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THE ROTUNDITY OF LARGE PLANETARY OBJECTS

ADESANWO MORADEYO

Department of Physics

	APPROVED:	
	Jorge A. L´opez, Ph.D., Chair	
	vorge in 2 opez, i in.b., chun	
	Harold Slusher, Ph.D.	
Charles H. Ambler, PH.D		
Dean of the Graduate School		

DEDICATION

To my late great grand father and all my mentors

THE ROTUNDITY OF LARGE PLANETARY OBJECTS

By

ADESANWO MORADEYO, B.S.

PROJECT

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

In Partial Fulfillment

Of the Requirements

For the Degree of

MASTER OF SCIENCE

Department of Physics

THE UNIVERSITY OF TEXAS AT EL PASO

December 2004.

ACKNOWLEDGEMENTS

First and foremost, I want to thank God for giving me the opportunity to still be alive and for seeing me through this project and my master degree program in this University. I would like to thank my family for their support and encouragement,

I also want to thank my supervisor, Dr. Harold Slusher for his support and assistance. I acknowledge the members of the committee, Dr. Jorge L'opez for taking his time to serve on my project committee and I also want to thank my brothers and sisters, all my friends i.e. my classmate of the Physics Department, friends at home, America, England, Africa for their love and affections.

December, 2004

Adesanwo Moradeyo

ABSTRACT

The most obvious structural feature of planetary bodies is their roundness. This arises because of the dominate roles of the two effects of gravity and case of the deformation of matter, whether gas, liquid or solid. This project is about proving and solving out the time scale (τ) of flow restoring the body to spherical shape when we consider a slightly deformed self-gravitating sphere and also checking for spherical of different planetary objects.

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CHAPTER ONE

1. Proving the Time Scale Equation

The theory of the gravitational oscillation of a mass of liquid about the spherical form is due to Kelvin. If we take the origin at the centre and denoting the radius vector at any point of the surface by $a+\zeta$ i.e. where a is the radius in the undisturbed state.

We assume
$$\zeta = \sum_{1}^{\infty} \zeta_{n} \cdot \cdots \cdot (i)$$

Where ζ_n is a surface-harmonic of integral order n.

The ideal fluid continuity equation is $\nabla^2 \phi = 0$ i.e.

$$\theta = \nabla . q = 0$$

$$\partial \phi = \frac{\partial \phi}{\partial x} \partial x + \frac{\partial \phi}{\partial y} \partial y + \frac{\partial \phi}{\partial z} \partial z$$

where
$$u = \frac{\partial \phi}{\partial x}$$
 $v = \frac{\partial \phi}{\partial y}$ $w = \frac{\partial \phi}{\partial z}$

the velocity leads to the Laplace equation in the velocity potential

$$\nabla . \nabla \phi = \nabla^2 \phi = 0$$

The potential for all possible irrotational motions of an incompressible fluid must satisfy the Laplace equation, most ideal fluid motions are irrotational, since rotation must commonly arises through the action of viscosity (friction).

$$\nabla \cdot q = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

therefore the equation of continuity is satisfied by

$$\phi = \sum_{n=1}^{\infty} \frac{r^{n}}{a_{n}} s_{n} \cdot \cdots \cdot (ii)$$

where s_n is a surface-harmonic and kinematical conditions,

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial \phi}{\partial r} \cdot \dots \cdot (iii)$$

$$\frac{\partial \phi}{\partial r} = \sum_{1}^{\infty} \frac{nr^{n-1}}{a_n} S_n$$

$$\frac{\partial \phi}{\partial r} = \sum_{1}^{\infty} \frac{nr^{n-1}}{a_n} S_n$$

Therefore to be satisfied when r = a gives

$$\frac{\partial \zeta_n}{\partial t} = -\frac{n}{a} s_n \cdot \dots \cdot (iv)$$

The gravitational potential at the free surface is

$$\Omega = -\frac{4\pi\gamma\rho a^3}{3r} - \sum_{n=1}^{\infty} \frac{4\pi\gamma\rho a}{2n+1} \zeta_n \cdot \dots \cdot (v)$$
 Where γ is the gravitational

constant.

Putting
$$g = \frac{4}{3}\pi \gamma a$$
, $r = a + \Sigma \varsigma_n$

Resolving the gravitational potential

$$\Omega = cons \tan t + g \sum_{1}^{\infty} \frac{2(n-1)}{2n+1} \varsigma_n \cdot \cdots \cdot (vi)$$

substituting from (ii) and (v) in the pressure equation

$$\frac{p}{\rho} = \frac{\partial \phi}{\partial t} - \Omega + cons \tan t \cdot \dots \cdot (vii)$$

since P must be constant over the surface

$$\frac{\partial \zeta_n}{\partial t} = \frac{2(n-1)}{2n+1} g \zeta_n \cdot \cdots \cdot (viii)$$

Eliminating S_n between (vii) and (viii) we have

$$S_n = -\frac{a}{n} \frac{\partial \varsigma_n}{\partial t}$$

$$\frac{\partial^2 \zeta_n}{\partial t^2} = -\frac{n}{a} \frac{2(n-1)}{2n+1} g \zeta_n \cdot \cdots \cdot (ix)$$

therefore this shows that $\varsigma_n \propto \cos(\sigma_n t + \varepsilon)$, where we have

$$\sigma_n^2 = \frac{2n(n-1)}{2n+1} \frac{g}{a} \cdots (x)$$

for the same density of liquid $g\alpha$ a and the frequency is therefore independent of the dimensions of the globe.

If n = 1, i.e. $\sigma = 0$ since in a small deformation expressed by a surface-harmonic of the first order the surface remains spherical and the period is therefore infinitely long.

In case of a highly viscous globe returning asymptotically to the spherical form under the influence of gravitation, the velocity potential is in the form

$$\phi = A \frac{r^{n+1}}{a^{n+1}} s_n \cdot \cos(\sigma t + \varepsilon) \cdot \cdots \cdot (xi)$$

i.e. s_n is a the surface harmonic.

Therefore the kinetic energy included within a sphere of radius r, we have the expression in the form

$$\rho \iint \phi \frac{\partial \phi}{\partial r} r^2 \partial \varpi = \rho (n+1) \frac{r^{2n+2}}{a^{2n+2}} \iint s_n^2 \partial \varpi . A^2 \cos^2(\sigma t + \varepsilon) \cdots (xii)$$

since we are dealing with the spherical form under the influence of gravitation the total kinetic energy

$$T = \frac{1}{2} \rho g n a \iint s_n^2 \partial \varpi . A^2 \cos^2(\sigma t + \varepsilon) \cdots (xiii)$$

and potential energy is

$$v = \frac{1}{2} \rho g n a \iint s_n^2 \partial \varpi . A^2 \sin^2(\sigma t + \varepsilon) \cdot \cdots \cdot (xiv)$$

Total Energy = T + V

$$=\frac{1}{2}\rho gna \iint s_n^2 \partial \varpi . A^2 \cos^2(\sigma t + \varepsilon) + \frac{1}{2}\rho gna \iint s_n^2 \partial \varpi . A^2 \sin^2(\sigma t + \varepsilon)$$

$$T + V = \frac{1}{2} \rho g n a \iint s_n^2 \partial \varpi . A^2 \cdot \cdots \cdot (xv)$$

again, the dissipation in a sphere of radius r, calculated on the assumption that the motion is irrotational under the gravitational influence,

$$\mu \iint \frac{\partial \cdot q^2}{\partial r} r^2 \partial \varpi = \mu r^2 \frac{\partial}{\partial r} \iint q^2 \partial \varpi$$

$$r^{2} \iint q^{2} \partial \varpi = \frac{\partial}{\partial r} \iint \phi \frac{\partial \phi}{\partial r} r^{2} \partial \varpi$$

and $\iint \phi \frac{\partial \phi}{\partial r} r^2 \partial \varpi = \rho (n+1) \frac{r^{2n+2}}{a^{2n+2}} \iint s_n^2 \partial \varpi . A^2 \cos^2(\sigma t + \varepsilon)$

$$= \rho(n+1)2(n+1)\frac{r^{2n+1}}{a^{2n+2}} \iint s_n^2 \partial \varpi$$

putting r = a, we have for the total dissipation

$$2F = (\rho 2(n+1)^2 + 1)\mu \iint s_n^2 \partial \varpi . A^2 \cos^2(\sigma t + \varepsilon) \cdot \cdot \cdot \cdot (xvi)$$

therefore, if the effects of inertia being disregarded and also if the effect of viscosity be represented by a gradual variation of the coefficient A,we have

$$\frac{\partial}{\partial t}(T+V) = 2F \cdot \cdots \cdot (xvii)$$

The mean value of which, per unit time is

$$2\overline{F} = \rho(n+1)^2 + 1.\mu \iint s_n^2 \partial \varpi . A^2$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho g n a \iint s_n^2 \partial \varpi . A^2 \right) = \rho (n+1) + 1 \cdot \mu \iint s_n^2 \partial \varpi . A^2$$

$$\frac{\partial A}{\partial t} = \frac{2(n+1)^2 + 1}{n} \frac{v}{ga} A \cdot \cdots \cdot (xviii)$$

from equation (xviii), we can shew $A\alpha \ell^{\frac{-t}{\tau}}$ and $\frac{\partial A}{\partial t} = \tau \rightarrow \text{time scale or time interval;}$

$$\tau = \frac{2(n+1)^2 + 1}{n} \frac{\nu}{ga} \cdots (xix)$$

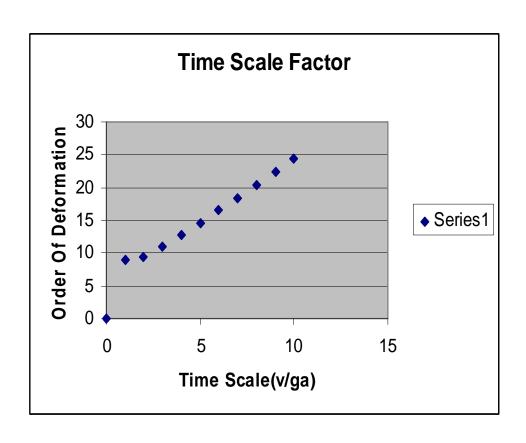
where n is the order of deformation, ν is the kinematic viscosity is the acceleration due to gravity

From the time scale or time interval, checking for different values of n, where n is the order of the deformation (of surface wavelength $\frac{2\pi a}{n}$)

$$\tau = \frac{2(n+1)^2 + 1}{n} \frac{v}{ga}$$

We can check different values of n and plot the time scale τ against the order of deformation and vice versa.

Order of Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$
0	$0\left(\frac{\nu}{ga}\right)$
1	$9\left(\frac{\nu}{ga}\right)$
2	$9.5\left(\frac{v}{ga}\right)$
3	$11\left(\frac{v}{ga}\right)$
4	$12.75 \left(\frac{v}{ga} \right)$
5	$14.6 \left(\frac{v}{ga} \right)$
6	$16.5\left(\frac{v}{ga}\right)$
7	$18.43 \left(\frac{v}{ga}\right)$
8	$20.375 \left(\frac{v}{ga} \right)$
9	$22.33 \left(\frac{v}{ga} \right)$
10	$24.3 \left(\frac{v}{ga} \right)$



We can calculate the time scale τ , for different values of (a) using the different values of (n). For a typical material of density $3000\,kg/m^3$

and kinematic viscosity $v = 10^{17} m^2 / s$ and for values of a = 1,100,200,300,400,500 and 1000 km.

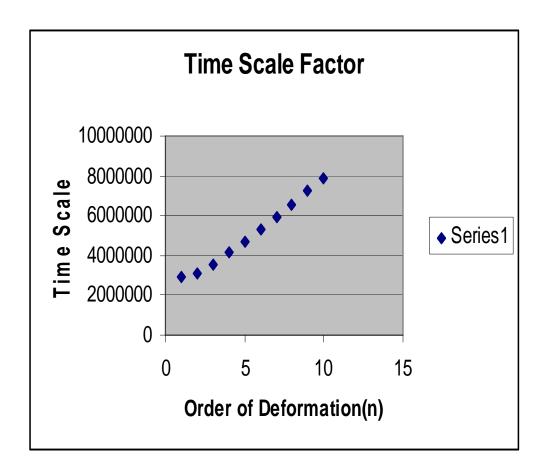
Kinematic viscosity is defined as the dynamic viscosity divided by density,

$$v = \frac{\eta}{\rho} = 10^{17} \, m^2 \, / \, s$$

And the acceleration due to gravity $g = 9.8m/s^2$

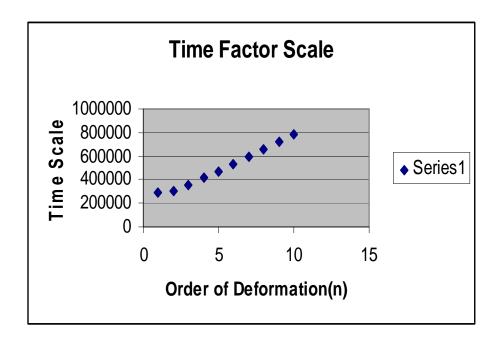
CASE 1. For $a = 1km, v = 10^{17} m^2 / s, g = 9.8m / s^2$

		<u></u>
Order of Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	2.9×10 ⁶ years
2	$9.5\left(\frac{v}{ga}\right)$	3.1×10 ⁶ years
3	$11\left(\frac{v}{ga}\right)$	3.6×10 ⁶ years
4	$12.75 \left(\frac{v}{ga}\right)$	4.1×10 ⁶ years
5	$14.6 \left(\frac{v}{ga}\right)$	4.7×10^6 years
6	$16.5 \left(\frac{v}{ga}\right)$	5.3×10 ⁶ years
7	$18.43 \left(\frac{v}{ga}\right)$	6.0×10 ⁶ years
8	$20.375 \left(\frac{v}{ga}\right)$	6.6×10 ⁶ years
9	$22.33 \left(\frac{v}{ga}\right)$	7.2×10^6 years
10	$24.3 \left(\frac{v}{ga}\right)$	7.9×10 ⁶ years



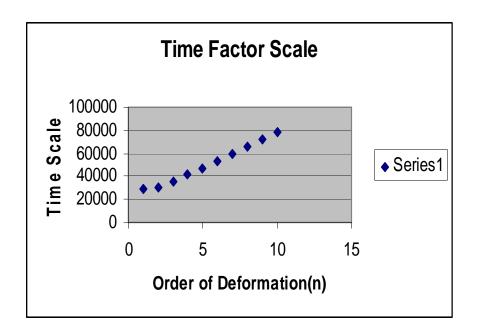
CASE 2. For $a = 10km, v = 10^{17} \, m^2 / s, g = 9.8m / s^2$

Order of Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	2.9×10 ⁵ years
2	$9.5\left(\frac{v}{ga}\right)$	3.1×10^5 years
3	$11\left(\frac{v}{ga}\right)$	3.6×10 ⁵ years
4	$12.75 \left(\frac{v}{ga}\right)$	4.1×10^5 years
5	$14.6 \left(\frac{v}{ga} \right)$	4.7×10^5 years
6	$16.5 \left(\frac{v}{ga}\right)$	5.3×10 ⁵ years
7	$18.43 \left(\frac{v}{ga}\right)$	6.0×10^5 years
8	$20.375 \left(\frac{v}{ga}\right)$	6.6×10 ⁵ years
9	$22.33 \left(\frac{v}{ga}\right)$	7.2×10^5 years
10	$24.3 \left(\frac{v}{ga}\right)$	7.9×10 ⁵ years



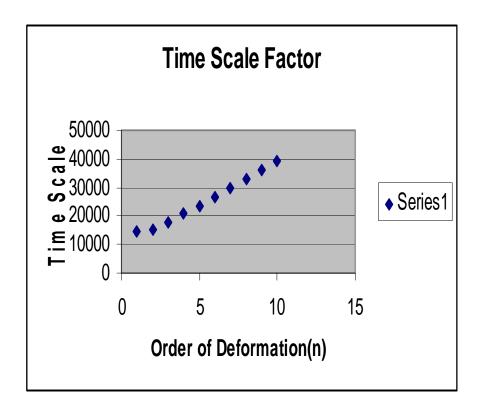
CASE 3. For $a = 100km, v = 10^{17} m^2 / s, g = 9.8m / s^2$

Order of Deformation(n)	Time Scale $\tau\left(\frac{v}{ga}\right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	2.9×10 ⁴ years
2	$9.5\left(\frac{v}{ga}\right)$	3.1×10^4 years
3	$11\left(\frac{v}{ga}\right)$	3.6×10^4 years
4	$12.75 \left(\frac{v}{ga}\right)$	4.1×10^4 years
5	$14.6 \left(\frac{v}{ga}\right)$	4.7×10^4 years
6	$16.5 \left(\frac{v}{ga}\right)$	5.3×10 ⁴ years
7	$18.43 \left(\frac{v}{ga}\right)$	6.0×10^4 years
8	$20.375 \left(\frac{v}{ga}\right)$	6.6×10^4 years
9	$22.33 \left(\frac{v}{ga}\right)$	7.2×10^4 years
10	$24.3 \left(\frac{v}{ga}\right)$	7.9×10^4 years



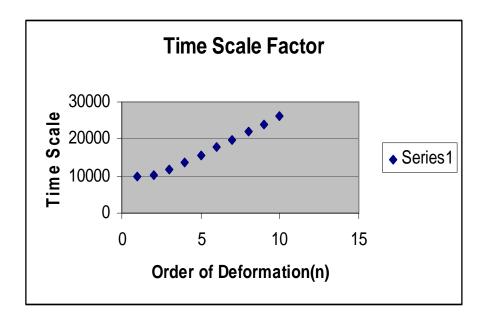
CASE 4. For $a = 200km, v = 10^{17} m^2 / s, g = 9.8m / s^2$

Order of Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	1.5×10^4 years
2	$9.5 \left(\frac{v}{ga} \right)$	1.54×10^4 years
3	$11\left(\frac{v}{ga}\right)$	1.8×10 ⁴ years
4	$12.75 \left(\frac{v}{ga}\right)$	2.1×10^4 years
5	$14.6 \left(\frac{v}{ga} \right)$	2.4×10^4 years
6	$16.5 \left(\frac{v}{ga}\right)$	2.7×10^4 years
7	$18.43 \left(\frac{v}{ga}\right)$	3.0×10^4 years
8	$20.375 \left(\frac{v}{ga}\right)$	3.3×10^4 years
9	$22.33 \left(\frac{v}{ga}\right)$	3.6×10^4 years
10	$24.3 \left(\frac{v}{ga}\right)$	3.9×10^4 years



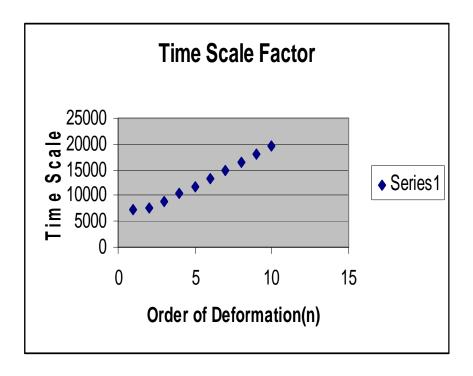
CASE 5. For $a = 300km, v = 10^{17} m^2 / s, g = 9.8m / s^2$

Order of Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	9.7×10^3 years
2	$9.5 \left(\frac{v}{ga} \right)$	1.0×10^4 years
3	$11\left(\frac{v}{ga}\right)$	1.2×10^4 years
4	$12.75 \left(\frac{v}{ga}\right)$	4.1×10^4 years
5	$14.6 \left(\frac{v}{ga} \right)$	1.6×10^4 years
6	$16.5 \left(\frac{v}{ga}\right)$	1.8×10^4 years
7	$18.43 \left(\frac{v}{ga}\right)$	2.0×10^4 years
8	$20.375 \left(\frac{v}{ga}\right)$	2.2×10^4 years
9	$22.33 \left(\frac{v}{ga}\right)$	2.4×10^4 years
10	$24.3 \left(\frac{v}{ga} \right)$	2.6×10^4 years



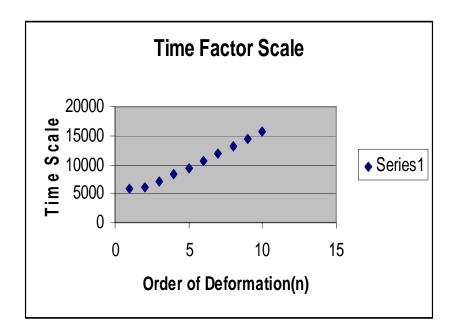
CASE 6. For $a = 400km, v = 10^{17} m^2 / s, g = 9.8m / s^2$

Order of	(11)	
Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	7.3×10^3 years
2	$9.5\left(\frac{v}{ga}\right)$	7.7×10^3 years
3	$11\left(\frac{v}{ga}\right)$	8.9×10^3 years
4	$12.75 \left(\frac{v}{ga}\right)$	1.0×10^4 years
5	$14.6 \left(\frac{v}{ga}\right)$	1.2×10^4 years
6	$16.5 \left(\frac{v}{ga}\right)$	1.3×10 ⁴ years
7	$18.43 \left(\frac{v}{ga}\right)$	1.5×10^4 years
8	$20.375 \left(\frac{v}{ga}\right)$	1.7×10^4 years
9	$22.33 \left(\frac{v}{ga}\right)$	1.8×10 ⁴ years
10	$24.3 \left(\frac{v}{ga}\right)$	2.0×10^4 years



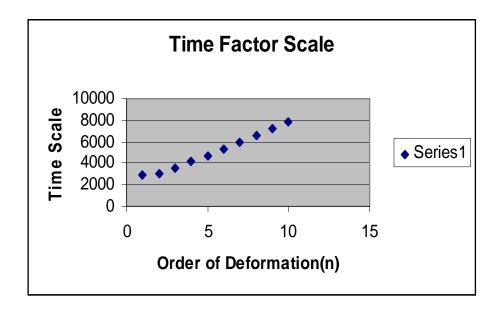
CASE 7. For $a = 500km, v = 10^{17} m^2 / s, g = 9.8m / s^2$

Order of	(,,)	
Deformation(n)	Time Scale $\tau \left(\frac{V}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	5.8×10^3 years
2	$9.5\left(\frac{v}{ga}\right)$	6.2×10^3 years
3	$11\left(\frac{v}{ga}\right)$	7.1×10^3 years
4	$12.75 \left(\frac{v}{ga}\right)$	8.3×10^3 years
5	$14.6 \left(\frac{v}{ga} \right)$	9.5×10^3 years
6	$16.5 \left(\frac{v}{ga}\right)$	1.1×10^4 years
7	$18.43 \left(\frac{v}{ga}\right)$	1.2×10^4 years
8	$20.375 \left(\frac{v}{ga}\right)$	1.3×10^4 years
9	$22.33 \left(\frac{v}{ga}\right)$	1.5×10^4 years
10	$24.3 \left(\frac{v}{ga}\right)$	1.6×10^4 years



CASE 8. For $a = 1000km, v = 10^{17} \, m^2 \, / \, s, g = 9.8 m / \, s^2$

Order of Deformation(n)	Time Scale $\tau \left(\frac{v}{ga} \right)$	Time Scale $\tau(years)$
0	$0\left(\frac{v}{ga}\right)$	0 years
1	$9\left(\frac{v}{ga}\right)$	2.9×10^3 years
2	$9.5\left(\frac{v}{ga}\right)$	3.1×10^3 years
3	$11\left(\frac{v}{ga}\right)$	3.6×10^3 years
4	$12.75 \left(\frac{v}{ga}\right)$	4.1×10^3 years
5	$14.6 \left(\frac{v}{ga} \right)$	4.7×10^3 years
6	$16.5 \left(\frac{v}{ga}\right)$	5.3×10^3 years
7	$18.43 \left(\frac{v}{ga}\right)$	6.0×10^3 years
8	$20.375 \left(\frac{v}{ga}\right)$	6.6×10^3 years
9	$22.33 \left(\frac{v}{ga}\right)$	7.2×10^3 years
10	$24.3 \left(\frac{v}{ga}\right)$	7.9×10^3 years



1.1 Checking for Spherical of different planets

We know clearly that objects of radius of order $10^2 \, km$ or less will not necessarily be at all spherical

SUN



Period: 0.00days Rotation: 30days

Distance: 1.015au Diameter: 1392000km

 $The Radius = \frac{diameter}{2} = \frac{1392000km}{2} = 686000km$

MERCURY

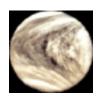


Period: 87.97days Rotation: 58.65days

Distance: 0.642au Diameter: 4878km

The Radius = $\frac{diameter}{2} = \frac{4878km}{2} = 2439km$

VENUS



Period: 224.70days Rotation: 243.01days

Distance: 1.252au Diameter: 12104km

$$The Radius = \frac{diameter}{2} = \frac{12104km}{2} = 6052km$$

MARS



Period: 1.88yrs Rotation: 24.62hrs

Distance: 2.095au Diameter: 6787km

The Radius = $\frac{diameter}{2} = \frac{6787km}{2} = 3393.5km$

JUPITER



Period: 11.86YRS Rotation: 9.84hrs

Distance: 4.68au Diameter: 142980km

 $The Radius = \frac{diameter}{2} = \frac{142980km}{2} = 71490km$

SATURN



Period: 29.46yrs Rotation: 10.66hrs

Distance: 9.55au Diameter: 120540km

 $The Radius = \frac{diameter}{2} = \frac{120540km}{2} = 60270km$

URANUS



Period: 84.01yrs Rotation: 17.23hrs

Distance: 18.829au Diameter: 51200km

 $The Radius = \frac{diameter}{2} = \frac{51200km}{2} = 25600km$

NEPTUNE



Period: 164.79yrs Rotation: 16.10hrs

Distance: 29.321au Diameter: 49530km

 $The Radius = \frac{diameter}{2} = \frac{49530km}{2} = 24765km$

PLUTO



Period: 247.69yrs Rotation: 153.28hrs

Distance: 28.932au Diameter: 2300km

 $The Radius = \frac{diameter}{2} = \frac{2300km}{2} = 1150km$

1.2 Calculating different time scale τ for the planetary objects,

$$\tau = \frac{2(n+1)^2 + 1}{n} \frac{v}{ga}$$

When the kinematic viscosity is $10^{20} m^2 / s$, n = 2 and radius (r) = a

PLANETS	n	r(km)	$v(m^2/s)$	$g(m/s^2)$	$\tau(years)$
SUN	2	686000	10 ²⁰	9.8	4.5×10^3
MERCURY	2	2439	10 ²⁰	9.8	1.3×10 ⁶
VENUS	2	6052	10 ²⁰	9.8	5.1×10 ⁵
MARS	2	3393.5	10 ²⁰	9.8	9.1×10 ⁶
JUPITER	2	71490	10 ²⁰	9.8	4.3×10 ⁴
SATURN	2	60270	10 ²⁰	9.8	5.1×10 ⁴
URANUS	2	25600	10 ²⁰	9.8	1.2×10 ⁵
NEPTUNE	2	24765	10 ²⁰	9.8	1.2×10 ⁵
PLUTO	2	1150	10 ²⁰	9.8	2.7×10 ⁶

CHAPTER TWO

2. CONCLUSION

The effects of gravity are completely dominant for bodies of radii larger than a few hundred kilometers, which even if they were severely damaged would return closely to spherical form in times of the order $10^3 - 10^6$ years; nevertheless, bumps and pits of heights typically of order 10km can be a permanent features.

2.1 NOTATIONS

 $C \cdots Wave (or phase) velocity$

 $D \cdot \cdots \cdot D$ iameter

F Force

 $M \cdot \cdots \cdot Mass$

Q(q)......Discharge

R(r) Radius, also the universal gas constant

 $X, Y, Z \cdots$ Volume of body force (gravity) along OX, OY, OZ, respectively.

 $d \cdot \cdots \cdot$ Depth (wave theory)

g ····· Gravity acceleration

 $n \cdot \cdots \cdot Element$

 $t, T \cdots Time$

OY and OZ respectively.

x, y, z......Coordinates of a point along OX, OY and OZ respectively

 μ ······Coefficient of viscosity also scale, vertical

v ····· Kinematic coefficient of viscosity

 ρDensity

 σ ····· Normal stress

 τTime scale also time interval or shearing stress

 ϕ ······Potential function

$$v = -grad\phi$$

$$\left(u = -\frac{\partial \phi}{\partial x}, v = \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)$$

 $\frac{\partial A}{\partial *}$Partial derivative (with respect to *)

 $\zeta_n \cdot \dots \cdot Surface$ -harmonic of integral order n

 s_n Surface harmonic Ω Gravitational-potential γ Gravitational constant P Pressure T Kinetic energy V Potential energy \overline{F} Total dissipation