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DETECTING CRACKS IN THIN PLATES BY USING LAMB WAVE SCANNING: GEOMETRIC APPROACH

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Abstract. *A crack in a thin plate reflects ultrasonic waves; therefore, it is reasonable to determine the location of the crack by measuring the reflected waves. The problem of locating the crack can be reformulated in purely geometric terms. Previously, time-consuming iterative numerical methods were used to solve the resulting geometric problem. In this paper, we show that explicit (and fast to compute) formulas can be used instead.*

Formulation of the engineering problem. One of the most common problems in aging aircraft structures is the presence of cracks. These cracks are often not visible because they are hidden inside the structure or covered with paint. It is therefore necessary to use techniques of non-destructive testing (NDT) such as ultrasonic Lamb waves.

Lamb waves in thin plates are very convenient in detecting cracks in large-scale structures because these waves can propagate long distances and thus, can help us explore large portions of the plate; see, e.g., (Viktorov 1967).

In a faultless plate, a Lamb wave can travel long distances without dispersion or reflection. Defects reflect and scatter these waves; as a result, the very presence of a reflected wave indicates a defect. It is reasonable to determine the location of the crack by measuring the reflected waves.

Reduction to a geometric problem. To locate the crack, we generate a wave pulse that is sent, via a transmitter T, to the plate. This pulse propagates through the plate and reaches a sensor S.

In a faultless plate, the only signal we receive at S is a signal that goes directly from T to S; this signal is received at a time $t_1 = t_0 + d_0/v$, where t_0 is the moment of time when the original signal was sent, d_0 is the distance between T and S, and v is the (known) velocity with which the Lamb waves propagate.

In a plate with defects, in addition to this direct signal, we also observe the signal reflected from a defect; this reflected signal arrives at S at a moment $t_2 = t_0 + d/v$, where d is the length of the path TFS = TF + FS from T to S via a reflecting point F on the fault. Since we measure t_2 and we know the values t_0 and v, we can therefore determine the distance d as $v \cdot (t_2 - t_0)$.

If we move the sensor a little bit, to a new location S′ at a small distance s from the old one, then the reflection point shifts a little bit to a new point F′, and the path length changes from d to a new value d'.

On a large scale, a crack is usually reasonably smooth. Therefore, between the two close points F and F′, the shape of a crack can be approximated by a straight line segment. Thus, we arrive at the following geometric problem (see Fig. 1):

- We know the location of three points T, S, and S′ on the plane.
- We know that there is a segment FF′ of a straight line ℓ on the same plane.
- We know the length d of the two-line-segment path that starts at T, gets reflected by ℓ at a point F $\in \ell$, and ends at S.
- We also know the length d' of the two-line-segment path that starts at T, gets reflected by ℓ at a point F′ $\in \ell$, and ends at S′.
- Our objective is to locate the points F and F′.

How this problem was solved before. For the (unknown) reflection point F, we know the sum TF + FS of the distances from two known points: T and S. It is a known geometrical fact that for any given two points T and S, the set of all points F with a given sum TF + FS is an ellipse. Due to Snell's law describing wave reflection, the angle between the incoming wave and the crack must be the same as between the crack and the outcoming wave.

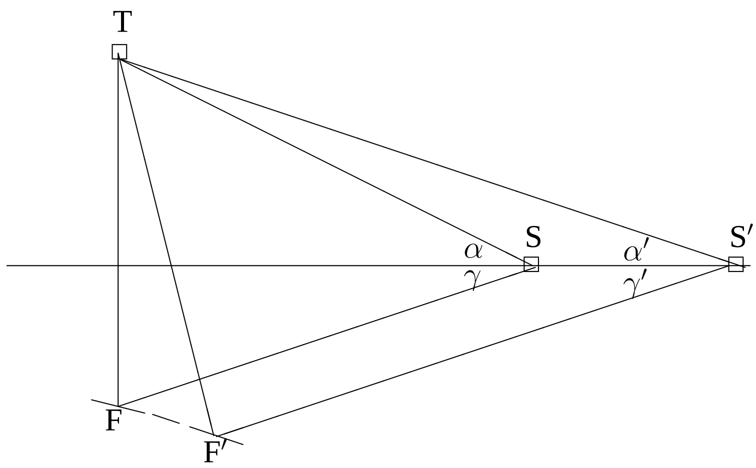

Fig. 1

Due to the properties of an ellipse, we can conclude that the crack is tangent to this ellipse at the reflection point F. Similarly, the crack is tangent to an ellipse of all the points F′ for which TF′ + F′S $= d'$. Thus, the crack can be determined as a common tangent to two known ellipses. In (De Villa et al., 2001), this idea was used to determine the crack location: explicit equation for tangents were written down, and the resulting system of equations was solved by a numerical technique.

What is main deficiency of the known solution. In (De Villa et al., 2001), a (time-consuming) iterative numerical methods were used to solve the resulting geometric problem. It is desirable to use (if possible) an explicit, faster-to-compute method instead. Such a method is presented in this paper.

Main ideas. The first idea is to take into consideration that the path $d =$ TF + FS is equal to the distance between the sensor S and the reflection R of the transmitter T in the straight line ℓ that extends the fault segment FF′ (see Fig. 2):

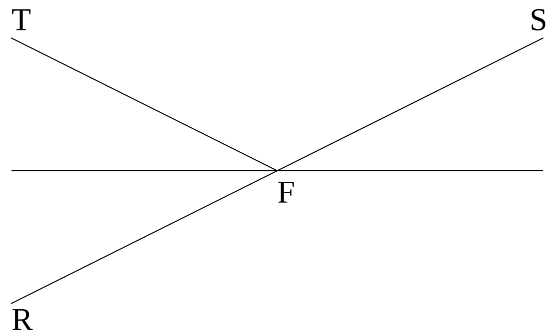

Fig. 2

Indeed, TF = RF, and due to Snell's law, RF is a continuation of FS, so RS = RF + FS = TF + FS $= d$. Similarly, RS$' = d'$. In the triangle $\triangle$RSS$'$, we thus know all three sides and hence, we can use the Law of Cosines to determine the angle $\angle$RSS$' = \pi - \gamma$:

$$(d')^2 = d^2 + s^2 - 2d \cdot s \cdot \cos(\pi - \gamma),$$

hence, since $\cos(\pi - \gamma) = -\cos(\gamma)$, we conclude that

$$\cos(\gamma) = \frac{(d')^2 - d^2 - s^2}{2d \cdot s}. \tag{1}$$

We now know the direction from the sensor S to the fault point F; to determine the distance r from S to F, we can apply the Law of Cosines to the triangle $\triangle$TFS. In this triangle, we know the angle $\angle$TSF $= \alpha + \gamma$ and we know that TS $= d_0$, SF $= r$, and TF $= d - r$ (see Fig. 3):

Therefore,

$$(d - r)^2 = r^2 + d_0^2 - 2r \cdot d_0 \cdot \cos(\alpha + \gamma).$$

Opening parentheses and canceling the terms r^2 in both sides, we get a linear equation for r, hence

$$r = \frac{d^2 - d_0^2}{2(d - d_0 \cdot \cos(\alpha + \gamma))}. \tag{2}$$

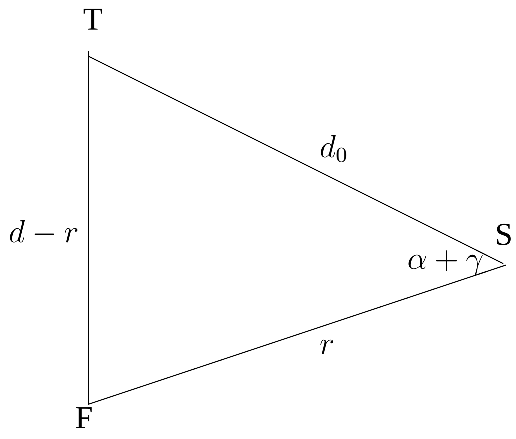

Fig. 3

New algorithm and results. The resulting new algorithm is as follows: We know the propagation speed v of the Lamb waves. Based on the known location of the points T, S, and S′, we compute the distance $d_0 =$ TS, the distance $s =$ SS′, and the angle α. We send a pulse signal at time t_0, we measure the time t_2 when the second pulse arrives at the sensor S, and we compute the distance $d = v \cdot (t_2 - t_0)$. We move S to a new location S′ at a known distance s from S, repeat the experiment and compute the new distance d'. Then, we use the formula (1) to compute the angle γ between the known line SS′ and the direction to the fault, and we compute the distance $r =$ SF by using the formula (2). Once we know the angle and the distance, we can find the location of the fault point F.

Similarly, we can find the location of F′. As we move the sensor along the line SS′, we can find several points on the fault and thus, the location and shape of the fault.

We have successfully used this algorithm to find cracks, in particular, to find cracks near rivet holes where other methods have difficulty finding these cracks; see, e.g., (Osegueda et al. 2002) and (Osegueda et al. 2003).

Alternative geometric set-up. In the previous set-up, we fix the location of the transmitter T, and moved the senor S. As we move the sensor further away from T, the signal fades, and the sensitivity of this method decreases. An alternative idea is therefore to fix the connection between T and S and to move both T and S at the same time (in the direction SS′ which is orthogonal to TS), so that TS = T′S′ $= d_0$. How can we now find the fault location?

Geometric analysis of the new set-up. In the new set-up, the path $d =$ TFS measured by the first sensor is equal to the distance SR between S and the reflection R of the point T in the fault FF′. Similarly, the path $d' =$ T′F′S′ measured by the second sensor is equal to the distance S′R′ between S′ and the reflection R′ of the point T′ in the fault FF′.

Let β denote the angle between the fault FF′ and the direction SS′ in which the sensor moves. We know that TS $\perp$ SS′, and, due to the properties of reflection, RT $\perp$ FF′; therefore, the angle $\angle$RTS between RT and TS is also equal to β. Similarly, $\angle$R′T′S′ $= \beta$.

Due to the properties of reflection, the distance RR′ is equal to TT′ $= s$. Since TT′ $\|$ SS′, the line TT′ is at angle β to the reflecting line FF′, hence RR′ is also at angle β from the reflecting line; see Fig. 4. If from R and T we draw the lines RR″ and TT″ that are parallel to FF′ (and which are hence orthogonal to RT and R′T′), then we conclude that R′R″ = T′T″ $= s \cdot \sin(\beta)$ hence

$$\mathrm{R'T'} - \mathrm{RT} = \mathrm{R'R''} + \mathrm{T'T''} = 2s \cdot \sin(\beta).$$

If we denote an average of RT and R′T′ by M, we can thus conclude that RT $= M - d \cdot \sin(\beta)$ and R′T′ $= M + d \cdot \sin(\beta)$.

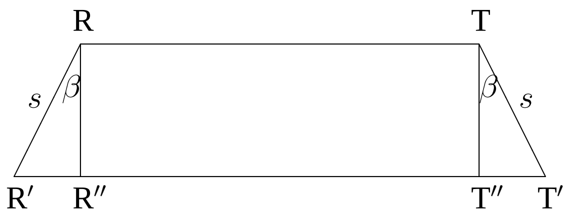

Fig. 4

In the triangle $\triangle$RTS, RT $= M - s \cdot \sin(\beta)$, $\angle$RTS $= \beta$, TS $= d_0$, and RS $= d$; therefore, the Law of Cosines leads to:

$$d^2 = (M - s \cdot \sin(\beta))^2 + d_0^2 - 2(M - s \cdot \sin(\beta)) \cdot d_0 \cdot \cos(\beta). \quad (3)$$

Similarly,

$$(d')^2 = (M + s \cdot \sin(\beta))^2 + d_0^2 - 2(M + s \cdot \sin(\beta)) \cdot d_0 \cdot \cos(\beta). \quad (4)$$

Subtracting (3) from (4), we conclude that:

$$(d')^2 - d^2 = 4M \cdot s \cdot \sin(\beta) - 4s \cdot d_0 \cdot \sin(\beta) \cdot \cos(\beta),$$

hence for

$$z_1 \stackrel{\rm def}{=} \frac{(d')^2 - d^2}{4s} \quad (5)$$

we get the formula

$$z_1 = \sin(\beta) \cdot (M - d_0 \cdot \cos(\beta)). \quad (6)$$

Averaging (3) and (4), we conclude that for

$$z_2 \stackrel{\rm def}{=} \frac{(d')^2 + d^2}{2}, \quad (7)$$

we get the formula

$$z_2 = M^2 + s^2 \cdot \sin^2(\beta) + d_0^2 - 2M \cdot d_0 \cdot \cos(\beta). \quad (8)$$

From (6), we conclude that

$$z_1^2 = \sin^2(\beta) \cdot (M^2 - 2M \cdot d_0 \cdot \cos(\beta) + d_0^2 \cdot \cos^2(\beta)), \quad (9)$$

and from (8), that

$$z_2 \cdot \sin^2(\beta) = \sin^2(\beta) \cdot (M^2 + s^2 \cdot \sin^2(\beta) + d_0^2 - 2M \cdot d_0 \cdot \cos(\beta)). \quad (10)$$

Subtracting (9) from (10), we conclude that

$$z_2 \cdot \sin^2(\beta) - z_1^2 = s^2 \cdot \sin^4(\beta) + d_0^2 \cdot \sin^4(\beta),$$

i.e., that

$$(s^2 + d_0^2) \cdot \sin^4(\beta) - z_2 \cdot \sin^2(\beta) + z_1^2 = 0.$$

This is a quadratic equation in terms of the unknown $\sin^2(\beta)$, so

$$\sin^2(\beta) = \frac{z_2 - \sqrt{z_2^2 - 4z_1^2 \cdot (s^2 + d_0^2)}}{2(s^2 + d_0^2)}. \tag{11}$$

One we know the angle β, we can use the formula (6) to determine $M = z_1/\sin(\beta) + d_0 \cdot \cos(\beta)$ and hence, $\mathrm{RT} = M - s \cdot \sin(\beta)$ as

$$\mathrm{RT} = \frac{z_1}{\sin(\beta)} + d_0 \cdot \cos(\beta) - s \cdot \sin(\beta). \tag{12}$$

Let us select the coordinate system in which the x-axis is parallel to SS′, and the y-axis is parallel to ST. We know the coordinates x_T and y_T of the point T, we know the angle β between TS (i.e., the y-axis) and the direction TR, and we know the distance RT; thus, we can find the coordinates (x_R, y_R) of the point R as

$$x_R = x_T - \mathrm{RT} \cdot \sin(\beta); \;\; y_R = y_T - \mathrm{RT} \cdot \cos(\beta). \tag{13}$$

The midpoint m between the point T and its reflection R in the line that extends FF′ is a point on this extended line ℓ; its coordinates are

$$x_m = \frac{x_T + x_R}{2}; \;\; y_m = \frac{y_T + y_R}{2}. \tag{14}$$

By definition of the angle β, the fault segment FF′ forms an angle β with the line SS′ (i.e., with the x-axis). Therefore, the line ℓ goes through this point m at the angle β with the x-axis, hence the line ℓ is described by the equation:

$$y = y_m - \tan(\beta) \cdot (x - x_m). \tag{15}$$

The fault point F is the intersection between the line ℓ and the line SR. The point S has coordinates

$$x_S = x_T; \;\; y_S = y_T - d_0; \tag{16}$$

therefore, the equations of the line SR can be described as:

$$y = y_R + \frac{y_S - y_R}{x_S - x_R} \cdot (x - x_R). \tag{17}$$

We can therefore find the coordinates x and y of the fault point F as the solution to the system of two linear equations (15) and (17) with two unknowns – a solution that can be obtained explicitly in terms of the coefficients.

Resulting algorithm for the alternative set-up. We know the propagation speed v of the Lamb waves, we know the distance d_0 between the transmitter T and the sensor S. We send a pulse signal at time t_0, we measure the times t_2 when the second pulse arrives at the sensor S, and we compute the distance $d = v \cdot (t_2 - t_0)$. Then, we move the combination of T and S to a new location T′S′ at a distance s from TS, repeat the experiment and compute the new distance d'.

We compute z_1 and z_2 by using the formulas (5) and (7), then the angle β by using the formula (11), then RT from the formula (12), and the coordinates of the points R, m, and S from formulas (13), (14), and (16). After that, we solve the system of two linear equations (15) and (17) with two unknowns x and y and find the coordinates of the point F on the fault.

Similarly, we can find the location of F′. As we move the transmitter and the sensor, we can find several points on the fault and thus, the location and shape of the fault.

Open problem. In the above algorithms, we approximated (locally) a smooth-shaped crack as a straight line segment. The closer the sensor locations S and S′ are to each other, the better this approximation. However, if we make these locations too close, then the difference between the signals received at these locations will get below the noise level and thus, we will be unable to locate the fault. So, to increase the approximation accuracy – and thus, to increase the accuracy of fault location – it is desirable to use a more accurate approximation to the fault shape.

A natural idea is to take second order (curvature) terms into consideration and represent a crack as a circular arc. For this representation, can we still get explicit formulas for reconstructing fault location?

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