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Vladik Kreinovich
University of Texas at El Paso, vladik@utep.edu

Olga Kosheleva
University of Texas at El Paso, olgak@utep.edu

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Paradox of Choice: A Possible Explanation

Vladik Kreinovich and Olga Kosheleva
University of Texas at El Paso
500 W. University
El Paso, TX 79968, USA
vladik@utep.edu. olgak@utep.edu

Abstract

At first glance, we would expect that the more choices we have, the happier we will be. Experiments show, however, then when the number of choices increases, customers become *less* happy. In this paper, we provide a possible explanation for this paradox.

1 Formulation of the Problem

Intuitively, the more choices we have, the better we should feel about it. In practice, however, if we add many additional choices to the current ones, the customers become less happy; see, e.g., [2] and references therein. The decrease of happiness is relatively small – so small that while in many cases, it is barely above the level of statistical significance, and often below this level [3], but in many cases, it is there. How can we explain this counterintuitive phenomenon?

2 Possible Explanation

How to describe customer choice? There are many possible settings of the user choice. For example, a customer has a fixed amount of money that he or she is willing to spend on a certain product, and the customer is looking for the best value for this amount. In this case, the customer is looking for the largest value per unit price.

Alternatively, a customer may be interested in buying a certain product, and he/she is looking for the cheapest option (among those option that satisfy his/her requirements). In this case, the customer is looking for the smallest value of the price per unit, i.e., equivalently, for the largest number of units per dollar.

There may be other possible setting. In all these setting, a customer wants to maximize his or her gain (or, equivalently, minimize his or her loss). In the general case, let us denote the quantity that we want to maximize by q, and the values of this quantity corresponding to different choices by $q_1, \ldots, q_n$.

Few choices vs. multiple choices: the main difference. When the number n of available choices if small, the customer can simply consider all the options and select the one with the largest value of the desired quantity $\max(q_1, \ldots, q_n)$.

The decrease is happiness starts when the number of choices becomes so large that it is not realistically possible to seriously consider all these choices in detail – a situation often happening in supermarkets. In such situations, since the customer cannot consider all the options, he/she considers only *some* of these options; among the considered options, the customer selects the one with the largest value q. Let us denote the number of considered options by k. Without losing generality, let us assume that the considered options are $q_1, \ldots, q_k$. In this case, the resulting quality is equal to $\max(q_1, \ldots, q_k)$.

Paradox of choice reformulated in precise terms. In these terms, the paradox of choice can be reformulated as follows. We consider two possible situations:

- in the first situation, the number n of choices is relatively small, so, among the options with values $q_1, \ldots, q_n$, the customer select the option with quality $Q = \max(q_1, \ldots, q_n)$;
- in the second situation, the number n of choices is large, so, among the options with values $q'_1, \ldots, q'_n$, the customer selects an option with quality $Q' = \max(q'_1, \ldots, q'_k)$ for some $k < n$.

The empirical fact is that even when $Q = Q'$ – i.e., when the selected product is of the same quality in both cases – a customer is usually less happy in the second situation.

Towards a possible explanation. In general, the values of the quality q corresponding to different products are bounded. Let us denote the lower bound by $\underline{q}$ and the upper bound by $\overline{q}$. The actual quality of different choices is randomly distributed in the interval $[\underline{q}, \overline{q}]$.

In general, we have no reason to believe that some values from this interval are more frequent than others. So, it is reasonable to assume that all the values from this interval are equally probable, i.e., that we have a uniform distribution on this interval; see, e.g., [1].

Similarly, we have no reason to believe that there is a correlation between different options, so all these quantities can be considered independent [1].

If we choose between n choices, then the resulting quality is equal to $Q = \max(q_1, \ldots, q_n)$. What is the expected value of Q? To find the expected value, let us find the corresponding cumulative distribution $F(q) \stackrel{\text{def}}{=} \text{Prob}(Q \le q)$.

The maximum Q of k values q_i is smaller than or equal to a given number q if and only if each of these values is $\le q$. Thus, due to independence assumption (see, e.g., [4]):

$$F(q) \stackrel{\text{def}}{=} \text{Prob}(Q \le q) = \text{Prob}((q_1 \le q)\,\&\,\ldots\,\&\,(q_n \le q)) =$$

$$\text{Prob}(q_1 \le q) \cdot \ldots \cdot \text{Prob}(q_k \le q).$$

For the uniform distribution, $\text{Prob}(q_i \le q) = \dfrac{q - \underline{q}}{\overline{q} - \underline{q}}$, so $F(q) = \left(\dfrac{q - \underline{q}}{\overline{q} - \underline{q}}\right)^n$. Thus, the corresponding probability density function $f(q)$ has the form

$$f(q) = \frac{dF(q)}{dq} = n \cdot \frac{(q - \underline{x})^{n-1}}{(\overline{q} - \underline{q})^n}.$$

Therefore, the mean grade $E[Q]$ is equal to

$$E[Q] = \int_{\underline{q}}^{\overline{q}} q \cdot f(q)\, dq = \int_{\underline{q}}^{\overline{q}} q \cdot k \cdot \frac{(q - \underline{q})^{n-1}}{(\overline{q} - \underline{q})^k}\, dq.$$

By introducing a new variable $x \stackrel{\text{def}}{=} q - \underline{q}$, for which $q = \underline{q} + x$, we can explicitly compute the corresponding integral, and get

$$E[Q] = \underline{q} + \frac{n}{n+1} \cdot (\overline{q} - \underline{q}).$$

Let us show that this formula enables us to explain the paradox of choice.

Resulting explanation. When we have a small number of choices, we select the option with the largest value Q. Since we did consider all available options, we know that this is the best choice we could have made.

When we have a large number of options n, then we select the value $Q = \max(q_1, \ldots, q_k)$ for some $k < n$. The expected value of this choice is equal to

$$Q = \underline{q} + \frac{k}{k+1} \cdot (\overline{q} - \underline{q}).$$

In this case, we did not consider all available options, and thus, we are not sure that the choice we made is the best possible one – maybe we could get a better result if we considered more options.

Theoretically, if we were able to consider all n options in detail, we would be able to get an option with the average quality of

$$Q_0 = \underline{q} + \frac{n}{n+1} \cdot (\overline{q} - \underline{q}).$$

Since $n > k$, we have $\dfrac{1}{n+1} < \dfrac{1}{k+1}$, thus,

$$\frac{n}{n+1} = 1 - \frac{1}{n+1} > 1 - \frac{1}{k+1} = \frac{k}{k+1},$$

and $Q_0 > Q$.

So, while we got exactly the same quality Q as in the first case, we also know that:

- in the first case, we did select the best of available options, while

- in the second case, we could have attained better quality if we tested more options.

Since our goal is to maximize quality, and in the second case, we know that we did not reach the maximum – so that we could, e.g., have gotten more value per dollar – we thus naturally feel less happy.

This also explains why the difference is small: the potential relative increase

$$\frac{Q_0 - Q}{Q} = \frac{\dfrac{1}{k+1} - \dfrac{1}{n+1}}{1 - \dfrac{1}{k+1}}$$

is small: e.g., for $k = 10$ and $n = 100$, it is about 10%, not much.

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