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A Supplement to "The Shape of the Solution Set for Systems of Interval Linear Equations with Dependent Coefficients"

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In our paper [AKM], we considered *systems of interval linear equations with dependent coefficients*, i.e., systems of the type

$$\sum_{j=1}^{n} a_{ij} x_j = b_i, \tag{0.1}$$

where

$$a_{ij} = a_{ij}^{(0)} + \sum_{\alpha=1}^{p} a_{ij\alpha} f_\alpha, \tag{0.2}$$

$$b_i = b_i^{(0)} + \sum_{\alpha=1}^{p} b_{i\alpha} f_\alpha, \tag{0.3}$$

$a_{ij}^{(0)}$, $a_{ij\alpha}$, $b_i^{(0)}$, and $b_{i\alpha}$ are given real numbers, $(1 \le i \le m,\ 1 \le j \le n,\ 1 \le \alpha \le p)$, and coefficients f_α can take arbitrary values from the given intervals $\mathbf{f}_\alpha$. These systems are common in practice, when due to measurement uncertainty, we do not know the exact values of the coefficients of linear equations. By a *solution set* of such a system, we mean the set of all solution corresponding to different values $f_\alpha \in \mathbf{f}_\alpha$.

In [AKM], we described the shape of the solution set. Namely, we showed that each solution set is *semialgebraic*, i.e., it can be represented as a finite union of subsets, each of which is defined by a finite system of polynomial equations $P_r(x_1, \ldots, x_q) = 0$ and inequalities of the types $P_s(x_1, \ldots, x_q) > 0$ and $P_t(x_1, \ldots, x_q) \ge 0$ (for some polynomials P_i). We also showed that for every subset $I = \{i_1, \ldots, i_q\} \subset \{1, \ldots, n\}$,

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the corresponding *projection* of a solution set, i.e., the set of all vectors $(x_{i_1}, \ldots, x_{i_q}) \in \mathbb{R}^q$ that can be extended to a solution $(x_1, \ldots, x_n)$ of a system, is also semialgebraic, and that, vice versa, every semialgebraic set can be represented as a projection of the solution set of some system of interval linear equations with dependent coefficients.

In this representation, however, we allowed intervals $\mathbf{f}_\alpha$ to be arbitrarily wide. In terms of measurements, wide intervals correspond to low accuracy. It is natural to ask the following question: if we only consider narrow intervals, which correspond to high measurement accuracy, will we still get all possible semialgebraic shapes or a narrower class of shapes? In this article, we show that even for narrow intervals, all possible shapes are possible.

Let us recall (see, e.g., [KLRK]) that for a given $\delta > 0$, an interval $\mathbf{x} = [\tilde{x} - \Delta, \tilde{x} + \Delta]$ is called *absolutely δ-narrow* if $\Delta \leq \delta$, and is *relatively δ-narrow* if $\Delta \leq \delta \cdot |\tilde{x}|$.

Theorem 0.1. *For every $\delta > 0$, every semialgebraic set can be represented as a projection of the solution set of some system of interval linear equations with dependent coefficients, whose intervals are both absolutely and relatively δ-narrow.*

Proof. We have already proven, in [AKM], that an arbitrary semialgebraic set can be represented as a projection of the solution set of some system of interval linear equations with dependent coefficients. To prove the new result, we will show how to transform the corresponding system with possibly wide intervals $\mathbf{f}_\alpha = [f^-_\alpha, f^+_\alpha]$ into a new system with narrow intervals.

It is easy to check that an interval $[1 - \delta, 1 + \delta]$ is both absolutely and relatively δ-narrow. Every coefficient $f_\alpha \in \mathbf{f}_\alpha$ can be represented as $k_\alpha \cdot g_\alpha + l_\alpha$, where $g_\alpha \in [1 - \delta, 1 + \delta]$, $k_\alpha = (f^+_\alpha - f^-_\alpha)/(2\delta)$, and $l_\alpha = f^-_\alpha - l_\alpha - k_\alpha \cdot (1 - \delta)$. Substituting the expression for f_α in terms of g_α into the equations (0.2) and (0.3), we get a new system of interval linear equations with dependent coefficients which has the same solution set as the old system, but for which all intervals are narrow. □

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