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Olga Kosheleva

*University of Texas at El Paso*, [olgak@utep.edu](mailto:olgak@utep.edu)

Vladik Kreinovich

*University of Texas at El Paso*, [vladik@utep.edu](mailto:vladik@utep.edu)

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# Why We Need Extra Physical Dimensions: A Simple Geometric Explanation

Olga Kosheleva and Vladik Kreinovich  
University of Texas at El Paso  
El Paso, TX 79968, USA  
olgak@utep.edu, vladik@utep.edu

## Abstract

It is known that a consistent description of point-wise particles requires that we add extra physical dimensions to the usual four dimensions of space-time. The need for such dimensions is based on not-very-intuitive complex mathematics. It is therefore desirable to try to come up with a simpler geometric explanation for this phenomenon. In this paper, we provide a simple geometric explanation of why extra physical dimensions are needed.

## 1 Need for Extra Physical Dimensions: Reminder

**Problems with the usual 4-dimensional space-time models.** In relativistic physics, elementary particles are points in space; see, e.g., [3]. Point-wise character of elementary particles makes many physical quantities infinite. For example, the energy density  $\rho(x)$  of the electric field  $\vec{E}(x)$  is known to be proportional to  $|\vec{E}(x)|^2$ , and the electric field of a point-wise particle decreases with the distance  $r$  to the particle according to the Coulomb law  $|\vec{E}(x)| \sim \frac{1}{r^2}$ . Thus, the energy density  $\rho(x)$  is proportional to  $|\vec{E}(x)|^2 \sim \frac{1}{r^4}$ .

The overall energy is equal to the integral  $\int \rho(x) dx$  and is, thus, proportional to the integral  $\int \frac{1}{r^4} dx$ . In polar coordinates, after integrating over angular coordinates, we get

$$I = \int_0^\infty \frac{2\pi \cdot r^2}{r^4} dr = 2\pi \cdot \int_0^\infty \frac{1}{r^2} dr.$$

This integral is equal to

$$I = -2\pi \cdot \frac{1}{r} \Big|_0^\infty = \infty.$$

Similar physically meaningless infinities appear when we compute other quantities related to a point particle [3].

*Comment.* The above computations use a non-quantum approximation, but similar infinities appear when we take into account quantum effects as well.

**Current solution.** It turns out that infinities can be avoided if we assume that the space-time has extra dimensions beyond the four usual ones. For example, string theory shows that we can get a consistent physical theory if we assume that the space-time is 10-dimensional; see, e.g., [4].

**Remaining challenge.** A problem with this solution is that it is heavily mathematical, there is no simple intuitive geometric explanation of why extra dimensions are needed.

*Comment.* It should be mentioned that:

- while there is no clear geometric explanation of *why extra dimensions are needed*,
- there are simple geometric explanations of *why namely 10 is a good dimension*; see, e.g., [5].

**What we do in this paper.** In this paper, we provide a possible geometric explanation of why extra space-time dimensions are needed.

## 2 Analysis of the Problem and the Resulting Explanation of Extra Physical Dimension(s)

**Natural idea: discrete space-time.** The infinities are caused by integration to  $r = 0$ . Thus, one possible way to avoid infinities is to assume that spatial coordinates – and other quantities – are discrete. This idea is ubiquitous in physics [3]:

- an electric charge cannot take any possible value, it must be proportional to some constant;
- quantum physics started with Planck's hypothesis that energy of light of a given wavelength cannot take any possible value, it must be proportional to some constant (dependent on this frequency), etc.

**Resulting description of space-time.** Let us apply the discreteness idea to variables that describe space-time geometry, namely,

- to the space-time coordinates  $x_1, \dots, x_n$ , and

- to the components  $g_{ij}$  of the metric tensor that describes the proper time  $s(x, x')$  between two points  $x = (x_1, \dots, x_n)$  and  $x' = (x'_1, \dots, x'_n)$  as follows:

$$s^2(x, x') = \sum_{i=1}^n \sum_{j=1}^n g_{ij} \cdot (x_i - x'_i) \cdot (x_j - x'_j). \quad (1)$$

For space-time coordinates, discreteness means that all the coordinates must be integer multiples of some fixed quantum  $q_x$ , i.e., that for every point  $x$  and for each coordinate  $i$ , we must have  $x_i = X_i \cdot q_x$  for some integer  $X_i$ . Similarly, for the components of the metric tensor  $g_{ij}$ , discreteness means that there exists some fixed quantum  $q_g$  for which, for each component  $g_{ij}$ , we have  $g_{ij} = G_{ij} \cdot q_g$  for some integer  $G_{ij}$ .

Under these two discreteness assumptions, the formula (1) that describes the square  $s^2(x, x')$  of the proper time between the points  $x = (X_1 \cdot q_x, \dots, X_n \cdot q_x)$  and  $x' = (X'_1 \cdot q_x, \dots, X'_n \cdot q_x)$  takes the form

$$s^2(x, x') = S^2(X, X') \cdot q_x^2 \cdot q_g, \quad (2)$$

where we denoted

$$S^2(X, X') \stackrel{\text{def}}{=} \sum_{i=1}^n \sum_{j=1}^n G_{ij} \cdot (X_i - X'_i) \cdot (X_j - X'_j). \quad (3)$$

**Empirical fact: there are light-like particles.** It is a known physical fact that:

- in addition to usual particles like electrons and protons that travel with speeds smaller than the speed of light, and for which, therefore,  $s^2(x, x') > 0$  for every two points  $x \neq x'$  on the particle's trajectory,
- there also exist "light-like" particles like photons that always travel with the speed of light and for which  $s^2(x, x') = 0$  for every two points  $x \neq x'$  on the particle's trajectory.

**In the continuous space-time, the possibility of light-like particles is mathematically trivial.** In the continuous space-time, when each coordinate  $x_i$  can take any real value, it is always possible to find pairs of points  $x \neq x'$  for which  $s^2(x, x') = 0$  – provided, of course, that the matrix  $g_{ij}$  is not positive or negative definite, i.e., provided that:

- there exist pairs  $(x, x')$  with  $s^2(x, x') > 0$ , and
- there exist pairs  $(x, x')$  with  $s^2(x, x') < 0$ .

**In discrete space-time, the existence of light-like particles is automatically guaranteed only if we have extra physical dimensions.** In the discrete space-time model (2)-(3), however, it is not always true that if a

quadratic form (3) with integer coefficients  $G_{ij}$  attains both positive and negative values, there exist integer values  $X_i - X'_i$  for which this form is equal to 0.

Such a general statement is true if and only if we have at least five variables, i.e., if and only if  $n \geq 5$ . This result was proven by A. Meyer in 1884 [6] and is known as Meyer's Theorem; see, e.g., [1, 7, 8].

**Resulting explanation.** Thus, to make sure that a discrete space-time is always consistent with the existence of light-like particles, we must assume that the dimension of space-time is at least five.

This explains the need for at least one extra physical dimension – in addition to the usual four dimensions of space-time.

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