Interval Methods in Non-Destructive Testing of Material Structures

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Abstract

In many practical situations, e.g., in aerospace applications and in mammography, it is important to test the structural integrity of material structures. We show that interval methods can help.

1 Introduction

Integrity testing is often very time-consuming and expensive. In many practical situations, it is very important to test structural integrity of material structures:

- For example, flight is a very stressful experience for materials and structures. As a result, even small faults in aerospace structures can lead to catastrophic results. It is therefore desirable to test the structural integrity of an airplane and/or a space structure before the flight (and ideally, in-flight as well) and detect potential faults.
- Mammography is another example where detecting small faults is extremely important for detecting possible breast cancer at the early stage, when it is still possible to cure it.

Several non-destructive testing techniques are currently used. One of the most useful is ultrasound testing, in which we send the ultrasound and detect the faults by measuring the ultrasound that passed through the tested structure or that was reflected by it.

One possibility is to have a point-by-point ultrasound testing, the so called *C-scan*. This testing detects the exact locations and shapes of all the faults. Its main drawback, however, is that since we need to cover every point, we get a very time-consuming (and therefore, very expensive) testing process.

A faster idea is to send waves through the material so that with each measurement, we will be able to test not just a single point, but the entire line between the transmitter and the receiver. To make this procedure work, we need special signals called *Lamb waves*.

There are other testing techniques. All these techniques aim at determining whether there is a fault, and if there are faults, what is the location and the size of each fault.

How can we save time and money? In spite of many time-saving ideas, for each of these methods, we must still scan a huge area for potential small faults. As a result, testing requires lots of time, and is very expensive. How can we save the time and cost of testing? Our main idea is this:

The existing testing procedures are very expensive and time-consuming because they try not only to check whether there is a fault, but also to find its location and size. If our only goal is to detect the fault, and we are not interested in its exact location, then the problem becomes much simpler and hopefully, easier to solve. Therefore, we suggest the following two-step testing:

- First, we apply a simpler test to check whether there is a fault.
- Only when the first test detects the presence of a fault, we run more expensive tests to locate and size this fault.

This two-step procedure is very similar to medical testing: In medical testing, first, the basic parameters are tested such as body temperature, blood pressure, pulse, etc. If everything is OK, then the person is considered healthy. Only if something is not OK, then the whole battery of often expensive and time-consuming tests is used to detect what exactly is wrong with the patient.

So the question is: How can we detect the presence of a fault?

2 Our main idea: checking non-linearity

General description. Let us first describe this idea in general terms. For testing, we send a signal and measure the resulting signal. The input signal can be described by its intensity $x_1, \ldots, x_n$ at different moments of time. The intensities $y_1, \ldots, y_m$ of the resulting signal depend on x_i: $y_j = f_j(x_1, \ldots, x_n)$, where the functions f_j depend on the tested structure.

Usually, we do not know the exact analytical expression for the dependency f_j, so we can use the fact that an arbitrary continuous function can be approximated by a polynomial (of a sufficiently large order). Thus, we can take a structure, try a general linear dependency first, then, if necessary, general quadratic, etc., until we find the dependency that fits the desired data.

If a structure has no faults, then the surface is usually smooth. As a result, the dependency f_j is also smooth; we can expand it in Taylor series. Since we are sending relatively weak signals x_i (strong signals can damage the plane), we can neglect quadratic terms and only consider linear terms in these series; thus, the dependency will be *linear*.

A fault is, usually, a violation of smoothness (e.g., a crack). Thus, if there is a fault, the structure stops being smooth; hence, the function f_j stops being smooth, and therefore, linear terms are no longer sufficient. Thus, *in the absence of fault, the dependence is linear, but with the faults, the dependence is non-linear.*

So, we can detect the fault by checking whether the dependency of y_j on x_i is linear.

Comment. The idea that non-linear terms can be helpful has been suggested some time ago; see, e.g., [2].

Non-linear terms simplify the detection of non-smoothness. When a smooth object acquires a fault, two changes occur in the dependence of y_i on x_j: first, linear terms change; second, non-linear terms appear. Thus, there are two possible methods of detecting non-smoothness: we can compare the linear response with the response of an ideal (smooth) object, or we can try to detect non-smoothness by finding non-linearity.

The first method works well if we do know the ideal response function f_i. However, in some cases (e.g., in mammography), we *do not know* the linear response of the fault-less object. Or, alternatively, we may know the original response, but we also know that this response can change not only because of faults, but also because of stress, material wariness, and other factors that do not necessarily mean that there is a dangerous fault inside. In such cases, we *cannot* detect the fault by comparing the current *linear* response with the ideal one; however, if we detect *non-linear* terms, it is a clear indication that there are some faults inside.

The resulting proposal: main idea. As a result of the above analysis, we propose the following way of detecting faults:

- We apply different signals x_j to the object, and measure the response y_i.
- If the measurement results are consistent with the linear dependence of y_i on x_j, this means that there are no faults, and no further testing is needed.

- If the measurement results are inconsistent with the linear model, this means that there is a fault, and so further thorough tests are needed.

Checking linearity is easy. As a result, for non-destructive testing of material structures, we get a simple test that enables us to save time and resources (necessary for the detailed solution of the inverse problem) by limiting this detalization only to the cases when the presence of the faults was revealed by non-linearity.

Let us confirm that non-smoothness leads to non-linearity. To show the above non-linearity is indeed practically detectable and thus, practically useful, we will present mechanical analysis and experimental results.

3 Interval methods are needed

After K measurements, we have K sets of data $\tilde{x}_1^{(k)},\ldots,\tilde{x}_n^{(k)},\tilde{y}_1^{(k)},\ldots,\tilde{y}_m^{(k)}$, $1 \le c \le K$. Often, we do not know the probabilities of different measurement errors, we only know the upper bounds for these errors. So, we know the *intervals* $X_1^{(k)},\ldots,X_n^{(k)},Y_1^{(k)},\ldots,Y_m^{(k)}$ of possible values of the measured quantities. We want to check whether this dependence can be linear, i.e., whether these exist coefficients c_{ij} for which, for every k and i, $\sum c_{ij}\cdot x_j^{(k)} \in Y_i^{(k)}$ for some $x_j^{(k)} \in X_j^{(k)}$. This is a known problem of interval computations: check whether the given system of interval linear equations is solvable (here, the unknowns are c_{ij}, interval coefficients are $X_j^{(k)}$ and $Y_i^{(k)}$).

Our main concern is not to miss the fault, so we need *guaranteed* methods. Thus, we need to use interval (guaranteed) methods for solving linear interval systems. For square matrices, methods of solving such systems are described, e.g., [1, 10, 11]; methods of solving systems with general (not necessarily square) matrices are described, e.g., in [9, 11]; a bibliography of such methods is given on p. 169 of [10].

From the mathematical viewpoint, this problem is similar to the problem of *identification* of linear systems under interval uncertainty. For a latest survey of interval identification problem and related algorithms, see, e.g., [15], Section 5.4.2.1, and references therein; practical applications of linear interval identification are described also, e.g., in [6, 7, 8, 12, 14]. The main difference between our problem and identification is as follows:

- in identification of linear systems, we assume that the dependence is linear, and we try to determine possible values of the coefficients c_{ij};
- in our problem, we are not directly interested in the exact values of the coefficients, only in knowing whether such coefficients exist.

4 Mechanical analysis of non-linearity

In this section, we present a simplified mechanical explanation of non-linearity. This explanation is too oversimplified to explain the *quantitative* experimental results, but it explains, on the *qualitative* level, why non-linearities do occur.

In order to understand how non-linear effects can occur, let us first describe how the signal travels through a fault-less plate. In this case, at the location of the transmitter, we send, at any given moment of time t, the signal $x(t) = A\cdot\cos(\omega\cdot t)$. This signal travels to the receiver (measuring device) with a velocity equal to the speed of sound. For simplicity, we can assume that the plate is homogeneous, so at any point, we have the same speed of sound v. Thus, while traveling from the transmitter to the receiver, the signal gets delayed by the amount of time $\Delta t = \dfrac{d}{v}$, where d is the distance between the transmitter and the receiver. As a result, at any moment of time t, the values of the observed signal $y(t)$ is proportional to value $x(t-\Delta t)$ that the input signal had Δt seconds ago: $y(t) = c\cdot x(t-\Delta t)$, where the coefficient c describes the loss of amplitude.

Thus, for a fault-less plate, we indeed have a linear dependence between the transmitted signal $x(t)$ and the measured signal $y(t)$.

Let us now consider the case when a fault lies between the transmitter and the receiver. This fault may be a crack or a hole. In this case, we can also use the formula $y(t) = c\cdot x(t-\Delta t)$, where Δt is the delay. However, this delay can no longer be computed simply as $\dfrac{d}{v}$, because, in addition to going straight through the material, the signal has to go either through or around the crack. In both cases, the presence of the crack changes the travel time:

- If the ultrasound has to travel through air, then it is delayed because the speed of sound in the air is smaller than the speed of sound in the solid body.
- If the ultrasound has to go around the crack, then the speed of sound stays the same, but the length of the path increases, and so the signal is also delayed.

In both cases, the delay Δt between the transmitter and the receiver can be computed as $\Delta t = \frac{d}{v} + c_f \cdot d_0$, where d_0 is the linear size of the fault, i.e., the distance between the front and the rear borders ("walls") of the fault area (front and rear with respect to the transmitter), and the coefficient c_f describes how fast the signal passes the fault area. As a result, the measured signal is equal to $y(t) = c \cdot x(t - \Delta t) = c \cdot A \cdot \cos(\omega \cdot t - \omega \cdot \Delta t)$. Since we are interested in detecting small faults, the value d_0 is small, so we can expand the expression for $y(t)$ in terms of d_0 and keep only the first few terms. As a result, we get the following formula

$$y(t) = c \cdot A \cdot \cos\left(\omega \cdot t - \omega \cdot \frac{d}{v}\right) + c \cdot c_f \cdot d_0 \cdot A \cdot \sin\left(\omega \cdot t - \omega \cdot \frac{d}{v}\right) + o(d_0). \tag{1}$$

Before we send the signal, the plate is immobile, and the distance d_0 stays constant: $d_0(t) = d_0^{(0)}$. However, as we transmit the signal $x(t)$, the plate starts vibrating, and this vibration changes the position of both borders and therefore, changes the distance d_0: $d_0 = d_0(t)$. In order to describe this change, let us denote the distance between the transmitter and the fault's front border by d_f. By the time the signal reaches this left border, it is delayed by the time $\frac{d_f}{v}$, i.e., takes the form $x_{\text{front}}(t) = c_{\text{front}} \cdot A \cdot \cos\left(\omega \cdot t - \omega \cdot \frac{d_f}{v}\right)$. This vibration causes the corresponding change in the location of this front border: instead of being equal exactly to d_f, this location oscillates around x_f. At any given moment of time, the change in location is proportional to the amplitude $x_{\text{front}}(t)$ of the oscillating signal:

$$d_{\text{front}}(t) = d_f + c_{\text{mov}} \cdot x_{\text{front}}(t) = d_f + c_{\text{mov}} \cdot c_{\text{front}} \cdot A \cdot \cos\left(\omega \cdot t - \omega \cdot \frac{d_f}{v}\right),$$

for some coefficient c_{mov}.

Similarly, the signal that passes to the rear border gets delayed by $\approx \frac{d_f}{v} + c_f \cdot d_0^{(0)}$. Thus, the location location of the rare border also changes, as

$$d_{\text{rear}}(t) = d_f + c_{\text{mov}} \cdot x_{\text{rear}}(t) = d_f + c_{\text{mov}} \cdot c_{\text{front}} \cdot A \cdot \cos\left(\omega \cdot t - \omega \cdot \frac{d_f}{v} - \omega \cdot c_f \cdot d_0^{(0)}\right).$$

As a result of these slightly different oscillations, the size $d_0(t) = d_{\text{rear}}(t) - d_{\text{front}}(t)$ also changes with time. We have already mentioned that the size d_0 is small, so we can expand the expression for $d_0(t)$ in terms of $d_0^{(0)}$ and keep only the first few terms. As a result, we get the following formula:

$$d_0(t) = d_0^{(0)} + c_{\text{mov}} \cdot c_{\text{front}} \cdot A \cdot \omega \cdot c_f \cdot d_0^{(0)} \cdot \sin\left(\omega \cdot t - \omega \cdot \frac{d_f}{v}\right) + o(d_0^{(0)}). \tag{2}$$

Substituting (2) into (1), we get, in $y(t)$, in addition to terms proportional to $\cos(\omega \cdot t)$, also quadratic terms $\sin^2(\omega \cdot t)$ which lead to double frequency terms in the Fourier transform of $y(t)$. These terms are proportional to A^2.

Similarly, we get cubic terms, etc.

5 Experimental confirmation of non-linearity

First experiments: pseudo-random signals. The first experimental confirmation that for an ultrasonic scan, faults do cause non-linear terms, was presented in [16]. Namely, it was known that for a fault-less plate, the dependence of the measured signal $y(t)$ on the transmitted signal $x(t)$ is linear, i.e., $y(t) = \int A(t-s) \cdot x(s)\, ds$ for some function $A(t)$. It turned out that for a plate with a fault, this dependence is non-linear: namely, cubic terms must be taken into consideration. To detect this non-linearity, the authors of [16] used *pseudo-random signals* that combine components of several different frequencies with pseudo-random amplitudes and pseudo-random phases.

The data from [16] shows that the amplitude of the cubic term is roughly proportional to the cube of the linear fault size. Thus, not only the non-linear terms indicate the *presence* of the fault, but also the value of the cubic term can be used to determine the *size* of the fault.

Pseudo-random signals are difficult to generate, so, it is preferable to use simpler test signals. In practice, it is difficult to generate pseudo-random signals. It is therefore desirable to confirm that non-linearity can be also observed for simpler signals, e.g., for sinusoid signals.

Experiment with sinusoid signals: a hardware part. In our experiment, as a signal x_j, we sent an ultrasound wave. To generate this wave, a sinusoid electric signal $x(t) = A \cdot \cos(\omega \cdot t)$ was sent to the transducer, which then generated an ultrasonic wave in the tested object. The transducer was set at an angle of incidence of 31° with the plate, so that a wave would go along the surface of the plate (such waves are called *Lamb waves*; see, e.g., [2, 3, 4, 5, 13]).

If the transducer was ideally linear, then we would get an ultrasonic wave of the exact same frequency and of the same sinusoid shape as the original electric signal. In this case, to detect the non-linearity of the plate, it would be sufficient to place a single sensor on the plate and check whether the signal $y(t)$ measured by this sensor depends linearly on $x(t)$.

In reality, however, the transducer is somewhat non-linear; as a result, the ultrasonic signal sent to the plate contained components at frequencies different from the original frequency ω: it has components which are slightly different from ω, and it also has higher harmonics, i.e., frequencies close to 2ω, 3ω, etc. We chose $\omega = 500KHz$; for this frequency, the ultrasonic signal is mainly located in the frequency area from 350 to 650 KHz.

To separate the non-linearity of the transducer from the non-linearity of the plate itself, we placed *two* sensors on the plate: the first sensor is located near the transducer, and it measures the ultrasonic wave $x_1(t)$ that the transducer generates; the second sensor is located at a distance from the transducer, and it measures the wave $x_2(t)$ changed after passing through the plate. Then, we check whether $x_2(t)$ linearly depends on $x_1(t)$.

How to check non-linearity: general discussion. The detection of non-linearity is based on the fact that the general linear time-invariant dependency has the form $x_2(t) = \int A(t-s) \cdot x_1(s)\,ds$ for some function $A(t)$. In terms of Fourier components, this dependency takes a simple form $\hat{x}_2(\omega) = \hat{A}(\omega) \cdot \hat{x}_1(\omega)$. Thus, to check whether the dependence is linear, it is sufficient to check whether, for each ω, the Fourier component $\hat{x}_2(\omega)$ is a linear function of the Fourier component $\hat{x}_1(\omega)$.

How to check non-linearity: ideal case. If the signal $x_1(t)$ is purely harmonic $x_1(t) = A \cdot \cos(\omega \cdot t)$, then it has only one Fourier component, and all we have to do to check non-linearity is to take different amplitudes A, and to plot the absolute value of the corresponding Fourier component $|\hat{x}_2(\omega)|$ of the signal measured by the second sensor as a function of $A = |\hat{x}_1(\omega)|$. Instead of the absolute values of the Fourier components, we could take their energies $E_2 = |\hat{x}_2(\omega)|^2$ and $E_1 = |\hat{x}_1(\omega)|^2$; if the dependency of $x_2(t)$ on $x_1(t)$ is linear, then E_2 is a linear function of E_1: $E_2 = c_1 \cdot E_1$.

Due to the inevitable noise, the measured energy at the second sensor also contains a noise component, i.e., $E_2 = c_1 \cdot E_1 + c_0$, where c_0 is the energy of the noise. Crudely speaking, if the dependence of E_2 on E_1 is linear, this means that $x_2(t)$ linearly depends on $x_1(t)$, otherwise, the dependence of $x_2(t)$ on $x_1(t)$ is non-linear.

How to check non-linearity: main idea of the practical method. In our case, the original electric signal has only one Fourier component with the frequency $\omega = 500$ KHz, but, as we have mentioned, due to the non-linearity of the transducer, the resulting ultrasound wave has components in a certain vicinity of this original frequency. As a result, the signal's energy is distributed over the resulting range of frequencies. Thus, instead of the value $|\hat{x}_2(\omega)|$ corresponding to a single frequency, we took the total energy $E_2 = \int |\hat{x}_2(\omega)|^2\,d\omega$ of the signal in the frequency range, where the integral is taken over the entire range of frequencies [350 KHz, 650KHz]. Similarly, for the first sensor, we take an integral $E_1 = \int |\hat{x}_1(\omega)|^2\,d\omega$. We then check whether E_2 is a linear function of E_1.

The choice of an object. In our experiments, as a sample object, we took an aluminum 6065 plate; its size is 36×18 in, its thickness is $1/16$ in. Initially, we performed the measurements on the undamaged plate. Then, we simulated a crack by sawing across the 18 in width of the plate with a fine tooth hand saw. The crack is at the middle of the plate. The two sensors were placed at an equal distance from the crack (or, for the un-damaged plate, at an equal distance from the center line where we later cut in a crack).

The choice of a signal. We wanted to make sure that the first sensor really measures the original ultrasound wave. Therefore, we restricted our signals only to the first moments of time after the beginning of the experiments, before the wave reflected from the plate's borders gets back to the location of the first sensor. To be able to separate the original signal from its later reflections, we generated only five cycles of the 500 KHz wave.

To check for non-linearity, we repeated this experiment at several different voltage levels of the original electric signal: 0V (pure noise), 6V, 7V, 8V, and 9V.

The choice of sampling frequency. We used a sampling frequency of 10 million samples per second (MSPS), i.e., 20 samples per cycle (we first tried 5 MSPS, but the noise was too high to make any conclusions, so we had to double the sampling frequency).

Filtering out reflections and the original noise. Based on the geometry of the plate and on the known speed of sound waves, we estimated (and later experimentally confirmed) that the reflection starts in at least 250 points after the original signal, and that the entire signal (before reflection) occurs in the first 2500 data points, so we only measured the first 2500 data points.

The entire 2500-point data starts as noise (no signal), then contains the signal, and then has the signal mixed with the reflections. To separate the signal from the original noise and from the following reflections, we selected 256 points out of the 2500 available. As a criterion for selecting the front edge of the data, it is natural to chose the first instance when the measured signal exceeds a certain portion of the maximum amplitude. Based on our observations, we have chosen 1/6 as this portion. So, for each sensor k ($k = 1, 2$), we computed the largest value $A_{\max}$ of all 2500 amplitudes, found the first point t_i at which the measured value $x_k(t_i)$ was larger than or equal to $A_{\max}/6$, and counted a total of 256 points $x_k(t_i), x_k(t_{i+1}), \ldots, x_k(t_{i+255})$. Then, we applied Fast Fourier Transform (FFT) to the selected data, and used this FFT to compute the total energy E_k of the signal in the frequency interval $[350\text{ KHz}, 650\text{KHz}]$.

We further decreased noise by repeating the measurements. To decrease the noise, we repeated each five-wave burst 200 times, and averaged the signals before processing them. To estimate the measurement accuracy, we repeated the same 200-burst experiment ten time. Then, as a result of measuring energy, we took an interval $[E_k^-, E_k^+]$ between the smallest and the largest of the resulting ten values.

How to check non-linearity: formulation of the problem. As a result of the measurements, we got several intervals $[E_1^-(V), E_1^+(V)]$ and $[E_2^-(V), E_2^+(V)]$ corresponding to different voltages V. We know that for each voltage, the actual (unknown) values of the energy $E_1(V)$ and $E_2(V)$ lie within the corresponding intervals. The question is: is this data consistent with the assumption that $E_2(V)$ is a linear function of $E_1(V)$? Or, in other words, it is possible to find real numbers $c_1 > 0$, c_0, and values $E_1(V) \in [E_1^-(V), E_1^+(V)]$ and $E_2(V) \in [E_2^-(V), E_2^+(V)]$ for which $E_2(V) = c_1 \cdot E_1(V) + c_0$?

How to check non-linearity: derivation of an algorithm. For each V, we want to have a value of $E_2(V)$ that satisfies the following two properties: first, it belongs to the interval $[E_2^-(V), E_2^+(V)]$, and second, it can be represented as $c_1 \cdot E_1(V) + c_0$ for some $E_1(V) \in [E_1^-(V), E_1^+(V)]$.

Let us first assume that the values $c_1 > 0$ and c_0 are given. Since $c_1 > 0$, the function $c_1 \cdot E_1(V) + c_0$ is increasing, and so, for each V, when $E_1(V)$ takes values from the interval $[E_1^-(V), E_1^+(V)]$, the expression $c_1 \cdot E_1(V) + c_0$ takes values from the interval $[c_1 \cdot E_1^-(V) + c_0, c_1 \cdot E_1^+(V) + c_0]$. Thus, the above two conditions on $E_2(V)$ mean that $E_2(V)$ must belong to two different intervals: $[E_2^-(V), E_2^+(V)]$ and

$$[c_1 \cdot E_1^-(V) + c_0, c_1 \cdot E_1^+(V) + c_0].$$

This is possible if and only if these two intervals have a non-empty intersection, i.e., if $E_2^-(V) \le c_1 \cdot E_1^+(V) + c_0$ and $c_1 \cdot E_1^-(V) + c_0 \le E_2^+(V)$.

Now, the question is: when is it possible to find $c_1 > 0$ and c_0 for which these inequalities hold for all V? Let us first assume that c_1 is given. Then, by moving c_0 into one side of each inequality, we can reformulate the above inequalities in the following way: $E_2^-(V) - c_1 \cdot E_1^+(V) \le c_0$ and $c_0 \le E_2^+(V) - c_1 \cdot E_1^-(V)$. Such a value c_0 exists if and only if all the lower bounds for c_0 are smaller than or equal to all the upper bounds for c_0, in other words, if $E_2^-(V) - c_1 \cdot E_1^+(V) \le E_2^+(V') - c_1 \cdot E_1^-(V')$ for all possible values of V and V'.

So, the original question can be reformulated as follows: does there exist a value c_1 for which this inequality is true for all V and V'? We can somewhat simplify this inequality by moving all terms which contain c_1 to one side and all other terms to another side. As a result, we get the inequality

$$c_1 \cdot (E_1^-(V') - E_1^+(V)) \le E_2^+(V') - E_2^-(V).$$

In our case, the energy of the wave monotonically increases with the voltage V, so that if $V < V'$, then $E_1^+(V) < E_1^-(V')$. Hence, when $V < V'$, the above inequality is equivalent to

$$c_1 \le \frac{E_2^+(V') - E_2^-(V)}{E_1^-(V') - E_1^+(V)},$$

and when $V > V'$, the above inequality turns into

$$\frac{E_2^-(V) - E_2^+(V')}{E_1^+(V) - E_1^-(V')} \le c_1.$$

Such a value c_1 exists if and only if all lower bounds for c_1 are smaller than or equal to all the upper bounds for c_1, i.e., when

$$\max_{V'<V} \frac{E_2^-(V) - E_2^+(V')}{E_1^+(V) - E_1^-(V')} \le \min_{V<V'} \frac{E_2^+(V') - E_2^-(V)}{E_1^-(V') - E_1^+(V)}. \qquad (3)$$

How to check non-linearity: the resulting algorithm. To check non-linearity, we must check the inequality (3).

Comments. As we have mentioned earlier, our problem is closely related to the problem of identifying a linear system under interval uncertainty. It is therefore not surprising that our inequality (3) is related to known methods of interval identification:

- In the particular case when the intervals $[E_1^-(V), E_1^+(V)]$ are degenerate, formula (3) turns into a formula from [8].
- In principle, we could deduce the formula (3) by applying a general method of solving linear interval identification problems described in [15], Section 5.4.2.1.

Experimental results.

V	$[E_1^-(V), E_1^+(V)]$ undamaged, 10^6	$[E_2^-(V), E_2^+(V)]$ undamaged, 10^6	$[E_1^-(V), E_1^+(V)]$ damaged, 10^5	$[E_2^-(V), E_2^+(V)]$ damaged, 10^4
0V	$[0.00, 0.01]$	$[0.00, 0.01]$	$[0.02, 0.03]$	$[0.06, 0.11]$
6V	$[2.65, 2.66]$	$[1.59, 1.61]$	$[0.69, 0.70]$	$[0.23, 0.28]$
7V	$[3.12, 3.14]$	$[1.86, 1.88]$	$[0.87, 0.92]$	$[0.14, 0.23]$
8V	$[3.62, 3.64]$	$[2.16, 2.18]$	$[1.05, 1.08]$	$[4.75, 4.84]$
9V	$[4.59, 4.69]$	$[2.70, 2.80]$	$[1.28, 1.32]$	$[5.57, 5.80]$

In the undamaged case, we clearly have a linear dependency ($E_2(V) \approx 0.6 \cdot E_1(V)$), while in the damaged case, the dependence is clearly non-linear.

Why did nobody notice this non-linearity before? Our experiments do not require very complicated and accurate equipment, they use standard sensors and transducers. So why did not anybody make these experiments before? The main reason is that before, people used just one signal level (e.g., the highest possible), to detect the faults. The use of only one signal level is justified if the response is linear: then, another input level will not lead to any new information. To detect non-linear terms, however, we must use at least two different input levels.

6 Practical recommendation: brief summary

To detect the faults, we must use at least *two* different signal levels. If the increase in the signal level x_j leads to a proportional increase in the measured values y_i, then most probably the object is smooth. If the dependence of y_i on x_j is non-linear, then, most probably, there is a fault, so further analysis is needed.

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