Assessment of Functional Impairment in Human Locomotion: A Fuzzy-Motivated Approach

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***Abstract*—Many neurological disorders result in disordered motion. The effects of a disorder can be decrease by an appropriate rehabilitation. To make rehabilitation efficient, we need to monitor the patient and check how well he or she improves. In our previous papers, we proposed a fuzzy-based semi-heuristic method of gauging how well a patient improved. Surprisingly, this semi-heuristic method turned out to be more efficient that we expected. In this paper, we provide a justification for this efficiency.**

I. Introduction

Medical problem. Neurological disorders – e.g., the effects of a stroke – affect human locomotion (such as walking). In most cases, the effect of a neurological disorder can be mitigated by applying an appropriate rehabilitation. **Resulting computational task.** For the rehabilitation to be effective, it is necessary to be able:

- to correctly diagnose the problem,
- to assess its severity, and
- to monitor the effect of rehabilitation.

At present, this is mainly done subjectively, by experts who observe the patient. This is OK for the diagnosis, but for rehabilitation, a specialist can see a patient only so often, and it is definitely desirable to have a constant monitoring of how well rehabilitation works. For such a monitoring, we need to be able to *automatically* gauge how well the patient progresses – based on an automatic observation (measurement) of the patient's gait. Measuring the gait is indeed possible. For that, we can attach different sensors to the patient, e.g.,

- inertial sensors that measure the absolute and relative location of different parts of the body during the motion, and
- electromyograph (EMG) sensors that measure the electric muscle activity during the motion.

We can then record the results $x(t)$ of each sensor during a gait cycle. Based on these observed signals, and on the signals corresponding to healthy patients, we need to:

- gauge how severe is the original gait disorder – by observing the measured gait signals $x(t)$, and
- gauge whether the current rehabilitation procedure is helping – by comparing the measured gait signal $x(t)$, the original gait signal, and the gait signal corresponding to healthy people.

First step: normalization. Motions differ by speed and by intensity: e.g., the same person can walk slower or faster. To reduce the effect of this difference on the observed signal $x(t)$, two normalizations are used.

First, to reduce the effect of different motion speed, we normalize the observed signal by re-scaling time so that it is now measured in terms of the gait cycle. In other words, instead of the original dependence $x(t)$, we consider the re-scaled dependence $x'(T) = x(t_0 + T \cdot T_0)$, where t_0 is the beginning of the gait cycle, T_0 is the gain cycle, and the new variable T describe the position of the sensor reading on the gait cycle. For example:

- the value $x'(0)$ describes the sensor's reading at the beginning of the gait cycle,
- the value $x'(0,5)$ describes the sensor's reading in the middle of the gait cycle,
- the value $x'(0.25)$ describes the sensor's reading at the quarter of the gait cycle.

Next, we reduce the effect of different intensity. Let $\underline{x}$ be the smallest possible value of the signal $x'(T)$ during the cycle, and let $\overline{x}$ be the largest possible value during the cycle. This means that the range of the signal $x'(T)$ is the interval $[\underline{x}, \overline{x}]$. Different intensities of the same motion correspond, in general, to different ranges. Thus, to reduce the effect of difference in intensities, we perform a linear re-scaling that reduce the original range into a standard range $[0, 1]$. Such a scaling has the form $x \to \dfrac{x - \underline{x}}{\overline{x} - \underline{x}}$. After such a re-scaling, we get a new signal

$$X(T) = \frac{x'(T) - \underline{x}}{\overline{x} - \underline{x}}.$$

Remaining problem. After re-scaling, all we have to do is compare the (re-scaled) observed signal $X(T)$ with a similarly re-scaled signal $X_0(T)$ corresponding to the average of normal behaviors.

When we observe gait of people with neurological disorders, even we non-specialists can easily see that something is not right with this gait. One would expect that the corresponding signals $X(T)$ and $X_0(T)$ are drastically different. However, surprisingly, these signals are very close to each other; see, e.g., [1], [5], [8]. This closeness make an automatic detection of motion disorders a difficult task.

II. Fuzzy Approach

Fuzzy approach. To formalize the way experts distinguish between the normal and abnormal gaits, in our previous papers, we proposed a semi-heuristic fuzzy-based method; see [1], [5], [8] for details.

First step: dividing the cycle into parts. An expert describes the gait by specifying how the motion looked like at different parts of the gait cycle. Correspondingly, in our method, we first divide the gait cycle into several equal parts.

For each part, we take all the measured values $X(T)$ obtained during this part, and form a triangular membership function $\mu(x)$ that best describes these values.

How to describe the gait on each part of the gait cycle. A triangular membership function is uniquely determined if we describe the range $[a, b]$ on which it is defined and the point m at which is attains the value 1:

- for x from the lower endpoint a to the point m, this function linearly increases from 0 to 1, and thus, has the form
$$\mu_{a,b,m}(x) = \frac{x-a}{m-a};$$
- for x from the point m to the upper endpoint b, this function linearly decreases from 1 to 0 and thus, has the form
$$\mu_{a,b,m}(x) = \frac{b-x}{b-m}.$$

In designing these functions, we used an approach described in [4], [6], [7]. In this approach, the goal is to satisfy two objectives:

- on the one hand, we would like to select a fuzzy sets that contains as many of the corresponding measured values $x_1, \ldots, x_n$ as possible;
- on the other hand, we would like to select a fuzzy set which is as specific as possible, i.e., for which the width $b-a$ of the range on which this triangular membership function is defined should be as small as possible.

Each element x_i belongs to the fuzzy set with a degree $\mu_{a,b,m}(x_i)$. If this fuzzy set was a crisp set, this degree would be simply 0 or 1, and to find the total number of elements belonging to this set, we could simply add up the degrees corresponding to all elements – this would give us exactly the number of elements. A similar approach is used to describe the number of elements in a fuzzy set (see, e.g., [2], [3]): we simply add up the membership values corresponding to different elements, i.e., consider the sum $\sum\limits_{i=1}^{n} \mu_{a,b,m}(x_i)$.

To combine the two goals of maximizing this sum and minimizing the width $b-a$, we maximize the ratio
$$\frac{\sum\limits_{i=1}^{n} \mu_{a,b,m}(x_i)}{b-a}.$$
Once this maximization problem is solved, we thus get the parameters a, b, and m that describe the signal on this part of the gait cycle.

Comparing two motions. For each motion, and for each part of the cycle, we have parameters describing this motion at this part of the cycle. The parameters corresponding to all parts form a tuple $g = (g_1, \ldots, g_N)$ describing the person's gait.

Now, we need to compare:

- the tuple $g = (g_1, \ldots, g_N)$ describing the observed gait with
- the tuple $n = (n_1, \ldots, n_N)$ describing the (average) normal gait.

We want to know how similar are the corresponding tuples. Since we are using a fuzzy-based approach, it is reasonable to take into account that each value from each tuple is a number from the interval $[0, 1]$, so we can view each tuple as a fuzzy set.

Thus, the problem of finding the similarity between tuples is reduced to the problem of finding the similarity between the corresponding fuzzy sets. How can we gauge the degree of similarity between two fuzzy sets?

For crisp sets A and B, the degree of similarity can be described as the ratio $\frac{|A \cap B|}{|A \cup B|}$, where $|A|$ denote the number of elements in a set A: this ratio is equal to 1 if and only if the two sets coincide, and if we add an element to one of the sets without adding it to another one, this degree decreases. It is reasonable to use a similar formula to describe the similarity of fuzzy sets.

For simplicity, we can use $\min$ to describe intersection and $\max$ to describe union. Then:

- the degree to which the i-th element belongs to the intersection is equal to $\min(g_i, n_i)$, and
- the degree to which the i-th element belongs to the union is equal to $\max(g_i, n_i)$.

Thus:

- the number of elements in the intersection is equal to $\sum\limits_{i=1}^{N} \min(g_i, p_i)$, while
- the number of elements in the union is equal to $\sum\limits_{i=1}^{N} \min(g_i, p_i)$.

So, we arrive at the following formula for the desired degree of similarity:

Resulting formula. The degree of similarity between the two tuples is equal to the ratio

$$\frac{\sum\limits_{i=1}^{N} \min(g_i, p_i)}{\sum\limits_{i=1}^{N} \min(g_i, p_i)}.$$

This formula is in good accordance with the expert opinions. Our preliminary results (see, e.g., [1], [5], [8]) show that this formula is in good accordance with the expert opinion about the severity of the patients's disorder.

Why is this semi-heuristic formula so good? Our objective was to come up with a reasonable formula based on expert opinions. We fuzzy expected that there would be a need to further tune the formula – as it happens in fuzzy control; see, e.g., [2], [3]. Surprisingly, this formula works well even without tuning.

Why? In this paper, we attempts to explain why the above formula turned out to be more empirically successful than we expected.

III. Towards an Explanation for the Above Semi-Heuristic Fuzzy Technique

Idea. To explain why the above semi-heuristic fuzzy technique works well, we will do the following:

- first, we will come up with a simplified equivalent formulation of this technique, and
- then, we will come up with an explanation which is based on this simplified equivalent formulation.

We need to divide the gait cycle into a large number of parts. In the above technique, we describe the signal on each part of the gait cycle by three numbers – the parameters of the corresponding membership function. When the part is large, three numbers are, in general, not sufficient to describe the signal $x(t)$ on this part, since we have many different types of behavior. However, when the part is small, we can expand the dependence $x(t)$ into Taylor series relative to the center $\widetilde{t}$ of this part:

$$x(t) = x\left(\widetilde{t}\right) + \frac{dx}{dt} \cdot \Delta t + \frac{1}{2} \cdot \frac{d^2x}{dt^2} \cdot \Delta t^2 + \ldots,$$

where $\Delta t \stackrel{\text{def}}{=} t - \widetilde{t}$, and keep only a few first terms in this expansion.

When the part is narrow, then the difference Δt is small, and we can ignore quadratic terms; in this case, the original signal is approximated by a linear function, and we only need two parameters to describe a general linear function of one variable. When the part becomes even smaller, i.e., when the difference Δt becomes even smaller, we can ignore linear terms as well, and assume that the signal $x(t)$ is constant throughout this part. To describe a constant, it is sufficient to have a single parameter.

In general, the narrower the part, the more accurate the 3-parameter description of the signal on this part. Thus, since we are interested in an adequate description of the signal, we will assume that the gait cycle is divided into a large number of parts.

Resulting description of the tuples. On each part, the corresponding values x_i are close to each other – and to the value $x(t_i)$ of the signal in the midpoint of this part. So, the parameters a, m, and b are also close to this midpoint value $x(t_i)$. Hence, the tuple describing the signal is approximately equal to the tuple consisting of the values $x(t_1)$, $x(t_2)$, $\ldots$, $x(t_n)$, each of which is repeated three times.

Similarly, the tuple corresponding to the gaits of the healthy persons consists of the values $x_0(t_1)$, $x_0(t_2)$, $\ldots$, $x_0(t_n)$, each of which is repeated three times.

Towards the equivalent description of the degree of similarity. Since the elements of the first tuple are approximately equal to $x(t_i)$ (with each element repeated three times) and the elements of the second tuple are approximately equal to $x_0(t_i)$ (with each element also repeated three times), the corresponding degree of similarity is approximately equal to the ratio

$$s = \frac{3 \cdot \sum\limits_{i=1}^{n} \min(x(t_i), x_0(t_i))}{3 \cdot \sum\limits_{i=1}^{n} \max(x(t_i), x_0(t_i))}.$$

Dividing both numerator and denominator by 3, we conclude that

$$s = \frac{\sum\limits_{i=1}^{n} \min(x(t_i), x_0(t_i))}{\sum\limits_{i=1}^{n} \max(x(t_i), x_0(t_i))}.$$

Now, we can use the above-mentioned fact that the actual signal $x(t)$ is close to the normal gain signal $x_0(t_i)$. This closeness means that the difference $\Delta x(t_i) \stackrel{\text{def}}{=} x(t_i) - x_0(t_i)$ is small, and so, we can safely ignore terms which are quadratic (or higher order) in terms of these differences $\Delta x(t_i)$.

Substituting the expression $x(t_i) = x_0(t_i) + \Delta x(t_i)$ into the above formula for the similarity degree s, we conclude that

$$s = \frac{\sum\limits_{i=1}^{n} \min(x_0(t_i) + \Delta x(t_i), x_0(t_i))}{\sum\limits_{i=1}^{n} \max(x_0(t_i) + \Delta x(t_i), x_0(t_i))}.$$

In this expression, both minimum and maximum are easy to compute. For minimum, we get:

- $\min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i)$ if $\Delta x(t_i) \geq 0$, and
- $\min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i) + \Delta x(t_i)$ if $\Delta x(t_i) < 0$.

Similarly, for maximum:

- $\max(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i) + \Delta x(t_i)$ if $\Delta x(t_i) \geq 0$, and
- $\min(x_0(t_i) + \Delta x(t_i), x_0(t_i)) = x_0(t_i)$ if $\Delta x(t_i) < 0$.

Substituting these expressions into the above formula for s, we conclude that

$$s = \frac{\sum\limits_{i=1}^{n} x_0(t_i) + \sum\limits_{i:\Delta x(t_i)<0} \Delta x(t_i)}{\sum\limits_{i=1}^{n} x_0(t_i) + \sum\limits_{i:\Delta x(t_i)\ge 0} \Delta x(t_i)}.$$

This expression can be simplified if we introduce the notation $s_0 \stackrel{\text{def}}{=} \sum\limits_{i=1}^{n} x_0(t_i)$, then we get

$$s = \frac{s_0 + \sum\limits_{i:\Delta x(t_i)<0} \Delta x(t_i)}{s_0 + \sum\limits_{i:\Delta x(t_i)\ge 0} \Delta x(t_i)}.$$

Dividing both the numerator and the denominator by s_0, we conclude that

$$s = \frac{1 + \sum\limits_{i:\Delta x(t_i)<0} \dfrac{\Delta x(t_i)}{s_0}}{1 + \sum\limits_{i:\Delta x(t_i)\ge 0} \dfrac{\Delta x(t_i)}{s_0}}.$$

Since $|\Delta x(t_i)| \ll x(t_i)$, we have

$$\sum_{i=1}^{n} |\Delta x(t_i)| \ll \sum_{i=1}^{n} x_0(t_i) = s_0,$$

so

$$\left|\sum_{i:\Delta x(t_i)<0} \frac{\Delta x(t_i)}{s_0}\right| \ll 1 \text{ and } \left|\sum_{i:\Delta x(t_i)\ge 0} \frac{\Delta x(t_i)}{s_0}\right| \ll 1.$$

In general, when $|a| \ll 1$ and $|b| \ll 1$, we have

$$\frac{1+a}{1+b} \approx (1+a)\cdot(1-b+\ldots) = 1+a-b+\ldots$$

Thus,

$$s \approx 1 + \sum_{i:\Delta x(t_i)<0} \frac{\Delta x(t_i)}{s_0} - \sum_{i:\Delta x(t_i)\ge 0} \frac{\Delta x_0(t_i)}{s_0},$$

i.e.,

$$s = 1 + \frac{1}{s_0}\cdot\left(\sum_{i:\Delta x(t_i)<0} \Delta x(t_i) - \sum_{i:\Delta x(t_i)\ge 0} \Delta x(t_i)\right).$$

One can easily check that these two sums can be equivalently described as a single one:

Resulting equivalent reformulation of the degree of similarity.

$$s \approx 1 - \frac{1}{s_0}\cdot \sum_{i=1}^{n} |\Delta x(t_i)|.$$

Thus, we arrive at the following conclusion: the degree of dissimilarity (i.e., the severity of the disorder) is proportional to the sum

$$S \stackrel{\text{def}}{=} \sum_{i=1}^{n} |\Delta x(t_i)|.$$

Comment. From the mathematical viewpoint, once we multiply this sum by the difference $\Delta t = t_{i+1} - t_i$, we get an integral sum $\sum\limits_{i=1}^{n} |\Delta x_0(t_i)| \cdot \Delta t$ for the interval $\int |\Delta x(t)|\,dt$. Since we have divided the gait cycle into a large number of parts, the above integral sum is practically indistinguishable from the interval and thus, the original sum S is approximately equal to $\dfrac{1}{\Delta t}\cdot \displaystyle\int |\Delta x(t)|\,dt$.

The value Δt does not depend on the patient, so we can conclude that the dissimilarity (i.e., the severity of the disorder) is proportional to the integral

$$I \stackrel{\text{def}}{=} \int |\Delta x(t)|\,dt.$$

Final step: explanation of the reformulated formula. Let us explain why the integral S is a good measure of the disorder's severity.

In general, the difference $\Delta x(t)$ between the actual and the ideal gaits affects many different types of behavior. For some behaviors, this effect may be minimal, but for others, the effect is drastic. It is therefore reasonable to gauge the severity of a disorder by the worst-case effect of this difference.

For each objective, the effectiveness of how well this activity can be performed with the given gait is a functional depending on the function $x(t)$. We describe the gait by the values $x(t_1)$, ..., $x(t_n)$, so we can say that the effectiveness E is a function of all these values:

$$E = F(x(t_1),\ldots,x(t_n)).$$

For the patient, as we have mentioned, we have

$$x(t_i) = x_0(t_i) + \Delta x(t_i),$$

where the differences $\Delta x(t_i)$ are small – so that terms quadratic in terms of these differences can be safely ignored. We can therefore substitute the expression $x(t_i) = x_0(t_i) + \Delta x(t_i)$ into the above formula for efficiency and get

$$E = F(x_0(t_1) + \Delta x(t_1),\ldots,x_0(t_i) + \Delta x(t_n)).$$

Expanding the dependence F in Taylor series and ignoring quadratic and higher order terms in this expansion, we conclude that

$$E = F(x_0(t_1),\ldots,x_0(t_n)) + \sum_{i=1}^{n} c_i \cdot \Delta x(t_i),$$

where c_i is the corresponding partial derivative $c_i \stackrel{\text{def}}{=} \dfrac{\partial F}{\partial x(t_i)}$. Thus, the loss of efficiency $\Delta E = E_0 - E$ in comparison with the efficiency $E_0 = F(x_0(t_1),\ldots,x_0(t_n))$ corresponding to the normal gait is equal to

$$\Delta E = -\sum_{i=1}^{n} c_i \cdot \Delta x(t_i).$$

The severity of a disorder is determined by the worst-case loss, i.e., by the largest possible value of this sum over all corresponding functions F. There should be a limit M_i on the (absolute value of) each derivative c_i – otherwise, this largest possible value will be infinite. It makes sense to assume that the limit M_i is the same for all the moments of time t_i. Indeed, the motion process is periodic, selecting the starting point of the cycle is reasonably arbitrary, and the upper bound should not depend on the (reasonably) arbitrary choice of the starting point. Thus, we arrive at the following problem:

- We know the values $\Delta x(t_i)$.
- We know the upper bound M on the absolute values of the coefficients c_i.
- We want to find the largest possible value of the sum

  $$-\sum_{i=1}^{n} c_i \cdot \Delta x(t_i)$$

  over all possible values c_i for which $|c_i| \le M$.

The sum attains the maximum when each term $-c_i \cdot \Delta x(t_i)$ is the largest possible.

When $\Delta x(t_i) > 0$, this term decreases with c_i and thus, its largest possible value is attained when c_i attains its smallest possible value $c_i = -M$. For this value c_i, this term takes the value $M \cdot \Delta x(t_i)$.

When $\Delta x(t_i) \le 0$, this term increases with c_i and thus, its largest possible value is attained when c_i attains its largest possible value $c_i = M$. For this value c_i, this term takes the value $-M \cdot \Delta x(t_i)$.

Both cases can be described by a single expression $M \cdot |\Delta x(t_i)|$. Thus, the largest value of the above sum is equal to

$$\sum_{i=1}^{n} M \cdot |\Delta x(t_i)| = M \cdot \sum_{i=1}^{n} |\Delta x(t_i)|.$$

So, the worst-case effect of a gait disorder is indeed proportional to the sum $\sum_{i=1}^{n} |\Delta x(t_i)|$ – which is equivalent to the above semi-heuristic fuzzy technique.

So, the above fuzzy technique has been justified.

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