather it requires theoretical thought.

Algorithms- a mathematical procedure that, if correctly completed, contains a logical solution

Conceptual understanding- the understanding that a student has as to why something is the way it is; logical relationships working internally and being created in the mind

Graphic representation- the physical model of someone's comprehension of a problem on paper through the use of symbols

Insufficient information- lacking of important information needed to solve a problem

Keywords- words in a mathematical word problem that may give a clue as to how to solve a problem

Mathematical manipulatives- tools used in math to aid in the understanding of a concept

Prior knowledge- something that was previously learned that may help in understanding a new concept

Problem solving- the act of solving a problem that is presented

Reading comprehension- the full understanding of what one reads

Reasoning- understanding the logic behind a concept

Relevant graphic representation- representation that is necessary for finding a solution to a problem

Schema-Based instruction- instructing with a rule or basis for placing something into a category

Student self-assessment- a student monitoring the progress of his/her own learning

Sufficient information- enough information for one to solve a problem

Traditional mathematics- a method of mathematics that was set decades ago with a strict "right-or-wrong" policy

Visual learners- learners who need something to look at in order to fully grasp a concept

Visual spatial ability- an ability to learn holistically rather than in a multi-step way and see more abstract concepts

Word problem- a short story that asks a question at the end usually requiring some form of a mathematical computation

H. Organization of study

This study was designed to focus on three students and analyze the way those students solved mathematical word problems. After giving two third-grade classes a pre-survey, which asked students about the difficulty of problem solving, the students were given a pre-test. Following this was three months of intensive practice of problem-solving strategies and techniques. After the three months, students were given a post-survey and a post-test. After looking at student work and scores, three students were chosen who showed progress and strategy on the tests. Those students were given an extensive interview, which displayed difficulties and helpful strategies.

Chapter II

LITERATURE REVIEW

This review of the literature relating to mathematical problem solving will focus on a definition of the phenomena, provide insight as to why certain students face difficulty with abstract mathematical situations, and then explore instructional strategies, including the use of graphical representations, which may help students be more successful in their acquisition and application of mathematical concepts.

A. Definition of the Problem

The phenomena of students having difficulty with mathematical problem solving can be attributed to the demands of developmental limitations, the need to integrate several different cognitive processes, a multi-step process, and the challenges of bridging word problems with knowledge of real-life situations. Piaget's categories of knowledge provide a means of understanding these difficulties and offer a framework upon which others have built an understanding of mathematical reasoning that incorporates varying uses and types of graphical representations.

1. A framework of knowledge.

Given the difficulties outlined above with mathematical problem solving, it is helpful to make sense of students' lack of comprehension by employing Piaget's conceptual framework of knowledge. Piaget defines three types of knowledge named physical, social, and logico-mathematical (Piaget, 1952). Physical knowledge is that which can be seen, touched, smelled, tasted, or heard. Students use this knowledge when exploring new concepts with concrete manipulatives or through observation. Piaget's social knowledge is that which is created. Examples of social knowledge include holidays and language. Finally, logico-mathematical knowledge is an abstraction, which is created, or interpreted in the minds of individuals.

Piaget identifies two types of abstraction. These are empirical and constructive. When students focus on one physical type of knowledge, such as the size of an object, the other physical properties are abstracted, such as color or weight, because they are known empirically through our senses. As empirical abstraction involves the use of physical knowledge, constructive abstraction involves logico-mathematical knowledge. Mental relationships that are made between two objects, such as "similar" or "different," can be abstracted from our thinking about objects. The abstraction takes place in the mind rather than being an actual physical property that is abstracted. According to Kamii, Kirkland, and Lewis (2001), when problem solving, students do this as a way of organizing information. This abstraction is important because a problem cannot be solved without both types of abstraction present. Different people can see one thing from his or her individual point of view. From this constructivist viewpoint, individual students, through their understanding and reasoning of their surroundings, create mathematical knowledge. Thus, according to Piaget, students need to discover math on their own in order to understand the concepts of addition, subtraction, multiplication and division.

Kamii et al. (2001) makes the point that physical knowledge could not be made if the abstractions were presented from the beginning. In other words, “we could not construct physical knowledge, such as ‘red,’ if we did not have the category of ‘color” (p. 26). Kamii et al. is using Piaget’s methodical thinking to explain a student’s rationale when working a word problem. The idea being presented is that a student cannot represent a problem on paper if he or she cannot make it exist in his or her mind. The representation on paper should be a representation of what the student is thinking. It should go from the mind to the paper. Kamii et al. (2001) stated, “Representation is what people do” (p. 32). It is reiterated that representation is not a picture or symbol acting alone, some reasoning has to be made out of the information given. If students are trying to represent an addition problem, the symbol “+” is used as a way to show that two parts are being combined to make a whole (addition). Otherwise, if there is no reasoning in the process, the symbol is irrelevant, as are the numbers. According to the research done by Kamii et al. (2001), if the child has a high level of abstraction, then he or she will have a better understanding of the representation and understanding of the mathematical process. Redundant writing of symbols in math emphasizes a low level of abstraction.

Having made the theoretical distinction between empirical and constructive abstraction, Piaget (1952) went on to say that in the psychological reality of the child, one could not take place without the other. For example, we could not construct the relationship “different” if all the objects in the world were identical. Similarly, the relationship “two” would be impossible to create if children thought that objects behave like drops of water, which can combine to become one drop.

Kamii et al. (2001) studied abstract representation. Representation occurs every day in any situation. In third-grade, students can reason on an abstract level, but it does not come easily. Teachers must emphasize children's thinking before representing children's thinking. Representation will follow if the child uses a high level of abstraction. By using textbooks and worksheets, signs are encouraged rather than representation. Kamii et al. have found that mathematical games promote abstraction, which will later develop representation. Worksheets and textbooks provide strategies involving signs and shortcuts, which will keep the students from using a higher level of abstraction.

2. A Developmental Process

Piaget's three types of knowledge develop the appropriate skills that students need when problem solving. Research has proved why logico-mathematical knowledge is important for children to develop at an early age, as it becomes more necessary when math concepts become more challenging. Often, math can seem to be a rote memorization process (Owen & Fuchs, 2002). Students need to find reasoning amongst mathematical procedures and develop a true understanding of mathematical concepts.

Fuchs, Fuchs, Prentice, Burch, and Paulsen (2002) conducted a study in which the definition of the word "transfer" was taught to the students before problem solving began. Once the students understood that information could transfer from one skill to another, (i.e. babies learn from a bottle, then a toddler cup to a real cup) the concept was related to math. After this is clear to students, the four types of word problems (addition, subtraction, multiplication, and division) are taught. Now that students understand how concepts “transfer” it is easier for students to understand how each word problem relates to one of the four basic problems that were previously taught. Students are trained to see that problems may look different, but still represent the same problem type. Transfer is encouraged outside the math classroom as well. One focus of transfer is to find how everyday problems have similar solutions that are represented in one of the four mathematical processes (Fuchs et al., 2002).

B. Representation and Problem Solving

1. Types of representation systems

According to Goldin and Shteingold (2001), in mathematics there are two systems of graphical representation. The first is the external system. This is what the students are able to take from the mathematics that are presented to them by the teacher. The second system is the internal system. This system involves the way the students internalize the information given to them and the plan that is made and carried out onto paper. It may be self-created symbols or representation. For effective teaching and conceptual learning to take place, both systems are necessary in the mathematics classroom.

While students listen and watch the mathematics instruction, which the teacher gives (external system), their internal system comprehends it in their heads. From that point, the student will develop the knowledge for the concept that is being taught and internalize it in a way that is specific to that child. One student might take in something that the teacher says and create reasoning from it that a different student might lack. Goldin and Shteingold (2001) believe this to be the reason why both systems are imperative to the learning process. Some students have problems understanding and reasoning. This is due to internal underdevelopment.

Goldin and Shteingold (2001) define representation as being “a sign or a configuration of signs, characters, or objects” (p. 3). It is used when something is needed to represent an object. To represent other objects, students create external representations. The representation is so clear that a teacher can point it out and explain what the student’s reasoning was.

Often, Goldin and Shteingold (2001) believe, students are able to use external representation, but not fully understand the concept being taught. For example, in Algebra, a student may be able to manipulate an equation, but not be able to explain it to someone with enough reasoning to where it makes sense. Students may memorize algorithms and rules, but still not have the conceptual development take place. This is where internal systems start becoming a useful tool.

The two systems relate easily to two popular psychological theories. Goldin and Shteingold (2001) argue that constructivists find internal representations to be the most relevant. Constructivists see the behaviorist theory as being too objective and that students should have more freedom and discovery in Math. These beliefs continue as children are encouraged to make their own discoveries in order to grasp concepts. Children’s thinking is valued more under this theory and open-ended questions are preferred in order to encourage critical thinking. The center of the classroom is more interactive and less teacher-driven.

The second theory that Goldin and Shteingold (2001) discuss is the behaviorist theory. Behaviorists tend to use the external representation as a base for learning. In this community, the constructivist theory is seen as too subjective, distracting students from the objective that is being taught. These philosophers place their focus on tests and having one correct answer, rather than discussion and open-ended questions. In a behaviorist classroom setting, the teacher would be instructing and the students would use the teacher's method, which may cause the student to lose the chance for conceptual understanding.

2. Representation: from teacher to student

It is the role of a student to take in (internalize) all the information that is presented to him/her. Then, the student must learn to represent the information in a way that makes sense (externalize). The sign or symbol created becomes the basis for how the student views a particular mathematical concept. Representation, as defined by Pape and Tchoshanov (2001), refers to the internal and external structure of all mathematical concepts. Representation is developed when a student is able to internalize external representations that have been obtained through a child's mind. Research has proven that mathematics manipulative materials help students grasp a conceptual meaning for the concept being taught. Pape and Tchoshanov (2001) argue that children must find a relation between the manipulative materials and the mathematics concept itself in order to fully understand what the manipulative materials represent. Generally, math manipulative materials are used to help students learn the concrete operation; however, Pape and Tchoshanov believe that teachers often tend to teach abstract mathematics procedures before allowing mastery of the concrete operation and, even more, the students making a connection between the two. In order for students to develop the appropriate skills for concrete reasoning, some sort of exploration must take place beforehand, allowing students to bridge the concrete with the abstract.

According to Pape and Tchoshanov (2001) one integral component to conceptualizing mathematics is the social interaction that takes place between teacher and student, as well as student and student. Through dialogue, knowledge is shared, and one can learn from what is said in a conversation without either person being aware of it. It is acknowledged that oftentimes teachers ask for representation as an end result rather than as a process in which the student reaches the end result. When students are told that representation must be on the final product, the representation becomes irrelevant and/or unnecessary to the student’s work. The representation should be used as a tool, which encourages the student to reason and conceptualize the concept being practiced.

Mathematics should be taught in a variety of contexts. When Pape and Tchoshanov tested three groups of students, it was found that the group, which received multiple instructional techniques performed higher than the other two groups who were taught using only one strategy. This most likely was due to the fact that the students who understood more than one technique were able to decipher which strategy was most appropriate for the mathematics problem. It became easier for the student to be more flexible in his/her approach to finding a solution. This also encouraged more interaction in the classroom, which also proved to be a helpful tool in mathematics problem solving.

Lesser and Tchoshanov (2005) stated that if just one method of representation is modeled during instruction, conceptual understanding will suffer. Students should have a variety of methods represented have flexibility to choose what is most appropriate for the problem at hand.

3. Schema-based instruction

Research has proven that when classifying word problems into one of the four mathematical operations, students will more likely find success. “Schema-based instruction” is recognized by Jitendra, Sczesniak, Griffin, and Deatline-Buchman (2007) as an advantage for students when problem solving as well as for student learning.

Fuchs, L.S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C.L., & Owen, R. (2003) define schema-based instruction as that which focuses on the structure and representation of a word problem. Students trained themselves to classify each word problem into a category, which allowed them to recognize patterns amongst different problems. This form of self-regulation forces students to be held responsible for finding the solution to story problems. By building the background of word problems, finding the solution was not as difficult. Overall, Jitendra et al. (2007) was able to “learn about effective ways to enhance the problem-solving curriculum as well as facilitate teacher implementation and student learning" (p. 294). Due to the results of the study, teachers were able to make appropriate changes to lessons involving schema-based instruction, proving that representation of a problem can either hinder or help a student. Jitendra et al. suggests that pushing students toward independence in problem solving is a necessary component to finding success. Jitendra et al. (2007) acknowledge the difficulty in teaching all students; however, another benefit of this type of instruction and self-regulation is that it can work for a wide range of students from high to low. Whatever the ability level, students all benefit from the visuals used in the schema-based instruction. With students self-assessing and choosing the appropriate diagram to match with each problem, it becomes more fluent and more reasoning is made. Jitendra et al. believe schema-based instruction requires self-regulation, which in turn improves students' problem solving abilities.

Problem schemata identification must occur first in order to stimulate conceptual understanding. This involves recognizing the problem's pattern. Once a schema is deciphered, a plan to solve the problem is created and hopefully followed through correctly (Jitendra, 2002).

Owen and Fuchs (2002) found that strategy instruction encourages students to analyze their thinking and develop a skill that will be useful in post-school life. The study done was geared toward students with a learning disability; however, a point was made that it is just as effective for all students. Not only is problem solving a life-long skill, it is necessary to improving learning while still in school.

Jitendra, Griffin, Deatline-Buchman, Dipipi-Hoy, Sczesniak, Sokol, and Xin (2005) find that strategy instruction is “integral to the concept of what constitutes learning in general education classrooms (p. 320). Jitendra et al. find that a traditional approach to mathematics education is not as intense or demanding as a problem-solving approach. Standards are created to encourage higher order thinking and conceptual understanding. National Council of Teachers of Mathematics (NCTM) acknowledges that reasoning is necessary in math rather than rote memorization. When Jitendra et al. conducted their study, which tested several basal textbooks to compare them with the NCTM standards; it was found that problem-solving opportunities were present more than any other standard from NCTM. In most textbooks, reasoning was only present half the time amongst all problem-solving activities. Also, Jitendra et al. found that students were less likely to have to create representation on his/her own. Because of their findings, Jitendra et al. suggest that teachers modify lessons from textbooks and gear them more closely toward the NCTM standards. Sense making and reasoning is necessary for problem solving. Textbooks alone, while good for certain tasks, are not enough for students to be successful in all the NCTM standards.

4. Graphic representations

Researchers agree that if students develop the skills to classify mathematical word problems into the proper schemata, assess whether or not the proper steps are being carried out, relate the stories in the problem to real-life experiences and use games as a discovery approach to learning mathematical operations, they will find that using graphic representations will come easier and lead to better problem-solving success. The development of relevant graphic representation use will become one of the most useful tools in third-grade problem solving.

Graphic representations are a good approach to help students comprehend when reading a word problem. Jitendra (2002) has done several studies that have proven the positive effects of using graphic representations to solve word problems. In fact, Jitendra has found this strategy to work the best with students with disabilities. The representations helped students to be successful as well as enjoy problem solving. Corter and Zahner (2007) found that graphic representations were useful to the students because, overall, they were used when the students were told to “show their work”. If the students did it on their own, then the representations proved to be necessary to solving the problems. One drawback was that the students were forced to choose the most appropriate representation, which is difficult for students to do. In fact, Corter and Zahner emphasize how difficult it can be to study and measure problem solving; however, the two scholars also emphasize its importance as well.

If students draw what they read, they will understand the problem and carry out the correct mathematical procedure. This is one of the best ways for students to express his/her thinking. Whether these representations are pictures or diagrams, students can organize their thoughts much easier than with words or numbers. Because elementary students are very visual learners, making graphic representations will lead to more success in mathematical problem solving.

Kamii, Kirkland, and Lewis (2001) remind us of Piaget's belief on the three kinds of knowledge. It was found that abstraction is a big process in the students' problem-solving process. Scholars suggest that abstraction ability is developed from good use of higher-order thinking, reasoning and good visual-spatial ability. In addition, Edens and Potter believe there are two basic ways to help students develop the necessary skills for successful problem solving. There is a choice, according to the scholars, of displaying examples of diagrams or allowing students to create his/her own diagrams. Students who used the diagrams in several research studies were found to have more success than those who did not (Edens and Potter, 2008). Giving students the diagrams rather than having them invent the diagrams led to more success. The reason for this, as Edens and Potter found, is because when students are given the freedom to invent representation, it becomes irrelevant or not meaningful to the problem solution.

If the student is able to create a visual representation that shows both concrete and spatial understanding, then he or she will have a better basis for solving the word problem correctly. Jitendra (2002) defines problem representation as students changing words into a "meaningful graphic representation" (p. 34). According to Yeo (2005), students who are still in the concrete-operational stage need visuals in order to help solve word problems. Different options should be available to students when solving word problems.

Van Garderen (2006) found that spatial visualization is an imperative part of the problem-solving process. Those students who performed well on the spatial visualization portion of the study also performed well on problem-solving measures as well. Edens and Potter (2008) state that in order to create a mental image, students need to have spatial visualization, or “the ability to picture the way an object will look after it has been moved to another position” (p. 185). Problem solving requires an accurate transfer from words to representation. Edens and Potter believe it to be one of the most important parts of the problem solving cycle; however, there is little instruction on how to take a word problem and create a visual representation that is meaningful to the problem solution.

To successfully answer a word problem, students need to have good visualization strategies. According to Edens and Potter (2008), graphic representations can range from numerals to those of patterns and algorithms. A good graphic representation can help the student to be more thorough and understand the problem and its concept easier.

In the above reference, Edens and Potter (2008) distinguish the difference between schematic and pictorial graphic representation. Because schematic representations focus on the spatial visualization and conceptual understanding of the problem, a student will seldom reach the correct answer using pictorial representations alone, though more often than not there is evidence of schematic representation to accompany the pictorial.

In fact it was found in a study done by Hegarty and Koshevinikov (1999) that success on word problems is positively correlated with schematic representations. There was no correlation between pictorial representations alone and problem solving. In addition, there was a significant difference in levels of schematic representation as well; the levels scored being high, medium, and low. Those students who used both schematic and pictorial representations were found to have significantly answered correctly, or, if incorrect, the reason being a computational error only. When students create their own representation, the problem will mean more to them than just reading it blindly. In addition, creating representation allows students to stop and think about what the problem is asking, which, with conceptual understanding of mathematic operations, will lead to success in solving the problem (Edens and Potter, 2008). In Van Garderen's study, it was found that there was more use of pictorial representation than schematic. Also, the use of schematic representation was positively correlated with spatial visualization ability. At the same time, pictorial representation was negatively related to spatial visualization ability. Those students that had low spatial-visualization ability tended to use pictorial representations and those with a high spatial-visualization ability used schematic representation. These students' representation were more detailed and advanced than others.

5. Schematic vs. Pictorial Framework

Scholars have suggested that students who have a more developed logico-mathematical knowledge will be more inclined to develop more relevant representations of mathematic problems. Edens and Potter (2008) claim that graphic representations can be either schematic or pictorial. A schematic representation is one that involves only the necessary information to solve the problem. All the information is pertinent to the problem solution. The details of the diagram are all valuable to the problem in some way. It is a focused picture that does not get the student off track and is simply created for problem solution. A schematic representation, as per Blatto-Valee, Kelly, Gaustad, Porter, and Fonzi (2007), is one in which objects drawn are representative of a solution to the problem.

Edens and Potter (2008) define a pictorial representation as one that contains “expressive and extraneous elements not necessary for problem solution” (p. 186). This would most likely be created by the student who spends significantly more time on the problem than the other students do. Also, this student somehow never manages to reach the “math” part of the problem. Pictorial representations seem to get students off track. The student who creates a pictorial representation focuses not on the pertinent information to solving the problem, but rather on the details that surround that information. According to Blatto-Valee et al. (2007), a pictorial representation is one that clearly represents images given in the word problem but have no relation to pieces of information necessary to solving the problem.

Edens and Potter (2008) conducted a study in which the degree of visual representation used was measured. Each student was measured not only on whether visual representations were used, but whether the representations were schematic or pictorial. Even further, the representations were measured to be either meaningful or irrelevant to the problem solution. First, Edens and Potter found that conducting the study in an art classroom encouraged students to use more pictorial representations and have “more art-like behaviors” (p. 195). This is believed to be one variable in the study that may have hindered the students’ use of schematic representation rather than pictorial. More important results were that there was a significant relationship between schematic representation use and problem-solving success; however, there was no correlation between success and the use of pictorial representation. Those students that used schematic representations either answered correctly or made a computational error. This was the same for those who used schematic and even incorporated a little bit of pictorial representation into his/her model. Students who used schematic representations were found to have better spatial ability than those who used pictorial representations. Overall, Edens and Potter discovered that schematic representations proved to be more successful than pictorial representations.

Blatto-Vallee et al. (2007) conducted a study in which schematic representations were compared to pictorial representations and how each correlated with problem-solving success. The study was done to assess the effects on deaf children, yet the results closely compared to previous research done. That research has proven that it is important for students to have a good grasp on spatial-visualization because it is integral to the problem-solving process. In order to perform at a successful spatial-visualization level, students must use schematic representation rather than pictorial.

Several scholars have brought constructivism into operation by introducing the concept of visual spatial mechanisms. Visual/Spatial ability is important in problem solving. Van Garderen (2006) found that students with low visual/spatial ability used pictorial representations, and those with high ability used schematic representations. Because pictorial representations involve unnecessary details to solve the problem, it makes sense that students without visual/spatial ability used them. In addition, it makes sense that students with good visual/spatial ability used schematic representations.

Edens and Potter (2008) studied how students “unpack” a word problem. Research was done to show that most students had more problems constructing a graphic representation for a word problem than actually carrying out the proper steps to solving the word problem.

Van Garderen (2006) found that spatial visualization positively affects students’ problem-solving ability. In fact, when Van Garderen did a study, it was found that students used some type of visual images on more than half of the problems given. Schematic imagery was positively correlated with spatial visualization; at the same time, spatial visualization was positively correlated with problem-solving performance. In other words, those who performed well on the problem-solving portion did as well on the spatial-visualization portion of the assessment.

C. Reading Comprehension and Graphic Representation

It has been found that one main reason for students having a hard time with mathematical word problems is that the student does not comprehend what he or she is reading. The problem does not necessarily lie with the student’s inability to read on grade level; rather, it is believed to be a problem with being in a mathematics frame of mind. Tchoshanov (2006) conducted a study in which one problem was given to two different groups of students. The first group received the problem in a mathematics setting by a mathematics teacher. The problem was set up just as a math problem would be, this was referred to as the short version. The second group received the problem from a reading teacher in a reading setting. The problem was referred to as the extended version due to the fact that it was presented in a story with comprehension questions following. Although the same information was given and the question had the same answer, students who received the extended version were found to have the most impressive performance when compared to those who received the mathematics version. Although both groups had the direction “Read everything carefully,” it was found to be a distraction for the students who were given the short version and preferred to simply perform mathematic operations, all the while lacking comprehension. Both versions had no reasonable answer to the problem being tested. The group with the extended version didn’t try to use mathematic operations to solve the problem; however, they realized that there was no solution to be reached. Tchoshanov also found students in the extended version group to be more inquisitive, as well as more engaged in their work.

Students who are presented with a mathematic word problem suddenly become the victims of a strenuous, multi-step process. According to Fuchs, L.S., Seethaler, P.M., Powell, S.R., Fuchs, D., Hamlett, C.L., & Fletcher, J.M. (2008), problem solving is a multi-step process which can lead to confusion for an elementary student. They claim that:

> A word problem requires students to construct a problem model by identifying the number sentence that incorporates the given and the missing information, and deriving the calculation problem for finding the missing information. (p. 156)

Multi-step processes are developmental and must be learned by practice.

1. Making sense of math

Reasoning is lost when math is rushed through. Brousseau and Gibel (2005) stated that math shouldn't be a "simple recitation of a memorized proof." These proofs are just "model reasoning," which is acceptable if it later leads to true reasoning; however, more often than not, it teaches students that math is simply "recipes and algorithms" with no flexibility (p. 14). This style of learning reduces live mathematical thinking and problem solving. According to Brousseau and Gibel, a math classroom should include the following:

- share with their peers the real reasons that have led each of them to construct reasoning from the models
- grasp the reasons why the steps are necessary
- share reasoning of the solution

Crespo (2003) emphasizes the importance of posing high-cognitive mathematical questions to students. It was found by Crespo that math should be used not as a means to make mathematics easier or to have shortcuts, but rather as a tool to make students think about math and make sense of it. Another tool that Crespo found involved student errors. When studied, it was discovered that errors can be used as a learning device to find out where students' work became incorrect.

Van de Walle (2004) encourages educators that keywords are an ineffective method for teaching problem solving. When keywords are taught as a tool, any reasoning that may have been concluded from the problem is lost. In order to make the necessary connections, it is important that students' prior knowledge is facilitated (Jitendra, Sczesniak, Griffin, and Deatline-Buchman 2007).

Math is not a subject to swiftly go through without looking back. Math takes time and patience. It is a discovery subject. Due to time constraints of the school year, math is taught as a question/answer subject. It should be everything but this. One main problem, according to Fiori (2007) is that "Teachers give problems; students give answers" (p. 696). This view is how students look at math, when in fact it should be viewed as an open-ended, discovery process. Also, telling students that they are incorrect when they have reached an answer is another mistake. A good way for students to take ownership of their work is to allow any question that a student has and, in addition, allow any solution that a student has. No student's question or solution to a problem should ever be underestimated. Students reach answers by different methods and whether they are wrong or right, they are definitely leading to something accurate. Students sometimes see things from a different perspective than teachers do and it is important not to dismiss what students are saying. Graphic representations help teachers to see their "thinking" on paper. There is no better way to help a student than to see exactly where he/she went wrong. Fiori asked, “Might a student have a richer mathematical experience if he is allowed to fumble around with a misconception for a few days than if he is steered promptly to the ‘truth’?” (p. 696). There is no better way to learn than by making mistakes, especially in math. When students lose the ability to discover, a disservice is being done to them. Without the concrete part of math, all sense making will be halted and eventually lost. Spatial reasoning is a big part of sense making. If students can reason, they are more apt to finding success.

Myers (2007) stated the following:

> Mathematics is more than just the steps needed to solve a problem, more than just a set of isolated rules and procedures. After all, if mathematics were just a set of rules and regulations, would we mathematicians waste our time exploring it? (p. 694).

Math is not a one-step process. Many steps are involved in successful understanding of all aspects of math. Graphic representations aid students in the success of problem solving. They help students organize their thinking and allow teachers to comprehend how and where the students were mistaken. In addition, teachers can gain ideas from students to help other struggling students. From a young age, students find ways to problem solve. Children should be asked how they will go about solving a problem and later how they solved it. Sarama and Clements (2007) said, “Young children have the ability to become powerful problem solvers” (p. 18). All children need is motivation and the ability to answer a problem in the manner they think is best. Problem solving is something that will not go away. After students are finished with school, problems will continue to arise in everyday life. Students need to develop good problem-solving skills according to how they learn.

Because of the difficulties that problem solving presents, teachers need to offer several strategies for students to choose from. Myers (2007) stated, “Since students grasp concepts in different ways and at different rates, helping individual students requires that teachers know several different paths to the same end” (p. 692). Different learning styles require different teaching styles. Especially in math, students need visual models. Teachers can make models for students; however, if the students make the models, better comprehension will take place. Yeo (2005) offer five strategies for one word problem. Yeo stated:

> It is essential to expose students to various strategies so as to enable them to deal with abstract mathematical concepts and to observe a wide range of possibilities and alternatives in solving mathematical problems (p. 30).

Two of the strategies that were offered are geared toward students in the concrete-operational stage. They are bar diagrams and graphic representations. Bar diagrams help students visualize the abstract concepts that are not given in the problem. Graphic representations allow students to use relevant information and reach new information that was not given. Yeo encourages multiple strategies because teachers should “choose the most suitable strategy to address individual differences in students” (p. 31). Jitendra (2002) stated, “Using graphic representations, students can develop their problem-solving skills, but also integrate addition and subtraction concepts” (p. 34).

Hohn and Frey (2002) believe a heuristic approach is necessary for students to be successful in problem solving. The training that was given to the students was called SOLVED. The steps are: State the problem; Options to use; Links to the past; Visual aid; Execute your answer and, Do check back. This acronym was created to help students organize the information given in a word problem and successfully carry out a problem-solving plan. Hohn and Frey had two groups -- the treatment group who used the SOLVED steps and the control group who followed standard guidelines for problem solving. The results showed a “greater improvement” from the treatment group than the control group. It was concluded that students did benefit from SOLVED.

Fuchs et al. (2002) believe that “explanations rely heavily on posters, for frequent student reference, and are laced with opportunities for choral responding to promote engagement” (p. 70). It is not enough to tell the student something; rather, a visual representation is beneficial to the student and will promote retention of the information. Often, math is focused on computations and algorithms, rather than the need to make sense of the math being performed. Math is too complex to lack sense; rather, it requires students to have a deep level of knowledge and cognitive processes, as Edens and Potter (2008) believe.

Several researchers have found that textbooks tend to give students the impression that mathematics is an over-simplified exercise in which a series of problems are gamed using a pallet of algorithmic techniques. The result of this belief gives the impression that mathematics problem solving, as provided in textbooks, is disconnected from real life situations. In an attempt to make mathematics more meaningful, word problems are constructed to contextualize mathematics problem solving. Van de Walle (2004) argues that word problems help students make connections with meanings, interpretations, and relationships to mathematical situations. Others have argued, however, that the attempt to make mathematical problem solving more meaningful has fallen short. Jitendra et al., make the point that a traditional problem-solving lesson from a textbook will require that one single operation (i.e. addition) be used on all problems on the page. This keeps students from being exposed to problems where different strategies are required to find the solution.

According to national standards, students need to view mathematics as an everyday process. Rather than presenting math at a specific time during the school day, it should be acknowledged as something that is present in everyday life.

National Council of Teachers of Mathematics, acknowledge that real-life situations allow students to make more sense out of the concept being taught (Borden and Geskus, 2001). In this study, a story was read to a third-grade class involving a situation in which the characters were challenged with a measurement problem when baking strawberry shortcake. After the story, the students were instructed to find the flaws in the characters' measurement of ingredients. First, the students identified the problems in the characters' procedures. The responses were very obvious problems, but as the students discussed the problems with each other, more detailed and accurate explanations for the problems were revealed. When the students were able to find and analyze the measurement errors, the details of the measurements that the students found were precise and close to the actual amounts needed. Later, reports found that each group had “at least one child who had a good understanding of fractional parts and the relationship of the parts to the whole” (p. 541). The students used any technique they could find in order to solve the problem. The problem became interesting to the students; therefore, they wanted to continue the learning process. This forced the students to make sense out of the math rather than perform routine math procedures. Students need to take ownership of math in order to fully understand the concept or concepts that it involves. When students do this, they will realize that it is not as frightening as it appears to be.

D. Problem-Solving Classroom Intervention Strategies

Several instructional strategies have been proposed which draw from the theoretical work cited above. These include the use of student self-assessment and using games as tool. As students begin to develop appropriate problem-solving skills that will lead toward success, mathematical reasoning will be made; in addition, higher-level mathematical thinking will be encouraged.

Problem solving, as per Van de Walle (2006), should be integrated into a daily routine, rather than something that is taught to find an answer to a test question. It should not be seen as a requirement to the math curriculum, but rather as a means for actually solving problems. One reason why math appears to be difficult for students is because it is often taught in only one way. Students need a variety of options when learning how to do math, because not all students learn the same way (Yeo, 2005). According to Jitendra (2002) third-grade students typically take the information given in a word problem and add the numbers, rather than try to understand the actual problem to be solved. This type of problem-solving strategy encourages mostly “unsuccessful problem-solvers” (p. 34). The traditional way of teaching math is an approach that many students do not understand. There is a small portion of students who will understand it and those that do, love it. Van de Walle (2006) says that problem solving and the “do-as-I-show-you” technique does not prove to develop positive results unless the results belong to a student who is proficient in math. Students fall into a trend where they are performing the mathematics tasks to reach an answer and not growing as a learner. If teachers could teach students with a variety of approaches of solving a math problem, more students would understand it and love it. Two pre-service teachers were used in a study done by Crespo (2003). One had a negative attitude toward math and did not feel competent in the subject. The other was highly competent towards math; however, both found it difficult to use mathematics questions with highly cognitive demand. Problem solving requires students to use the type of knowledge that will require students to make sense of math. Sarama and Cements (2007) believe the same – that if children are encouraged by adults to solve problems at a young age, they will become great problem solvers. The way to develop these skills in children is by modeling. Also, exposing children to math problems, manipulatives, questioning their method of solution and offering help when needed are all methods, according to Sarama, of developing good problem solvers.

Leinwand and Ginsburg (2007) state Singapore's success in the math program known as Singapore Math. According to the Trends in International Mathematics and Science Studies (TIMSS), Singapore has always been ranked at the top of the list of countries proficient in mathematics. At the center of this country's math program is problem solving. The program consists of less material in a year's span; however it requires more higher-level thinking. It provides representations relevant to solving the problem rather than only pictures that relate to the words in the problem. Also, the text may give information about the problem or even encourage the student to ask questions about the problem. The representations provided help students build their conceptual understanding of math compared to programs that encourage memorization of math operations. This gives students a wider range of mathematics knowledge as well as more conceptual understanding.

1. Schema based instruction and student self-assessment

Van de Walle (2006) refers to Cognitively Guided Instruction (1999) when he identifies four schemata in which mathematical word problems fall. These are join problems, separate problems, part-part-whole problems and compare problems. The join problems have three quantities involved that must be joined together to find the problem solution. These are the initial amount, a change amount, and the part being added or joined. The result of joining the amounts is the whole. The problems classified as separate begin with a whole and separates one part to find the remaining part. In a part-part-whole problem, there are two parts that are grouped together to make a whole. Compare problems involve comparing two quantities in order to find a difference.

With schema-based instruction in place, students are able to take ownership of his/her learning in a way that encourages student self-assessment. Students need motivation to learn to love something. Brookhart, Andolina, Zuza, and Furman (2004) found that student self-assessment encourages ownership of learning. A student should reflect on the progress that he/she is making in order to reach success. Fuchs et al. (2003) believes that self-monitoring leaves a positive effect on students' learning in mathematical problem solving. Lan (1996) defines self-monitoring as deliberately placing emphasis on one specific part of a behavior. In addition, Lan believes it is an integral part of the learning process.

Owen and Fuchs (2002) agree that training students to see strategies in different word problems is the most beneficial approach to developing better problem solvers. Jitendra (2002) finds problem schemata to be one of the most important elements to problem solving. In fact, Jitendra suggests that students classify problems into groups in order to recognize patterns and find relationships amongst problems.

Self-monitoring involves two basic components. In the first, the student reflects on his/her progress toward a goal. The second involves the student predicting the amount of time that will be spent to reach that goal. Lan also stated that this should encourage the student to determine what strategy is most appropriate. When Lan (1996) did a study where self-monitoring was the manipulated variable, he found that "students in the self-monitoring condition performed better than students in non-self-monitoring groups on each course examination” (p. 5). When students were told to monitor their own progression of memorizing times tables, Brookhart et al. (2004) discovered that a deeper lesson was taught while students were self-monitoring. The theorists defined two categories of self-monitoring. These categories are motivational and cognitive. It was found that self-assessment would encourage students to take ownership of their own learning. Brookhart et al. also stated that taking ownership of learning would give students self-worth, which in turn lead to more desire to continue the learning process. In addition, they believed in self-assessment so much that a study was conducted to prove that it stimulated the learning being done. The study was called Minute Math. Students were given five minutes to answer as many multiplication facts as possible. In order to self-monitor, the students would fill out a survey before each assessment. Also, a student would look at his/her data and predict how many facts would be answered the following week. In addition, students would create a visual representation, as they would graph the results and predictions. To measure motivation, students were classified into different categories dependent upon their expressed goals being Master, Performance or both. As far as understanding, the students were classified by use of strategy. Basically, three levels were described: whether their reflections indicated a clear pattern of study strategy use and problem-solving strategy use or whether strategy use was inconsistent. The mean of the five tests that were given was taken from each student. Furthermore, descriptive statistics were used to analyze the prediction accuracy. It was found that over the ten-week course, students became more accurate of their predictions. In both classrooms in which the study was performed, it was apparent that self-assessment increased the students' meta-cognition in lessons that involved rote memorization such as memorizing multiplication facts. Also, it was observed that students liked seeing their progress on their graphs. Overall, a self-assessment involves the ownership of one's learning. Students are more motivated to be successful and, therefore, learn long-term (Brookhart et al.). Fuchs, Fuchs, Prentice, Burch, and Paulsen (2002) use self-regulated learning practices as well. "Goal setting helps motivate children to focus their effort and work hard" (p. 72). When students set their own goals, they are better able to use the skills and strategies to reach their goal.

To build the appropriate skills for self-assessment, it is important to encourage questioning. Children need to be skeptical of math and know when and what kind of questions to ask. Myers (2007) suggests that teachers are the ones who can help children develop these skills simply by allowing them to explore mathematics on their own. "Teachers can nurture or negate the innate curiosity young children bring with them into the elementary school classroom" (p. 692). In fact, children, according to Myers, are more inclined to ask good questions than adults. Through exploration, students will begin to understand the concepts of math, and from there the procedures of math strategies will make sense. The confidence that students will get from their success will be a motivator for them to persevere. If one approach does not work, students will not be afraid to attempt newer, more difficult problems. Myers emphasizes that although math is made up of rules and procedures, and although there is a right or wrong answer, math does make sense. There are reasons for why certain steps have to be followed and, by teaching tricks and shortcuts, we are giving our students the contradictory message.

Fuchs, Fuchs, Prentice, Burch, and Paulsen (2002) found that it takes three steps for students to be successful in problem solving. Without the three things necessary, students will become overwhelmed, frustrated, or even not be concerned with whether or not the problem makes sense. First the students must master the expectations of problem solving that are set by the state or district. Word problems can be placed into so many different categories so it becomes difficult for students to decide which strategy is most appropriate. Because of this, students need to be able to categorize word problems together that require the same solution. In other words, word problems should begin to look familiar and be classified as addition, subtraction, multiplication, or division. Finally, students need to connect new problems to ones that they had previously solved. Fuchs et al. describe this process, as “transfer” (p. 70). Each type of problem should be introduced to the students in a way that they will succeed in solving a problem in the same category in the future. Gradually, students work on more parts of each problem independently and become less dependent on the teacher modeling each step. Soon, the students will begin working in pairs with the stronger students helping the weaker students. Finally, each student completes one word problem independently. These steps take place over one lesson.

Jitendra, Sczesniak, Griffin, and Deatline-Buchman (2007) reached similar conclusions. Four types of problems were identified into categories. They are change, combine, compare, and equalize. Jitendra et al. found that recognizing this relationship allows students to group word problems into categories and use prior knowledge to successfully answer them.

2. Games as a problem-solving strategy

It was noted by Kamii et al. (2001) that one teacher had her students play math games alone all year, giving out only four worksheets. It was found that students who could not complete the worksheets could not play the games, nor could they show accurate or relevant representations. The reason is because students who understood the concept of the operation could easily compute the numbers in his/her head, while those who did not understand the concept were unable to complete any computations because they did not know the procedural steps in which to do so. Because children like to play games, they will relate math to something fun and, in turn, think of math as being fun, and, more importantly, make sense. At the same time, teachers have to facilitate this type of thinking. If the option of using counters over paper and pencil is given but not encouraged, it will not likely happen. Ralston and Willoughby (1997) state that children will learn much more by playing math games than “drill and kill” worksheets, because the games will be played for a long amount of time after they are introduced. Games are teaching important skills as they are being played and students are solving real problems. Students are actually thinking instead of memorizing. Although finding strategies is occurring, it is not what the children are focusing on when they are playing. Instead, children are focusing on winning; but, all the while, students are actually developing very deep and necessary skills. Reasoning is used and teachers are not giving any valuable information away that students must discover on their own. Another benefit of the games is that students put forth more effort into the math than they would through a traditional method of mathematics instruction.

E. Synthesis of Literature Review

Piaget defines three types of knowledge in order to explain students' developmental process. Empirical abstraction is taken from physical knowledge, while constructive abstraction is related to logico-mathematical knowledge. Math, according to Piaget, should be learned through hands-on activities and a discovery process. When students lack the proper development of these three types of knowledge, it becomes difficult for students to understand abstract concepts. Representation of a problem is developed through logico-mathematical knowledge. Visual-spatial development allows students to represent a problem on paper. Development is a very important factor because students must find reasoning in the mathematical procedures they perform and truly develop an understanding of mathematical concepts. Part of reasoning is the ability to "transfer," which is how students categorize problems by common traits. Students should group a word problem into one of the four mathematical operations.

Research has found that there are two types of representation systems. The first stems from the information presented to the student called the external system. The internal system is personal to the student because it is an interpretation of the information presented. Both systems are imperative for conceptual learning to take place. Internal underdevelopment can hinder students from connecting the proper mathematical operation to a word problem. Constructivists believe that students should discover math on their own. Behaviorists have a “right or wrong” policy. Constructivists view the behaviorist theory as being too objective while the behaviorists see the constructivist theory as being too subjective.

There are several strategies that may help students with the internalizing process. Mathematical manipulative materials have been proven to help students grasp a conceptual meaning. Researchers have proven that in order for concrete reasoning to take place. It is after this happens that students can bridge the concrete with the abstract. Other strategies include peer-to-peer discussions. Conversing with peers often encourages learning. In addition to discovery in mathematics, students need to be offered multiple strategies to choose the most appropriate for his/her learning style. Conceptual understanding will suffer if only one method of representation is modeled during instruction.

Self-regulation improves students’ problem-solving abilities. Schema-based instruction encourages students to self-regulate. It is when self-regulation occurs that students become motivated.

Students who have developed the appropriate level of knowledge, who use both representation systems, and are encouraged with self-regulation will be able to successfully use graphic representations. The development of relevant graphic representation use will become one of the most useful tools in third-grade problem solving. If students can represent what they read, they will understand the problem and carry out the correct mathematical procedure. Because elementary students are such visual learners, the use of graphic representation will lead to more success in mathematical problem solving. Several studies were conducted in which the students who use graphic representations were compared to the students who did not use graphic representations. In every study that was reviewed, graphic representations proved to be a most useful tool in problem solving. Graphic representations were found to help the student to be more thorough and understand the problem and its concept easier. Although it is good to model different representations, a student's creation of representation is found to be more meaningful to that individual.

Representations can be either schematic or pictorial. Schematic representation has been found to be more useful to students while problem solving because these representations contain only the necessary information to solving the word problem.

Reading comprehension is a factor in students' problem solving abilities because word problems rely heavily on the students' understanding of the problem. Also, problem solving is determined to be a multi-step process that is developmental and can only be learned through practice. Often in Math, teachers encourage short-cuts as a tool; however, this cuts out any sense-making capabilities. Keywords are also taught as a tool, which in fact discourage reasoning or any connection with the word problem. When real-life experiences are integrated into a math lesson, students take ownership of it. If students find

Math relates to everyday life, students will become more interested in understanding it. Textbooks tend to give students enough practice on one topic at a time; however, the do not give students the opportunity to comprehend and carry out the proper mathematical procedure.

Students may be motivated by self-regulation and learning through games. Games teach important skills as they are being played and students are solving real problems. Again, students put forth more effort into math when playing games than they would with traditional mathematic instruction.

There are many reasons for students' inabilities to perform well on mathematics word problems. At the same time, there are many strategies offered to help students. It is necessary to expose students to these strategies in order for them to become life-long problem solvers in and beyond the mathematics classroom.

Chapter III

METHODOLOGY

A. Introduction

Problem solving becomes crucial in third grade. Students are faced with new standardized test requirements with little or no prior test-taking experience. Different teachers find different techniques to be helpful, and students have to figure out which is best for him/her. Because every student learns differently, it is important to provide students with multiple strategies to choose from. This study will look at three different students and what strategies helped them the most. Overall, the same conclusions can be made as to where teachers should place focus when applying problem-solving strategies to daily routines in the classroom.

1. Pilot study

In the same school year, a pilot study was conducted. This study was intended to prove that the use of graphic representations would aid the students in solving mathematical word problems. There was a sample of 39 students (n=39). This sample is the same as the sample in which the three case studies were drawn. The first classroom was the treatment group. After the pre-survey and pre-test were given a three month intervention began. The students learned different techniques when solving mathematical word problems. Each technique was specific to the use of graphic representations. After the intervention the students were given the post-survey and post-test. The second classroom was classified as the comparison group. This class received the same pre-survey and pre-test; however, there was no specific intervention that focused on the use of graphic representations. It was simply under the direction of the teacher. This class received the post-survey and post-test as well. A sample t-test comparing growth from the pre-test to the post-test was conducted. In addition, the same test was done comparing the correctness on the post-test between the treatment group and the comparison group was taken. It was found that only 28% of students in either group showed improvement from the pre-test to the post-test. After the results of this study were analyzed, the question of "why" became a mystery. The study presented in this paper was developed in order to answer the questions the pilot study revealed.

2. Research design

The students in two third-grade classrooms were given a pre-survey, as well as a pre-test. Each teacher was given three months to teach as many problem-solving strategies as possible. After the three months, the students were given a post-survey and post-test. From the thirty-three students, three were chosen to give extensive interviews as to why he or she performed the strategies that were chosen. The choice of the three participants was due to improvement of scores on the two tests and the work that he or she did on each paper.

This study took place in two third-grade classrooms. The school is in a suburban border city. The city borders Texas and Mexico and is situated with a large Army post right in the center of the city. The school in which the study takes place is located in an area of the city with a large military population. The Academic Excellence Indicator System (AEIS) report is provided by the state in order to display all demographics for individual campuses in Texas. The report for 2008-2009 was not released in time to be included in this paper. The school demographics will be presented from 2007-2008. That year there were 730 students at this campus. The third-grade accounted for 15% of the school with 111 students. 46% of the third grade was Hispanic, 31% were white, 20% were African-American, 0.3% was Native American and 3% were Asian-American. In the third grade, 42% were Economically Disadvantaged, 8.6% were Limited English Proficient, 0.5% of the third-grade population was labeled as Disciplinary Placements, and 33.3% were coded as At-Risk. On average, each teacher had 16.5 students. Again, this data is a representation of the school the year previous to when the study took place.

The three students that were identified to be studied were all in the third grade. After choosing the three using the criterion-based sample as described above, two of the students were assigned to my general education class for the 2008-2009 school year and the third student was assigned to another general education classroom. The teacher of the second classroom was specifically chosen because of similar teaching styles. After collecting and analyzing the data from the pilot study, all three students showed growth from the pre-test to the post-test and had interesting changes to their survey #1 and #2. Each of the participants was enrolled in general education with no supplemental resources or classes. The names of all three participants were changed in this paper for the purpose of confidentiality.

B. Purpose of Study

The main purpose of this study was to find strategies that students find helpful when solving mathematical word problems. These strategies should encourage students to use graphic representations when solving word problems. By teaching and modeling different techniques, it was believed that students would find use of the representations and become successful in answering word problems. I was aware that a solution to all students' struggles would not be discovered; rather, an option for teachers to use when teaching students to problem solve would become available. The purpose of the study was to understand the challenges students in third grade face when solving mathematical word problems as well as what strategies students find helpful. As a pilot study was performed prior to the three case studies, it was discovered that modeling and encouraging the use of graphic representations alone was not enough to have students comprehend and solve unusual word problems containing sufficient, insufficient, and extra information. Only 28% of third graders were able to show improvement determined by the pre- and post-tests, which begs the question "What strategies do third-grade students find helpful when solving mathematic word problems?"

Participants

Because all of the participants were minors, consent forms signed by the parents were necessary. Prior to the study, consent forms were collected from each participant. There were two types of consent forms used, as one (Appendix A) was used for the students enrolled in my class. It explained the treatment used in the pilot study. In addition, because I was the teacher of the class, the students (participants), as well as their parents, were very knowledgeable as to what the study entailed. A more general consent form (Appendix B) was given to the third student. The study began in early November and was completed in late April.

C. Design of Survey

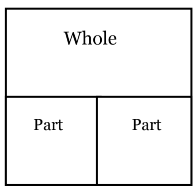
First a pre-survey (Appendix C) was given to each participant. This survey allowed me to gain information on how the students felt about problem solving. Gaining insight into the students' feelings toward the topic being researched is a good method of learning about the participants and their way of thinking. The first question was designed specifically to find out what the student believed was the most difficult part of problem solving. Question two simply looks into what helpful strategy the student has had previously. This was important because when scoring the pre-test, I must know of the participant's knowledge beforehand. Therefore, the measurements of the intervention will be accurate. Finally, the third question allowed the student to express the difficulties that he or she incurs in problem solving. This gave insight as to what I should focus on while instructing the students and what should be considered when developing lessons.

D. Design of Tests and Teaching Techniques

Following the survey was the pre-test, which is a compilation of four questions (Appendix D). The questions are designed specifically for this study. The first is a problem in which the solution is stated in the question. This question is designed to include three different numerals. In order to answer correctly, the student must correctly comprehend what the problem is asking. Since the answer is stated in the problem, the student merely must take the information from the problem. No mathematical operation or number manipulation is needed. Extra numbers are given in the problem purposely to check the student's comprehension of the problem. The second and fourth questions have insufficient information, meaning the problem cannot be solved. In both problems there is information that is both irrelevant and inapplicable in solving the problem. The third problem has extra information; however, there is a solution to be reached after performing a one-step mathematical operation. Again, the problem has irrelevant information that students can easily find misguiding when trying to solve the problem. Comprehension of the problem is the first step to answer these problems correctly. The main purpose of the problems was to teach the participants that using graphic representations would help to comprehend the problems and, in turn, answer them correctly.

For the next three months, the students and I worked on strategies including the use of graphic representations. The strategies used in the classroom include but are not limited to underlining the question, circling the unit, unit box, part-part-whole chart, arrays, t-chart for organizing patterns, organized list, and making a number line. The strategies listed are outlined in table 2.1. Students worked alone and in pairs. Students used mathematics manipulative materials, as well as acting out the problem. Each student tried each technique more than once to become familiar with it. Students were highly encouraged to

Table 3.1

Underlining the question	The student underlines the question in the word problem in order to understand what the problem is asking of them.
Circle the unit	The unit is the object that the student is counting
Unit box	The student writes the unit in a box to the side to serve as a reminder as to what they should be counting
Part-part-whole chart 	The student reads the word problem and determines where the information fits appropriately in the chart.
Array	The array is used for multiplication problems. The students use dots, circles, x's, or any other symbol and organize the information in an orderly arrangement
T-chart	Students place information in two separate columns in order to compare the information with each other.
Organized list	Helps students view the information given and review it in order to understand what still needs to be done.
Number line	A line with numbers in correct numerical order

read the problem as a story rather than a mathematics word problem. I asked questions such as: who are the characters, what is the setting, what are the characters doing, what is the characters problem in the story, and finally, what is the solution to the problem? Working on problem solving every day for thirty minutes was found to be overwhelming, so it was adjusted down to only three times per week. The materials that the students used consisted of a collection of word problems from the 2002, 2003, and 2004 Sharon Wells Mathematics Curriculum, Inc. The district purchased this program to be used in subsequent years and teachers continue to utilize it to supplement and allow more practice on problem solving. In the entire intervention was 36 problems chosen from the second six-weeks of the curriculum. These problems were chosen simply because it was available in its entirety. In addition, I found it necessary to design four problems, which I incorporated into the intervention, as two days' worth of work was needed. Although the curriculum was being used, it was not that which was being measured. The problems were simply used to model using a variety of mathematical strategies to the students. The focus of the study was on the students' use of graphic representations and how it affected problem-solving success.

The two students assigned to my classroom sustained many days of problem-solving strategies. The students were taught different techniques for each problem. First, the student was instructed to take pertinent information and pull it out of the problem. Van de Walle (2006) part-part-whole charts were a major tool for students organizing information. This strategy was introduced to students as a pertinent part of the problem-solving process. Students were trained to extract the information from the problem and place it into the part-part-whole chart. Often, the students discovered that a part-part-whole chart wouldn't be appropriate; however, students soon learned how to adjust. Symbols were encouraged rather than drawing the literal interpretation of what the problem involved. Letters were a recommended technique, as it took less time than writing out a full name. Students were taught to organize information in a table as well. In order for concrete understanding, the students were told to act out what was happening in the word problem. In addition, counters were used as a physical representation of an object and finally, the students were expected to take these concrete representations and transfer them into a graphic representation on paper. The representations that were modeled and encouraged were all of schematic nature. Students were discouraged from drawing pictures as would be done in an artful manner. In fact, the word "pictures" was never used; rather, the term "show your thinking" was used to encourage the use of graphic representations. Signs, symbols, and any other visuals were acceptable. Another skill that students used was classifying problems into familiar groups as much previous research has suggested. It became routine to determine the schema in which the problem belonged.

The teacher that the third case study was assigned to used similar teaching techniques as much of the planning and training the teachers receive is done together. Her strategies include, but are not limited to, underlining the question, circling the unit, circling the important information, a total box, t-chart for patterns, number line, arrays, drawing a picture of what the problem said, and identifying any keywords.

After an ample amount of teaching, modeling, and practicing, a post-test (Appendix E) was given to each student. The post-test was arranged the same way as the pre-test in the sense that one question contained sufficient information, one question had the answer in the problem, and two of the questions had insufficient information; however, they were not the same. In addition, students were given a post-survey, which was identical to the pre-survey. All students received the post-test and post-survey.

E. Choice of Case Studies

The students who stood as case studies were chosen with a criterion based sample. This is defined by Miles and Huberman (1994) as choosing cases which fall into predetermined criteria. This particular criterion was having an improved score from the pre-test to the post-test. The three students in this case study were placed into this criterion and therefore chosen as the participants.

F. Design of Interview

The goal of the interview was to discover what strategies helped the student. The interviews were structured in a way that would help students explain each step and hopefully realize where he or she steered in the wrong direction. The student who was the speaker, would explain the steps while the interviewer would just listen, not prompt or provoke. The interpretation of the interview came from not only answers to questions, but also the tone of voice and intention of the answers. Often the questions that were created would lead to a second similar question.

1. Case study #1: Jeremiah

Jeremiah is an AB honor-roll student who has always scored at the top of the class. He is of Hispanic descent and eight years of age. He was born into a military family and has many experiences to share with his classmates and teacher. Jeremiah was assigned to the classroom in which I, the lead researcher, was the teacher for the 2008-2009 school year. Immediately, it was apparent what type of student Jeremiah was. He was able to draw conclusions and make observations that other students do not always see. His comments are most valuable to the topic being discussed and he is quick to correct a mistake that others do not see.

Considering all of these attributes that Jeremiah possesses, he does not receive A honor roll on any of his report cards. He is neither a perfectionist nor a master of test-taking. His strengths are in how he views concepts and how he perceives knowledge. He is very patient with his work and makes certain to put all of his effort into everything he does. It is these attributes that put him above the others.

In second grade, Jeremiah completed the year with a 90% average. Because the 2008-2009 school year was his third-grade year, he had no TAKS (Texas Assessment of Knowledge and Skills) data coming from second grade. His received a score of 2436 on his 2009 TAKS-Mathematics, missing only two questions out of the total forty questions. This put Jeremiah at commended status. Only 31 out of 137 third graders reached this status (23%). The Texas Primary Reading Inventory (TPRI) test measures reading fluency, comprehension, vocabulary, and spelling. Jeremiah was ranked 45 out of 144 readers in the third grade in January. This tells us that he is close to the top third of his class in reading. This includes comprehension, fluency, vocabulary, and spelling. He answered six out of eight comprehension questions correctly on the TPRI. Currently in fourth grade, Jeremiah has an 89% average and received a 94% on his latest Mathematics benchmark. These are both graded on the same 100% scale.

Jeremiah declared on his pre-survey (Appendix F.1) that he first looks at the numbers in the problem and reads to decide if he should add or subtract them. He stated that what helps him when working on a math word problem is to think of the number question in his head, then do it on paper to make sure that it is correct. He also said that hardest part of a math word problem is to solve the question.

After the intervention, Jeremiah's answers changed on two of the questions (Appendix F.3). Instead of first looking at the numbers and deciding if he should add or subtract, he now underlines the question. The thing that helps him the most to solve the math word problems is to create a part-part-whole chart and organize the information inside the chart. One thing that remains the same is what he considers the most difficult part of the math word problem, which he said is getting your answer.

After sitting down with Jeremiah at length and discussing his use of representation and his comprehension, I found one of Jeremiah's most helpful strategies to be the use of graphic representations because they helped him to organize the information given in the word problem.

2. Case study #2: Roman

Roman is an eight-year-old boy of Hispanic descent. He is an extremely artistic young boy who can draw pictures with so much detail. His masterpieces look like those of a professional, though he says he does not enjoy drawing because everyone always asks it of him. He does well in school, though his grades are those of an average student. His final average in Mathematics for his second-grade school year was 91. On his TAKS-Mathematics he scored a 2323. He missed four out of forty questions. On the TPRI he ranked 8 out of 144 students tested. Also, he answered seven out of eight questions correctly. Roman is doing very well in fourth grade, as his average is 91%. On his latest Mathematics benchmark, he received an 88%.

When Roman was asked what he does first in a math word problem, he also answered that he looks at the number, then adds or subtracts (Appendix G.1). He said the addition or subtraction sign on the bottom left is what helps him most to solve a math word problem. It can only be assumed because of the description that he gave, that Roman is referring to the mathematical symbol in a vertically structured number sentence. Also confusing is his answer to what is hardest about math word problems. He states that the hardest part is when he has to add five numbers. Although Roman is not very specific as to what he is referring to, a reasonable assumption would be a five-number addition problem, which is rarely given to a third-grade student; however, can is an intimidating problem.

Roman shows a lot of growth as he reflects on his work after the intervention (Appendix G.3). Now his first step when solving a math word problem is to read the question, see what his choices are and choose the one he thinks is correct. Showing his work helps him the most in reaching the correct answer. This astute answer shows that he needs some sort of visual representation on his paper in order for him to organize the information and reach the correct answer. Again, the hardest part of a word problem for him involves the actual task of choosing the correct answer. Roman is referring to a TAKS formatted test with multiple choice answers. He states that it is most difficult when he has three good answer choices and he has to select one of them.

3. Case study #3: Benjamin

Benjamin is a student that was assigned to the class by a pre-existing school-based placement. His class was the comparison group from the pilot study. He is a white male at eight years of age. Benjamin is an enlightening learner, as he shares all of his observations with his peers. He is focused and likes learning new things while being unafraid to question when he is confused. On his TAKS Mathematics he received a score of 2222, missing seven out of forty questions. Benjamin is ranked 68 out of 144 third graders on the TPRI test. He answered seven out of eight comprehension questions accurately. Currently, he holds an 82% in fourth-grade Math and received a 71% on his last Math benchmark.

When Benjamin was asked initially what he does first when answering a word problem, he stated that he reads it first (Appendix H.1). The thing that helps him most when solving a word problem is when he has to work it out. Benjamin also said that the hardest part for him is when he has to do multiplication.

Benjamin did not receive the intervention, but was given the second survey (Appendix H.3) at the same time as the treatment group was given the second survey. Again, he said the first thing he does when solving word problems is to read the problem. What helps him to solve the problem is to circle the numbers, and the hardest part about word problems is everything.

F. Data Collection

As two of the participants were enrolled in my class, I became very familiar with their individual learning styles and personalities. I was familiar with their needs and how to best help them understand a difficult concept. This was a benefit for me as a researcher because I was able to fully analyze the work of the students. I was not as familiar with the third student, Benjamin; however, through my analysis, as well as interviews and observations, I could collect the most important data, which helped clarify my research questions.

The qualitative data that was taken will help me by revealing variables that took place throughout the study that may have been overlooked when designing it. This data will reveal what modifications and adjustments are necessary so that future researchers can benefit and use this study as a tool.

I kept an observation log each day so as to refresh the memory of the work we did. In addition, the students collected their work in a specified notebook. This assisted me as I can go back and reflect on what technique I taught the students to use for specific problems. Both surveys that were given are elements of the qualitative research. Students' answers to why something is difficult or what is so difficult about a concept become a very useful tool in analyzing student work.

After the students had completed the study, I interviewed a select few. The goal was simply to determine if the students could verbally explain their answer choices and reasoning behind them.

In education, it becomes difficult to conduct an accurate study. The reason behind the study was to simply monitor the frequency of the students' use of graphic representation and determine which strategy would help more students most often. This study is relevant to a regular third-grade mathematics lesson. It has helped both the students and me to become better at problem-solving skills. The study was beneficial to the students, as it was 100% academic training for mathematical word problems.

Chapter IV

RESULTS

Research by Fuchs et al. (2003) and Brookhart et al. (2004) has shown that students learn best when they take ownership of their learning. Through looking at student work and interviewing these students at length, I am able to understand what particular strategies work best for these students. It is refreshing to listen to explanations of what does and does not make sense when working mathematical word problems. Graphic representations are one good method that all three students found to be helpful when critical thinking is necessary.

A. Transcriptions of Case Studies

1. Case study #1: Jeremiah

The following is the conversation that took place when interviewing Jeremiah (Appendix F.5):

Teacher: What is your favorite subject?

Jeremiah: Science.

Teacher: Do you like reading?

Jeremiah: Yes, I love non-fiction books the most, biographies.

Teacher: Why do you like non-fiction?

Jeremiah: Because you learn more.

Teacher: Do you like math?

Jeremiah: Yes, because it is challenging.

Teacher: What is your favorite thing about math?

Jeremiah: Multiplying.

Teacher: When you have to do a word problem, what is it that you look for?

Jeremiah: The question because it tells you what to do, kind of like how to figure it out.

Teacher: What do you do first?

Jeremiah: Look at the question and circle the information to figure out the operation.

Teacher: How do you know that you understand the problem?

Jeremiah: I reread to make sure and rewrite the information from the problem.

Teacher: What helps you to solve the problem?

Jeremiah: I make a chart or use pictures.

Teacher: How do pictures help you?

Jeremiah: They help me because I like them more than words. I relate more to them because I love to draw.

Teacher: Do you draw a picture, an exact picture, or do you use symbols for the pictures?

Jeremiah: I use symbols.

Teacher: Which kind? Could you give an example of what you would use to represent apples? Or Tuesday?

Jeremiah: I would use circles to represent apples, or the letter T to represent Tuesday.

Teacher: Let’s look at this problem (#2 on pre-test, see Appendix F.2). Can you read it and tell me who and what this problem is about?

Jeremiah: (Reads the problem). It's about Cara. She has two shoes with red shoelaces and three shoes with blue shoelaces. How many green? I know she can wear them all.

Teacher: How can you know that she wears them? What are your clues?

Jeremiah: They're shoes, but I guess it doesn't say that.

Teacher: What does the problem want you to find out?

Jeremiah: How many green. It should tell you, but it doesn't. It cannot be figured out.

Teacher: How did you get four before?

Jeremiah: I don't know. I thought it said just one green.

Teacher: Okay, lets look at this one (#2 on the post-test, see Appendix F.4). Can you read it and then tell me who and what it is about?

Jeremiah: (Reads the problem). It's about Julia and her homework. She is doing 30 minutes of homework and 30 minutes of reading. How long is the TV show?

Teacher: I noticed on your paper, you wrote "you can't figure that out without any evidence." What kind of evidence do you need?

Jeremiah: The problem doesn't mention anything about a TV show.

Teacher: You're right. These two problems are very similar. What helped you on this one (post-test), but tricked you on this one (pre-test)?

Jeremiah: The first one is talking about the same thing, but different color (shoelaces). The second one is completely different things.

Teacher: What could you have done on the first one to help you not to be tricked?

Jeremiah: If I drew the shoe with the letter of the color (i.e. blue=b) above the shoe, then I would have known there were no green shoelaces.

The first thing that stood out in my conversation with Jeremiah was when he said he looks at the question first because it tells him what to do. He uses the question as a source of information, which is a strategy for problem solving. He is able to recognize the structure of a word problem. He differentiates a mathematics word problem from a story because it has a question and he knows he needs that question to help him solve whatever problem he has. This adds to the difficulty and hinders students from viewing mathematics word problems as stories. They are constructed in a way that gives information, asks a question, and poses a problem with no solution. Most stories that are being read in Reading class have problems, but state the solution as the student continues to read. That is one of the enjoyments of reading. Being forced to find the solution to the problem takes out the enjoyment and leaves students feeling overwhelmed.

Jeremiah stated that he always underlines the question and circles the important information first. This is a strategy that many teachers use to help their students. If we train students to do this, does it help them to understand the problem? Jeremiah says he rereads the problem to make sure that he fully understands it. Also, he extracts the pertinent information from the problem and writes it down. This strategy helps him because he is taking more ownership of the problem by writing it down on his own. The information is becoming more familiar to him and he can use it to manipulate it in the most appropriate way. Jeremiah goes on to make a chart or picture of the problem. One of the most interesting things that he says is that he likes the pictures more than words. He says he relates to them better. This is true among so many third graders and students alike. Pictures are kid-friendly. A story with pictures is much more appealing to most students than just words. Even adults sometimes understand things better when there is a picture or diagram to supplement the words. Just as students are trained to underline the question, teachers should encourage some type of graphic representation to be built by the student. It will reinforce understanding and conceptual growth.

As Jeremiah explained his confusion in the problem that deals with different colored shoelaces, I realized something about the way third-graders think. He assumed that the person in the story was wearing these shoelaces. I didn't ask him if he thought Cara was wearing the shoes, nor was it at all important to the problem solution. When I asked him how he knew she wore them, he just said he knew she did because they are shoes. People wear shoes and he knows that. He also realized that there was not enough information given in the problem. To a child, this becomes somewhat stressful. He was trying to figure out why there was no information about green shoelaces, yet the question was asking about them. Because of his uneasy reaction to the problem, he created a number that he "thought" he saw and solved the problem anyway. It did not make sense to have a problem without sufficient information. In a child's mind, there should be sufficient information or else there is no reasonable solution. Therefore, the missing information was created.

Jeremiah did answer one of the two questions correctly. They were constructed the same way, leaving out the information needed to solve the problem. He was very quick to explain why he thought he missed one but not the other. His reason is both interesting and logical. He blames the problem set up for his confusion. Because the objects being used in the first problem were the same, except for the color, he was more easily confused than when the objects were all different. This is an interesting thing to think about when writing problems in the future.

Jeremiah finished the interview by acknowledging that some sort of representation would have helped him develop a better understanding of what the problem was asking him to do. He says he would have drawn the shoe with a letter representing the color of the shoelaces. This is helpful to know because representing the problem with graphics can help other students as well when faced with the challenge of mathematics word problems.

Jeremiah recognized his errors in that on the first problem he tried to add two numbers together. In fact, he realized that he tried to add three and one, when one wasn't even mentioned in the problem. He said he misread and thought it said one. Many students do this, which causes them to lose the conceptual understanding which is necessary to solve the problems. It is often a first reaction. In fact, Jeremiah actually created a number even though it was not stated in the problem. He added the two numbers together and was content in using that as his answer. This reaffirms my hypothesis that students see word problems and automatically look to see what they can add. The problem has no value to them as far being a story. Reading comprehension strategies are useful even in mathematics, as students should put more focus on the content of the story problem rather than the numbers. After Jeremiah was taught several strategies, he was able to view the same type of problem as a story and conclude that it had no solution due to lack of evidence. He did make the connection that the second problem might have been less tricky, because all of the units being combined were different things (i.e., homework, reading, and television show). He said the first problem was more confusing because all the objects were shoelaces, just different colors. Although this might have been a variable, he concluded that he could have answered it correctly had he represented his thinking on paper with symbols.

Jeremiah is just like many other third-graders in that he is a visual learner. He needs visual strategies to reinforce reading comprehension and the completion of the mathematical operation. Graphic representations are/were the perfect tool for Jeremiah to solve the word problems he was given. Hopefully, in the future he will take these strategies with him and implement them as needed.

2.Case study 2: Roman

Roman came into the interview (Appendix G.5) remembering the pre- and post-test very clearly. He was excited to sit and discuss it with me and had quick, justified answers for each question. The following is the conversation we had:

Teacher: What is your favorite subject?

Roman: Science. I like it because we do a lot of experiments and get to play around with stuff.

Teacher: Do you like reading? Why or why not?

Roman: Yes, I like reading fiction chapter books. I like Captain Underpants and another one, but I can't remember what it's called.

Teacher: What is your favorite thing about reading?

Roman: It's fun, because sometimes it makes you laugh or cry.

Teacher: Do you like math? Why or why not?

Roman: Yes, because you work with people. You can combine brains.

Teacher: What is your favorite thing about math?

Roman: Multiplication, because I like to be challenged.

Teacher: What do you look for first in a word problem?

Roman: It depends on how hard it is. The ones that are kind of hard are the ones that have to be converted. I like them though because I like the challenge. I also look to see if I'm going to add, multiply, and check to see if it makes sense.

Teacher: What do you do first?

Roman: See if it sounds right or not.

Teacher: What helps you to solve the problem?

Roman: Working with a partner and comparing work.

Teacher: Do you write anything down after you read the problem?

Roman: The number sentence. I try all the operations until I find the right answer.

Teacher: You try all of the operations?

Roman: Yes, then I find the matching answer.

Teacher: What if two answers match?

Roman: Then I have to figure out which one makes the most sense.

Teacher: How do you figure out if it makes sense?

Roman: I like to make a key on the side.

Teacher: What do you write in the key?

Roman: I write whatever it is that I have to count.

Teacher: Oh, okay. So, it's your unit? Almost like a unit box?

Roman: Yes, I forgot that we called it that. I just call it a key.

Teacher: What does the key help you to do?

Roman: To figure out how much something is worth and if I have to add or multiply.

Teacher: Do you draw anything else?

Roman: Drawing takes too much time. It is a waste of time.

Teacher: What about symbols? Do they help you?

Roman: Symbols help me to concentrate on what to do to solve the problem.

Teacher: Let's look at this problem (#1 on pre-test, see Appendix G.2). Can you read it to yourself? (Roman reads the problem). Can you tell me who and what this is about?

Roman: It is about Jenna and her brothers. She is making cookies, they are gingerbread and tree cookies.

Teacher: What are the brothers doing?

Roman: They will eat the tree cookies. Jenna will eat the gingerbread.

Teacher: Does it say that the brothers will eat the tree cookies and Jenna will eat the gingerbread?

Roman: Well, it says the brothers will eat the tree, so that means Jenna will eat the gingerbread. Those are the only ones left.

Teacher: Okay, good. I like how you organized your information. What did you notice about this problem?

Roman: I noticed that the answer is in the problem.

Teacher: That was very smart of you, good job. Now look at this problem (#1 on post-test, see Appendix G.4). Can you explain these symbols that you drew over here?

Roman: Well, he made two Valentines, so I drew a two with a heart. Then he used three sheets of pink paper, so I drew a three with three squares. He used four sheets of red paper, so I drew a four with four squares.

Teacher: And why did you cross out the two with the heart?

Roman: Well, I knew that the question says to count paper, so the heart doesn't matter because it is the cards.

Teacher: Wow, so the symbols helped you realize that?

Roman: Yes.

Teacher: That's very smart. Look at this one now (#3 on pre-test, see Appendix G.2). What is it about?

Roman: There are 15 clownfish swimming and two starfish distract four of the clownfish.

Teacher: What does that mean? Can you explain what exactly is happening in the story?

Roman: Well, two of the starfish take four clownfish away.

Teacher: I like the way you explain that. I like your words "take…away". What do you think you did wrong when you answered this problem?

Roman: Oh, I added the clownfish together instead of subtracting.

Teacher: That's good that you realize that. Let's compare that with #3 on this one (post-test, see Appendix G.4). What is this story about?

Roman: It is about Oscar sending postcards.

Teacher: Where is Oscar sending them?

Roman: Hmm…oh wait, he isn't sending them, he's buying them.

Teacher: Yes, you're right. How did your symbols help you over here?

Roman: I drew a two and then a stick person to show that it was two brothers, not postcards. Then I drew a three and a square to show three postcards and a six and a square to show six more postcards. Then I could cross out "two brothers" and I realized that I needed to add six plus three.

Teacher: How did you know to add those two numbers?

Roman: Well, I knew I was counting the postcards he's collected so I knew to add.

Teacher: What was different from the problem we were looking at here? Why do you think this one (#3 on pre-test, see Appendix G.2) tricked you, but not this one (#3 on post-test, see Appendix G. 4)?

Roman: Because this one (#3 on post-test, see Appendix G.4) was adding and this one (#3 on pre-test, see Appendix G. 2) was subtracting.

Teacher: Did the symbols you drew help you to answer this problem?

Roman: Yes, they helped me to tell apart brothers from postcards.

One of the first things Roman said was that he liked reading because of the emotions that he feels when he's engrossed in a story. He enjoys laughing or crying with the story, which is not something that many third-graders have experienced. Roman is a very mature reader. One of his favorite series is Captain Underpants. This is called a graphic novel. A graphic novel is a narrative story that is structured in comic form. Rather than a page of words, the words are conveyed through pictures with word bubbles. Something else Roman enjoys is the opportunity to work with his classmates during math time. For Roman and the others enrolled in my class, much of math was structured through group work. The students should feel that what they have to contribute is valuable to each other. Again, Roman mentions group work when he was asked what helps him to solve a problem. He reaffirms that working with a partner and comparing work is a good way to solve a problem.

An interesting thing regarding the process in which Roman solves a problem is that he attempts every operation until he ultimately finds an answer to match the options given. This is one of his first steps. It can be concluded that

Roman shows no interest in making sense of the problem from the onset. He does that only when it is becomes necessary. Attempting each of the four mathematical operations is easier for him than reading the problem for comprehension and making the best choice. This is the way that students are trained to view mathematical word problems. Basically, they are problems that need a solution – not stories.

Finally, when asked how he confronts a problem where two answers match, Roman explained that he tries to make sense out of the problem. His strategies include making a “key,” as he refers to it. In class, it was called a “unit box,” with the unit being the object being counted. This strategy is helpful because students can visually see the object, which they are looking for information on. Often a word problem contains extra information and the unit box will help students to eliminate unnecessary information from the onset. Roman continues to explain that drawing takes too much time. Roman is an avid artist in his everyday life. It is interesting that he finds drawing to be too time consuming in math. His response, though, included the use of symbols. He finds that symbols help him to understand the problem. As stated in Chapter 2, pictorial representations are not found to be useful, as they are replicas of actual objects (i.e., drawing a house to represent a house). Roman is referring to schematic representation, which is when a symbol is used to represent an object (i.e., using a square to represent a house). Roman finds that symbols help him concentrate on what to use to solve the problem. He proves that this is a viable tool when he used this strategy on his post-test. He drew a heart to represent Valentine’s cards and a square to represent paper. Once he realized he was counting paper (per his unit box) he was able to cross out the information pertaining to the Valentine's cards. That is very mature and very useful for a third grader to do when organizing information from a word problem.

Much like Jeremiah, Roman made a few assumptions when reading the word problems. One problem stated that Jenna made two types of cookies, trees and gingerbread. The problem explained that her brothers liked the tree cookies the most. Roman stated that the brothers would eat the tree cookies and Jenna would eat the gingerbread. The problem never stated that any of the cookies would be eaten; however, Roman used common sense to piece that together. This is a good reading strategy, yet has nothing to do with solving the problem. In fact, it can cause confusion for the reader because it never mentions who will eat which cookies. Students make assumptions because that's what good readers do.

As Roman explained another problem to me that involved clownfish and starfish, he was able to find that his verbiage gave him clues as to what to do to solve this particular problem. The problem involved 15 clownfish in which four of the clownfish were distracted by two starfish. When I asked him what that meant, he said the starfish "took away" the four clownfish. At that moment, he realized that he should have subtracted rather than adding. On a similar problem, Roman used symbols to represent the objects being used. He answered correctly. When I asked him why he thought he soled that problem correctly and the other one incorrectly he said it was because the first problem required him to subtract and the second one required him to add. Most students will answer the problem correctly. That is a default operation to many of them. It doesn't, however, always mean that they understand the problem completely.

Roman shows understanding of the mathematical concepts when he shows a graphic representation. He is able to transfer the information from words to a concrete representation, and that prevents any confusion when solving the problem. When Roman didn't use representation, he was easily confused on what information was pertinent to solving the problem. Roman likes to learn from his peers in a cooperative group setting. That is a concrete way to learn because the students are able to discuss among themselves until they can make sense of the problem. Putting the information into graphic representations, encouraging cooperative learning and conceptual understanding are all strategies that helped Roman to develop the necessary skills to solve these problems correctly.

3. Case study 3: Benjamin

Benjamin was not in my class; however, he fully remembered taking the pre- and post-test. I sat and talked with him for a while because I didn't really know him or his learning style (Appendix H.5). After a few minutes, I realized that he was a very articulate, intelligent boy who was very comfortable speaking with me, although he never had before. I enjoyed my conversation with him and it cleared up many questions that I had regarding his tests. The following is my conversation with him:

Teacher: Benjamin, what is your favorite subject?

Benjamin: Social Studies, because I like reading about the United States.

Teacher: That's great. What about reading, do you like reading?

Benjamin: Yes, I like Hank Zipzer books. I've read a lot of them and I'm reading My Dog the Scaredy Cat right now.

Teacher: Those are great books. What is your favorite thing about reading?

Benjamin: I like finishing the book to take an A.R. (Accelerated Reader) test.

Teacher: What about math? Do you like math?

Benjamin: Yes, I like adding and subtracting.

Teacher: What is your favorite thing about math?

Benjamin: Multiplication. I even know some by heart.

Teacher: Let's think about word problems. What do you look for when you first start a word problem?

Benjamin: I look at the question and information to circle. I cross out what I don't need.

Teacher: Is there always information to cross out?

Benjamin: No, only sometimes.

Teacher: What do you do first?

Benjamin: I look for important stuff to circle. I underline the question and cross out the trash.

Teacher: How do you know that you understand the problem?

Benjamin: I reread it.

Teacher: What helps you to solve the problem?

Benjamin: I write the numbers that I need to add.

Teacher: How do you know if you have to add the numbers or subtract them?

Benjamin: The question will say “altogether” which means add, or “how many more” which means subtract.

Teacher: Oh, so the words in the problem can guide you to solve the problem?

Benjamin: Yes, that’s what I look for.

Teacher: Do you write anything down after you read the problem?

Benjamin: I sometimes draw a picture.

Teacher: What would you draw if the problem were about stickers?

Benjamin: Maybe a circle to represent a sticker.

Teacher: How do the pictures help you to solve the problem?

Benjamin: Well, I draw the pictures because the words sometimes confuse me.

Teacher: Oh, I understand. I feel that way too sometimes. Do you like to draw pictures with a lot of details or simple pictures?

Benjamin: Simple pictures, because we don’t have time. I would use a stick figure to show a clown.

Teacher: That is very smart. Let’s look at this problem (#3 on pre-test, see Appendix H.2). Can you tell me what this problem is about?

Benjamin: There are 15 clownfish and two starfish. The starfish distract four of the clownfish. Now there are, um…11.

Teacher: Can you tell me what happened to the starfish? What does “distract” mean?

Benjamin: Distract means to keep them from doing something. Four of the clownfish floated away.

Teacher: Look at your answer. What did you do wrong, because you just said the answer was 11, but on your paper you put 13.

Benjamin: Oh, I took out the two starfish instead of the four clownfish. I didn't read it carefully before.

Teacher: That is a very good observation, Benjamin. Let's look at this one now (#3 on post-test, see Appendix H.4). What did you do differently on this one? They are the same problem, just different words, so why didn't this one trick you?

Benjamin: I think because I circled the information on this one (post-test) and I crossed out the trash.

Teacher: So, it helps you when you write stuff, cross things out, or draw pictures?

Benjamin: Yes.

Benjamin first stated that he enjoyed social studies the most because he liked reading about history. A first reaction to this would be to wonder why reading is enjoyed in other subjects, but not in math. The best conclusion I could reach is that problem posing is what scares students from enjoying the reading. Benjamin also said that he enjoys finishing a book to take an Accelerated Reader (AR) test. This is somewhat a motivation program for students to encourage good readers. Math doesn't have quite the motivational strategies as other subjects may. In fact, math tends to become a mystery to students who do not perform well, which can lead to frustration and animosity toward the subject.

First, Benjamin said that he always looks at the information in the question and crosses out what he doesn't need. He made it seem as though there is always something to cross out. When asked if there was always "trash," he said that it was only there some of the time. To fully understand the problem, Benjamin likes to reread the story. After that, he extracts the information onto his paper. This is a good technique because he takes ownership of the problem by writing down the given information. One thing that Benjamin made a note of that the others did not was key words. Benjamin even listed a few key words that he looks for -- "altogether" and "how many more". In Chapter 2, it was found that key words are not a good strategy because it leaves math as a mystery. Students need to look for content understanding rather than shortcuts. In addition, state tests tend to purposely add key words that will test students' understanding. One positive thing about Benjamin's use of key words is that he uses them to guide his representation. He stated that he uses symbols to represent the information given because the words confuse him. Symbols are more appropriate for Benjamin because he says there isn't enough time to draw the entire picture.

When Benjamin was asked to explain what the story involving clownfish and starfish was about, he was able to do so; however, he used mental math to solve the problem right away. When asked what "distract" means, he explained that four of the fish "floated away". Using his own key words, he was able to find the flaws in his work and see where he went wrong. When comparing the two problems that were structured the same way, Benjamin found that when he circled the information from each problem he understood what it was asking better. He made a connection with the problem, which gave him ownership of the problem. From his use of representation here, he reached the correct answer easily.

From a researcher's point of view, Benjamin had a very good observation when he said that the words in a math word problem confuse him. He, along with many other third graders, has a difficult time translating a math word problem into a story. When he views a word problem as a story rather than a mathematics problem, he understands it better.

B. Analyzing students' attitudes and problem solving strategies

Given the circumstances of the classroom, design of tests, and structure of teaching techniques, the three students in the study showed significant progress and possessed the ability to articulate the strategies that worked best for each of them. Also, the students' problem-solving strategies stood out above the rest of the class, because each showed the work understanding that I anticipated in all of the students. In the following section, I will compare and contrast the students' attitudes and problem-solving strategies with each other to find similarities and differences in the work they did. The comparison chart can be found in Appendix I, table 4.1.

All three students showed an exciting interest in reading. Jeremiah likes reading non-fiction biographies while Roman and Benjamin enjoy fiction books. It is interesting that all three students had very different reasons for their love of reading. Jeremiah likes non-fiction because he learns more from it. Roman likes reading fiction because it can make you laugh or cry and Benjamin is motivated to read by the Accelerated Reader program. Reading is important to each and comprehension is their goal. If the three students disliked reading, or showed poor comprehension, word problems would be even more of a struggle. All three students showed an interest in math as well. When asked what their favorite part was about math, the answer was unanimous. In my opinion this is because multiplication is viewed as a mystery. Students study to memorize their facts and either remember them or don't. Memorizing multiplication facts is discouraged because there is no conceptual understanding behind rote memorization. Students need concrete knowledge for each concept in order for their level of mathematics to improve. Roman emphasized that the level of difficulty in a word problem determines his success. Jeremiah and Benjamin both expressed and that it becomes difficult to know what information is needed. They use the question to guide them. All three of the students answered at least one problem incorrectly. This was due to either a computational error or the use of the wrong numbers.

The three students shared some information to give insight as to what strategies were helpful. Because all three students enjoy reading and have decent comprehension levels, it can be assumed that reading comprehension is a factor. Roman likes reading because of the emotions that are felt. He directly relates to the stories. All three students were able to tell me who and what each story (word problem) was about.

As they all three enjoy multiplication the most, it can be concluded that the competition and self-assessment play a part. Multiplication is often presented in a classroom through a timed test, a race against a neighboring class, a flash card race, etc. Students only memorize facts so it becomes a mystery to them. Students can also monitor their own progress by how well they do in class. This allows the students to take ownership and forces it to become personal. Self-assessment works because it makes students responsible for their learning. Roman enjoys working in groups with his peers. He is able to talk about the problem with other people and that helps him to understand how he is to proceed. He made it clear that comparing work helps him to understand the problem better.

Jeremiah and Benjamin both said that the question is the first thing they look at when they see a word problem. Jeremiah went as far as to say that it tells you what to do. He is making a very dignified observation because the question is guiding the problem. Benjamin circles information in the question that he knows he will need. Also, Benjamin uses the question to figure out what information he can cross out. He knows that problems often give too much information and it can confuse students. It is because of this that he utilizes the question to clarify what information he does or does not need.

Both Jeremiah and Benjamin use some type of graphic to help them solve the problem. Jeremiah says he makes a chart or some type of picture. Benjamin writes the numbers down that he needs. Jeremiah says he likes pictures because he can relate to them more since he loves to draw. Benjamin says words confuse him. Both students find pictures easier to understand and helpful in the problem-solving process. All three students said they would not draw an exact picture of what object the problem contains. They all agreed that symbols would be just as sufficient and more reasonable for time purposes. Roman believed that his representation helped him to know what information was not necessary in solving the problem. He said drawing pictures takes too much time, but he likes to use simple symbols. Jeremiah believed symbols would have assisted him on one of the problems. Benjamin had the same thoughts when he said he did better on the problem in which he actually crossed out and circled information. All three students believed that extracting information with symbols and marking information on paper was a good tool in solving mathematical word problems. Looking at Roman's post-test (Appendix G.4), we can see that he represented the objects in the word problem with a symbol. Now, he could cross out unnecessary information and visually see what numbers he needed to solve the problem. It was agreed by all students that when graphic representations of the word problems were used, better understanding of the problem took place.

C. How do the Results of the Study Address Research Questions?

1. Challenges with problem solving

When each of the students were asked to discuss what may have confused them on a problem that they answered incorrectly, all referred to the words in the problem. Each student made an error not in mathematics, but in reading comprehension. Benjamin assumed that there were four green shoelaces when, in fact, the problem never mentioned green shoelaces. Roman realized that his problem was that he added when he should have subtracted. When he explained the problem to me, he caught the error in his work. He read it so fast and assumed that his choice to add was correct that he missed the comprehension part of the problem. Benjamin caught his mistake when explaining the problem to me as well. He realized that he added the clownfish to the starfish when he should have been taking away some of the starfish from the group. All students failed to retain comprehension of the problem, which caused the incorrect answers.

Another problem pointed out by Roman was the difficulty in subtracting versus adding. He said when it is necessary to add, the problems are easier and he makes less mistakes. For a problem that required subtracting, he used the wrong numbers. This is his explanation. However, when analyzing his work, the choice of subtraction is not the problem; rather, it is the objects, which he used to do the math. He focused on the wrong numbers. This shows that numbers take precedence to the student. The words in the problem don't have to make sense in order for the student to perform the math. The students' errors prove how important reading comprehension in math actually is.

Jeremiah brought up an interesting point when he said one of the problems was that one story had a consistent object but different colors. To him that was more confusing than the problem that had very different objects to compare. He was confused not with what the problem was asking but which type of object he had to focus on. Again, this is a reading comprehension issue. Without the student understanding the problem, it is easy for him to focus on incorrect information or to believe that the problem has information in it which does not.

2. Helpful strategies for problem solving

Two of the three students believe that the main focus should be in the question. Jeremiah and Benjamin stated that the question would give you the information necessary to tell you which operation to use as well as tell you what information is not needed at all. Roman said the question helps him to see if the problem sounds right or not. When starting to work the problem, Jeremiah likes to make a chart or draw pictures. If he represents the problem on paper in a way that he understands it, then his next steps are more visible to him. He says that he relates more to pictures because he likes to draw. Another thing he does to save time is use symbols rather than actual pictures of the objects. He might use a circle to represent an apple. Roman says working with a partner helps because they can talk about the next step to decide if what they do makes sense. He starts with a number sentence for each operation and chooses which best fits the problem. He says drawing takes too much time, so he uses symbols to help him concentrate on what to do to solve the problem. Benjamin likes to write down the numbers that he will use in his calculation. He says he sometimes draws a picture because the words confuse him. Again, he is attempting to relate to the problem by creating what makes sense to him. He says he would use simple figures because he doesn't have enough time to draw it exactly as how he envisions it would look.

On the actual tests that were given, the three students showed the best strategies that made sense to them. Jeremiah did not show any representation on one problem that he missed. After reviewing it with him, he said he should have drawn circles with a letter inside to represent the object's color. Had he done that, he would have realized that there was no solution to the problem due to insufficient information. Roman had one problem involving people and postcards. He needed to count postcards only. On the side, he drew a stick person to represent the people and a rectangle to represent the postcards. After he read the question, he said he realized he should be counting postcards only and crossed out the stick person. It was then that he knew what to do and found the correct answer. Roman said the symbols that he made helped him to tell apart the people from the postcards. Prior to this, he underlined his question and circled his unit (postcards) so that he would remember what he had to count. Benjamin had a similar strategy. He said when he writes information down, crosses information out, or draws pictures, it helps him to understand what he is expected to do. His work confirmed his belief because on one problem that he answered correctly he did just that. He wrote down the information he needed on the side and crossed out information in the problem.

3. The assistance of graphic representation

These three students were chosen by a criterion based sample, to be a part of this study. The criteria being that they each missed one problem on the pre-test that was answered correctly on the post-test. This begs the question, what did they do differently? After sitting and talking with each student, it was found that they all showed some type of representation on their paper that was not previously done. Jeremiah realized that had he drawn symbols to show his work on one problem that he missed, he would have easily found the correct answer. When he was talking about the problem, he was easily able to see where his mistake was. Often it is the case that students do not find reasoning on their own or in a small group. Roman expressed his enjoyment of talking with a partner. When students talk about a math word problem, they are encouraging making sense out of what they read and finding reasoning behind it. Students are trained to read and compute without ever being encouraged to reason first. Roman was also the student who likes to read because it makes him laugh or cry. He enjoys reading graphic novels because he can relate to the pictures. Graphic novels have become popular in the third grade and beyond because children find enjoyment in reading through pictures. They will also find more enjoyment in math through pictures. Roman transfers word problems into pictures for this exact reason. It makes more sense to him. He has a better understanding of reading if it is displayed through pictures. This is true for math as well. On his paper, he did use symbols and organize his thinking in a way that allowed him to see what information he needed to solve the problem. He acknowledged that symbols helped him to decipher between people and postcards. Benjamin said that simple figures help him to answer a problem. He says the words sometimes confuse him. When he transfers the words into something he understands (graphic representation) he is better able to answer the problem correctly.

All three students used similar techniques to work out the word problems. A teacher can train a student to use these techniques, but, ultimately, the student will use whatever is helpful to him/her. These three students were not told to do anything in particular but choose what would help them to become successful. Students' work is the most valuable teaching tool, because it shows what is helpful and what is not. Teachers can build lessons around helpful strategies. If the student can relate to something as Jeremiah, Roman, and Benjamin said they relate to pictures, a richer learning experience will take place.

Chapter V

DISCUSSION

A. Answering the research questions

According to the literature review outlined in this paper, the difficulties with problem solving include, but are not limited to, reading comprehension, teachers training students to solve word problems rather than emphasizing conceptual understanding, and students' desire to quickly manipulate numbers without full comprehension of a problem. The literature review also provides many helpful strategies for students and teachers. These include schema-based instruction, student self-assessment, games, bridging real-life with mathematics textbooks, and, most of all, graphic representation. Researchers have agreed that graphic representation can aid a student through a mathematical word problem. Representing a problem has proven to assist students in reading comprehension as well as successfully answering word problems.

B. Suggestions for practice

As students in each grade will face the challenge of solving mathematical word problems, it is important for teachers to use strategies that will fit the students' learning styles. Students must have a say in how to organize thoughts and carry out a problem solution. Teachers should model techniques to give students the external information allowing each student to internalize the information in his/her own way.

It is important for teachers to value the thinking of his/her students and allow students to share that thinking with others. In addition, students need the responsibility of monitoring their own progress and striving to reach a goal. Conceptual understanding must be encouraged throughout each mathematical concept in order for students to develop the proper base. Memorization, short cuts, and tricks sometimes work for the present, but rarely stick with students throughout their school career or even adulthood.

Lastly, mathematics is a developmental understanding of concepts and is a helpful strategy for students solving mathematical word problems. It should be viewed as such and conveyed as such to each student.

C. Reflection and limitations

As in many education action research projects, it became difficult to perfect this study. Working with children, I found there to be an abundance of variables, such as structure of the classroom, the proper exposure to all types of problems, and time constraints to name a few. These became too difficult to control. After the pilot study I realized that, though my beliefs hadn't changed, my desire to "fix" something I believed in grew dim. As a teacher you have to allow room for error and be flexible with change. Not every problem that students' find in mathematics has one solution; rather, these problems become learning experiences for both the students and the teacher. Once I had the three students whom I felt would add the most to this study, I began to realize how much there is to learn from the students. A conversation with one student can be sometime more helpful than merely observing an entire class at once. The children were able to express the difficulties they were having so eloquently. These difficulties that were expressed were not as visible to me through observation alone. This proves that test scores do not always reflect the extent of knowledge that a student possesses.

As a teacher I often feel as though I missed something or should have explained a concept with more detail. In this study, I feel that the students should have had more opportunity to provide strategy to the entire class and let them take the role of the teacher. Knowing this and actually doing it become very different things when in the classroom. Time does not allow for lengthy conversations regarding one word problem alone. Had there been more time to devote to problem solving, more strategies would have been offered to the students and more conceptual understanding would have taken place. Unfortunately for all of us, there is just not enough time in the school day because of time constraints.

D. Directions for future research

Future research will help to answer the questions that still linger. I would like to have a study that can back-up the qualitative findings of this study with quantitative data. Both the pre- and post-survey, as well as the pre- and post-test will be given, but there will be a very detailed intervention between the two. I will make sure to expose students to every type of problem that are displayed in the pre-and post tests and allow the students to share their findings with each other. In problem solving, student self-assessment will take place, as well as schema-based instruction. During the intervention students will become teachers and aid in their peers' learning experiences. Most of all, students will be exposed to all types of graphic representation. Schematic representation will be encouraged, as symbols and letters will be modeled and displayed. After the data is collected, I will find correlations between the use of representation with test scores and the effects of the intervention on the test scores.

Lesser and Tchoshanov (2005) found that representational sequence plays a major role in students' problem solving abilities. In a future investigation, I would place emphasis on this concept giving students the opportunity to be both the problem solver and the problem creator. Students would be responsible for finding a problem solution when given a question, and in turn be given a problem solution, when expected to find the problem question. Finally, students will be given a graphic representation and be expected to find both the problem question and problem solution. This would help ensure students' problem solving skills.

To further investigate reading comprehension, I will compare reading levels with scores on both tests. A correlation between reading comprehension and test scores will be found. A second classroom will be used as a comparison group. A correlation between comparison group and treatment group will be found as well. Hopefully, in the next study the results will significantly support the conclusions of this study.

E. Conclusion

This study found that with mathematical word problems come many difficulties for elementary third-grade students. The challenges that children face were found to be reading comprehension, conceptual understanding, unnecessary information in a word problem, and choosing the appropriate mathematical operation. Through the literature reviewed in this paper, many helpful strategies were revealed to aid students in solving mathematical word problems. These strategies include group discussion on problem solving strategies, student self-assessment, incorporating games, learning that math makes sense rather than using short cuts and keywords, and representing information using a type of graphic. The study found a steady theme throughout. This was the use of graphic representation. When this was done, each student was able to show better understanding of the word problem. As each student has a different ability to internalize information and understand concepts, graphic representation serves as a constant in the learning experience. Students will always be able to utilize this strategy as it is self-created and the student can show ownership of his/her learning. In addition to graphic representation, cooperative learning groups were found to be helpful when solving mathematical word problems. Students often learn from each other what teachers have a hard time relaying.

Problem solving is a life-long skill that will continue to be useful and develop well after schooling is completed. Teachers can provide assistance through the strategies suggested in this paper, which will allow students more options to choose from. The challenges of problem solving will remain; however, the strategies and interventions suggested can be implemented in order for students to feel successful and as though they have developed the proper conceptual understanding of each mathematical concept, including problem solving.

REFERENCES

Blatto-Vallee G., Kelly, R.R., Gaustad, M.G., Porter, J., & Fonzi, J. (2007). Visual-spatial representation in mathematical problem solving by deaf and hearing students. *The Journal of Deaf Students and Deaf Education*, 12(4), 432-448.

Borden, J., & Geskus, E. (2001). The most magnificent strawberry shortcake. *Teaching Children Mathematics*, 7(9), 538-541.

Brookhart, S.M., Andolina, M., & Zuza, M. (2004). Minute math: An action research study of student self-assessment. *Educational Studies in Mathematics*, 57(2), 213-227.

Brousseau, G., & Gibel, P. (2005). Didactical handling of students' reasoning processes in problem solving situations. *Educational Studies in Mathematics, 59*(1/3), 13-58.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (1999). *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann

Corter, J.E., & Zahner, D.C. (2007). Use of external visual representation in probability problem solving. *Statistics Education Research Journal,* 6(1), 22-50.

Crespo, S. (2003). Learning to pose mathematical problems: exploring changes in pre-service teachers' practices. *Educational Studies in Mathematics,* 52(3), 243-270.

Edens, K., & Potter, E. (2008). How students "unpack" the structure of a word problem: graphic representations and problem solving. *School of Science and Mathematics,* 108(5), 184-96.

Fiori, Nick. (2007). Four practices that math classrooms could do without. *Phi Delta Kappan,* 88(9), 695-696.

Fuchs, L.S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C.L., & Owen, R. (2003). Enhancing third-grade students' mathematical problem solving with self-regulated learning strategies. *Journal of Educational Psychology*, 95, 306-315.

Fuchs, L.S., Fuchs, D., Prentice, K., Burch, M., & Paulsen, K. (2002). Hot math: promoting mathematical problem solving among third-grade students with disabilities. *The Council for Exceptional Children*, 35, 70-73.

Fuchs, L.S., Seethaler, P.M., Powell, S.R., Fuchs, D., Hamlett, C.L., & Fletcher, J.M. (2008). Effects of preventative tutoring on the mathematical problem solving of third grade students with math and reading difficulties. *Exceptional Children,* 74 (2), 155-173.

Goldin, G., & Shteingold, M. (2001). Systems of representations and the development of mathematical concepts. In Cuoco, A., & Curcio, F.R. (Ed.), *The Roles of Representation in School Mathematics.* (p. 1-23). Reston, VA: NCTM.

Hegarty, M., & Koshevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal of Educational Psychology,* 91 684-689.

Hohn, R.L., & Frey, B. (2002). Heuristic training and performance in elementary mathematical problem solving. *The Journal of Educational Research,* 95(6), 374-380.

Jitendra, Asha (2002). Teaching students math problem-solving through graphic representations. *Teaching Exceptional Children*, 34(4), 34-38.

Jitendra, A.K. Griffin, C.C., Deatline-Buchman, A., Dipipi-Hoy, C., Sczesniak, E., Sokol, N.G., & Xin, Y.P. (2005). Adherence to mathematics professional standards and instructional design criteria for problem solving in mathematics*. Council for Exceptional Children*, 71(3) 319-337.

Jitendra, A.K., Sczesniak E., Griffin, C.C., & Deatline-Buchman, A. (2007). Mathematical word problem solving in third-grade classrooms. *The Journal of Educational Research,* 100(5), 283-302.

Kamii, C., Kirkland, L., & Lewis, B. A. (2001). Representation and abstraction in young children's numerical reasoning. In Cuoco, A., & Curcio, F.R. (Ed.), *The Roles of Representation in School Mathematics.* (p. 24-34). Reston, VA: NCTM.

Lan, W. (1996). The effects of self-monitoring on students’ course performance, use of learning strategies, attitude, self-judgment ability, and knowledge representation. *The Journal of Experimental Education,* 64, 101-115.

Leinwand, S. & Ginsburg, A.L. (2007). Learning from Singapore math. *Educational Leadership*, 65(3) 32-36.

Lesser, L. & Tchoshanov, M. (2005). The effect of representation and representational sequence on students' understanding, research report. In G. M. Lloyd, M.R. Wilson, J.L.M. Wilkins, & S.L. Behm (Eds.), *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

Miles, M.B. & Huberman, M. (1994). *Qualitative Data Analysis*. Thousand Oaks: Sage Publications.

Myers, P. (2007). Why? Why? Why? Future teachers discover mathematical depth. *Phi Delta Kappan,* 88(9), 691-694, 696.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston Va.: NCTM.

Owen, R.L., & Fuchs, L.S. (2002), Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and Special Education, 23(5), 268-278.*

Pape, S. J., & Tchoshanov, M.A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice, 40(2), 118-127.*

Piaget, J. (1951). *Play, Dreams, and Imitation in Childhood.* New York: W.W. Norton and Co., 1951.

Ralston, A., & Willoughby, S.S. (1997). Realistic problem formulation and problem solving. *Mathematics Teacher*, 90, 430-434.

Sarama, J. & Clements, D.H. (2007). How children problem solve. *Scholastic Early Childhood Today,* 21(7), 16, 17-19.

Sharon Wells Consulting, Inc. (n.d.). *Sharon wells mathematics curriculum, inc.: grade 3 problem solving, 2nd six weeks.*

Tchoshanov, M. (2006). Impact of instructional setting, problem posing, and language on elementary school students' performance in solving mathematical word problems. *Proceedings of the International Conference on the Psychology in Mathematics Education – North America,* Merida, Mexico.

Van de Walle, J.A. (2004). *Elementary and middle school mathematics.* Boston: Allyn and Bacon.

Van de Walle, J.A. (2006). *Teaching student centered mathematics k-3.* Boston: Allyn and Bacon.

Van de Walle, J.A. (2006). *Teaching student centered mathematics 3-5.* Boston: Allyn and Bacon.

Van Garderen, D. (2006). Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. *Journal of Learning Disabilities,* 39 (6), 496-506.

Yeo, K.K.J. (2005). Five strategies for one arithmetic problem. *Mathematics in School*. 34(1), 30-31.

APPENDIX A

Form of Consent

Dear Parent/Legal Guardian,

I, Elizabeth Bernadette, ask for your consent to use your child in an action research study for my Masters program. The study is titled Graphic Solutions to Problem Solving. It will consist of two surveys and a pre-test and a post-test. In addition, I will maintain an observation log as well as several interviews that your child may be a part of. It is possible that a photograph of your child will be taken and used for the observation data only. The research intervention will take place from December to March. The entire class is asked to participate.

I am studying the difficulties students have with mathematical word problems. It is my belief that if students use graphic representations to organize their thinking, the child will more accurately answer the word problem. A number and/or pseudonym will represent your child. The name of the school will remain anonymous. Everything being done in the study is something that will not hinder; rather enhance your child's learning experience. Each student will receive equal benefits from the study. Should you not consent, your child will still receive the intervention, I will simply not include him/her in the data; however, I ask that you please consent as my data will reflect more accurate results with the entire sample present. With your help the study will be more beneficial to both the students and to me. The study will not create any more or less work than would normally be given. If at any time, you wish to withdraw your child, feel free to do so.

Thank you for helping me to help my students,

Elizabeth Bernadette
Dr. Nixon Elementary
11141 Loma Roja Dr.
(915) 849-5700

----------I DO consent

----------I DO NOT consent

---	-------------------------------------	-----------------
Name of participant	Signature of student	Date
--	-------------------------------------	-----------------
Name of parent (printed)	Signature of parent	Date
--	-------------------------------------	-----------------
Name of supervisor	Signature of supervisor	Date

Appendix B

General Form of Consent

Dear Parent/Legal Guardian,

I, Elizabeth Bernadette, ask for your consent to use your child in an action research study form my Masters program. The study is titled, Graphic Solutions to Problem Solving. It will consist of two surveys and a pre-test and a post-test. The entire class is asked to participate.

A number and/or pseudonym will represent your child. The name of the school will remain anonymous. Everything being done in the study is something that will not hinder; rather, enhance your child's learning experience. Each student will receive equal benefits from the study. I ask that you please consent as my data will reflect more accurate results with the entire sample present. The study will not create any more or less work than would normally be given. If at any time you wish to withdraw your child, feel free to do so.

Thank you for helping me to help the students,

Elizabeth Bernadette
Dr. Nixon Elementary
11141 Loma Roja Dr.
(915) 849-5700

----------I DO consent

----------I DO NOT consent

--	-------------------------------------	-----------------
Name of participant	Signature of student	Date
--	-------------------------------------	-----------------
Name of parent (printed)	Signature of parent	Date
--	-------------------------------------	-----------------
Name of supervisor	Signature of supervisor	Date

Appendix C

Survey

Number:____________ Date:_________________

1. What do you do first when you are solving a math word problem?

2. What helps you the most to solve a math word problem?

3. What is the hardest part about math word problems?

Appendix D

Pre-test

Number: _____________ Date:_____________________

1. Every Christmas Jenna bakes cookies for her 4 brothers. This year, Jenna baked 12 tree cookies and 8 gingerbread cookies. Her brothers like the tree cookies the most. How many tree cookies did Jenna bake?

2. Cara has 2 pairs of shoes with red shoelaces and 3 pairs with blue shoelaces. How many pairs of shoes does Cara have with green shoelaces?

3. In a school of fish swimming by, there are 15 clownfish. 2 starfish floating by distract 4 of the clownfish and they follow them. How many clownfish remain in the group?

4. Every week the zoo has to buy food for the animals. Last week the zoo bought 44 bunches of bananas and 83 pounds of peanuts. About how many bushels of apples does the zoo purchase every week?

Appendix E

Post-test

1. Jimmy made 2 Valentine cards out of different colored paper. He used 3 sheets of pink paper and 4 sheets of red paper. How many sheets of red paper did Jimmy use?

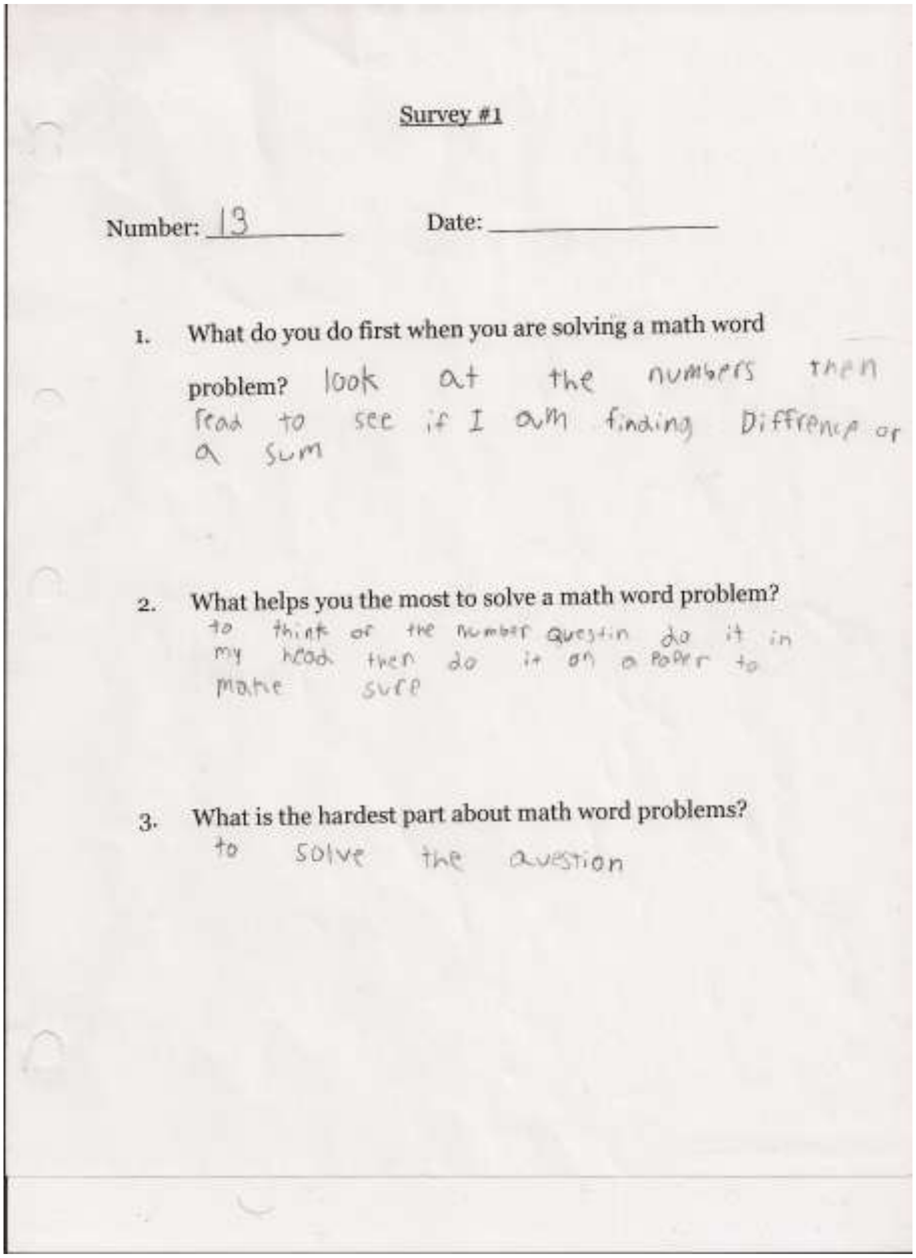
2. Every night Julia has to do 30 minutes of homework and 30 minutes of reading. How long is Julia's favorite television show?

3. Oscar and his 2 brothers love to collect post cards. When he went to Colorado, Oscar bought 3 new postcards. In Texas, he bought 6 more. How many postcards does Oscar have in his collection now?

4. The school basketball team scored 6 baskets before half time. The tea won the game by 9 points. How many baskets did the team score in the second half?

Appendix F.1

Jeremiah's pre-survey



Survey #1

Number: 13 Date: ________

1. What do you do first when you are solving a math word problem? look at the numbers then read to see if I am finding Diffrence or a sum

2. What helps you the most to solve a math word problem?
to think of the number questin do it in my head then do it on a paper to make sure

3. What is the hardest part about math word problems?
to solve the question

Appendix F.2

Jeremiah's Pre-test

Pre-Test

Number: 13 Date: ______________

1. Every Christmas Jenna bakes cookies for her 4 brothers. This year, Jenna baked 12 tree cookies and 8 gingerbread cookies. Her brothers like the tree cookies the most. How many tree cookies did Jenna bake?

12 tree cookies

does not make scence becouse it is not asking you for do anything

2. Cara has 2 pairs of shoes with red shoelaces and 3 pairs with blue shoelaces. How many pairs of shoes does Cara have with green shoelaces?

3 + 1 = 4

4 Pairs of shoes

3. In a school of fish swimming by, there are 15 clownfish. 2 starfish floating by distract 4 of the clownfish and they follow them. How many clownfish remain in the group? 11 Clown fish

15 − 4 = 11

4. Every week the zoo has to buy food for the animals. Last week the zoo bought 44 bunches of bananas and 83 pounds of peanuts. About how many bushels of apples does the zoo purchase every week?

44 83 −1

122

122 apples

Appendix F.3

Jeremiah's post-survey

Survey ~~#1~~ #2

Number: 13

1. What do you do first when you are solving a math word problem? underline the Question

2. What helps you the most to solve a math word problem? the W.P.P. chart

3. What is the hardest part about math word problems? getting your anwser

Appendix F.4

Jeremiah's post-test

Number: 13

Post-Test

1. Jimmy made 2 Valentine's cards out of different colored paper. ~~He used 3 sheets of pink paper and~~ 4 sheets of red paper. How many sheets of red paper did Jimmy use?

$4 \div 2 = 2$

4	
2	?

2 cards
~~3 pink~~
4 red

2. Every night Julia has to do 30 minutes of homework and 30 minutes of reading. How long is Julia's favorite television show?

You cant firgure thout with out any evdence

3. Oscar and his 2 brothers love to collect post cards. When he went to Colorado, Oscar bought 3 new postcards. In Texas he bought 6 more. How many postcards does Oscar have in his collection now?

3 new
6 new

$3+6 = 9$ more

?	
3	6

4. The school basketball team scored 6 baskets before half time. The team won the game by 9 points. How many baskets did the team score in the second half?

3 Points

1 6 Points
2 3 Points

Appendix F.5

Jeremiah's interview

13

Formal Interview

1. What is your favorite subject?
Science

2. Do you like Reading? Why or why not?
yes, I love non fiction books the most
Biographies

3. What is your favorite thing about Reading?
non fiction because you learn more

4. Do you like Math? Why or why not?
yes. Because it is challenging

5. What is your favorite thing about Math?
Multiplying

6. What do you look for in a word problem?
question tells you what to do
how to figure it out

7. What do you do first?
question
circle info to figure out the operation

8. After you read it, what do you do?

9. How do you know that you understand the problem?
Reread to make sure
Rewrite info

10. What helps you to solve the problem?
make a ~~St~~ chart
use pictures. They help because I like them more than words. I relate more b/c I love to draw.

11. Do you write anything down after you read the problem?
The info

12. Does it help you to draw anything after reading the problem?

Appendix F.5

Jeremiah's interview continued

13. Do you draw a picture or do you use symbols? If so, which symbols do you use?

- symbols
- letters
- shapes

14.

Pre:

2 Cara

- Has 2 shoes w/ red
- 3 shoes w/ blue
- Green shoelaces?

What could have done:
Draw shoes write color on shoelaces

One can wear them (How can you know?)
The problem should tell you but it doesn't
Cannot be figured out.

Post:

2 Julia's homework

- 30 min. of homework
- 30 min. of reading
- TV show?

The first is talking about the same thing but diff. color (shoelaces) The second is completely different.
Pictures would have helped me to not be tricked.

Appendix G.1

Roman's pre-survey

Survey #1

Number: 7 Date: Mon, Dec, 1, 2008-07

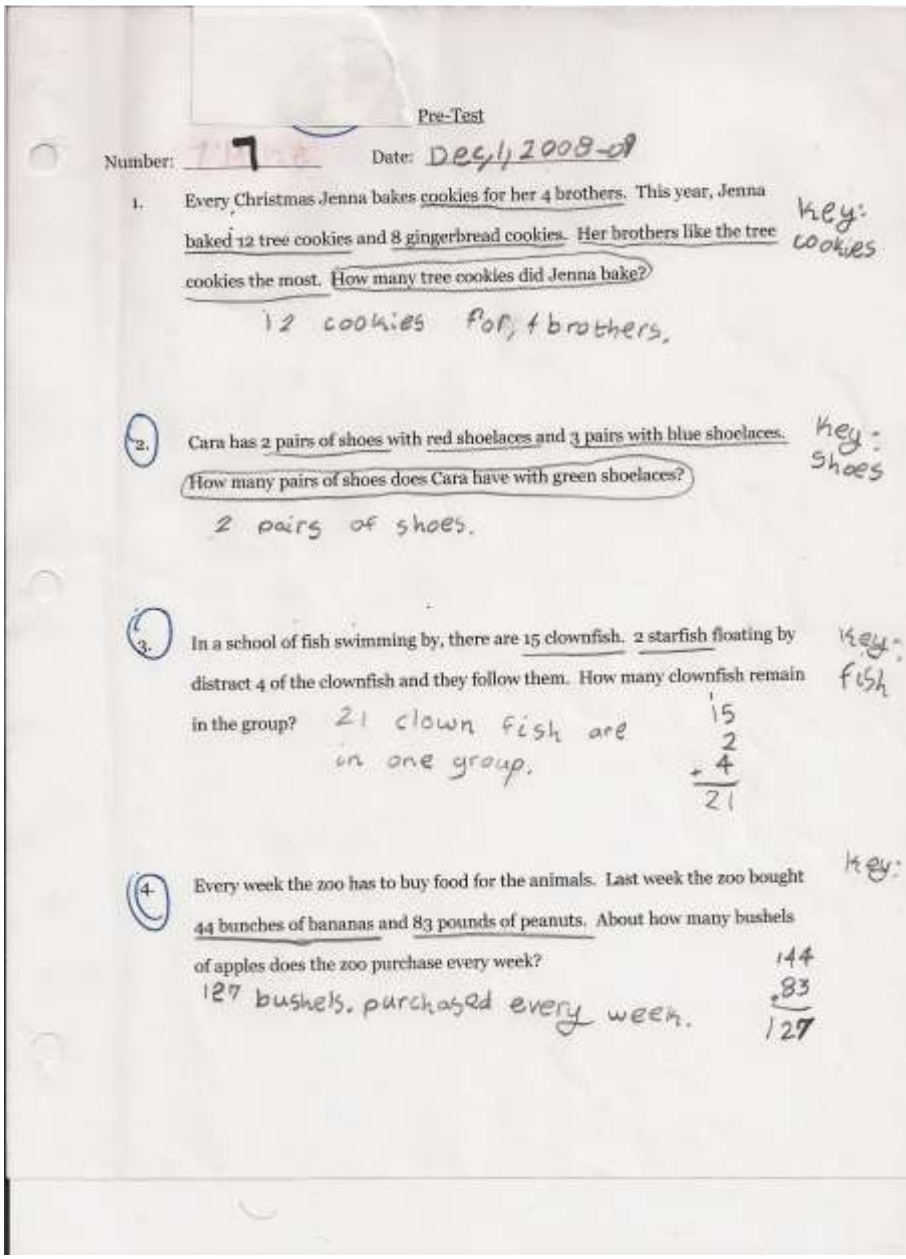
1. What do you do first when you are solving a math word problem? I see what numbers I'll subbract, add.

2. What helps you the most to solve a math word problem?
The additon or subbraction sign on the bottom left.

3. What is the hardest part about math word problems?
When I do 5 numbers.

Appendix G.2

Roman's Pre-test



Pre-Test

Number: 7 Date: Dec,1,2008-09

1. Every Christmas Jenna bakes cookies for her 4 brothers. This year, Jenna baked 12 tree cookies and 8 gingerbread cookies. Her brothers like the tree cookies the most. How many tree cookies did Jenna bake?

Key: cookies

12 cookies for, 4 brothers.

2. Cara has 2 pairs of shoes with red shoelaces and 3 pairs with blue shoelaces. How many pairs of shoes does Cara have with green shoelaces?

Key: Shoes

2 pairs of shoes.

3. In a school of fish swimming by, there are 15 clownfish. 2 starfish floating by distract 4 of the clownfish and they follow them. How many clownfish remain in the group?

Key: fish

21 clown fish are in one group.

$$\begin{array}{r} 15 \\ 2 \\ +\ 4 \\ \hline 21 \end{array}$$

4. Every week the zoo has to buy food for the animals. Last week the zoo bought 44 bunches of bananas and 83 pounds of peanuts. About how many bushels of apples does the zoo purchase every week?

Key:

127 bushels. purchased every week.

$$\begin{array}{r} 144 \\ +\ 83 \\ \hline 127 \end{array}$$

Appendix G.3

Roman's post-survey

Survey #~~1~~2

Number: 7

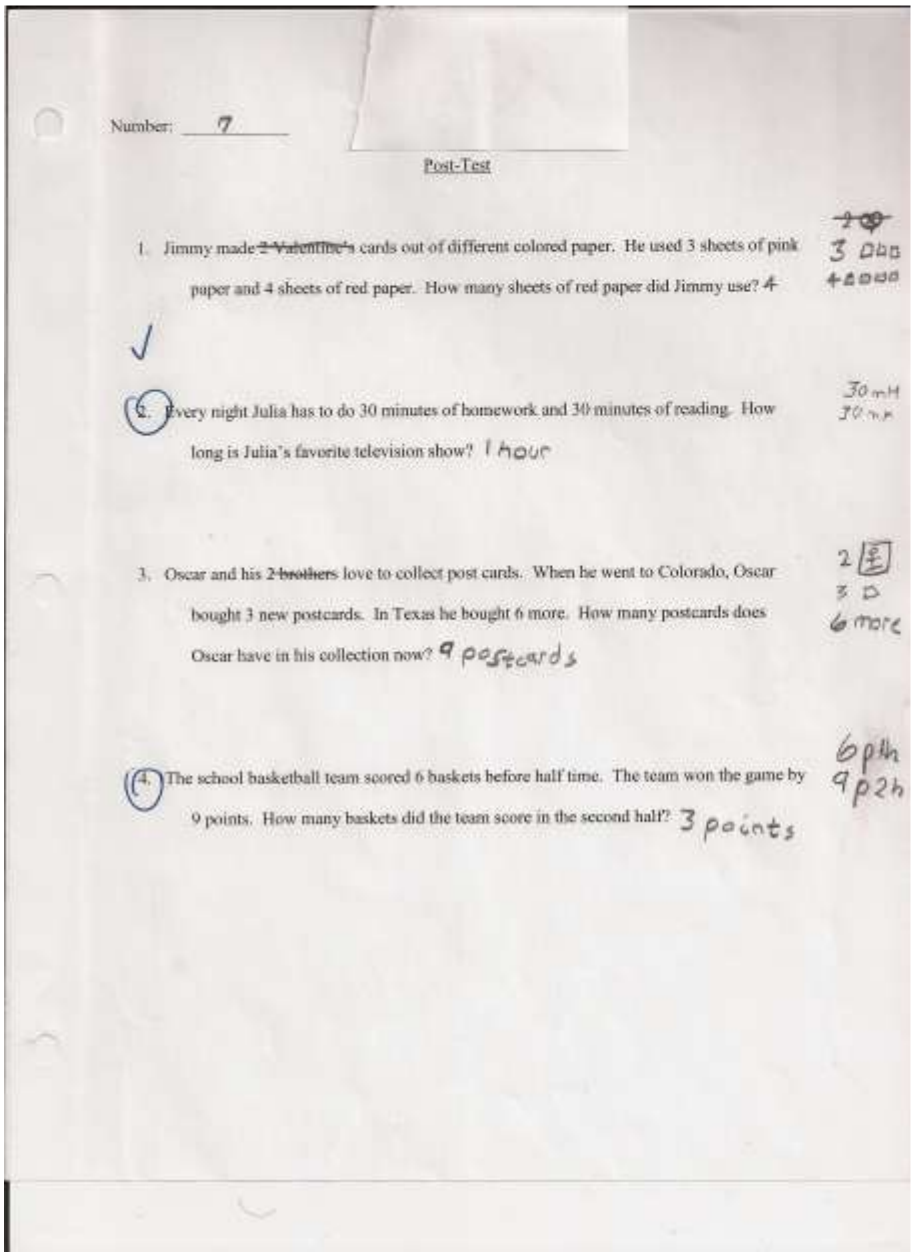
1. What do you do first when you are solving a math word problem? I read the question, see what my choises are than I Choose the one I think is right.

2. What helps you the most to solve a math word problem?
by Showing my work

3. What is the hardest part about math word problems?
When theres three numbers, and you haveto elliminate it.

Appendix G.4

Roman's post-test



Number: 7

Post-Test

1. Jimmy made ~~2 Valentine's~~ cards out of different colored paper. He used 3 sheets of pink paper and 4 sheets of red paper. How many sheets of red paper did Jimmy use? 4

~~2~~
3
4

2. Every night Julia has to do 30 minutes of homework and 30 minutes of reading. How long is Julia's favorite television show? 1 hour

30 mH
30 mR

3. Oscar and his ~~2 brothers~~ love to collect post cards. When he went to Colorado, Oscar bought 3 new postcards. In Texas he bought 6 more. How many postcards does Oscar have in his collection now? 9 postcards

2
3
6 more

4. The school basketball team scored 6 baskets before half time. The team won the game by 9 points. How many baskets did the team score in the second half? 3 points

6 p 1h
9 p 2h

Appendix G.5

Roman's interview

7

Formal Interview

1. What is your favorite subject?
Science. a lot of experiments
Play around.

2. Do you like Reading? Why or why not?
Yes. Captain Underpants, etc...
Fiction

3. What is your favorite thing about Reading?
Sometimes it makes you laugh or cry

4. Do you like Math? Why or why not?
Yes. To work with people, combine brains

5. What is your favorite thing about Math?
Multiplication to be challenged.

6. What do you look for in a word problem?
Kind of hard ~~because~~ sometimes they have to be converted. Like the challenge.

7. What do you do first? What to add or multiply, check to see if it sounds right or not | makes sense

8. After you read it, what do you do?

9. How do you know that you understand the problem?

10. What helps you to solve the problem?
working with a partner and comparing work

11. Do you write anything down after you read the problem?
Number sentence
Try all operations

12. Does it help you to draw anything after reading the problem?
To make a key to figure out how much something is worth to add or multiply

Appendix G.5

Roman's interview continued

13. Do you draw a picture or do you use symbols? If so, which symbols do you use?

Symbol. drawing would waste time. Concentrate on what to do to solve the problem.

14.

Pre: - Jenna & brothers - Brothers will eat tree
Key - making cookies - She'll eat ginger
- gingerbread & tree - answer in problem

Post:
Pictorial #

Pre:
3 15 C
- 2 S distract 4 C
- I added instead of subtracting

Post:
3 Oscar
- Sending postcards - where? Oh wait, he isn't sending them he's buying them.
- Drew stick person to show that it was bros. not p.c.
- 3 ☐ = new p.c.
- 6 more = added 6 to 3
- Adding not sub.
- Rep. helped to tell apart p.c. not bros.

Appendix H.1

Benjamin's pre-survey

Survey #1

Number: number #19 Date: 12-05-2008

1. What do you do first when you are solving a math word problem? I read it first

2. What helps you the most to solve a math word problem?
When I work it out

3. What is the hardest part about math word problems?
If it sois times

Appendix H.2

Benjamin's pre-test

Pre-Test

Number: # 19 Date: 12-05-2008

1. Every Christmas Jenna bakes cookies for her 4 brothers. This year, Jenna baked 12 tree cookies and 8 gingerbread cookies. Her brothers like the tree cookies the most. How many tree cookies did Jenna bake? 12

2. Cara has 2 pairs of shoes with red shoelaces and 3 pairs with blue shoelaces. How many pairs of shoes does Cara have with green shoelaces? 4

3. In a school of fish swimming by, there are 15 clownfish. 2 starfish floating by distract 4 of the clownfish and they follow them. How many clownfish remain in the group? 13

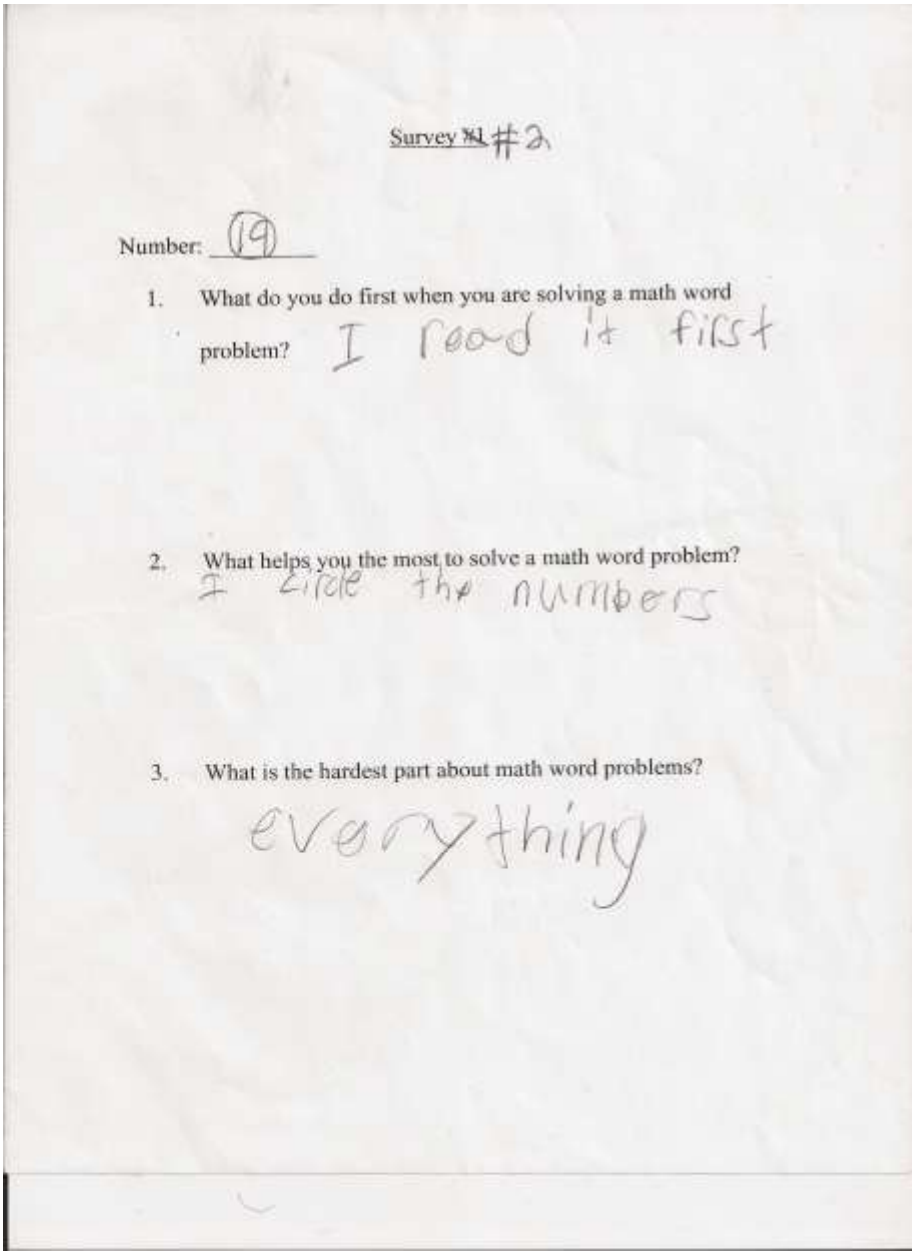
$$\begin{array}{r} 15 \\ -\ 2 \\ \hline 13 \end{array}$$

4. Every week the zoo has to buy food for the animals. Last week the zoo bought 44 bunches of bananas and 83 pounds of peanuts. About how many bushels of apples does the zoo purchase every week? 127

$$\begin{array}{r} 44 \\ +\ 83 \\ \hline 127 \end{array}$$

Appendix H.3

Benjamin's post-survey



Survey ~~#1~~ #2

Number: 19

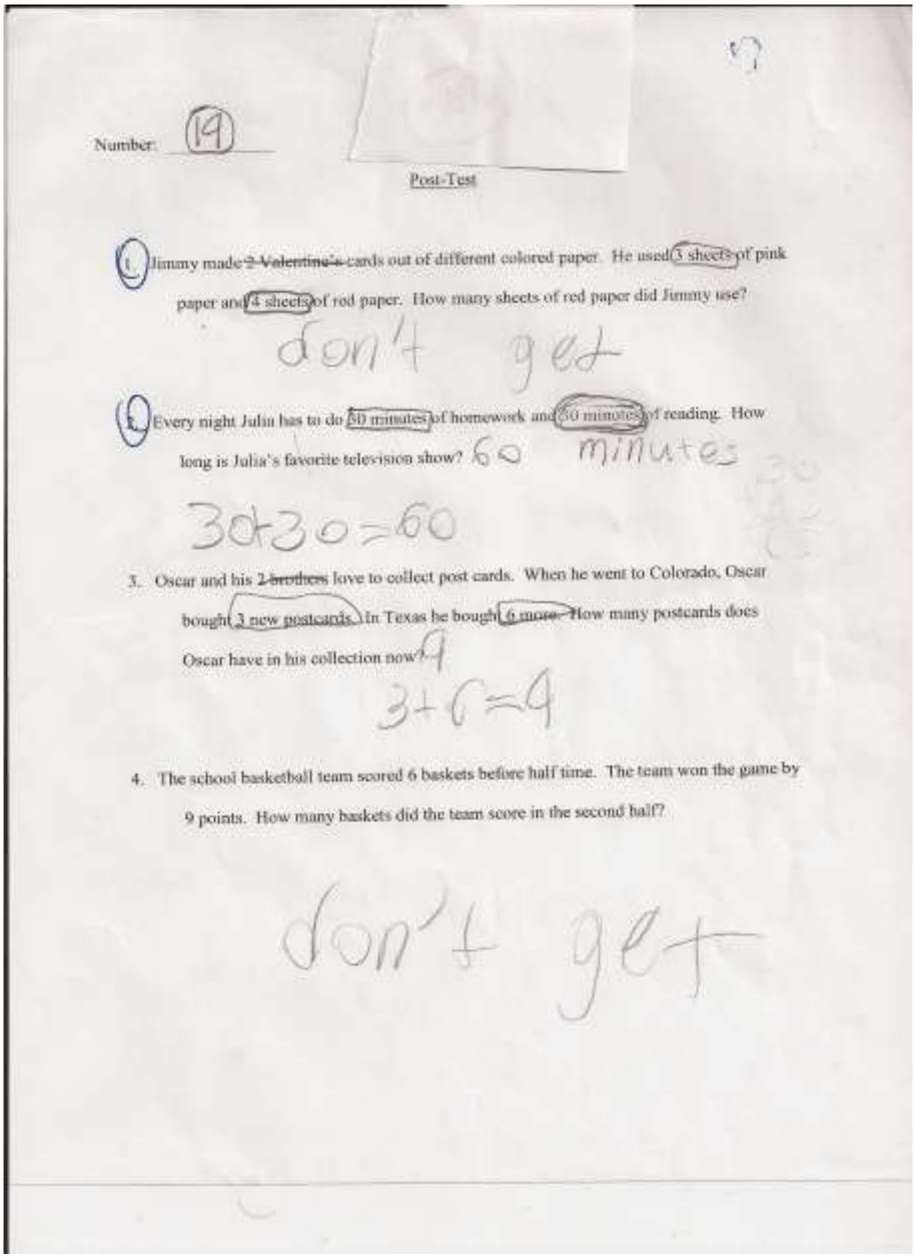
1. What do you do first when you are solving a math word problem? I read it first

2. What helps you the most to solve a math word problem?
I circle the numbers

3. What is the hardest part about math word problems?
everything

Appendix H.4

Benjamin's post-test



Number: 19

Post-Test

1. Jimmy made ~~2 Valentine's~~ cards out of different colored paper. He used 3 sheets of pink paper and 4 sheets of red paper. How many sheets of red paper did Jimmy use?

don't get

2. Every night Julia has to do 30 minutes of homework and 30 minutes of reading. How long is Julia's favorite television show? 60 minutes

30+30=60

3. Oscar and his ~~2 brothers~~ love to collect post cards. When he went to Colorado, Oscar bought 3 new postcards. In Texas he bought 6 more. How many postcards does Oscar have in his collection now? 9

3+6=9

4. The school basketball team scored 6 baskets before half time. The team won the game by 9 points. How many baskets did the team score in the second half?

don't get

Appendix H.5

Benjamin's interview

(19)

Formal Interview

1. What is your favorite subject?
Social Studies, reading about the U.S.

2. Do you like Reading? Why or why not?
yes, chapter books.
My Dog the ~~Scad~~ Scardy Cat - Hank Zipzer

3. What is your favorite thing about Reading?
Finishing the book to take an AR test

4. Do you like Math? Why or why not?
yes. adding and subtracting

5. What is your favorite thing about Math?
multiplication
some by heart

6. What do you look for in a word problem?
question. look for info to circle.
Cross out what I don't need. sometimes.

7. What do you do first?
important stuff to circle
underline question, cross out trash

8. After you read it, what do you do?

9. How do you know that you understand the problem?
Reread

10. What helps you to solve the problem?
Write the numbers to add. add = altogether
Sub = how many more

11. Do you write anything down after you read the problem?
draw a picture
i.e. circle to represent a ~~circle~~ sticker

12. Does it help you to draw anything after reading the problem?
draw pictures because the words sometimes confuse me.

Appendix H.5

Benjamin's interview continued

13. Do you draw a picture or do you use symbols? If so, which symbols do you use?

draw a simple picture (stick figure i.e. to rep. clown)

14.

Pre-

3 starfish (2)

Clownfish (15)

What happened? What does "distract" mean

(To keep them from doing something)

Post. (what did I do differently)

3 Circled info on post.

Crossed out trash)

Appendix I

Interview summary sheet

Table 4.1

	Jeremiah	**Roman**	**Benjamin**
1. What is your favorite subject?	Science	Science	Social Studies
2. Do you like reading?	Yes, I love non-fiction books the most, biographies.	Yes, I like reading fiction chapter books. I like Captain Underpants and another one but I can't remember what it's called.	Yes, I like Hank Zipzer books. I've read a lot of them and I'm reading My Dog the Scaredy Cat right now.
3. What is your favorite thing about reading?	I learn more from non-fiction	It's fun because sometimes it makes you laugh or cry	I like finishing the books to take an A.R. (Accelerated Reader) test
4. Do you like Math?	Yes, because it is challenging	Yes, because you work with people, you can combine brains.	Yes, I like adding and subtracting
5. What is your favorite thing about Math?	Multiplying	Multiplication because I like to be challenged	Multiplication, I even know some by heart.
6. When you have to do a word problem, what is it that you look for?	The question because it tells you what to do, kind of like how to figure it out.	It depends on how hard it is. The ones that are kind of hard are the ones that have to be converted. I like them thought because I like the challenge. I also look to see if I'm going to add, multiply, and check to see if it makes sense.	I look at the question and information to circle. I cross out what I don't need.
7. What do you do first?	Look at the question and circle the	See if it sounds right or not.	I look for important stuff to circle. I underline

	Jeremiah	Roman	Benjamin
	information to figure out the operation.		the question and cross out the trash.
8. What helps you to solve the problem?	I make a chart or use pictures.	Working with a partner and comparing work.	I write the numbers that I need to add.
9. What do you write down on your paper?	Pictures. They help me because I like them more then words. I relate to them because I love to draw.	The number sentence. I try all the operations until I find the right answer.	I sometimes draw a picture because the words sometimes confuse me.
10. Do you draw an exact picture or do you use symbols for the pictures?	I use symbols. A circle to represent apples, or a T to represent Tuesday.	Drawing takes too much time it is a waste of time. Symbols help me to concentrate on what to do to solve the problem.	I would use simple figures because we don't have time to draw. I would use a stick figure to draw a clown or a circle to represent stickers.
12. Lets look at problem…what is the story about?	#2 on pre-test: It's about Cara. She has two shoes with red shoelaces and three shoes with blue shoelaces. How many green? I know she can wear them all.	#3 on pre-test. There are 15 clownfish swimming and two starfish distract four of the clownfish.	#3 on pre-test: There are 15 clownfish and two starfish. The starfish distract four of the clownfish. Now there are, um…11.
13. Why do you think…	They're shoes, but I guess it doesn't say that. I need to know how many green, it should tell you but it doesn't. It cannot be figured out. I don't know how I got four. I thought it said just one green.	Well, two of the starfish take four clownfish away. Oh, I added the clownfish together instead of subtracting.	Distract means to keep them from doing something. Four of the clownfish floated away. Oh I took out the two starfish instead of the four clownfish before. I didn't read it carefully before.

	Jeremiah	Roman	Benjamin
14. Let's look at problem…what is the story about?	# 2 on the post-test It's about Julia and her homework. She is doing 30 min. of homework and 30 min. of reading. How long is the TV show?	#3 on post-test: It is about Oscar sending postcards.	*Benjamin looked at # 3 on post-test but was not asked questions 14 and 15.*
15. Why do you think…	This problem doesn't mention anything about a TV show so you can't figure that out without any evidence.	Hmmm…oh wait, he isn't sending them, he's buying them. I drew a 2 and then a stick person to show that it was two brothers not postcards. Then I drew a 3 and a square to show three postcards and a 6 and a square to show six more postcards. Then I could cross out "two brothers" and I realized that I needed to add six plus three. I knew I was counting the postcards he's collected so I knew to add.	
16. Why do you think the other problem tricked you but not this one?	The first one is talking about the same thing but different colors. The second one is completely different things so it was easier.	Because this one was adding and the other one was subtracting. They symbols helped me to tell apart brothers from postcards.	I think because I circled the information on this one and I crossed out the trash. When I write stuff, cross things out, or draw pictures it

	Jeremiah	Roman	Benjamin
			helps.

CURRICULUM VITA

Elizabeth Bernadette was born in El Paso, Texas. She is the second of four children, daughter of Henry and Laura Bernadette. Elizabeth graduated from Irvin High School in El Paso, Texas in May of 2001 and entered The University of Texas at El Paso in the fall of 2001. Throughout her undergraduate career, Elizabeth worked at several law offices. The year before graduating, Elizabeth was a participant in NASA's Pre-Service Teachers Institute. For two weeks in July, the participants endured extensive training in teaching Math, Science, and Technology at Langley Space Center in Langley, Virginia. During the last two semesters of undergraduate school, she completed her student teaching in both first and third grade at Dr. Nixon Elementary School in El Paso, Texas. After graduation in December 2005, she was offered a third grade position at the same school that she student taught and the same in which she is currently employed. To further her education, Elizabeth participated in several Teacher Quality Grants at the University of Texas at El Paso. With the influence of positive mentors, it was here that she decided to enter Graduate school in the fall of 2007 and receive her Master of Arts in Education with a specialization in Mathematics Education.

Permanent Address: 4649 Loma Del Sur, #2803
El Paso, Texas 79934