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UFuzzy Prediction Models in Measurement

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Fuzzy Prediction Models in Measurement

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***Abstract*-The paper investigates a feasibility of fuzzy models application in measurement procedures. It considers the problem of measurement information fusion from different sources, when one of the sources provides predictions regarding approximate values of the measured variables or their combinations. Typically this information is given by an expert but may be mined from available data also. This information is formalized as fuzzy prediction models and is used in combination with the measurement results to improve the measurement accuracy. The properties of the modified estimates are studied in comparison with the conventional ones. The conditions when fuzzy models application can achieve a significant accuracy gain are derived, the gain value is evaluated, the recommendations on fuzzy prediction model production and formalization in practical applications are given.**

Keywords: fuzzy prediction, measurement science, accuracy gain

1.INTRODUCTION

Measurement is the most fundamental method of science in obtaining knowledge and in controlling systems. The current philosophical viewpoint on measurement defines its task as a parameter adjustment of a system model formulated mentally (presumably by an expert), so it is able to predict the same results actually supplied by observation [9]. In complex engineering and science systems some information might come from sensors while another part may be supplied by the experts. Due to their nature, these sources differentiate in reliability and uncertainty of the information produced. Uncertainty also can be influenced by the characteristics of procedures and tools applied in information acquisition and processing. Experts may formulate their estimates based on their previous experience and general and domain knowledge. While lagging to instruments in accuracy of particular measurement results, the expert's estimates may complement them by providing information regarding relationships and associations between sensor signals that could be applied for improving overall accuracy and reliability. This situation is typical for many traditional scientific applications. For example, in geological and geophysical sciences in the past fifteen years earthquake studies have grown from the collection of seismic data on frequency-limited seismometers (often in analog form) and mapping of surface faulting using geodetic techniques, to routine collection of digital seismic data on seismometers with broad frequency responses and mapping of surface deformation using a combination of geodetic, GPS and radar interferometry. In novel applications related to measurements in cyberspace the necessity of fusion of different models becomes even more important. In applications related to computer security and software quality one has to deal with the measurements inside very complex, unbounded and usually not strictly defined systems. For example, the number of possible metrics included in the guidelines for security evaluation [2] exceeds a few dozen. Some of the attributes used to calculate the corresponding metrics could be obtained from software and other sensors and counters while others are to be provided by the experts.

Models of various kinds are used extensively to provide representations of some aspects of the real life system under measurement (see fig. 1). Among main types, linguistic and mathematical models are listed [15]. The linguistic model uses the natural language to express sufficient parameters and their interactions. In the empirical science and the arts this is the prime form for presenting models of situations and relationships. While it usually serves as the first level of modeling, it often plays a complimentary role, providing descriptions of parameters and it is commonly supplied by an expert. Imprecision of natural language is a major obstacle to application of conventional methods of computing with information decribed in natural language [22]. Fuzzy logic, according to its founder L.Zadeh, was specifically designed as a methodology for processing linguistic and other imprecise models the way that no other methodology can serve this purpose [21]. Starting with an introduction of the concepts of a linguistic variable and granulation [17] with further development of the theory of a fuzzy constraint and fuzzy constraint propagation [18-20] fuzzy logic has developed an all inclusive mechanism for expressing, formalization and processing of linguistic models, which may become an invaluable tool for formalizing initial information about the object or a process and its application in measurement procedures. Information, which we consider in this paper, represents a high level model of the underlying concepts of the objects or processes under measurement. The model could be provided or derived based upon either of the following or their combinations:

1. knowledge of the physical, biological, chemical, mechanical or other natural laws, according to which an underlying system or process operates,
2. knowledge of the design and operational characteristics of the sensors, communication networks and other equipment,
3. expert estimates,
4. data mining and knowledge acquisition methods including
    a. statistical methods based on regression analysis and other techniques,
    b. intelligent data-driven methodologies, such as fuzzy logic, neural networks, genetic algorithms.

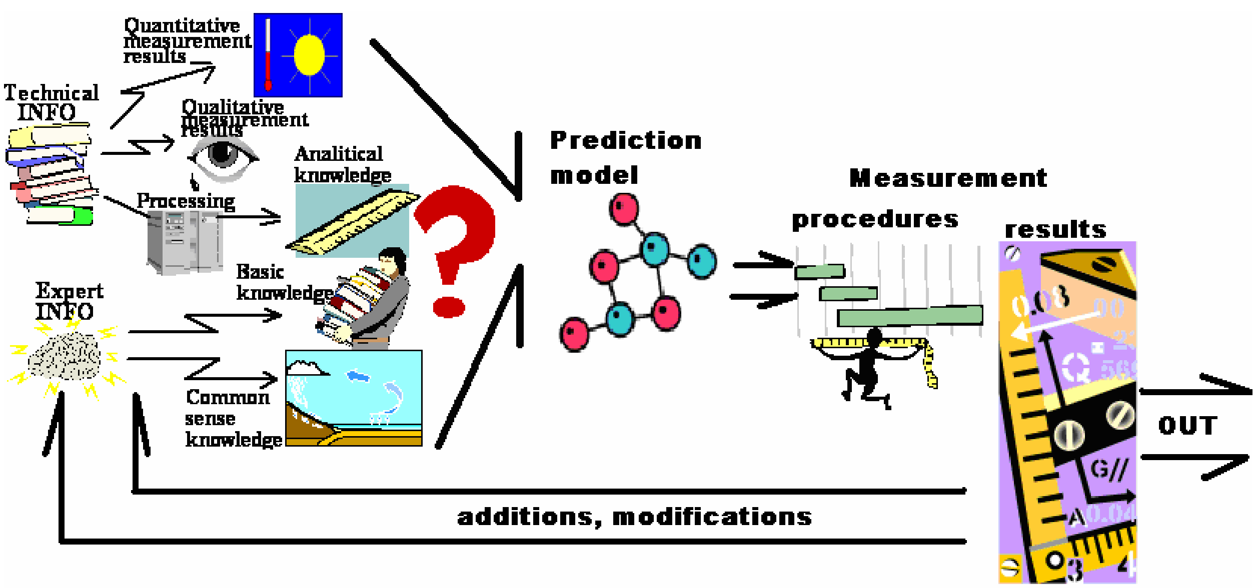


Figure 1. Measurement as a process of scientific cognition with various information sources

This model could be expressed in a form of functional or order relationships, may be approximate, stochastic or fuzzy. The following examples of possible functional relationships and their applications may be given:

1. in the measurement of flow rates in a variety of pipelines, which converge into one we have $Q_1 + Q_2 + \ldots + Q_n = Q$, where $Q_1, Q_2, \ldots, Q_n$ are the measurement results of the flow rates in converging pipelines and Q is the measured flow rate in the common pipeline.
2. measurement techniques applied in the fault tolerant skewed inertial measurement units, which are currently used in many aircraft and space systems [1], in which parity residuals indicative of the sensor errors are derived and then compared to calculated thresholds.

Measurement procedures actually implement modifications of the system models and adjustment of their parameters. Historically, they relied mainly on probability and statistics formalisms for their implementation. The probabilistic and statistical models and methods are widely applied in measurement procedures and processing of the measurement results. These models have been promoted into national and international standards [3]. Despite new research results appearing over the last decade, which are based on possibilistic models, traditional approaches are used almost exclusively in measurement practice.

This paper aims at demonstrating feasibility of fuzzy models application along with stochastic and statistical models for implementing such fundamental procedures of scientific cognition as measurement, defining the conditions, when these models can be applied and estimating the possible gain one can achieve from their realization. Its goal is three-fold. It attempts:

a) to develop a general method allowing fusing the prediction models of the values of the measured variables and their linear combinations typically received from experts with the measurement observations received from sensor systems,

b) to evaluate the accuracy of the measurements obtained with the prediction models use and the possible gain, which a prediction application can deliver,

c) to derive conditions when a prediction application can produce the gain in accuracy as opposed to possible losses and to develop practical recommendations on how to achieve it in applications.

In section 2 the prediction model is formalized as fuzzy. The problem of its application along with the measurement results is formulated as an optimization problem with fuzzy constraints. The properties of the estimates received as this problem solution are examined in the next section 3. Section 4 investigates the gain in accuracy while section 5 researchers the conditions necessary to achieve a gain in versus a loss and to maximize the gain achieved by fuzzy prediction application. Section 6 provides a practical example and recommendations on how to attain the gain in complex networked sensor systems.

2. Mathematical problem formulation

The general goal of a measurement procedure is to find estimates of the parameter vector X of some available system model. Also we assume that a prediction model regarding those parameter values is available. The model predicts the values of some functions f(X) of the parameter values. The functions f() are introduced to generalize a case where the prediction could be made about some association of a few parameters. The prediction is usually expressed as a model of type "the fuction f of the parameter vector X is R", where R is a constraining relation. One can see that this prediction belongs to the class of a generalized constraint as determined by L.Zadeh [22]. This model could be formalized as a fuzzy model and described with the set of membership functions μ(f(X)). A conventional way of finding the model parameter estimates based on the measurements would go through its definition as a mathematical programming problem and search for the parameter $\hat{X}$ estimates by maximizing some criterion

(1) $$\hat{X} = \arg\max_{x} F(Y_1, Y_2, ..., Y_n, X),$$

where F( ) is a functional, whose shape is determined by the estimation methods,

Yi (i=1,n) is a set of m_i measurements of the ith variable.

Let us consider a prediction model information as a fuzzy constraint for the parameter vector X and given by the set of membership functions $\mu_i(f_i(X))$, i = 1,m, where m is the number of predictions made by the experts. Each membership function describes one prediction model, typically about the possible value of the measurement result or a linear combination (for example, an average or a weighted average) of the measurement results. In this paper we will consider two commonly applied by experts predictions: an interval prediction, where an expert's prediction could be formalized with a trapezoidal membership function (see fig.2) and an approximate value prediction, where an expert's prediction could be formalized with a bell-shaped membership function (see fig.3).

1) trapezoidal or interval where

$$\mu(x) = 1,\ f - s \le x \le f + s;\ or\ 0,\ otherwise$$

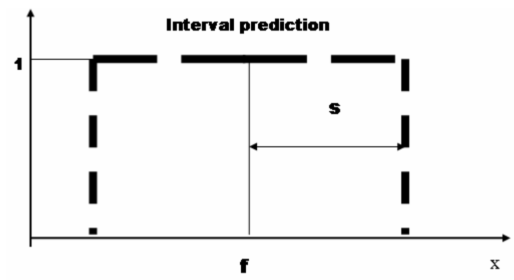


Figure 2. Membership function for an interval prediction

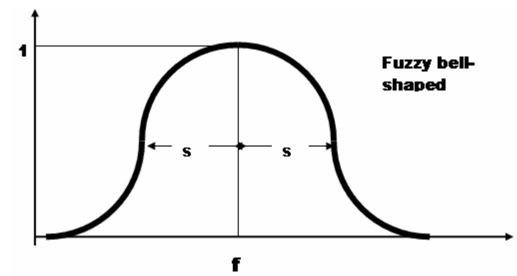


Figure 3. Membership function for an approximate value prediction around f

2) bell- shaped or Gaussian where $\mu(x) = e^{-\frac{(x-f)^2}{s^2}}$, which we will call an approximate value prediction.

In both membership functions the parameter ***f*** constitutes the expert's prediction of the measurement result or the results combination while a parameter ***s*** represents the degree of uncertainty or fuzziness in an expert's prediction. We will call this parameter in the text and figures below the prediction interval width. Note that an expert may predict values of the variables other than measured ones but related to them, where the relationship is given by a set of functions or equations. In this case the membership functions for the measurement predictions could be derived from original ones. The methods of a fuzzy model's information acquisition and its uncertainty propagation can be found in a number of publications (see, e.g. [6,12,16]).

Given a fuzzy prediction, the problem (1) can be considered as an optimization problem with fuzzy constraints. By now research of fuzzy constraints (see [11] for one application) has developed a variety of methodologies of solving such problems. One of the simplest and the most obvious ways is merging of the functional criteria and the constraints into one synergetic criterion and search for a global solution by the way of this criterion optimization. The problem can be re-formulated as search for the estimate maximizing the combination criterion F(.)

(2) $$\widetilde{X} = \arg\max_{X} F(Y_1, Y_2, \ldots, Y_n, X) \times \mu(f(X))$$

This problem could be tried with conventional or intelligent methods. The method choice should depend on the estimation techniques applied as well as on the membership function shapes (see [13,14] for more detail). We will call the solution of this optimization problem a modified estimate. This approach will allow fusing of stochastic methods of measurement results processing and fuzzy models of expert's information.

3. INVESTIGATION OF THE MODIFIED ESTIMATES VERSUS THE CONVENTIONAL ONES

3.1. VARIOUS COMBINATIONS OF MEASUREMENT DISTRIBUTIONS AND FUZZY PREDICTIONS

In this section we will compare the modified estimates of the measurements received with the application of fuzzy prediction models and the conventional estimates, which do not take the predictions into account. We will consider both interval and approximate value prediction models, introduced in the previous section in combinations with two most popular measurement results distributions: uniform distribution and normal distribution that will create the following cases:

a) uniform measurements distribution and interval prediction,

b) normal measurements distribution and interval prediction,

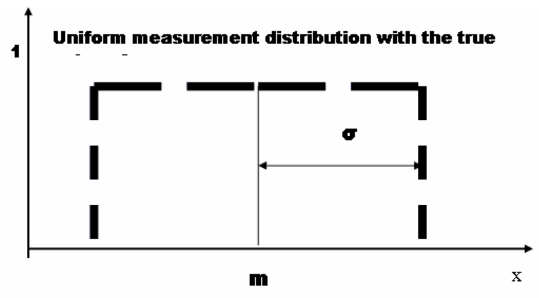
c) uniform measurements distribution and approximate value prediction,

d) normal measurements distribution and approximate value prediction.

Uniform measurements distribution whose probability density function is given by the formula [7]

$$f(y) = 1/2\sigma \ , x - \sigma \le y \le x + \sigma \ ; or \ \ 0 \ , otherwise$$

is presented on fig. 4



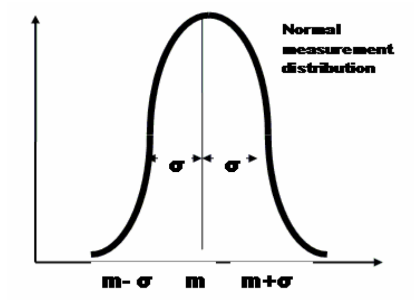


Figure 4. Uniform measurements distribution with the true value of m and measurement error of σ

Figure 5. Normal measurement distribution with the true value of m and measurement error of σ

The maximum likelihood estimator of the distribution parameter m, which is considered as the conventional measurement results estimator in a case of a uniform distribution, is given by the middle-of-the range [5]:

$$\hat{x} = \frac{1}{2}[\min(Y_1, Y_2, ..., Y_n) + \max(Y_1, Y_2, ..., Y_n)] = mid - range(Y_1, Y_2, ..., Y_n)$$

The variance, which is traditionally used as the accuracy indicator of the unbiased estimates will be $\frac{range^2}{12}$.

The second measurement distribution to be considered is the normal distribution whose probability density function (see fig. 5) is given by the formula $f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-m)^2}{2\sigma^2}}$ [8].

The maximum likelihood estimate of the measurements with the normal distribution is the mean value m estimated by the sample mean [10]. As this estimate is unbiased, the measurement accuracy is commonly calculated as the variance parameter of the normal distribution and is equal to σ^2. The formulae for the conventional estimates in the case of a normal distribution presented in Table 1 were retrieved from [8]

Let us start our consideration with a simple case of one measurement and one prediction only, which could be described as follows:

$y = x + \varepsilon_Y$

$x \approx f,$

where y is a measurement result of the variable with the true value of x ,

ε_Y is a total measurement error, which might include errors introduced over measurement, communication and processing of information,

f is a prediction value of the measurement .

Table 1. Measurement estimates and their variances for different cases of measurement result distributions (uniform and normal) and predictions (interval and approximate value)

	Measurement estimate	Variance
Conventional ML estimator of the uniform distribution of measurement	$\hat{x}$ =mid-range $(Y_1, Y_2, ..., Y_n)$	$\frac{range^2}{12}$
Modified ML estimator of the uniformly distributed measurements plus an interval prediction	$\tilde{x}$ =Mid-range $(\mathrm{Y}_1, \mathrm{Y}_2, ..., \mathrm{Y}_n \subset [f-s, f+s])$	$\frac{truncatedrange^2}{12}$
Conventional ML estimator of the normal distribution of measurements	$\hat{x} = \frac{1}{n}\sum_{i=1}^{n} Y_i$	$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \hat{x})^2$
Modified ML estimator of the normally distributed measurements plus an interval prediction	$\hat{x} + \frac{Z(\frac{f-s-\hat{x}}{\sigma}) - Z(\frac{f+s-\hat{x}}{\sigma})}{\Phi(\frac{f+s-\hat{x}}{\sigma}) - \Phi(\frac{f-s-\hat{x}}{\sigma})}$ where $\hat{x}$ and σ are defined as above	$[1 + \frac{(\frac{f-s-\hat{x}}{\sigma})Z(\frac{f-s-\hat{x}}{\sigma}) - (\frac{f+s-\hat{x}}{\sigma})Z(\frac{f+s-\hat{x}}{\sigma})}{\Phi(\frac{f+s-\hat{x}}{\sigma}) - \Phi(\frac{f-s-\hat{x}}{\sigma})} - \{\frac{Z(\frac{f-s-\hat{x}}{\sigma}) - Z(\frac{f+s-\hat{x}}{\sigma})}{\Phi(\frac{f+s-\hat{x}}{\sigma}) - \Phi(\frac{f-s-\hat{x}}{\sigma})}\}^2]\sigma^2$
Modified ML estimator of the normally distributed measurements plus an approximate equality prediction	$\tilde{x} = \frac{\frac{g^2}{2}\sum_{i=1}^{n}\frac{y_i}{n} + \frac{1}{n}f}{\frac{g^2}{2} + \frac{1}{n}}$	$\frac{\sigma^2}{(1 + \frac{2}{g^2})^2}$

In the notation above :

$Y_1, Y_2, ..., Y_n$ are measurement results,

$Z(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$

$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{t^2}{2}} dt$ is a cumulative distribution function

σ is the parameter of the measurement result distribution,
f and s are parameters of the prediction membership function,
$g = s/\sigma$ is the prediction uncertainty factor

Case A : (uniform measurement results distribution and interval prediction) picks up only measurements, which are in the certain range given by the boundaries ***f-s*** and ***f+s***. This is a typical part in measurement procedures, where the results taken from the sensors are compared with the thresholds. The procedure creates a truncated measurement distribution , whose parameters depend on the shift of the prediction interval against the true value. Depending on the prediction error that is the difference between the true value and the middle of the prediction interval f as well as fuzziness of the prediction (modeled by the width of the prediction interval) the following cases are possible:

1) the interval where the measurement results are distributed is completely covered by the prediction interval, i.e. $f - s \leq m - \sigma \leq m + \sigma \leq f + s$, in this case the prediction does not change the measurement as all measurements results will be taken into account in calculating the estimate
2) the measurement results distribution interval and the prediction interval do not overlap at all – this case does not produce any estimate as all measurement results will be rejected
3) the measurement results distribution interval and the prediction interval partially overlap – in this case the distribution interval gets truncated, the variance becomes smaller but the estimate becomes biased [4] toward the prediction

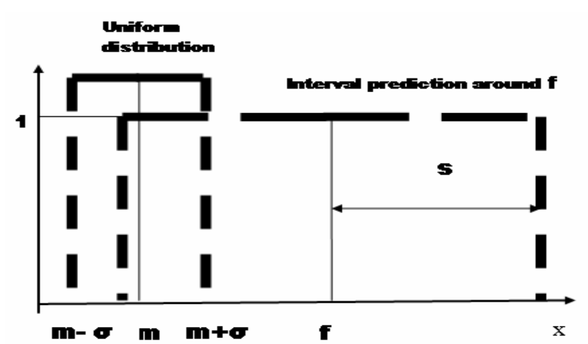


Figure 6. Uniform measurement distribution and interval prediction (partial overlap case)

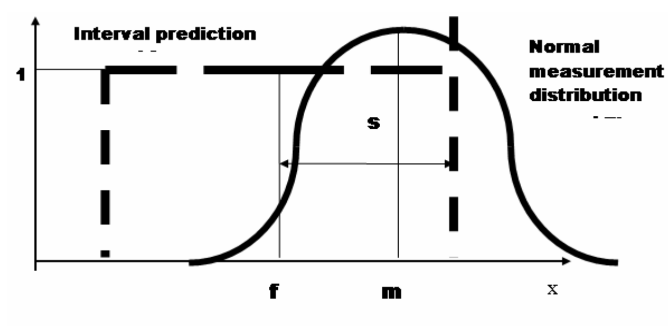


Figure 7. Normal measurement distribution and interval prediction

Further comparison of the modified and conventional estimates has been done by computer simulation with the results presented in sections 4 and 5.

Case B (normal measurement results distribution and interval prediction) again picks up only measurements, which are in the certain range given by the boundaries ***f - s*** and ***f + s*** (see fig. 7). This procedure creates a doubly truncated normal distribution [4] with the probability density function

$$f(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(y-m)^2}{2\sigma^2}} \left[\frac{1}{\sqrt{2\pi}\,\sigma} \int_{f-s}^{f+s} e^{-\frac{(y-m)^2}{2\sigma^2}} dt \right]^{-1} =$$

$$= \sigma^{-1} Z\left(\frac{y-m}{\sigma}\right)\left[\Phi\left(\frac{f+s-m}{\sigma}\right) - \Phi\left(\frac{f-s-m}{\sigma}\right)\right]^{-1}$$

where $Z(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$ and $\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{t^2}{2}} dt$ is a cumulative distribution function

The expected value of the truncated normally distributed variable, which will become a modified estimate and its variance are taken from [4] and given in the Table 1. As their calculations require tabulated values of the normal distribution accumulation function, further comparison is done by computer simulation with the results presented in sections 4 and 5.

Case C (uniform measurement results distribution and an approximate value prediction) weights all measurements results according to the specified membership function – see fig. 8

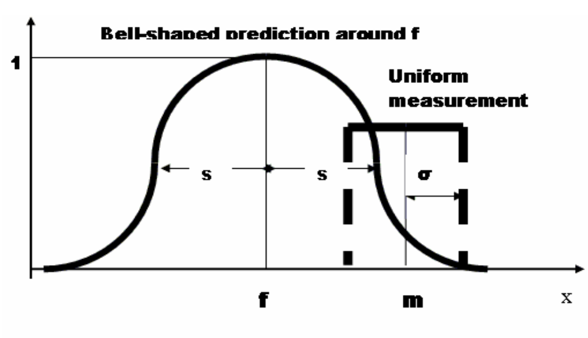


Figure 8. Uniform measurement results distribution and approximate equality prediction

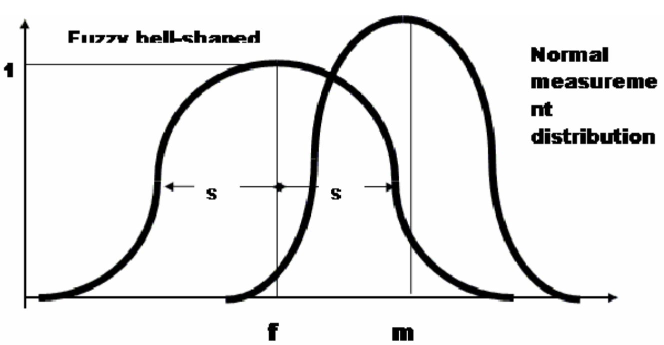


Figure 9. Normal measurement results distribution and approximate equality prediction

In this case the modified estimate $\tilde{x}$ can be found as the solution of the optimization problem (2), which will be the measurement result having the maximum weight against the fuzzy prediction

$$\tilde{x} = \arg \max_{m-\sigma \le y \le m+\sigma} [\mu(y_1), \mu(y_2), ..., \mu(y_n)]$$

Further comparison of the conventional and modified estimates has been conducted by computer simulation with the results presented in section 4 and 5.

Case D (normal measurement results distribution and approximate equality prediction) weights all measurements results according to the specified membership function –see fig. 9

The modified maximum likelihood problem (2) in this case could be written as

$$L(x) = \frac{1}{(2\pi)^{\frac{n}{2}} \prod_{i=1}^{n} \sigma_i} e^{-\sum_{i=1}^{n} \frac{(y_i - x)^2}{2\sigma_i^2}} \times e^{-\frac{(x-f)^2}{s^2}} \to \max$$

Taking the log from likelihood function given above we will get

$$l(x) = -\sum_{i=1}^{n} \log \sigma_i - \frac{n}{2} \log 2\pi - \sum_{i=1}^{n} \frac{(y_i - x)^2}{2\sigma_i^2} - \frac{(x-f)^2}{s^2} \to \max$$

Differentiating and equating the result to 0, we will get the algebraic equation below and the modified estimate $\tilde{x}$ in this case will be found as the solution of the algebraic equation:

$$(\sum_{i=1}^{n} \frac{1}{\sigma_i^2} + \frac{2}{s^2})\tilde{x} = \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2} + \frac{2f}{s^2}$$

Assuming that all measurements have the same accuracy, $\sigma_i = \sigma, \forall i$ one can see that the modified estimate becomes a weighted sum of the conventional one and the prediction value:

$$\tilde{x} = \frac{\frac{g^2}{2} \sum_{i=1}^{n} \frac{y_i}{n} + \frac{1}{n} f}{\frac{g^2}{2} + \frac{1}{n}}$$ where $g = s/\sigma$ is the ratio of prediction uncertainty (the parameter of the membership function used to describe an expert's prediction) to measurement error, which is called later a prediction uncertainty factor.

3.2. NORMAL MEASUREMENT DISTRIBUTION AND APPROXIMATE VALUE PREDICTION

We will investigate the properties of the modified estimate in this case analytically. We will consider a more general case of indirect measurements, where the measurement result could be obtained as a linear combination of a few sensor results, quite a typical situation in multisensor systems where the measurement includes averaging of a few sensor results, for example. We will continue to assume the probabilistic model of a normal distribution, which is the most widely applied in practice for measurement results (equation (3)) and the prediction model, in which the approximate values of linear combinations of a few variables are given (equation (4)), which is again the mostly anticipated feasibility in a case of sensor networks:

(3) $Y = AX + \varepsilon_Y$

(4) $f \approx BX$

where Y is a n×1 vector (under the condition of n > 1) of measurement results,

X is a k×1 vector (under the condition of k > 1) of true values of the measured variables,

ε_Y is a n×1 vector (under the condition of n > 1) of measurement errors,

f is is a m×1 vector (under the condition of m > 1) of the prediction values,

A, B are matrices giving the structures of measurement schemes and prediction models.

(3) is a classical measurement equation applied in measurement theory and standards. We will consider measurement results normally distributed with no bias and the covariance matrix Σy.

(4) describes the prediction made that m linear combinations of the measured variables (the combinations are given with the matrix B) approximately have values given by the vector f components. The predictions are described mathematically by using the membership functions of the type $\mu(x) = e^{-\frac{(x-f)^2}{s^2}}$ with the parameters of fuzziness given with the matrix Σb, which is a diagonal matrix of the rank m with elements calculated as squares of the prediction uncertainty parameters. For example, if an expert makes a prediction that five measured variables have certain

approximate values, this is modeled with five bell-shaped membership functions where the ith membership function has parameters f_i and s_i. The parameter f_i represents the predicted value of the measurement while the parameter s_i describes the prediction fuzziness. After that the matrix Σb is composed as a diagonal matrix of rank 5 and elements s_i^2.

With the direct measurements and predictions, matrices A and B become unit matrices and the equations (3) and (4) become simpler:

$$Y = X + \varepsilon_Y$$

$$X \approx f \quad \text{or} \quad x_1 \approx f_1,\ x_2 \approx f_2,\ \dots,\ x_k \approx f_k$$

meaning that the model predicts approximate values of all k measured variables.

We are ready now to attack optimization problems (1) and (2) formulated in section 2. We will be looking for a solution maximizing the likelihood function, which is composed from normal distribution probability functions and in a case of a prediction model the fuzzy model membership function is added. Under general conditions, a conventional maximum likelihood estimate, which does not take into account a fuzzy prediction model and is based only on measurement results could be found as a solution of the optimization problem (1) as

$$\hat{X} = (A^T \Sigma_y^{-1} A)^{-1} A^T \Sigma_y^{-1} Y \, ;$$

The modified estimate, which does take into account a fuzzy prediction model along with measurement results could be found as a solution of the optimization problem (2) as

$$\tilde{X} = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} (A^T \Sigma_y^{-1} Y + 2B^T \Sigma_b^{-1} f)\, ;$$

or in one variable case when we measure and predict just one variable one time

$$\hat{X} = Y \text{ ; and}$$

$$\tilde{X} = (Y/\sigma^2 + 2f/s^2)/(1/\sigma^2 + 2/s^2) = (Y + 2f/g^2)/(1 + 2/g^2)\,,$$

where $g = s/\sigma$ is the ratio of prediction uncertainty to measurement error, which is called later a prediction uncertainty factor.

The bias and the generalized dispersion of these estimates are equal correspondingly:

(5) $M(\hat{X}-X)=0$; $\quad \mathrm{cov}(\hat{X})=M[(\hat{X}-M\hat{X})(\hat{X}-M\hat{X})^T]=(A^T\Sigma_y^{-1}A)^{-1}$

(6) $M(\tilde{X}-X)=2(A^T\Sigma_y^{-1}A+2B^T\Sigma_b^{-1}B)^{-1}B^T\Sigma_b^{-1}(f-BX)$

(7) $\mathrm{cov}(\tilde{X})=M[(\tilde{X}-M\tilde{X})(\tilde{X}-M\tilde{X})^T]=(A^T\Sigma_y^{-1}A+2B^T\Sigma_b^{-1}B)^{-1}A^T\Sigma_y^{-1}A(A^T\Sigma_y^{-1}A+2B^T\Sigma_b^{-1}B)^{-1}$

where M() serves as a mean operator.

Within the framework given above the following statements describing the role of a fuzzy prediction model in measurement are formulated. They are directly derived from the formulas given above for normal measurement distribution and an approximate value predictions. Other cases presented in Table 1 concur to these statements as well.

Property 1. The modified measurement estimate that takes into account a fuzzy prediction model along with measurement results coincides with a conventional one that does not, if and only if the prediction value coincides with the conventional estimate.

As shown above, the modified estimate is the weighted sum of a conventional one and the prediction value. If and only if they coincide, the modified estimate will coincide with them also.

Corollary 1. An application of fuzzy prediction models shifts the measurement result.

Property2. The modified estimate sits between the conventional estimate and the prediction value.

This property is directly derived from the modified estimate being the weighted sum of the conventional estimate and the prediction value.

Corollary 2. An application of fuzzy prediction models shifts the measurement result towards the prediction value.

The dependence of a modified estimate's bias on the prediction uncertainty factor under different prediction errors in the case of one measured variable and one prediction made is given on fig. 10. One may see that the bias becomes pretty small when the prediction error is still about ten times higher than the prediction uncertainty. It means that the prediction model should be able to make reliable, practically unbiased predictions under certain conditions to be investigated further.

Property 3. Analyzing the formulae above one may see that a modified estimate's bias is proportional to the prediction error (f-BX).

This property could be directly seen in formula (6) above.

Property 4. One also can see from formula (6) that when a prediction is absolutely correct (f-BX=0) the modified estimate becomes unbiased as in this case $M(\tilde{X} - X) = 0$

Property 5.On the other hand, the same result can be achieved when the prediction fuzziness is very big.

The modified estimate becomes unbiased when the prediction model gives a correct prediction or refuses to make any prediction at all. Actually, the bias of the modified estimate mainly depends on the ratio between the prediction error and the prediction fuzziness parameter or in other words on the ratio between the prediction model correctness and the forecaster confidence in the prediction made. One can observe the properties introduced in this section on plots presented on figure 10, which demonstrates the bias of the modified estimate in the case of normally distributed measurement results and an approximate value prediction.

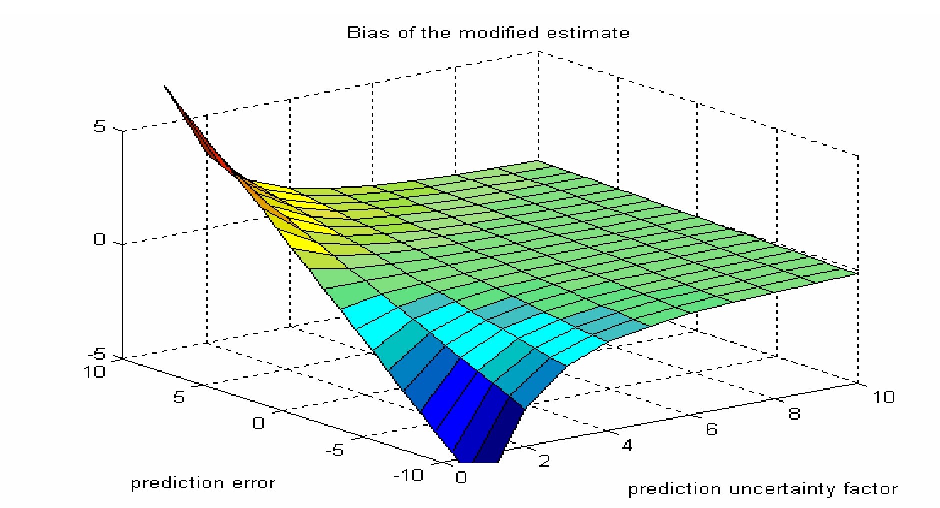


Figure 10. Dependence of the modified estimate's bias on the prediction uncertainty factor (the ratio of the prediction interval width to the measurement error) and the prediction error

4. Investigation of the gain/loss due to the prediction or what can we gain from using fuzzy prediction models?

Estimate's accuracy traditionally is taken as its variance. However, the variance can not be considered a good comparison base in the case when the modified estimate is biased due to the prediction. We have to consider another accuracy indicator, the mean square error (MSE), which is the mean of squares of deviations between the estimate and the true value. This indicator takes into account both the estimate's bias and its deviation from it. MSE of the considered estimates will equal correspondingly:

$$E_{\hat{X}} = M[(\hat{X} - X)(\hat{X} - X)^T] = (A^T \Sigma_y^{-1} A)^{-1};$$

$$E_{\tilde{X}} = M[(\tilde{X} - X)(\tilde{X} - X)^T] = (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1} (4B^T \Sigma_b^{-1} (f - BX)(f - BX)^T \Sigma_b^{-1} B +$$
$$+ A^T \Sigma_y^{-1} A)(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)^{-1};$$

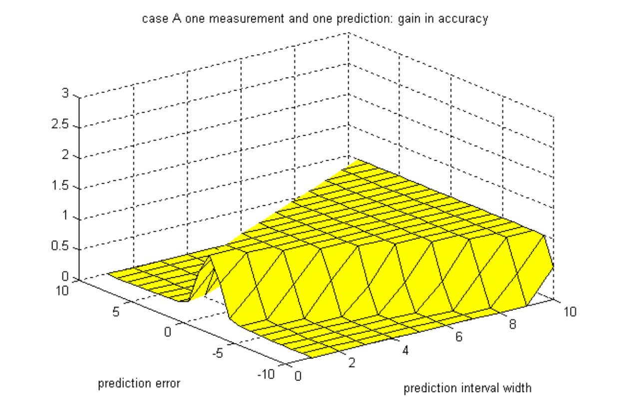
The problem of this gain evaluation deserves a special consideration. To evaluate the gain provided by an expert's information application, let us choose the projection of the estimate's MSE, which can be written as

$$(8)\quad T = \frac{1}{K} Tr(E_{\hat{X}} E_{\tilde{X}}^{-1}) =$$

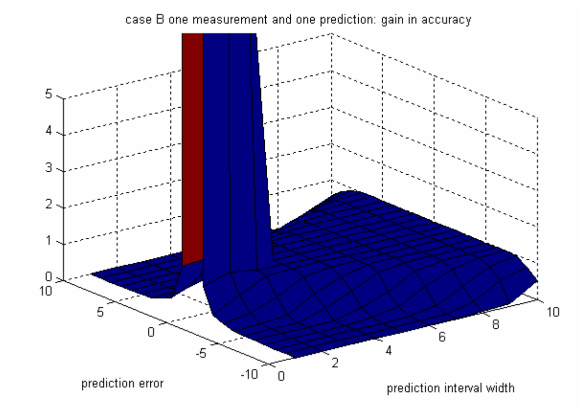
$$= \frac{1}{K} Tr\,[\,(A^T \Sigma_y^{-1} A)^{-1} (A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)(4B^T \Sigma_b^{-1} (f - BX)(f - BX)^T \Sigma_b^{-1} B + A^T \Sigma_y^{-1} A)^{-1}$$
$$(A^T \Sigma_y^{-1} A + 2B^T \Sigma_b^{-1} B)]$$

where K is the number of measured variables.

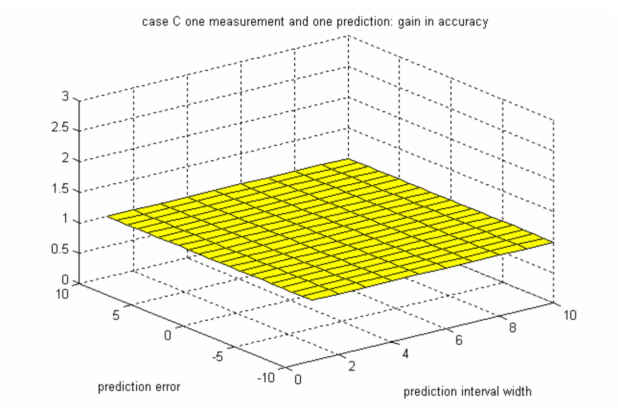
The gain depends on the measurement system structure and errors as well as prediction model errors and its uncertainty factors (actually the ratio between the measurement error to the prediction interval width). Figure 11 demonstrates the change in the gain value depending on the prediction interval width and the prediction errors when the mean measurement errors are fixed. The plots were constructed by computer simulation for four different combinations of measurement distributions and prediction types studied in this paper. The maximum gain could be achieved when the prediction is absolutely accurate (prediction error is zero). One can see that with the increase in prediction errors the gain goes down and transforms into loss when the error in prediction becomes considerably bigger than its fuzziness. This gain/loss transformation is more dramatic in the cases of normal measurement distribution in comparison to the uniform one, however, with the uniform distribution the maximum gain value is much smaller also.



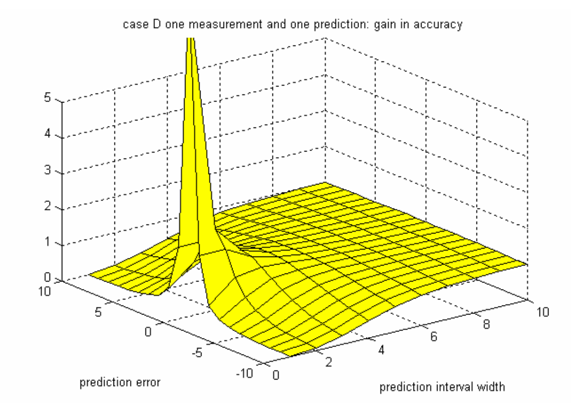
Uniform measurement distribution and interval prediction



Normal measurement distribution and interval prediction



Uniform measurement distribution and approximate prediction



Normal measurement distribution and approximate prediction

Figure 11. Relationship between the gain received by a modified estimate application and prediction interval widths and prediction errors (prediction errors and interval widths are scaled in measurement error units):

In a case of one measured value and one prediction the gain can be expressed in terms of a prediction uncertainty factor as

(9) $T = \sigma^2 (g^2 + 2)^2 / (4(f - x)^2 + g^4 \sigma^2)$

The biggest gain can be achieved with the correct predictions, i.e. f=BX, when

$T_{\max} = Tr[(U + 2(A^T \Sigma_y^{-1} A)^{-1} B^T \Sigma_b^{-1} B)^2]$,

where U is a unit matrix or in the case of direct measurements and predictions

$T_{\max} = Tr[(U + 2(\Sigma_y \Sigma_b^{-1})^2]$,

from where one may see that the maximum gain value depends on the ratio of the measurement errors to the prediction fuzziness. In another notation the maximum gain could be written as

(10) $T_{\max} = (1 + 2\sum_{i=1}^{K} \sigma_i^2 / \sum_{i=1}^{K} s_i^2)^2$, which in the case of one measured variable only becomes

(11) $T_{\max} = (1 + 2 / g^2)^2$.

One may see that in order to increase the maximum gain, the prediction fuzziness should be decreased in comparison with the measurement errors. One may also conclude that if the prediction fuzziness is much higher than the measurement errors, the use of a prediction model becomes doubtful as any possible gain value could be just a few percent. It means that in order to achieve a meaningful gain it is necessary to develop prediction models with a lower prediction interval width parameter. However, this strategy might be risky as it may result in loosing any gain at all as one can conclude from the formulae above and plots on figure 11.

5. INVESTIGATION OF THE GAIN AREAS OR UNDER WHICH CONDITIONS CAN WE GAIN FROM USING PREDICTION MODELS?

Property 6. Fuzzy prediction model application results in accuracy gain if the prediction errors are smaller than the weighted sum of measurement errors and prediction fuzziness – see (8) and (9)

Corollary. If the prediction errors are smaller than the prediction model fuzziness, than definitely the prediction model application will result in accuracy gain.

Let us clarify conditions when the modified estimate superiors a conventional one against the MSE. Mathematically the condition $E_{\tilde{X}} < E_{\hat{X}}$ can be shown equivalent to the conditions:

$B^T \Sigma_b^{-1} (f - BX)(f - BX)^T \Sigma_b^{-1} B < B^T \Sigma_b^{-1} B + B^T \Sigma_b^{-1} B (A^T \Sigma_y^{-1} A)^{-1} B^T \Sigma_b^{-1} B$;

or in other notation

(12) $(f - BX)(f - BX)^T < \Sigma_b + B(A^T \Sigma_y^{-1} A)^{-1} B^T$.

The left side of the inequality (12) constitutes the prediction error square, the right side combines the prediction fuzziness with measurement errors and depends on the structures of matrices A and B and values of Σ_y and Σ_b also. One may see that in order to increase the estimate accuracy, the prediction error should be not bigger than its fuzziness. Actually, it may be even larger by some value, which depends on the measurement errors. This relationship becomes clearer in the case of direct measurements and predictions when the matrices A and B are unit matrices, and matrices Σ_b and Σy are diagonals. In this case the condition (9) becomes more transparent as

(13) $\sum_{i=1}^{K}(f_i - x_i)^2 / K < \sum_{i=1}^{K} s_i^2 / K + \sum_{i=1}^{K} \sigma_i^2 / K$

where K is the number of measured variables,

σi, i=1, K is the root mean square error (RMSE) of the i-th variable measurement errors, and δi, i=1, K is the fuzziness of the ith prediction.

(13) in a case of one measurement and one prediction only becomes $(x - f)^2 \leq s^2 + \sigma^2$

Summarizing, one can conclude that use of prediction models improves accuracy if the prediction error square is less than the sum of the prediction fuzziness square and the measurement error square. Figure 12 presents the plots of gain areas calculated for other cases considered. The plots concur with this conclusion. One can see that while the narrow prediction fuzziness requires a precise prediction in terms of prediction error allowance, widening the prediction interval leaves significantly more space for prediction errors.

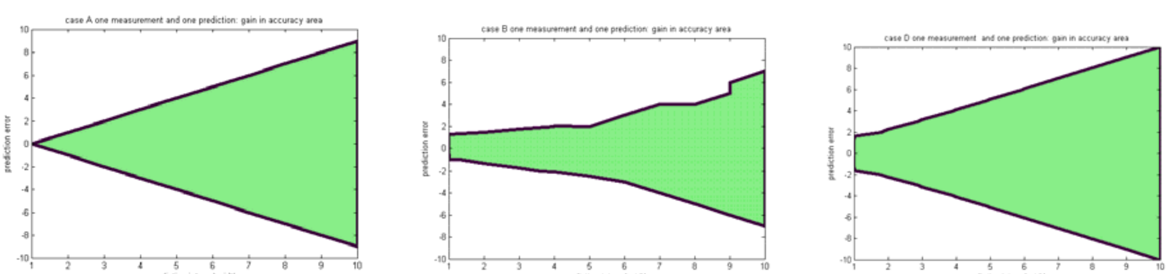


Figure 12. Gain areas for different combinations of measurement distributions and prediction types

6. PRACTICAL RECOMMENDATIONS AND EXAMPLES OF THE PREDICTION MODEL APPLICATION IN MEASUREMENT.

Or in other words what gain could be achieved with a rather inaccurate prediction? The answer to this question could be obtained by application of the formulas (8) in a general case and (9) in a case of one measurement and one prediction, which give the gain value as well as formulas (10) and (11), which present the maximum gain values under the condition of correct predictions. From these formulae one can see that the maximum gain value depends mainly on the prediction uncertainty factor or the ratio between the prediction interval width to the measurement error. The typical measurement accuracy for the modern measurement instruments could be in the vicinity of 1-2%. In this case, of say 2% measurement error, with the prediction fuzziness of 10% (for example, it might mean the prediction like "the measured variable has a value of around 10 units or actually somewhere roughly between 9 and 11 units", could achieve the gain up to 17%. In complex measurement systems exploiting sensor networks, accuracy could actually be much lower. In a case of around 10% measurement errors, the same predictions as in the previous example could achieve 900% gain. One should understand that such values of gain could be achieved when the prediction model gives an absolutely correct prediction.

However, even in a case when a prediction model includes a prediction error, there could still be some gain. Speaking very roughly, in order to get any gain the error value should be smaller than a sum of the prediction fuzziness and the measurement errors. This relationship allows a forecaster to develop a strategy to avoid any loss. If a forecaster is confident about the prediction value, a fuzzy prediction model with low prediction fuzziness should be applied and a high gain could be achieved. However, when the confidence level decreases, the prediction fuzziness could be increased that might make the gain value lower but allows avoiding losses.

Sensor networks example. Sensor networks, which converge the Internet, communications, and information technologies with miniaturization techniques have provided new opportunities for acquiring and communicating huge arrays of information coming from heterogeneous sensors and sensor arrays.

The Wireless Sensor Network (WSN) research platform used in this research includes the TelosB Berkeley Motes which were originally manufactured by Crossbow Technologies Inc. In this example we consider an application of three motes, each of which includes three sensors: illumination, temperature and humidity. Those sensors have the following graduation characteristics.

Humidity and Temperature sensors are located in the external Sensirion sensor. Their readings can be converted to SI units as follows:

for Temperature, Oscilloscope (software utility used in this project to collect and transmit measurement results) returns a 14-bit value that can be converted to degrees Celsius according to the formula:

temperature = -39.60 + 0.01*SOt , where SOt is the raw output of the sensor.

Humidity is a 12-bit value that is not temperature compensated and is calculated according to the formula: humidity = -4 + 0.0405*SOrh + $(-2.8 * 10^{-6})*(SOrh^2)$, where SOrh is the raw output of the relative humidity sensor. Using this calculation and the temperature measurement, one can correct the humidity measurement with temperature compensation

humidity_true = (Tc - 25) * (0.01 + 0.00008*SOrh) + humidity

where Tc is the temperature measured in degrees Celcius, SOrh is the raw output of the relative humidity sensor, and humidity is the uncompensated value calculated in equation above. We conducted experiments with a few motes (up to four) in a particular case measuring temperature and humidity indoors (college building, student dorm). The temperature measurements vary in 6 -7 bits, producing a measurement error of up to 110 sensor units. These measurement errors could be substituted into formula (3), where the matrix A is composed of the coefficients calculated from the above graduation parameters and the measurement scheme applied. However, assuming that the measurements of different sensors are not correlated as they were received from different motes, we may recalculate the measurement errors in degrees, which will be 1.1 degree and apply it to all four sensors. Let us suppose that an expert may give the approximate value temperature prediction with a prediction interval up to +- 5 degrees that is a very reasonable assumption. Fig. 13 gives the calculation of a possible gain from the application of an expert's prediction, while fig. 14 presents the possible gain area with four sensors measuring and transmitting the temperature. One can see that with a rather high confidence (the prediction fuzziness is about one degree) one can achieve a gain in accuracy of up to 50% under the condition that the prediction is accurate. Here the prediction error should be less than one degree, other wise the loss will occur. However, widening the prediction fuzziness to about 5 degrees will allow tolerating prediction errors of up to three degrees.

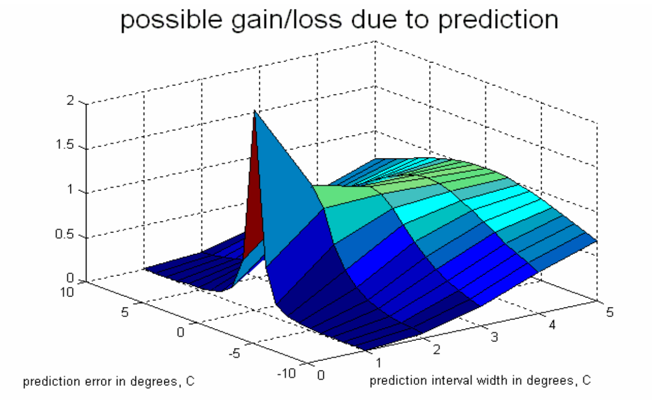


Figure 13. Dependence of the possible gain with four temperature sensors on the prediction error and the prediction fuzziness (prediction interval width in degress Celcius)

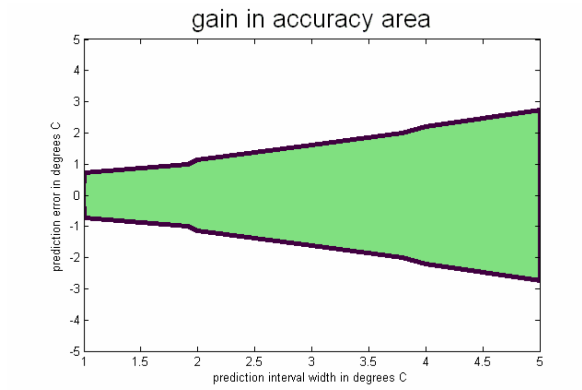


Figure 14. Dependence of the possible gain area with four temperature sensors on the prediction error and the prediction fuzziness (prediction interval width in degress Celcius)

7. CONCLUSION

Measurement as an underlying concept of scientific cognition is the process of formulating and modifying models of the systems under measurement and the environment based on observation results received. Different model types are applied in measurement. Their relationship and their place in the model hierarchy introduced by L.Zadeh in his theory of generalized definability [22] as well as the benefits we can get from their application call for a further study.

The problem of a fuzzy prediction model use for improvement the measurement procedures quality and the accuracy of received estimates may be solved by fusion of information from different sources, which are characterized by different degrees of uncertainty. The problem has been formalized mathematically as an optimization problem with fuzzy constraints and the solution has been found for the normally distributed measurement results and a specific expert's information.

The properties of the modified estimates have been investigated in comparison with the conventional ones. The modified estimates have been found more efficient under the condition when the prediction error does not overcome the sum of the average measurement error and the prediction fuzziness. The possible efficiency gain in practical applications was estimated. The procedures improving reliability of the modified estimates have been offered, which include recommendations on formulating prediction models: when the confidence in predicted value is high, a prediction fuzziness should be made low in order to achieve a high gain in accuracy; however, with a decrease in confidence a fuzziness could be made wider in order to avoid the estimate corruption and loss in accuracy.

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